$(\omega, \phi)P^-$ decays of tau leptons

A. Flores-Tlalpa* and G. López Castro+

Departamento de Física, Cinvestav, Apartado Postal 14-740, 07000 México, D.F., México

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The $\tau^- \to (\omega, \phi)P^-\nu_\tau$ decays, where $P^- = \pi^-$ or K^-, are considered within a phenomenological model with dominance of meson intermediate states. We assume SU(3) flavor symmetry to fix some of the unknown strong interaction couplings. Our predictions for the $\tau^- \to \phi(\pi^-, K^-)\nu_\tau$ branching fractions are in good agreement with recent measurements of the *BABAR* and BELLE Collaborations.

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I. INTRODUCTION

Tau lepton decays into a charged pseudoscalar P^- and an isoscalar vector meson V, generically denoted by $\tau^- \to VP^-\nu_\tau$, can occur in four possible ways:

$$\tau^- \to \omega\pi^-\nu, \tag{1}$$

$$\tau^- \to \phi\pi^-\nu, \tag{2}$$

$$\tau^- \to \omega K^-\nu, \tag{3}$$

$$\tau^- \to \phi K^-\nu. \tag{4}$$

Owing to the quark mixing angle factors, one naively expects that processes (1) and (2) [$\Delta S = 0$] would have larger branching fractions than decay modes involving a K^- meson ($\Delta S = -1$). However, the rich resonance structure of intermediate states combined with the high thresholds for the above processes will produce an interesting pattern worth to be investigated.

The study of $\tau^- \to VP^-\nu_\tau$ decays is interesting for several reasons. As is well known, tau decays into several pseudoscalar mesons are dominated by the production of intermediate resonant states [1]. A good quantitatively description of the decay modes shown in Eqs. (1)–(4) is important to better understand the dynamics of three and four pseudoscalar mesons produced in tau lepton decays. On the other hand, the study of such decays allows a direct access to the $\langle VP|J_\mu|0\rangle$ hadronic matrix element in the intermediate energy regime. Since $\tau \to VP\nu$ and $B, D \to Vl\nu$ decays are related by crossing, they can be useful to provide further tests of either nonrelativistic [2] or relativistic [3] quark model predictions. Finally, the $\tau^- \to (\omega, \phi)\pi^-\nu$ decays are related to the $e^+e^- \to (\omega, \phi)\pi^0$ processes via isospin symmetry, and their measurements can be useful to provide another test of the conserved vector current hypothesis [4–6].

In Table I, we display the experimental values for the branching ratios of $\tau^- \to VP^-\nu$ decays. The $\omega\pi^-$ final state is the most favored, and its branching fraction and spectral function were the first to be measured [1,7,8].

Because of their smaller branching fractions, the decay modes in Eqs. (2)–(4) were measured only very recently [9–11]. Previous upper bounds on τ decays involving ϕ mesons were reported in Ref. [12], where the upper limits $B(\tau \to \phi\pi\nu) \leq (1.2 \sim 2.0) \times 10^{-4}$ and $B(\tau \to \phi K\nu) \leq (5.4 \sim 6.7) \times 10^{-4}$ were set at the 90% c.l. [12].

Earlier theoretical estimates for some of these decays were considered in references [4–6]. Based on the conserved vector current hypothesis and using bounds for the cross section of $e^+e^- \to \phi\pi^0$, the loosely limit $B(\tau^- \to \phi\pi^-\nu) \leq 9.0 \times 10^{-4}$ at 90% C.L. was derived in [5]. On the other hand, by assuming that the form factor of this decay is dominated by the contribution of two vector resonances (ρ and ρ') and that flavor SU(3) is a good symmetry, the value $B(\tau^- \to \phi\pi^-\nu) = (1.20 \pm 0.48) \times 10^{-5}$ was obtained in Ref. [6]. This prediction clearly underestimates the measured fraction of $\phi\pi^-$ (see Table I). Concerning the ϕK^- modes, only rough estimates are available based on phase-space and quark mixing angles considerations [9].

In this paper, we revisit this subject and provide a unified description of the four VP^- decays shown in Eqs. (1)–(4) in the framework of a meson dominance model. We consider the possibility that a meson dominance model with a few intermediate states can account, in a unified way, for

TABLE I. Measured branching fractions of $\tau^- \to VP^-\nu_\tau$ decays. The result of Belle for the $\phi\pi$ channel $(**)$ is only preliminar as long as background subtraction and a complete study of systematics are not included.

VP^- mode	Branching fraction	Reference
$\omega\pi^-$	$(1.95 \pm 0.08) \times 10^{-2}$	[7,8]
$\phi\pi^-$	$(6.05 \pm 0.71) \times 10^{-5**}$	[9]
	$(3.42 \pm 0.55 \pm 0.25) \times 10^{-5}$	[10]
ωK^-	$(4.1 \pm 0.9) \times 10^{-4}$	[11]
ϕK^-	$(4.05 \pm 0.25 \pm 0.26) \times 10^{-5}$	[9]
	$(3.39 \pm 0.20 \pm 0.28) \times 10^{-5}$	[10]

*afflores@fis.cinvestav.mx
+glopez@fis.cinvestav.mx

the observed branching fractions of reactions (1)–(4). As a simplifying assumption we will rely on SU(3) flavor symmetry for the strong couplings that can not be fixed from experiment, and we will assume an ideal value $\tan\theta_V = 1/\sqrt{2}$ of the $\omega - \phi$ mixing angle (i.e., ϕ is assumed to be a pure $\bar{s}s$ state). In order to account for SU(3) symmetry breaking effects, we will assign a $\pm 15\%$ uncertainty to these strong coupling constants. On the basis of these assumptions, we conclude that present data on $\tau^- \to VP^-\nu$ decays can be easily accommodated within the meson dominance model.

II. MESON DOMINANCE MODEL FOR TAU DECAYS

Thus, let us first consider the decay $\tau^-(p_\tau) \to V(p_V)P^-(p_P)\nu(p_\nu)$, where p_i denotes the four momenta of the i-th particle. The hadronic matrix element can be decomposed in terms of four form factors ($Q = s$ or d) [13]:

$$\langle VP|\bar{Q}\gamma^\alpha(1-\gamma_5)u|0\rangle = ig\varepsilon^{\alpha\beta\mu\nu}\epsilon^*_\beta q_{+\mu}q_{-\nu} + f\epsilon^{*\alpha} + [a_+q_+^\alpha + a_-q_-^\alpha]\epsilon^*\cdot q_+, \quad (5)$$

where ϵ^*_β is the polarization four-vector of the outgoing vector meson ($p_V\cdot\epsilon^* = 0$), and $q_\pm = p_V \pm p_P$. The vector (g) and axial ($f, a_\pm$) form factors are functions of $s = q_+^2$ only.

If we define $\Sigma^2 = m_V^2 + m_P^2$, $\Delta^2 = m_V^2 - m_P^2$, and $\beta_{VP} = (1 - 2\Sigma^2/s + \Delta^4/s^2)^{1/2}$, the differential decay rate can be written in the simple form

$$\frac{d\Gamma}{ds} = \frac{G_F^2|V_{uQ}|^2m_\tau^3}{128\pi^3}\beta_{VP}\left(1-\frac{s}{m_\tau^2}\right)^2\left\{\frac{1}{2}\beta_{++} + \frac{1}{2}\beta_{--}\left[\frac{\Delta^4}{s^2} + \frac{1}{3}\left(1+\frac{2s}{m_\tau^2}\right)\beta_{VP}^2\right] + \frac{\Delta^2}{s}\mathrm{Re}[\beta_{+-}] + \frac{\alpha}{m_\tau^2}\right\}, \quad (6)$$

where G_F is the Fermi constant, V_{uQ} is the uQ entry of the Cabibbo-Kobayashi-Maskawa matrix and,

$$\begin{aligned}
\beta_{++} &= \frac{1}{4m_V^2}\{|f|^2 + 4m_V^2|g|^2(s-2\Sigma^2) + |a_+|^2s^2\beta_{VP}^2 + 2\,\mathrm{Re}[fa_+^*](s-3m_V^2-m_P^2)\}\\
\beta_{--} &= \frac{1}{4m_V^2}\{|f|^2 - 4m_V^2s|g|^2 + |a_-|^2s^2\beta_{VP}^2 + 2\,\mathrm{Re}[fa_-^*](s+\Delta^2)\}\\
\beta_{+-} &= \frac{1}{4m_V^2}\{|f|^2 + 4m_V^2\Delta^2|g|^2 + (fa_+^*)^*(s+\Delta^2) + (fa_-^*)(s-3m_V^2-m_P^2) + (a_+a_-^*)s^2\beta_{VP}^2\}\\
\alpha &= |f|^2 + s^2|g|^2\beta_{VP}^2.
\end{aligned} \quad (7)$$

The Feynman diagrams with the intermediate mesons that connect the weak current and the strong vertex in $\tau \to VP\nu$ decays are shown in Fig. 1.

Using the Feynman rules for the elements of these diagrams, we get the following expressions for the form factors (V_j, A_j, and P_j denote vector, axial, and pseudoscalar intermediate meson states, respectively):

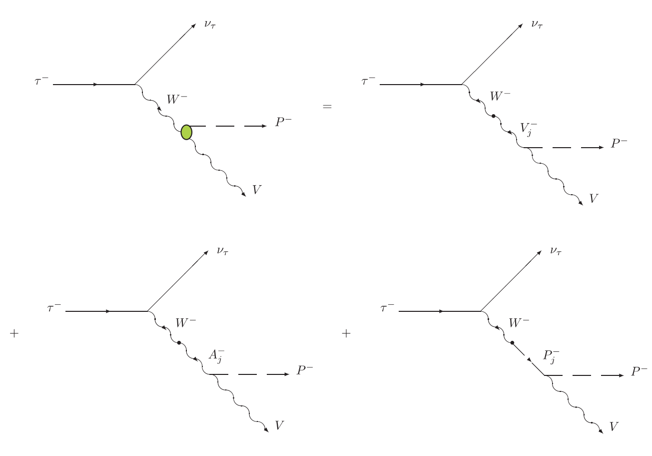

FIG. 1 (color online). Intermediate virtual meson contributions to $\tau^- \to VP^-\nu$ decays.

$$\begin{aligned}
g &= \frac{1}{2}\sum_j \frac{f_{V_j}g_{V_jVP}}{D_{V_j}(s)}, \qquad f = -\frac{1}{2}(s+\Delta^2)\sum_j\frac{f_{A_j}g_{A_jVP}}{D_{A_j}(s)},\\
a_+ &= \frac{1}{2}\sum_j\frac{f_{A_j}g_{A_jVP}}{D_{A_j}(s)} + 2\sum_j\frac{f_{P_j}g_{P_jVP}}{D_{P_j}(s)},\\
a_- &= \frac{1}{2}\sum_j\frac{f_{P_j}g_{P_jVP}}{D_{P_j}(s)},
\end{aligned} \quad (8)$$

where f_{M_j} denotes the weak coupling of the M_j intermediate meson, g_{M_jVP} is its strong coupling to the VP final state, and $D_{M_j}(s) \equiv s - m_{M_j}^2 + im_{M_j}\Gamma_{M_j}$, where $m_{M_j}(\Gamma_{M_j})$ is the mass (width) parameter of the corresponding intermediate state.

III. STRANGENESS-CONSERVING DECAYS

The G-parity properties of the weak currents and $V\pi^-$ system in this case, impose $f = a_- = a_+ = 0$. As in previous papers [4,6], we will assume that the nonvanishing vector form factor is saturated by the exchange of two vector resonances [the $\rho(770)$ and the $\rho'(1523)$]. Thus, we get ($V = \omega, \phi$)

$$g(s) = \frac{f_\rho g_{\rho V\pi}}{2D_\rho(s)}\left\{1 + \alpha_{V\pi}\frac{D_\rho(s)}{D_{\rho'}(s)}\right\}, \quad (9)$$

where $\alpha_{V\pi} = f_{\rho'} g_{\rho' V\pi}/f_\rho g_{\rho V\pi}$ is the only free parameter of the model at this stage. We chose the $\rho'(1523)$ state ($m_{\rho'} = 1523$ MeV and $\Gamma_{\rho'} = 400$ MeV [8,14,15]) as the second vector resonance, instead of the $\rho(1450)$ [1], because its larger width allows for a better fit to data on the spectral function [8,14,15].

As usual [15], we can define a vector spectral function whose expression becomes very simple in this case:

$$v(s) = \frac{s\beta^3_{V\pi}}{12\pi}|g(s)|^2. \quad (10)$$

This spectral function has been measured by the ALEPH [7] and CLEO [8] Collaborations for the dominant $\omega\pi^-$ final state. In order to fit the data on the spectral function, we use $f_\rho = (170.0 \pm 3.4) \times 10^3$ MeV^2 and $g_{\rho\omega\pi} = (15.2 \pm 1.9) \times 10^{-3}$ MeV^{-1} (this value is a bit larger than the average value $g_{\rho\omega\pi} = (12.3 \pm 1.2) \times 10^{-3}$ MeV^{-1} obtained from $\rho^\pm \to \pi^\pm\gamma$, $\rho^0 \to \pi^0\gamma$, and $\omega \to \pi^0\gamma$ decays, but they agree within their error bars). A fit to the spectral function reported in Ref. [7] gives us $\alpha_{\omega\pi} = -0.57 \pm 0.11$. The data points, taken from Ref. [8], and the fitted curves of the spectral function are shown in Fig. 2.

Using the above value of $\alpha_{\omega\pi}$, we can derive the following branching fraction by integration of Eq. (6):

$$B(\tau^- \to \omega\pi^-\nu_\tau) = (1.95 \pm 0.60)\%, \quad (11)$$

which is in very good agreement with the experimental value shown in Table I.

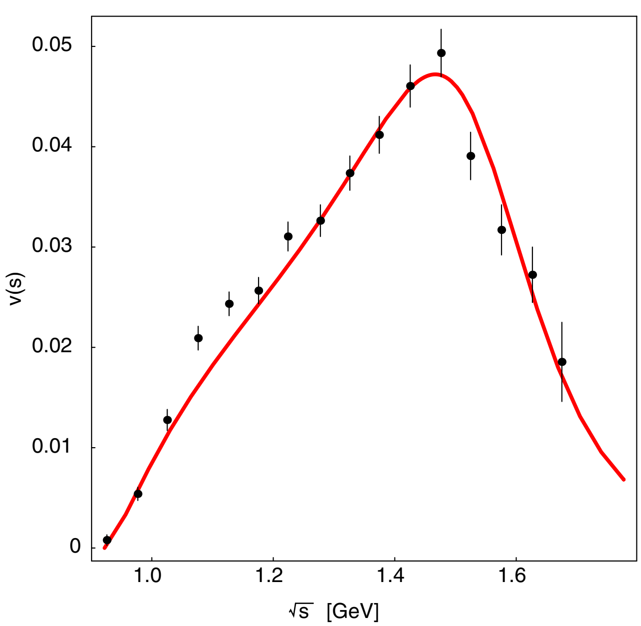

FIG. 2 (color online). Spectral function $v(s)$ of the $\omega\pi^-$ system: the best fit is represented by the solid line; the experimental data from CLEO [8] are shown with solid dots.

Now, we focus on the $\phi\pi$-decay channel. We will assume that flavor SU(3) is a good symmetry for the VVP couplings of the octet of vector mesons and of their radial excitations. Under this assumption, we can get $\alpha_{\phi\pi} = \alpha_{\omega\pi}$ for the relative weights of ρ and ρ' contributions in Eq. (9). In addition, we use $g_{\rho\phi\pi} = -(1.57 \pm 0.03) \times 10^{-3}$ MeV^{-1}, which is obtained from the $\phi \to \rho\pi \to \pi^+\pi^-\pi^0$ decay rate [1]. Now, if we insert these parameters into Eq. (6), we get

$$B(\tau^- \to \phi\pi^-\nu_\tau) = (3.64 \pm 1.15) \times 10^{-5}, \quad (12)$$

which seems to favor the result reported by *BABAR* [10] (see Table I). Note, however, that the result of the Belle Collaboration (see Table I) for this channel is only preliminar, because it has not been corrected by background subtractions and systematic uncertainties. The error bar in Eq. (12) includes an additional uncertainty of 15% in $\alpha_{\phi\pi}$, which we estimate as the breaking of SU(3) symmetry in deriving this coupling.

IV. COUPLING CONSTANTS FOR $\Delta S = -1$ DECAYS

The case of $\Delta S = -1$ tau decays is more difficult to deal with because the vector and axial weak currents can contribute to the decay amplitude. Consequently, more independent information is needed to specify the input coupling constants. We assume that the vector form factor g is dominated by the $K^* = K^*(892)$, and the $K'^* = K^*(1410)$ intermediate vector mesons (the contribution of the $K''^* = K^*(1680)$ can be neglected, because its coupling to the VK^- system is suppressed [1], and it is located at the end of the phase-space in tau decays; the argument also applies to higher-mass resonances). The axial form factors f, $a_\pm$ will originate from the exchange of the K^- pseudoscalar and the $K_1 = K_1(1270)$, $K_1' = K_1(1400)$ axial mesons. The expressions for the form factors were given in Eq. (8), and the numerical values of coupling constants will be discussed in the following subsections.

A. Weak couplings

The value of the K^- weak coupling is known with good precision, $f_K = (159.8 \pm 1.5)$ MeV [1], and is extracted from $K \to \mu\nu$ decays by including the effects of radiative corrections. The weak couplings of the other mesons can be extracted from the measurements of $\tau \to \mathcal{K}\nu$ decays (where $\mathcal{K}$ denotes either state among K^*, K'^*, K_1, K_1' mesons). We get

$$\begin{aligned} f_{K^*} &= (188.9 \pm 4.1) \times 10^3 \text{ MeV}^2, \\ f_{K'^*} &= (170^{+80}_{-57}) \times 10^3 \text{ MeV}^2, \\ f_{K_1} &= (215 \pm 25) \times 10^3 \text{ MeV}^2, \\ f_{K_1'} &= (170 \pm 130) \times 10^3 \text{ MeV}^2. \end{aligned} \quad (13)$$

The weak coupling with the largest uncertainty corresponds to the K_1' meson due to the poor quality of measurements. Since the K_1' intermediate state provides an important contribution to the $\tau \to \phi K \nu$ decay, we will rely on a quark model calculation to fix its value (see Sec. V below).

B. *KVK* couplings

The determination of the strong couplings of the intermediate resonances to the VK^- system in a reliable way is also a difficult task, either because some decays are not allowed by kinematics or, because there are not independent processes where their contribution can be studied. Thus, we will strongly rely on SU(3) flavor symmetry to fix their values when necessary.

From the experimental value of the $\phi \to K^+K^-$ branching fraction [1], we get $g_{K^+\phi K^-} = (4.48 \pm 0.04)$. Now, using the SU(3) symmetry and assuming an ideal value for the $\omega - \phi$ mixing angle ($\tan\theta_V = 1/\sqrt{2}$), we get

$$g_{K^+\omega K^-} = g_{K^+\phi K^-} \tan\theta_V = (3.17 \pm 0.03). \tag{14}$$

An alternative calculation of the KVK couplings can be obtained by assuming the vector-meson dominance model of the kaon electromagnetic form factors at zero momentum transfer

$$F_{K^+}(0) = \frac{g_{K^+\rho^0K^+}}{\gamma_\rho} + \frac{g_{K^+\omega K^+}}{\gamma_\omega} + \frac{g_{K^+\phi K^+}}{\gamma_\phi} = 1, \tag{15}$$

$$F_{K^0}(0) = \frac{g_{K^0\rho^0K^0}}{\gamma_\rho} + \frac{g_{K^0\omega K^0}}{\gamma_\omega} + \frac{g_{K^0\phi K^0}}{\gamma_\phi} = 0, \tag{16}$$

where em_V/γ_V defines the coupling of the neutral vector meson V to the photon. Now, we can use the SU(3) symmetry relations between the PVP' couplings (we assume again $\tan\theta_V = 1/\sqrt{2}$)

$$\begin{aligned} g_{K^+\rho^0K^-} &= -g_{K^0\rho^0\bar{K}^0} = \frac{1}{2}G^8_{PVP'}, \\ g_{K^+\omega K^-} &= g_{K^0\omega\bar{K}^0} = \frac{1}{2}G^8_{PVP'}, \\ g_{K^+\phi K^-} &= g_{K^0\phi\bar{K}^0} = \frac{1}{\sqrt{2}}G^8_{PVP'}. \end{aligned} \tag{17}$$

Solving the set of Eqs. (15) and (16) and using (17), we finally get

$$\begin{aligned} g_{K^+\omega K^-} &= \frac{\gamma_\omega\gamma_\phi}{2(\gamma_\phi + \sqrt{2}\gamma_\omega)} = 2.99 \pm 0.13 \\ g_{K^+\phi K^-} &= \frac{\gamma_\omega\gamma_\phi}{\sqrt{2}(\gamma_\phi + \sqrt{2}\gamma_\omega)} = 4.24 \pm 0.19, \end{aligned} \tag{18}$$

which are quite similar to the values computed from the $\phi \to KK$ decays (see above). In our computations of branching fractions, we will use the constants given in Eq. (14). We note that a departure of 15% from the SU(3) symmetry relation (14) produces a change of only 1.1% in the branching ratio of $\tau \to \omega K \nu$ decay.

C. $V'VP$ strong couplings

Flavor SU(3) symmetry predicts the following relations among $V'VP^-$ couplings (see for example [6]):

$$g_{K^*\omega K} = -\frac{1}{2\sqrt{3}}G^8_{VV'P}[\sin\theta_V - 2r\sqrt{2}\cos\theta_V], \tag{19}$$

$$g_{K^*\phi K} = -\frac{1}{2\sqrt{3}}G^8_{VV'P}[\cos\theta_V + 2r\sqrt{2}\sin\theta_V], \tag{20}$$

$$g_{\rho\omega\pi} = \frac{1}{\sqrt{3}}G^8_{VV'P}[\sin\theta_V + \sqrt{2}r\cos\theta_V], \tag{21}$$

$$g_{\rho\phi\pi} = \frac{1}{\sqrt{3}}G^8_{VV'P}[\cos\theta_V - \sqrt{2}r\sin\theta_V], \tag{22}$$

where $r \equiv G^0_{V'VP}/G^8_{V'VP}$ is the ratio of SU(3) singlet and octet $V'VP$ couplings [6]. The value of r can be obtained from the ratio of Eqs. (21) and (22) using the values of $g_{\rho\omega\pi}$ and $g_{\rho\phi\pi}$ given in Sec. III. In this way, we get $r = 1.256 \pm 0.038$. Now, if we insert this value of r into Eqs. (19) and (20) and use the experimental value of $g_{\rho\omega\pi}$ (see previous section) and the ideal value of the $\omega - \phi$ mixing angle, we get

$$\begin{aligned} g_{K^*\omega K} &= \left[\frac{4r-1}{4r+2}\right]g_{\rho\omega\pi} \\ &= (8.71 \pm 0.95) \times 10^{-3}\ \text{MeV}^{-1}, \end{aligned} \tag{23}$$

$$g_{K^*\phi K} = -\frac{1}{\sqrt{2}}g_{\rho\omega\pi} = -(10.7 \pm 1.3) \times 10^{-3}\ \text{MeV}^{-1}. \tag{24}$$

D. *AVP* couplings

The couplings of axial-vector mesons (A) to the VK^- system are the most difficult to determine. One may attempt to compute them from the measured branching fractions of K_1, K_1' into the ωK^- channel (which is the only allowed by kinematics). We get from this[1]

$$g_{K_1\omega K} = -(3.17 \pm 0.46) \times 10^{-3}\ \text{MeV}^{-1}, \tag{25}$$

$$g_{K_1'\omega K} = (4.8 \pm 2.4) \times 10^{-4}\ \text{MeV}^{-1}. \tag{26}$$

However, the $K_1\phi K$ couplings can not be obtained by means of this procedure, because K_1, $K_1' \to \phi K$ decays are not allowed by kinematics.

[1]In order to extract $g_{K_1\omega K}$ we have taken the maximum values for the mass and width of K_1 that are allowed by their error bars [1].

Given the poor quality of the measurements used to extract the above couplings, we can resort again to the SU(3) flavor symmetry. Since the (K_1, K_1') physical states are a mixture of the (K_{1A}, K_{1B}) states that belong to different 1^3P_1 (A) and 1^1P_1 (B) multiplets of axial mesons, we propose as the starting point the following $\mathcal{A}VP$ interaction Lagrangian:

$$\mathcal{L}_{\mathcal{A}VP} = ig^8_{AVP} f_{abc} P^a (\partial^\alpha A^{b\beta})(\partial_\alpha V^c_\beta - \partial_\beta V^c_\alpha) + g^8_{BVP} d_{abc} P^a (\partial^\alpha B^{b\beta})(\partial_\alpha V^c_\beta - \partial_\beta V^c_\alpha) + \sqrt{\frac{2}{3}} g^0_{BVP} \delta_{ab} P^a (\partial^\alpha B^{b\beta})(\partial_\alpha V^0_\beta - \partial_\beta V^0_\alpha). \tag{27}$$

The physical strange axial mesons are defined in terms of flavor SU(3) states as follows:

$$\begin{aligned} K_1 &= K_{1B}\cos\alpha - K_{1A}\sin\alpha, \qquad (\text{for } K_1^+, K_1^0) \\ K_1' &= K_{1B}\sin\alpha + K_{1A}\cos\alpha, \qquad (\text{for } K_1'^+, K_1'^0) \end{aligned} \tag{28}$$

and

$$\begin{aligned} \bar{K}_1 &= -\bar{K}_{1B}\cos\alpha - \bar{K}_{1A}\sin\alpha, \qquad (\text{for } K_1^-, \bar{K}_1^0) \\ \bar{K}_1' &= -\bar{K}_{1B}\sin\alpha + \bar{K}_{1A}\cos\alpha, \qquad (\text{for } K_1'^-, \bar{K}_1'^0). \end{aligned} \tag{29}$$

The determination of the $K_{1A} - K_{1B}$ mixing angle is still controversial [16]. According to different authors, its value can be in the range $30° \leq \alpha \leq 60°$ [16]. For illustrative purposes, in the present paper we will use $\alpha = 45°$ [1]. If in addition we assume a nonet symmetry for the 1^1P_1 couplings, namely $g^8_{BVP} = g^0_{BVP}$, and the ideal value for the $\omega - \phi$ mixing angle ($\tan\theta_V = 1/\sqrt{2}$), we get the following simplified expressions for the couplings that involve the ϕ and ω mesons (the expressions of the couplings constants for arbitrary values of the α and θ_V mixing angles are given in the appendix):

$$g_{K_1^+\omega K^-} = g_{K_1^0\omega\bar{K}^0} = -g_{K_1^-\omega K^+} = -g_{\bar{K}_1^0\omega K^0} = \frac{1}{2\sqrt{2}}\Sigma_+, \tag{30}$$

$$g_{K_1'^+\omega K^-} = g_{K_1'^0\omega\bar{K}^0} = -g_{K_1'^-\omega K^+} = -g_{\bar{K}_1'^0\omega K^0} = -\frac{1}{2\sqrt{2}}\Sigma_-, \tag{31}$$

$$g_{K_1^+\phi K^-} = g_{K_1^0\phi\bar{K}^0} = -g_{K_1^-\phi K^+} = -g_{\bar{K}_1^0\phi K^0} = \tfrac{1}{2}\Sigma_-, \tag{32}$$

$$g_{K_1'^+\phi K^-} = g_{K_1'^0\phi\bar{K}^0} = -g_{K_1'^-\phi K^+} = -g_{\bar{K}_1'^0\phi K^0} = -\tfrac{1}{2}\Sigma_+, \tag{33}$$

where we have defined $\Sigma_\pm \equiv g^8_{AVP} \pm g^8_{BVP}$.

We can fix the values of the effective couplings $\Sigma_\pm$ by using the decay rates of K_1, K_1' axial mesons in the same limit where Eqs. (30)–(33) were obtained. Using the expressions given in the appendix and comparing with the measured rates of $K_1' \to K^*\pi$ decays [1], we obtain $\Sigma_+ = (5.50 \pm 0.27) \times 10^{-3}$ MeV^{-1}. Similarly, from the measured rate of $K_1' \to \omega K$ we get $\Sigma_- = (1.36 \pm 0.67) \times 10^{-3}$ MeV^{-1}. Finally, if we insert these values into Eqs. (30)–(33), we get

$$g_{K_1^-\omega K^-} = -(1.94 \pm 0.10) \times 10^{-3}\ \text{MeV}^{-1}, \tag{34}$$

$$g_{K_1'^-\omega K^-} = (4.8 \pm 2.4) \times 10^{-4}\ \text{MeV}^{-1}, \tag{35}$$

$$g_{K_1^-\phi K^-} = -(6.8 \pm 3.4) \times 10^{-4}\ \text{MeV}^{-1}, \tag{36}$$

$$g_{K_1'^-\phi K^-} = (2.75 \pm 0.14) \times 10^{-3}\ \text{MeV}^{-1}. \tag{37}$$

Observe that the $K_1\omega K$ coupling in Eqs. (25) and (34) have similar sizes despite the different sources used for their determination. In our calculations, we will use the numerical values shown in Eqs. (34)–(37), and we will add an uncertainty of 15% as an estimate of SU(3) symmetry breaking effects.

V. STRANGENESS-CHANGING DECAYS

With the information on the coupling constants given in the previous section, the only free parameters to our disposal are the relative contributions between the K^* and K'^* vector mesons in the form factor $g(s)$

$$\alpha_{\omega K} \equiv \frac{f_{K'^*} g_{K'^*\omega K}}{f_{K^*} g_{K^*\omega K}}, \quad \text{and} \quad \alpha_{\phi K} \equiv \frac{f_{K'^*} g_{K'^*\phi K}}{f_{K^*} g_{K^*\phi K}}. \tag{38}$$

We can further attempt the use of SU(3) symmetry to derive such couplings. Instead, we will fix the values of $\alpha_{\omega K}$ by requiring that it reproduces the experimental branching fraction of the well-measured $\tau^- \to \omega K^- \nu$ decay (see Table I). Using this method, we obtain two possible values: $\alpha_{\omega K} = 0.54 \pm 0.38$ and $\alpha_{\omega K} = -0.77 \pm 0.40$. Both are consistent with SU(3), since they have similar sizes to the value $\alpha_{\omega\pi} = -0.57 \pm 0.11$, which reproduces the $\tau^- \to \omega\pi^-\nu$ decay data (see Sec. III).

Now, if we assume that $\alpha_{\phi K} \approx \alpha_{\omega K}$, which is also expected on the basis of SU(3), we can predict

$$B(\tau^- \to \phi K^- \nu) = \begin{cases} (2.2 \pm 2.6) \times 10^{-5}, & \text{for } \alpha_{\phi K} = 0.54 \pm 0.38, \\ (1.6 \pm 2.5) \times 10^{-5}, & \text{for } \alpha_{\phi K} = -0.77 \pm 0.40 \end{cases} \tag{39}$$

which are consistent with the experimental values measured by *BABAR* [10] and BELLE [9] Collaborations (see Table I), although with large uncertainties.

The large error bars quoted in Eq. (39) are dominated by the uncertainty in the $K_1' = K_1(1400)$ weak decay constant given in Eq. (13), which was extracted from the poorly measured $\tau \to K_1(1400)\nu$ decay. The error bar in Eq. (39) can be reduced if we use a weak coupling constant obtained from a phenomenological quark model. Thus, for example, if we assume $\alpha = 45°$ from the covariant quark model of Ref. [16], we obtain $f_{K_1'}^{\rm cqm} = (242 \pm 25) \times 10^3$ MeV^2. This value of the weak coupling does not affect in a sensitive way the ratio of K'^*/K^* couplings extracted from $\tau^- \to \omega K^- \nu$ branching fraction, which now become $\alpha_{\omega K} = 0.55 \pm 0.38$ or $\alpha_{\omega K} = -0.78 \pm 0.39$ (actually, the important sensitivity of this decay mode is upon the contribution of K^* and K_1 intermediate states). Using the value of $f_{K_1'}^{\rm cqm}$ obtained above, we get

$$B^{\rm cqm}(\tau^- \to \phi K^- \nu) = \begin{cases} (4.0 \pm 1.8) \times 10^{-5}, & \text{for } \alpha_{\phi K} = 0.55 \pm 0.38, \\ (3.3 \pm 1.8) \times 10^{-5}, & \text{for } \alpha_{\phi K} = -0.78 \pm 0.39 \end{cases} \tag{40}$$

whose central values are in better agreement with experimental data of *BABAR* and BELLE (see Table I) and with a better accuracy. The error bars in Eq. (40) are largely dominated by the uncertainties in the strong and weak coupling constants of the K_1' axial meson. In addition to the uncertainties in the quark model calculation of $f_{K_1'}^{\rm cqrm}$ (see above) and the error bars quoted in Eqs. (34)–(37), we have allowed for a 15% uncertainty in both couplings as an estimate of the SU(3) symmetry breaking effects.

In Fig. 3, we compare the invariant mass distribution of the ϕK^- system, with the measurements reported by the *BABAR* Collaboration [10]. As we can observe, our model (with our numbers multiplied by an arbitrary scale) nicely reproduces the data on the invariant mass distribution. Although it is difficult to discriminate between values of the two-fold ambiguity in $\alpha_{\phi K}$, data seems to favor the solution with $\alpha_{\phi K} = 0.55$. Following the warning note in the caption of Fig. 3, this comparison should be taken with due care.

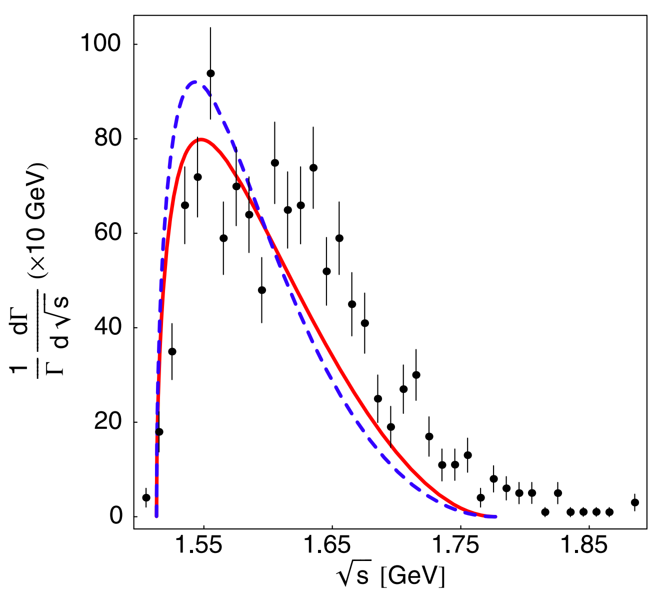

FIG. 3 (color online). Invariant mass distribution of the ϕK^- system in tau decays. The solid (dashed) line corresponds to $\alpha_{\phi K} = 0.55 (\alpha_{\phi K} = -0.78)$. Data points are taken from Ref. [10]; note, however, that these points are raw data and have not been unfolded by detector effects.

VI. CONCLUSIONS

The decays $\tau^- \to (\omega, \phi)P^-\nu_\tau$, where P is a charged pseudoscalar meson, are studied in a phenomenological model where the form factors are dominated by the exchange of meson states with the appropriate quantum numbers. We rely on SU(3) flavor symmetry to fix the strong interaction coupling constants that are not accessible to experimental determination, and we assign to them a $\pm 15\%$ uncertainty as an estimate of SU(3) symmetry breaking effects. In addition, we assume an ideal value for the $\omega - \phi$ mixing angle. Our predictions for the decay modes involving a ϕ vector meson

$$B(\tau^- \to \phi\pi^-\nu_\tau) = (3.64 \pm 1.15) \times 10^{-5},$$

$$B(\tau^- \to \phi K^-\nu_\tau) = \begin{cases} (4.0 \pm 1.8) \times 10^{-5}, & \text{for } \alpha_{\phi K} = 0.55 \pm 0.38, \\ (3.3 \pm 1.8) \times 10^{-5}, & \text{for } \alpha_{\phi K} = -0.78 \pm 0.39 \end{cases} \tag{41}$$

are in very good agreement with measurements reported recently by the *BABAR* [10] and BELLE [9] Collaborations. More refined experimental data on these tau decays, both in branching ratios and hadronic mass distributions, will be useful to provide a validity test of this model. In our view, this is the first calculation of the $\phi\pi^-$, ϕK^- decay modes, which include a full dynamical consideration of the hadronic system.

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Note added in proof.—In order to predict the $\tau \to \phi\pi\nu$ branching ratio, in the present paper we have extracted the $g_{\rho\phi\pi}$ coupling constant from $\phi \to 3\pi$ by assuming that this decay proceeds via the $\rho(770)$-meson intermediate state. However, it has been argued that the $\phi \to 3\pi$ decay amplitude can receive an additional contribution from a contact term [17] due to the Wess-Zumino anomaly [18]. In that case, the decay $\phi \to \pi\gamma$ would become a most reliable process to determine the $g_{\rho\phi\pi}$ coupling. Using this radiative ϕ decay we get $|g_{\rho\phi\pi}| = (0.66 \pm 0.002) \times 10^{-3}$ MeV^{-1} which is smaller than the value used in Sec. III. The corresponding prediction for the branching ratio of the relevant τ lepton decay would become $B(\tau \to \phi\pi\nu) = (6.5 \pm 1.7) \times 10^{-6}$, namely, almost a factor of 6 below the value given in Eq. (12). Once the final results from B factories become available, the correct prediction for this decay mode will be decided.

APPENDIX

There are two octets of axial mesons corresponding to the 1^3P_1 (denoted by A) and 1^1P_1 (denoted by B) quantum number configurations (see [1], p. 166). The A (B) octet is composed of one isotriplet a_1 (b_1), two isodoublets K_{1A} (K_{1B}), and one isosinglet f_1^8 (h_1^8) states. The axial-vector-pseudoscalar interaction is governed by the Lagrangian shown in Eq. (27), with physical strange mesons $K_1(1270)$ and $K_1(1400)$ defined in Eqs. (28) and (29).

From the interaction Lagrangian (27) we can derive the following strong coupling constants that involve the K_1, K_1' axial mesons and the ω, ϕ vector mesons of our interest:

$$g_{K_1^+\omega K^-} = g_{K_1^0\omega\bar{K}^0} = -g_{K_1^-\omega K^+} = -g_{\bar{K}_1^0\omega K^0} = \frac{\sqrt{3}}{2}\left[g_{AVP}^8 \sin\alpha \sin\theta_V - \frac{1}{3}g_{BVP}^8 \cos\alpha(\sin\theta_V - 2\sqrt{2}r_B\cos\theta_V)\right],$$

$$g_{K_1'^+\omega K^-} = g_{K_1'^0\omega\bar{K}^0} = -g_{K_1'^-\omega K^+} = -g_{\bar{K}_1'^0\omega K^0} = -\frac{\sqrt{3}}{2}\left[g_{AVP}^8 \cos\alpha \sin\theta_V + \frac{1}{3}g_{BVP}^8 \sin\alpha(\sin\theta_V - 2\sqrt{2}r_B\cos\theta_V)\right],$$

$$g_{K_1^+\phi K^-} = g_{K_1^0\phi\bar{K}^0} = -g_{K_1^-\phi K^+} = -g_{\bar{K}_1^0\phi K^0} = \frac{\sqrt{3}}{2}\left[g_{AVP}^8 \sin\alpha \cos\theta_V - \frac{1}{3}g_{BVP}^8 \cos\alpha(\cos\theta_V + 2\sqrt{2}r_B\sin\theta_V)\right],$$

$$g_{K_1'^+\phi K^-} = g_{K_1'^0\phi\bar{K}^0} = -g_{K_1'^-\phi K^+} = -g_{\bar{K}_1'^0\phi K^0} = -\frac{\sqrt{3}}{2}\left[g_{AVP}^8 \cos\alpha \cos\theta_V + \frac{1}{3}g_{BVP}^8 \sin\alpha(\cos\theta_V + 2\sqrt{2}r_B\sin\theta_V)\right],$$

where we have defined the ratio of singlet and octet couplings $r_B \equiv g_{BVP}^0/g_{BVP}^8$. In the above expressions, θ_V (respectively α) denotes the $\omega - \phi$ ($K_1 - K_1'$) mixing angle.

Other useful couplings involving the K_1 and K_1' axial mesons are

$$g_{K_1^+\rho^0K^-} = g_{\bar{K}_1^0\rho^0K^0} = -g_{K_1^-\rho^0K^+} = -g_{K_1^0\rho^0\bar{K}^0} = \frac{1}{2}(g_{AVP}^8 \sin\alpha + g_{BVP}^8 \cos\alpha),$$

$$g_{K_1^+\rho^-\bar{K}^0} = g_{K_1^0\rho^+K^-} = -g_{K_1^-\rho^+K^0} = -g_{\bar{K}_1^0\rho^-K^+} = \frac{1}{\sqrt{2}}(g_{AVP}^8 \sin\alpha + g_{BVP}^8 \cos\alpha),$$

$$g_{K_1'^+\rho^0K^-} = g_{\bar{K}_1'^0\rho^0K^0} = -g_{K_1'^-\rho^0K^+} = -g_{K_1'^0\rho^0\bar{K}^0} = -\frac{1}{2}(g_{AVP}^8 \cos\alpha - g_{BVP}^8 \sin\alpha),$$

$$g_{K_1'^+\rho^-\bar{K}^0} = g_{K_1'^0\rho^+K^-} = -g_{K_1'^-\rho^+K^0} = -g_{\bar{K}_1'^0\rho^-K^+} = -\frac{1}{\sqrt{2}}(g_{AVP}^8 \cos\alpha - g_{BVP}^8 \sin\alpha),$$

$$g_{K_1^+K^{*-}\pi^0} = g_{\bar{K}_1^0K^{*0}\pi^0} = -g_{K_1^-K^{*+}\pi^0} = -g_{K_1^0\bar{K}^{*0}\pi^0} = -\frac{1}{2}(g_{AVP}^8 \sin\alpha - g_{BVP}^8 \cos\alpha),$$

$$g_{K_1^+\bar{K}^{*0}\pi^-} = g_{K_1^0K^{*-}\pi^+} = -g_{K_1^-K^{*0}\pi^+} = -g_{\bar{K}_1^0K^{*+}\pi^-} = -\frac{1}{\sqrt{2}}(g_{AVP}^8 \sin\alpha - g_{BVP}^8 \cos\alpha),$$

$$g_{K_1'^+K^{*-}\pi^0} = g_{\bar{K}_1'^0K^{*0}\pi^0} = -g_{K_1'^-K^{*+}\pi^0} = -g_{K_1'^0\bar{K}^{*0}\pi^0} = \frac{1}{2}(g_{AVP}^8 \cos\alpha + g_{BVP}^8 \sin\alpha),$$

$$g_{K_1'^+\bar{K}^{*0}\pi^-} = g_{K_1'^0K^{*-}\pi^+} = -g_{K_1'^-K^{*0}\pi^+} = -g_{\bar{K}_1'^0K^{*+}\pi^-} = \frac{1}{\sqrt{2}}(g_{AVP}^8 \cos\alpha + g_{BVP}^8 \sin\alpha).$$

[1] W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).
[2] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D **39**, 799 (1989).
[3] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C **29**, 637 (1985).
[4] R. Decker, Z. Phys. C **36**, 487 (1987).
[5] S. I. Eidelman and V. N. Ivanchenko, Phys. Lett. B **257**, 437 (1991).
[6] G. López Castro and D. A. López Falcón, Phys. Rev. D **54**, 4400 (1996).
[7] D. Buskulic *et al.* (ALEPH Collaboration), Z. Phys. C **74**, 263 (1997).
[8] K. W. Edwards *et al.* (CLEO Collaboration), Phys. Rev. D **61**, 072003 (2000); R. Balest *et al.* (CLEO Collaboration), Phys. Rev. Lett. **75**, 3809 (1995).
[9] K. Abe *et al.* (Belle Collaboration), Phys. Lett. B **643**, 5 (2006); K. Inami *et al.* (Belle Collaboration), arXiv: hep-ex/0609018.
[10] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **100** 011801 (2008); I. M. Nugent, Nucl. Phys. B, Proc. Suppl. **169**, 98 (2007).
[11] K. Arms *et al.* (CLEO Collaboration), Phys. Rev. Lett. **94**, 241802 (2005).
[12] P. Avery *et al.* (CLEO Collaboration), Phys. Rev. D **55**, R1119 (1997).
[13] D. Scora and N. Isgur, Phys. Rev. D **40**, 1491 (1989); W. Roberts, arXiv:hep-ph/0204260; A. V. Manohar and M. B. Wise, *Heavy Quark Physics*, Cambridge Monogr. Part. Phys., Nucl. Phys., Cosmol. Vol. 10 (Cambridge University Press, Cambridge, England, 2000).
[14] R. R. Akhmetshin *et al.* (CMD-2 Collaboration), Phys. Lett. B **562**, 173 (2003).
[15] M. Davier, A. Hocker, and Z. Zhang, Rev. Mod. Phys. **78**, 1043 (2006).
[16] H. Y. Cheng, C. K. Chua, and C. W. Hwang, Phys. Rev. D **69**, 074025 (2004); H. Y. Cheng and K. C. Yang, Phys. Rev. D **76**, 114020 (2007).
[17] M. Visinescu, Prog. Theor. Phys.**42**, 136 (1969); S. Rudaz Phys. Lett. B **145**, 281 (1984); M. Benayoun, M. Feindt, M. Girone, A Kirk, P. Leruste, J. L. Narjoux, and K. Safarik, Z. Phys. C **58**, 31 (1993); E. A. Kuraev and Z. K. Silagadze, Phys. At. Nucl. **58**, 1589 (1995); [Yad. Fiz. **58N9**, 1687 (1995)]; J. L. Lucino-Martinez, M. Napsuciale, M. D. Scadron, and V. M. Villanueva, Phys. Rev. D **61**, 034013 (2000).
[18] J. Wess and B. Zumino, Phys. Lett. B **37**, 95 (1971).