Majorana neutrinos, neutrino mass spectrum, and the $|\langle m \rangle| \sim 10^{-3}$ eV frontier in neutrinoless double beta decay

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(Received 15 January 2008; published 6 June 2008)

If future neutrino oscillation experiments show that the neutrino mass spectrum is with normal ordering, $m_1 < m_2 < m_3$, and the searches for neutrinoless double beta $[(\beta\beta)_{0\nu}]$ decay with sensitivity to values of the effective Majorana mass $|\langle m \rangle| \gtrsim 10^{-2}$ eV give negative results, the next frontier in the quest for $(\beta\beta)_{0\nu}$-decay will correspond to $|\langle m \rangle| \sim 10^{-3}$ eV. By assuming that massive neutrinos are Majorana particles and their exchange is the dominant mechanism generating $(\beta\beta)_{0\nu}$-decay, we analyze the conditions under which $|\langle m \rangle|$, in the case of three-neutrino mixing and a neutrino mass spectrum with normal ordering, would satisfy $|\langle m \rangle| \geq 0.001$ eV. We consider the specific cases of (i) a normal hierarchical neutrino mass spectrum, (ii) a relatively small value of the CHOOZ angle θ_{13}, as well as (iii) the general case of a spectrum with normal ordering, a partial hierarchy, and a value of θ_{13} close to the existing upper limit. We study the ranges of the lightest neutrino mass m_1 and/or of $\sin^2\theta_{13}$ for which $|\langle m \rangle| \geq 0.001$ eV and discuss the phenomenological implications of such scenarios. We provide also an estimate of $|\langle m \rangle|$ when the three-neutrino masses and the neutrino mixing originate from a neutrino mass term of the Majorana type for the (left-handed) flavor neutrinos and $\sum_j^3 m_j U_{ej}^2 = 0$, but there does not exist a symmetry which forbids the $(\beta\beta)_{0\nu}$-decay.

DOI: 10.1103/PhysRevD.77.113003 PACS numbers: 14.60.Pq, 11.30.Fs, 14.60.Lm

I. INTRODUCTION

The experiments with solar [1–3], atmospheric [4], reactor [5,6], and accelerator neutrinos [7,8] have provided during the past several years compelling evidence for the existence of neutrino oscillations caused by nonzero neutrino masses and neutrino mixing. The neutrino oscillation data (see also [9,10]) imply the presence of three-neutrino mixing in the weak charged lepton current (see, e.g., [11]):

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\nu_{j\mathrm{L}}, \qquad l = e, \mu, \tau, \tag{1.1}$$

where $\nu_{l\mathrm{L}}$ are the flavor neutrino fields, $\nu_{j\mathrm{L}}$ is the field of neutrino ν_j having a mass m_j, and U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [12] $U \equiv U_{\mathrm{PMNS}}$.

In spite of the remarkable progress made, first, in demonstrating experimentally the existence of neutrino oscillations and, second, in determining the pattern of neutrino mixing and the values of the two neutrino mass squared differences, responsible for the solar and atmospheric neutrino oscillations, our knowledge in what concerns most of the basic aspects of neutrino mixing is very limited at present (see, e.g., [11]). We still do not know (i) what the nature of neutrinos with definite mass is—Dirac or Majorana—(ii) what type of spectrum neutrino masses obey, (iii) what the absolute scale of neutrino masses is, (iv) whether the *CP* symmetry is violated in the lepton sector by the neutrino mixing matrix U_{PMNS}, (v) what the value of the CHOOZ angle is—being the smallest mixing angle in the PMNS matrix, it controls (together with the Dirac *CP*-violating phase) the magnitude of *CP*-violation effects in neutrino oscillations—(vi) whether the observed patterns of neutrino mixing is related to the existence of a new symmetry in Nature, etc.

Establishing whether the neutrinos with definite mass ν_j are Dirac fermions possessing distinct antiparticles, or Majorana fermions, i.e., spin 1/2 particles that are identical with their antiparticles, is of fundamental importance for making progress in our understanding of the origin of neutrino masses and mixing and of the symmetries governing the lepton sector of particle interactions (see, e.g., [11]). It is well known that the presence of massive Dirac neutrinos is associated with the existence of a conserved additive lepton number, which can be, e.g., the total lepton charge $L = L_e + L_\mu + L_\tau$. If the particle interactions do not conserve any lepton charge, the massive neutrinos ν_j will be Majorana fermions (see, e.g., [13]).

The only feasible experiments having the potential of establishing the Majorana nature of massive neutrinos at present are the $(\beta\beta)_{0\nu}$-decay experiments searching for the process $(A, Z) \to (A, Z+2) + e^- + e^-$ (for reviews see,

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e.g., [13–16]). The observation of $(\beta\beta)_{0\nu}$-decay and the measurement of the corresponding half-life with sufficient accuracy would not only be a proof that the total lepton charge is not conserved but might also provide unique information on the (i) type of neutrino mass spectrum [17,18] (see also [19,20]), (ii) absolute scale of neutrino masses (see, e.g., [19]), and (iii) Majorana *CP*-violating (CPV) phases [18,21–23] (see also the related discussions in, e.g., [24–27]).

Under the assumptions of $3-\nu$ mixing, of massive neutrinos ν_j being Majorana particles, and of $(\beta\beta)_{0\nu}$-decay generated only by the (vector-axial) charged current weak interaction via the exchange of the three Majorana neutrinos ν_j having masses $m_j \lesssim$ a few MeV, the $(\beta\beta)_{0\nu}$-decay amplitude has the form (see, e.g., [13,18]) $A(\beta\beta)_{0\nu} \cong \langle m\rangle M$, where M is the corresponding nuclear matrix element (NME), which does not depend on the neutrino mixing parameters, and $\langle m\rangle$ is the $(\beta\beta)_{0\nu}$-decay effective Majorana mass:

$$|\langle m\rangle| = |m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\alpha_{21}} + m_3|U_{e3}|^2 e^{i\alpha_{31}}|. \quad (1.2)$$

Here $|U_{ej}|$, $j = 1, 2, 3$, are the absolute values of the elements of the first row of the PMNS mixing matrix, $|U_{e1}| = c_{12}c_{13}$, $|U_{e2}| = s_{12}c_{13}$, $|U_{e3}| = s_{13}$, $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, $\theta_{12} \equiv \theta_\odot$, $\theta_{23} \equiv \theta_A$, and θ_{13} being the solar neutrino, atmospheric neutrino, and CHOOZ mixing angles in the standard parametrization of U_{PMNS} (see, e.g., [18]), and α_{21} and α_{31} are the two Majorana *CP*-violation phases in U_{PMNS} [28,29].

The experimental searches for $(\beta\beta)_{0\nu}$-decay have a long history [14]. The best sensitivity was achieved in the Heidelberg-Moscow ^{76}Ge experiment [30]: $|\langle m\rangle| <$ (0.35–1.05) eV (90% C.L.), where a factor of 3 uncertainty in the relevant NME (see, e.g., [31]) is taken into account. The IGEX Collaboration has obtained [32] $|\langle m\rangle| <$ (0.33–1.35) eV (90% C.L.). A positive signal at $>3\sigma$, corresponding to $|\langle m\rangle| =$ (0.1–0.9) eV, is claimed to be observed in [33], while a recent analysis reports evidence at 6σ of neutrinoless double beta decay with $|\langle m\rangle| = 0.32 \pm 0.03$ eV at 68% C.L. [34]. Two experiments, NEMO3 (with ^{100}Mo and ^{82}Se) [35] and CUORICINO (with ^{130}Te) [36], designed to reach a sensitivity to $|\langle m\rangle| \sim$ (0.2–0.3) eV, set the limits $|\langle m\rangle| <$ (0.7–1.2) eV [35] and $|\langle m\rangle| <$ (0.19–0.68) eV [36] (90% C.L.), respectively, where estimated uncertainties in the NME are accounted for. Most importantly, a large number of projects aim at a sensitivity to $|\langle m\rangle| \sim$ (0.01–0.05) eV [37]: CUORE (^{130}Te), GERDA (^{76}Ge), SuperNEMO, EXO (^{136}Xe), MAJORANA (^{76}Ge), MOON (^{100}Mo), COBRA (^{116}Cd), XMASS (^{136}Xe), CANDLES (^{48}Ca), etc. These experiments, in particular, will test the positive result claimed in [33].

The predicted value of $|\langle m\rangle|$ depends strongly on the type of ν-mass spectrum [17,18], more precisely, on the type of hierarchy neutrino masses obey. Let us recall that the neutrino mass spectrum (in a standardly used convention) can be with *normal ordering*, $m_1 < m_2 < m_3$, or with *inverted ordering*, $m_3 < m_1 < m_2$. The first corresponds to $\Delta m^2_A \equiv \Delta m^2_{31} > 0$, $|\Delta m^2_A| \sim (0.05)^2$ eV^2 being the neutrino mass squared difference responsible for the (dominant) atmospheric neutrino oscillations; the second is realized if $\Delta m^2_A \equiv \Delta m^2_{32} < 0$. Depending on the $\text{sgn}(\Delta m^2_A)$ and the value of the lightest neutrino mass, i.e., the absolute neutrino mass scale $\min(m_j) \equiv m_{\min}$, the neutrino mass spectrum can be (i) *normal hierarchical* (NH): $m_1 \ll m_2 < m_3$, $m_2 \cong (\Delta m^2_\odot)^{1/2}$, $m_3 \cong (\Delta m^2_A)^{1/2}$, $\Delta m^2_\odot \equiv \Delta m^2_{21} \sim 0.009$ eV being the neutrino mass squared difference driving the solar ν_e oscillations; (ii) *inverted hierarchical* (IH): $m_3 \ll m_1 < m_2$, with $m_{1,2} \cong |\Delta m^2_A|^{1/2}$, $\Delta m^2_\odot = \Delta m^2_{21}$; (iii) *quasidegenerate* (QD): $m_1 \cong m_2 \cong m_3 \cong m_0$, $m^2_j \gg |\Delta m^2_A|$, $m_0 \gtrsim$ 0.10 eV.

The existence of significant and robust lower bounds on $|\langle m\rangle|$ in the cases of IH and QD spectra [17] (see also [19]), given respectively[1] by $|\langle m\rangle| \gtrsim 0.01$ eV and $|\langle m\rangle| \gtrsim$ 0.03 eV, which lie either partially (IH spectrum) or completely (QD spectrum) within the range of sensitivity of the next generation of $(\beta\beta)_{0\nu}$-decay experiments, is one of the most important features of the predictions of $|\langle m\rangle|$. At the same time, we have $|\langle m\rangle| \lesssim 5\times10^{-3}$ eV in the case of the NH spectrum [23]. The fact that $\max(|\langle m\rangle|)$ in the case of the NH spectrum is considerably smaller than $\min(|\langle m\rangle|)$ for the IH and QD spectrum opens the possibility of obtaining information about the type of ν-mass spectrum from a measurement of $|\langle m\rangle| \neq 0$ [17]. More specifically, a positive result in the future generation of $(\beta\beta)_{0\nu}$-decay experiments with $|\langle m\rangle| > 0.01$ eV would imply that the NH spectrum is strongly disfavored (if not excluded). For $\Delta m^2_A > 0$, such a result would mean that the neutrino mass spectrum is with normal ordering but is not hierarchical. If $\Delta m^2_A < 0$, the neutrino mass spectrum would be either IH or QD.

If the future $(\beta\beta)_{0\nu}$-decay experiments show that $|\langle m\rangle| < 0.01$ eV, both the IH and the QD spectrum will be ruled out for massive Majorana neutrinos. If, in addition, it is established in neutrino oscillation experiments that the neutrino mass spectrum is with *inverted ordering*, i.e., that $\Delta m^2_A < 0$, one would be led to conclude that either the massive neutrinos ν_j are Dirac fermions or that ν_j are Majorana particles but there are additional contributions to the $(\beta\beta)_{0\nu}$-decay amplitude which interfere destructively with that due to the exchange of light massive Majorana neutrinos. However, if Δm^2_A is determined to be positive in

[1]Up to small corrections, we have in the cases of two spectra [17]: $|\langle m\rangle| \gtrsim \Delta m^2_A \cos 2\theta_\odot$ (IH) and $|\langle m\rangle| \gtrsim m_0 \cos 2\theta_\odot$ (QD). The possibility of $\cos 2\theta_\odot = 0$ is ruled out at $\sim 6\sigma$ by the existing data [38,39], which also imply that $\cos 2\theta_\odot \gtrsim 0.26$ at 2σ [39]. We also have $\Delta m^2_A \gtrsim 2.0 \times 10^{-3}$ eV^2 at 3σ (see further).

neutrino oscillation experiments, the upper limit $|\langle m\rangle| < 0.01$ eV would be perfectly compatible with massive Majorana neutrinos possessing a NH mass spectrum, or a mass spectrum with normal ordering but partial hierarchy, and the quest for $|\langle m\rangle|$ would still be open.

If indeed in the next generation of $(\beta\beta)_{0\nu}$-decay experiments it is found that $|\langle m\rangle| < 0.01$ eV, while the neutrino oscillation experiments show that $\Delta m^2_{\mathrm{A}} > 0$, the next frontier in the searches for $(\beta\beta)_{0\nu}$-decay would most probably correspond to values of $|\langle m\rangle| \sim 0.001$ eV. By taking $|\langle m\rangle| = 0.001$ eV as a reference value, we investigate in the present article the conditions under which $|\langle m\rangle|$ in the case of a neutrino mass spectrum with normal ordering would be guaranteed to satisfy $|\langle m\rangle| \gtrsim 0.001$ eV. We consider the specific cases of (i) a normal hierarchical neutrino mass spectrum, (ii) a relatively small value of the CHOOZ angle θ_{13}, as well as (iii) the general case of a spectrum with normal ordering, a partial hierarchy, and a value of θ_{13} close to the existing upper limit. We study the ranges of the lightest neutrino mass m_1 and/or of $\sin^2\theta_{13}$ for which $|\langle m\rangle| \gtrsim 0.001$ eV and discuss the phenomenological implications of such scenarios.

In the present analysis we do not include the effect of the uncertainty related to the imprecise knowledge of the $(\beta\beta)_{0\nu}$-decay nuclear matrix elements (see, e.g., [31]). We hope that by the time it will become clear whether the searches for $(\beta\beta)_{0\nu}$-decay will require a sensitivity to values of $|\langle m\rangle| < 0.01$ eV, the problem of sufficiently precise calculation of the $(\beta\beta)_{0\nu}$-decay nuclear matrix elements will be resolved.[2]

The paper is organized as follows. In Sec. II we present predictions for $|\langle m\rangle|$ using the present 2σ experimentally allowed ranges of values of the neutrino oscillation parameters and future prospective uncertainties in their values. In Sec. III we analyze the conditions under which $|\langle m\rangle|$ in the case of ν mass spectrum with normal ordering would be guaranteed to satisfy $|\langle m\rangle| \gtrsim 0.001$ eV. We consider the cases of (i) a normal hierarchical spectrum, (ii) small θ_{13}, and (ii) a spectrum with a partial hierarchy. In Sec. IV we give an estimate of $|\langle m\rangle|$ when the three ν masses and the neutrino mixing originate from neutrino mass term of Majorana type for the (left-handed) flavor neutrinos and $\sum_{j=1}^{3} m_j U_{ej}^2 = 0$, but $(\beta\beta)_{0\nu}$-decay is allowed. Section V contains the conclusions of the present analysis.

[2]Encouraging results, in what regards the problem of calculation of the NME, were reported in [31]. A possible test of the NME calculations is discussed in [40]. Let us note that nuclear matrix elements uncertainties do not affect the predictions for the effective Majorana mass parameter directly but induce a spread on the values of the $(\beta\beta)_{0\nu}$-decay half-life times which correspond to the predicted values of $|\langle m\rangle|$. Conversely, if a measurement of the half-life time is performed or a stringent bound is obtained, they would affect the experimentally determined value of $|\langle m\rangle|$ and the constraints following from the latter.

II. NEUTRINO OSCILLATION DATA AND PREDICTIONS FOR $|\langle m\rangle|$

The existing neutrino oscillation data allow us to determine the parameters which drive the solar neutrino and the dominant atmospheric neutrino oscillations— $\Delta m^2_{\odot} = \Delta m^2_{21}$, $\sin^2\theta_{12} \equiv \sin^2\theta_{\odot}$, and $|\Delta m^2_{\mathrm{A}}| = |\Delta m^2_{31}| \cong |\Delta m^2_{32}|$, $\sin^2 2\theta_{23}$—with a relatively good precision and to obtain rather stringent limits on the CHOOZ angle [41] θ_{13} (see, e.g., [38,39]). The best fit values and the 2σ allowed ranges of $|\Delta m^2_{\mathrm{A}}|$, $\Delta m^2_{\odot}$, and $\sin^2\theta_{\odot}$ read [39]:

$$\begin{gathered}(|\Delta m^2_{\mathrm{A}}|)_{\mathrm{BF}} = 2.4 \times 10^{-3}\ \mathrm{eV}^2,\\ 2.1 \times 10^{-3}\ \mathrm{eV}^2 \leq |\Delta m^2_{\mathrm{A}}| \leq 2.7 \times 10^{-3}\ \mathrm{eV}^2,\end{gathered} \tag{2.1}$$

$$\begin{gathered}(\Delta m^2_{\odot})_{\mathrm{BF}} = 7.6 \times 10^{-5}\ \mathrm{eV}^2,\\ 7.3 \times 10^{-5}\ \mathrm{eV}^2 \leq \Delta m^2_{\odot} \leq 8.1 \times 10^{-5}\ \mathrm{eV}^2,\end{gathered} \tag{2.2}$$

$$(\sin^2\theta_{\odot})_{\mathrm{BF}} = 0.32, \qquad 0.28 \leq \sin^2\theta_{\odot} \leq 0.37. \tag{2.3}$$

A combined 3-ν oscillation analysis of the global neutrino oscillation data gives [39]

$$\sin^2\theta_{13} < 0.033\ (0.050) \quad \text{at } 2\sigma\ (3\sigma). \tag{2.4}$$

The existing data allow a determination of $\Delta m^2_{\odot}$, $\sin^2\theta_{\odot}$, and $|\Delta m^2_{\mathrm{A}}|$ at 3σ with an error of approximately 8%, 22%, and 17%, respectively [39]. Future oscillation experiments will improve considerably the precision on these basic parameters: The indicated 3σ errors could be reduced to 4%, 12% [42,43], and better than 5% [43–45] (see also the discussion in [11,23], and references therein), and even to $\sim$1% for Δm^2_{A} [46]. "Near" future experiments with reactor $\bar{\nu}_e$ can improve the current sensitivity to the value of $\sin^2\theta_{13}$ by a factor of (5–10) (see, e.g., [47]), while future long baseline experiments will aim at measuring values of $\sin^2\theta_{13}$ as small as 10^{-4}–10^{-3} (see, e.g., [43,45]).

The type of neutrino mass hierarchy, i.e., $\mathrm{sgn}(\Delta m^2_{\mathrm{A}})$, can be determined by studying oscillations of neutrinos and antineutrinos, say, $\nu_\mu \leftrightarrow \nu_e$ and $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$, in which matter effects are sufficiently large. This can be done in long baseline ν-oscillation experiments (see, e.g., [43,45,48]). If $\sin^2 2\theta_{13} \gtrsim 0.05$ and $\sin^2\theta_{23} \gtrsim 0.50$, information on $\mathrm{sgn}(\Delta m^2_{31})$ might be obtained in atmospheric neutrino experiments by investigating the effects of the subdominant transitions $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ of atmospheric neutrinos which traverse the Earth [49]. For $\nu_{\mu(e)}$ *(or* $\bar{\nu}_{\mu(e)}$) crossing the Earth core, a new type of resonancelike enhancement of the indicated transitions takes place due to the *(Earth) mantle-core constructive interference effect* *[neutrino oscillation length resonance (NOLR)]* [50].[3] For $\Delta m^2_{31} > 0$, the neutrino transitions $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ are enhanced, while, for $\Delta m^2_{31} < 0$, the enhancement of antineutrino transitions $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ takes place, which might allow one to determine $\mathrm{sgn}(\Delta m^2_{31})$. If $\sin^2\theta_{13}$ is sufficiently large, the sign of Δm^2_{A} can also be determined by studying the oscillations of reactor $\bar{\nu}_e$ on distances of $\sim$(20–40) km [53]. An experiment with reactor $\bar{\nu}_e$, which, in particular, might have the capabilities to measure $\mathrm{sgn}(\Delta m^2_{\mathrm{A}})$, was proposed recently in [54]. According to [54], this experiment can provide a determination of $|\Delta m^2_{\mathrm{A}}|$ with an uncertainty of (3–4)% at 3σ.

As is well known, neutrino oscillations are not sensitive to the absolute scale of neutrino masses. Information on the absolute neutrino mass scale can be derived in ^{3}H β-decay experiments [55–57] and from cosmological and astrophysical data. The most stringent upper bounds on the $\bar{\nu}_e$ mass were obtained in the Troitzk [56] and Mainz [57] experiments:

$$m_{\bar{\nu}_e} < 2.3 \text{ eV} \quad \text{at 95\% C.L.} \tag{2.5}$$

We have $m_{\bar{\nu}_e} \cong m_{1,2,3}$ in the case of the QD ν-mass spectrum. The KATRIN experiment [57] is planned to reach a sensitivity of $m_{\bar{\nu}_e} \sim 0.20$ eV; i.e., it will probe the region of the QD spectrum. Information on the type of neutrino mass spectrum can also be obtained in β-decay experiments having a sensitivity to neutrino masses [58] $\sim\sqrt{|\Delta m^2_{\mathrm{A}}|} \cong 5 \times 10^{-2}$ eV (i.e., by a factor of $\sim$4 better sensitivity than KATRIN [57]).

The cosmic microwave background radiation data of the WMAP experiment [59], combined with data from large scale structure surveys (2dFGRS and SDSS), lead to the following upper limit on the sum of neutrino masses (see, e.g., [60]):

$$\sum_j m_j \equiv \Sigma < (0.4\text{–}1.7) \text{ eV} \quad \text{at 95\% C.L.} \tag{2.6}$$

Data on weak lensing of galaxies, combined with data from the WMAP and PLANCK experiments, may allow Σ to be determined with an uncertainty of $\sim$0.04 eV [60,61].

It proves convenient to express [62] the three-neutrino masses in terms of $\Delta m^2_{\odot}$ and Δm^2_{A}, measured in neutrino oscillation experiments, and the absolute neutrino mass scale determined by $\min(m_j) \equiv m_{\min}$.[4] In both cases of ν-mass spectrum with normal and inverted ordering one has (in the convention that we use): $\Delta m^2_{\odot} = \Delta m^2_{21} > 0$, $m_2 = (m_1^2 + \Delta m^2_{\odot})^{1/2}$. For normal ordering, $m_{\min} \equiv m_1$, $\Delta m^2_{\mathrm{A}} = \Delta m^2_{31} > 0$, and $m_3 = (m_1^2 + \Delta m^2_{\mathrm{A}})^{1/2}$, while if the spectrum is with inverted ordering, $m_{\min} = m_3$, $\Delta m^2_{\mathrm{A}} = \Delta m^2_{32} < 0$, and $m_1 = (m_3^2 + |\Delta m^2_{\mathrm{A}}| - \Delta m^2_{\odot})^{1/2}$. For the elements of the PMNS matrix $|U_{\mathrm{e}j}|^2$, $j = 1, 2, 3$, as we have already indicated, the following relations hold: $|U_{\mathrm{e}1}|^2 = \cos^2\theta_{\odot}(1 - \sin^2\theta_{13})$, $|U_{\mathrm{e}2}|^2 = \sin^2\theta_{\odot}(1 - \sin^2\theta_{13})$, and $|U_{\mathrm{e}3}|^2 \equiv \sin^2\theta_{13}$. Thus, given $|\Delta m^2_{\mathrm{A}}|$, $\Delta m^2_{\odot}$, $\theta_{\odot}$, and θ_{13}, $|\langle m \rangle|$ depends on the lightest neutrino mass (absolute neutrino mass scale) $m_{\min}$, the two Majorana phases α_{21} and α_{31}, present in the PMNS matrix, and the type of neutrino mass spectrum (see, e.g., [18]). For a neutrino mass spectrum with normal ordering we have

$$\begin{aligned} |\langle m \rangle| = {} & |m_{\min}\cos^2\theta_{\odot}(1 - \sin^2\theta_{13}) \\ & + \sqrt{m^2_{\min} + \Delta m^2_{\odot}}\,\sin^2\theta_{\odot}(1 - \sin^2\theta_{13})e^{i\alpha_{21}} \\ & + \sqrt{m^2_{\min} + \Delta m^2_{\mathrm{A}}}\,\sin^2\theta_{13}e^{i\alpha_{31}}|, \\ m_{\min} \equiv {} & m_1. \end{aligned} \tag{2.7}$$

For a spectrum with inverted ordering a different expression is valid [18,21]:

$$\begin{aligned} |\langle m \rangle| = {} & |\sqrt{m^2_{\min} + |\Delta m^2_{\mathrm{A}}| - \Delta m^2_{\odot}}\cos^2\theta_{\odot}(1 - \sin^2\theta_{13}) \\ & + \sqrt{m^2_{\min} + |\Delta m^2_{\mathrm{A}}|}\,\sin^2\theta_{\odot}(1 - \sin^2\theta_{13})e^{i\alpha_{21}} \\ & + m_{\min}\sin^2\theta_{13}e^{i\alpha_{31}}| \end{aligned} \tag{2.8}$$

$$\begin{aligned} & \cong \sqrt{m^2_{\min} + |\Delta m^2_{\mathrm{A}}|}\,|\cos^2\theta_{\odot} \\ & \quad + \sin^2\theta_{\odot}e^{i\alpha_{21}}|(1 - \sin^2\theta_{13}),\ m_{\min} \\ & \equiv m_3. \end{aligned} \tag{2.9}$$

In Eq. (2.9) we have neglected $\Delta m^2_{\odot}$ with respect to $(m^2_{\min} + |\Delta m^2_{\mathrm{A}}|)$ and the term $m_{\min}\sin^2\theta_{13}$. According to the existing data, we have $\Delta m^2_{\odot}/(m^2_{\min} + |\Delta m^2_{\mathrm{A}}|) \lesssim 0.032$, and $m_{\min}\sin^2\theta_{13} \ll (m^2_{\min} + |\Delta m^2_{\mathrm{A}}|)^{1/2}\cos 2\theta_{\odot}$. Actually, the term $m_{\min}\sin^2\theta_{13}$ can always be neglected provided $\sin^2\theta_{13} \ll \cos 2\theta_{\odot}$. The expression for $|\langle m \rangle|$ in the case of the IH spectrum follows from Eq. (2.9) if $m^2_{\min} \ll |\Delta m^2_{\mathrm{A}}|$ and $m^2_{\min}$ is neglected with respect to $|\Delta m^2_{\mathrm{A}}|$. For the QD spectrum we get

$$\begin{aligned} |\langle m \rangle| = {} & m_0|(\cos^2\theta_{\odot} + \sin^2\theta_{\odot}e^{i\alpha_{21}})(1 - \sin^2\theta_{13}) \\ & + \sin^2\theta_{13}e^{i\alpha_{31}}| \end{aligned} \tag{2.10}$$

$$\cong m_0|\cos^2\theta_{\odot} + \sin^2\theta_{\odot}e^{i\alpha_{21}}|(1 - \sin^2\theta_{13}), \tag{2.11}$$

where $m_0 \equiv m_{\min}$, $m_1 \cong m_2 \cong m_3$. Evidently, as long as $\sin^2\theta_{13} \ll \cos 2\theta_{\odot}$, the terms $\propto \sin^2\theta_{13}$ in $|\langle m \rangle|$ play an insignificant role in the cases of a neutrino mass spectrum with inverted ordering (i.e., $\Delta m^2_{\mathrm{A}} < 0$) or of the QD type [for any $\mathrm{sgn}(\Delta m^2_{\mathrm{A}})$]. In what concerns the spectrum with normal ordering, the term $\sqrt{m^2_{\min} + \Delta m^2_{\mathrm{A}}}\sin^2\theta_{13}$ can be

[3] As a consequence of this effect, the corresponding $\nu_{\mu(e)}$ (*or* $\bar{\nu}_{\mu(e)}$) transition probabilities can be maximal [51] [for the precise conditions of the mantle-core (NOLR) enhancement, see [50,51]]. Let us note that the Earth mantle-core (NOLR) enhancement of neutrino transitions differs [50] from the Mikheev-Smirnov-Wolfenstein one. It also differs [50,51] from the parametric resonance mechanisms of enhancement discussed in [52].

[4] For a detailed discussion of the relevant formalism, see, e.g., [16,18].

crucial for determining the magnitude of $|\langle m\rangle|$ if massive neutrinos are not QD, i.e., if $m^2_{\rm min} \lesssim \Delta m^2_{\rm A}$, and $\sin^2\theta_{13}$ is sufficiently large (see further).

If CP invariance holds, we have [63] $\alpha_{21} = k\pi$ and $\alpha_{31} = k'\pi$, $k, k' = 0, 1, 2, \ldots$. In the case of CP invariance the phase factors

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \qquad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1, \qquad \eta_{32} \equiv e^{i\alpha_{32}} = \pm 1, \tag{2.12}$$

as is well known, have a simple physical interpretation [13,63]: η_{ik} is the relative CP parity of Majorana neutrinos ν_i and ν_k. Obviously, $|\langle m\rangle|$ depends strongly on the Majorana CPV phase(s): The CP-conserving values of $\alpha_{21} = 0, \pm\pi$ determine, for instance, the range of possible values of $|\langle m\rangle|$ in the cases of IH and QD spectra.

We recall that the neutrino oscillation experiments are insensitive to the two Majorana CP-violation phases in the PMNS matrix [28,64]—the latter do not enter into the expressions for the probabilities of flavor neutrino oscillations. It is interesting to note, however, that, in addition to playing an important role in the predictions for $|\langle m\rangle|$ and, correspondingly, of the $(\beta\beta)_{0\nu}$-decay half-life, the Majorana phase(s) in $U_{\rm PMNS}$ can provide the CP violation necessary for the generation of the baryon asymmetry of the Universe [65,66] (see also [67]). The Majorana phases α_{21} and α_{32} can also affect significantly the predictions for the rates of (lepton flavor-violating) decays $\mu \to e + \gamma$, $\tau \to \mu + \gamma$, etc., in a large class of supersymmetric theories with the seesaw mechanism of ν-mass generation [68].

TABLE I. The maximal values of $|\langle m\rangle|$ (in units of meV) for the NH and IH spectra and the minimal values of $|\langle m\rangle|$ (in units of meV) for the NH, IH, and QD spectra, obtained by using the 2σ allowed values of the neutrino oscillation parameters. The results for the NH and IH spectra are for $m_{\rm min} = 10^{-4}$ eV, while those for the QD spectrum correspond to $m_{\rm min} = 0.2$ eV.

$\|\langle m\rangle\|^{\rm NH}_{\rm min}$	$\|\langle m\rangle\|^{\rm NH}_{\rm max}$	$\|\langle m\rangle\|^{\rm IH}_{\rm min}$	$\|\langle m\rangle\|^{\rm IH}_{\rm max}$	$\|\langle m\rangle\|^{\rm QD}_{\rm min}$
0.7	4.8	11.3	51.5	44.2

First, we will update the predictions for $|\langle m\rangle|$ as a function of $m_{\rm min}$, using as input the 2σ ranges of values of $\Delta m^2_{\rm A}$, $\Delta m^2_\odot$, $\sin^2\theta_\odot$, and $\sin^2\theta_{13}$, obtained from the latest available set of neutrino oscillation data [see Eqs. (2.1), (2.2), and (2.3)]. Since α_{21} and α_{31} cannot be determined in independent experiments, we treat them as free parameters taking values $0 \leq \alpha_{21,31} \leq 2\pi$. The results of this analysis are shown in Fig. 1. We report in Table I the maximal and minimal values of $|\langle m\rangle|$ for the NH spectrum, $m_1 \ll m_2 < m_3$, for the IH spectrum, $m_3 \ll m_1 \simeq m_2$, and for the QD one, $m_{\rm min} = 0.2$ eV.

In Fig. 2, we show the predicted ranges of $|\langle m\rangle|$ using the present best fit values of the neutrino oscillation parameters and their prospective errors as discussed above. We as-

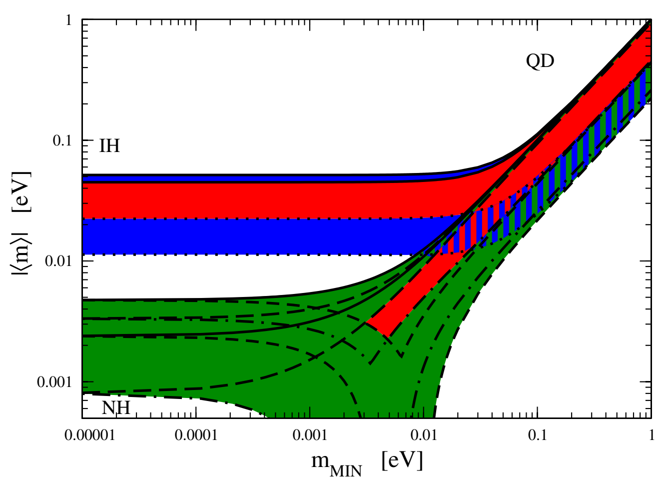

FIG. 1 (color online). The predicted value of $|\langle m\rangle|$ as a function of $m_{\rm min}$, obtained by using the 2σ allowed ranges of $\Delta m^2_{\rm A}$, $\Delta m^2_\odot$, $\sin^2\theta_\odot$, and $\sin^2\theta_{13}$. For the NH and QD (and interpolating) spectra, the green regions within the black lines of a given type (solid, short-dashed, long-dashed, and dashed-dotted) correspond to the four different sets of CP-conserving values of the two phases α_{21} and α_{31} and thus to the four possible combinations of the relative CP parities (η_{21}, η_{31}) of neutrinos $\nu_{1,2}$ and $\nu_{1,3}$: $(+1, +1)$, solid; $(-1, -1)$, short-dashed; $(+1, -1)$, long-dashed; and $(-1, +1)$, dashed-dotted lines. For the IH spectrum, the blue regions delimited by the black solid (dotted) lines correspond to $\eta_{21} = +1$ $(\eta_{21} = -1)$, independently of η_{31}. The regions shown in red correspond to violation of CP symmetry.

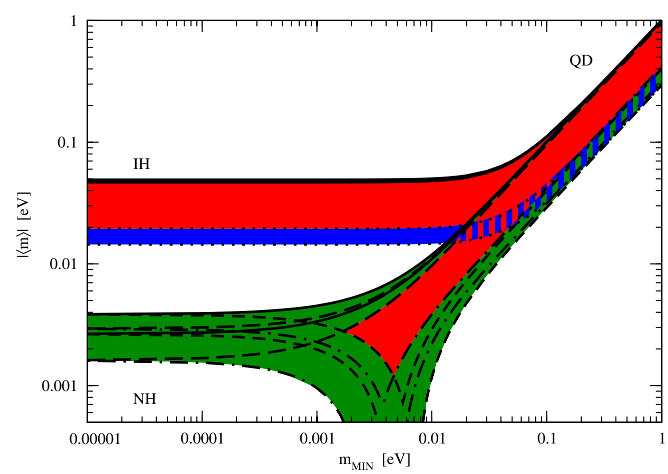

FIG. 2 (color online). The predicted value of $|\langle m\rangle|$ (including a prospective 2σ uncertainty) as a function of $m_{\rm min}$ for $\sin^2\theta_{13} = 0.01$. See text for further details. For the NH and QD (and interpolating) spectra, the regions within the black lines of a given type (solid, short-dashed, long-dashed, and dashed-dotted) correspond to the four different sets of CP-conserving values of the two phases α_{21} and α_{31} and thus to the four possible combinations of the relative CP parities (η_{21}, η_{31}) of neutrinos $\nu_{1,2}$ and $\nu_{1,3}$: $(+1, +1)$, solid; $(-1, -1)$, short-dashed; $(+1, -1)$, long-dashed; and $(-1, +1)$, dashed-dotted lines. For the IH spectrum, the regions delimited by the black solid (dotted) lines correspond to $\eta_{21} = +1$ $(\eta_{21} = -1)$, independently of η_{31}. The regions shown in red correspond to violation of CP symmetry.

TABLE II. The maximal values of $|\langle m\rangle|$ (in units of meV) for the NH and IH spectra and the minimal values of $|\langle m\rangle|$ (in units of meV) for the NH, IH, and QD spectra, at 2σ, for the best fit values of the oscillation parameters and using the prospective errors discussed in the text. We take $\sin^2\theta_{13} = 0.0$ [0.01] (0.02). The results for the NH and IH spectra are obtained for $m_{\rm min} = 10^{-4}$ eV, while those for the QD spectrum correspond to $m_{\rm min} = 0.2$ eV.

$\|\langle m\rangle\|^{\rm NH}_{\rm min}$	$\|\langle m\rangle\|^{\rm NH}_{\rm max}$	$\|\langle m\rangle\|^{\rm IH}_{\rm min}$	$\|\langle m\rangle\|^{\rm IH}_{\rm max}$	$\|\langle m\rangle\|^{\rm QD}_{\rm min}$
2.1 [1.5] (1.0)	3.5 [3.9] (4.4)	14.6 [14.4] (14.3)	49.6 [49.1] (48.6)	61.1 [58.5] (55.8)

sumed a 1σ experimental error of 2%, 2%, and 4% on $\Delta m^2_\odot$, $\Delta m^2_{\rm A}$, and $\sin^2\theta_\odot$, respectively. For $\sin^2\theta_{13}$, we take $\sin^2\theta_{13} = 0.01$, and we consider the 1σ uncertainty in the absolute value of 0.006. In Table II, we give the maximal and minimal values of $|\langle m\rangle|$ for the three spectra NH, IH, and QD.

III. THE $|\langle m\rangle| \sim 10^{-3}$ eV FRONTIER IN $(\beta\beta)_{0\nu}$ DECAY

In the present section we will analyze the conditions under which $|\langle m\rangle| \gtrsim 10^{-3}$ eV in the case of a neutrino mass spectrum with normal ordering. Before discussing the general case of arbitrary m_1 and $\sin^2\theta_{13}$ satisfying the presently existing experimental limits, we will consider two specific but physically interesting cases: (i) negligibly small m_1 (NH spectrum) and (ii) relatively small $\sin^2\theta_{13}$, such that the term $\sqrt{m_1^2 + \Delta m^2_{\rm A}}\sin^2\theta_{13}$ in Eq. (2.7) is strongly suppressed, $\sqrt{m_1^2 + \Delta m^2_{\rm A}}\sin^2\theta_{13} \lesssim 10^{-4}$ eV.

A. Normal hierarchical spectrum

In the case of the normal hierarchical spectrum, we have $m_1 \ll m_{2,3}$, and therefore only the two heavier neutrinos ν_2 and ν_3 contribute to the effective Majorana mass parameter. In this case $m_2 \cong \sqrt{\Delta m^2_\odot}$, $m_3 \cong \sqrt{\Delta m^2_{\rm A}}$, and the sum of neutrino masses reads:

$$m_1 + m_2 + m_3 \cong 0.058 \text{ eV}. \tag{3.1}$$

The effective Majorana mass is given by

$$|\langle m\rangle| \simeq \left|\sqrt{\Delta m^2_\odot}\sin^2\theta_\odot(1 - \sin^2\theta_{13}) + \sqrt{\Delta m^2_{\rm A}}\sin^2\theta_{13}e^{i\alpha_{32}}\right|, \tag{3.2}$$

where $\alpha_{32} \equiv \alpha_{31} - \alpha_{21}$ is the difference of the two Majorana CP-violating phases in $U_{\rm PMNS}$. We will refer to the first term in the right-hand side of Eq. (3.2) as the "solar term" due to its dependence on $\Delta m^2_\odot$, while to the second as the "atmospheric" one. The two terms in the expression for $|\langle m\rangle|$ add constructively if $0 \le \alpha_{32} \le \pi/2$, while for $\pi/2 < \alpha_{32} \le \pi$ partial or complete cancellation between the solar and atmospheric terms can take place. The cancellation is most effective in the case of CP invariance and $\alpha_{32} = \pi$. The degree of cancellation is controlled by $\sin^2\theta_{13}$. For sufficiently small values of θ_{13}, $\sin^2\theta_{13} \lesssim 0.01$, the solar term dominates, and $|\langle m\rangle|$ is predicted to be in the few meV range, $|\langle m\rangle| \sim (2\text{–}3) \times 10^{-3}$ eV. If $\sin^2\theta_{13}$ is close to the present 3σ bound [39] $\sin^2\theta_{13} < 0.05$, the solar and the atmospheric terms in Eq. (3.2) are of the same order, and a substantial cancellation can take place. We will analyze this possibility first qualitatively.

Consider the "extreme" case of $\alpha_{32} = \pi$ and $|\langle m\rangle| = 0$.[5] This requires [18,19,27]

$$|\langle m\rangle| = 0:\ \sin^2\theta_{13} = \frac{\sqrt{\Delta m^2_\odot}}{\sqrt{\Delta m^2_{\rm A}}}\sin^2\theta_\odot, \tag{3.3}$$

where we have neglected $\Delta m^2_\odot$ with respect to $\Delta m^2_{\rm A}$. By taking the best fit values of $\Delta m^2_\odot$, $\sin^2\theta_\odot$, and $\Delta m^2_{\rm A}$, determined from the analysis of the currently existing neutrino oscillation data, we get $\sin^2\theta_{13} = 0.057$, which is ruled out by the data. By using the 2σ and 3σ ranges of allowed values of the same three parameters, we find, respectively, $\sin^2\theta_{13} = 0.046$, which is close to the current 3σ upper limit on $\sin^2\theta_{13}$, and $\sin^2\theta_{13} = 0.041$. Thus, in order for $|\langle m\rangle|$ to be strongly suppressed, $|\langle m\rangle| \ll 10^{-3}$ eV, $\sin^2\theta_{13}$ should have a value close to the existing 3σ upper limit. If we use the current 2σ (3σ) upper limit on $\sin^2\theta_{13}$, $\sin^2\theta_{13} < 0.033$ (0.050), and the present best fit values of $\Delta m^2_\odot$, $\sin^2\theta_\odot$ and $\Delta m^2_{\rm A}$, we find for $\alpha_{32} = \pi$ that $|\langle m\rangle| \gtrsim 1.1(0.2) \times 10^{-3}$ eV. If $0 \le \alpha_{32} \le 5\pi/6$, we obtain $|\langle m\rangle| \gtrsim 1.5(1.3) \times 10^{-3}$ eV. It follows from this simple analysis that if, in the future high precision measurements of $\Delta m^2_\odot$, $\sin^2\theta_\odot$, and $\Delta m^2_{\rm A}$, the currently determined best fit values of these parameters will not change and $\sin^2\theta_{13}$ is found to have a value $\sin^2\theta_{13} \lesssim 0.01$ (0.03), the effective Majorana mass will satisfy $|\langle m\rangle| \gtrsim 2.2\ (1.2) \times 10^{-3}$ eV for any α_{32}. For, e.g., $0 \le \alpha_{32} \le 5\pi/6$, we have $|\langle m\rangle| \gtrsim 1.3 \times 10^{-3}$ eV for any $\sin^2\theta_{13}$ allowed at 3σ by the existing data. Values of $\alpha_{32} \ne 0$ in the indicated range are required for the generation of the baryon asymmetry of the Universe in the "flavored" leptogenesis scenario, in which the requisite CP violation is provided exclusively by the Majorana phase (difference) α_{32} [65].

We will perform next a similar analysis of the conditions under which $|\langle m\rangle| \gtrsim 10^{-3}$ eV, taking into account the

[5] We postpone the discussion of the $(\beta\beta)_{0\nu}$-decay in the case of $|\langle m\rangle| = 0$ to Sec. IV.

current and prospective uncertainties in the measured values of the relevant neutrino oscillation parameters. The minimal predicted value of $|\langle m\rangle|$, $|\langle m\rangle|_{\rm min}$, is obtained in the case of CP conservation and opposite CP parities of the two relevant neutrinos and can be evaluated as

$$|\langle m\rangle|_{\rm min} = |\langle m\rangle|_- - n\sigma(|\langle m\rangle|_-), \quad (3.4)$$

where $|\langle m\rangle|_-$ is the predicted value of $|\langle m\rangle|$ obtained by using the best fit values of the oscillation parameters, $\sigma(|\langle m\rangle|_-)$ is the error on $|\langle m\rangle|$, and $n = 1, 2, 3\ldots$.

By using the propagation of errors and assuming that the errors on the oscillation parameters of interest are small and independent, we obtain the 1σ error on $|\langle m\rangle|$ for any α_{32}:

$$\begin{aligned}\sigma(|\langle m\rangle|) \simeq \frac{1}{2|\langle m\rangle|}&(\sin^4\theta_{13}\Delta m^2_{\rm A}(\sin^2\theta_{13}\sqrt{\Delta m^2_{\rm A}} + \sqrt{\Delta m^2_\odot}\sin^2\theta_\odot\cos\alpha_{32})^2\delta^2(\Delta m^2_{\rm A}) + \Delta m^2_\odot\sin^4\theta_\odot(\sqrt{\Delta m^2_\odot}\sin^2\theta_\odot \\ &+ \sqrt{\Delta m^2_{\rm A}}\sin^2\theta_{13}\cos\alpha_{32})^2(4\delta^2(\sin^2\theta_\odot) + \delta^2(\Delta m^2_\odot)) + 4(\sin^2\theta_{13}\Delta m^2_{\rm A} - \Delta m^2_\odot\sin^4\theta_\odot \\ &+ \sqrt{\Delta m^2_\odot\Delta m^2_{\rm A}}\sin^2\theta_\odot\cos\alpha_{32})^2\sigma^2(\sin^2\theta_{13}))^{1/2}. \end{aligned} \quad (3.5)$$

Here $\delta(\sin^2\theta_\odot)$, $\delta(\Delta m^2_\odot)$, and $\delta(\Delta m^2_{\rm A})$ are the relative errors on the oscillation parameters $\Delta m^2_\odot$, $\sin^2\theta_\odot$, and $\Delta m^2_{\rm A}$, respectively, $\sigma(\sin^2\theta_{13})$ is the absolute error on $\sin^2\theta_{13}$, and we have used the fact that $\sin^2\theta_{13} \ll 1$. We have assumed (see Sec. II and Fig. 2) and will use in our further analysis (see Sec. II) the following values of the errors: $\delta(\sin^2\theta_\odot) = 4\%$, $\delta(\Delta m^2_\odot) = 2\%$, and $\delta(\Delta m^2_{\rm A}) = 2\%$. For the chosen values $\delta(\sin^2\theta_\odot)$, $\delta(\Delta m^2_\odot)$, and $\delta(\Delta m^2_{\rm A})$, the error on $\Delta m^2_\odot$ gives a subdominant contribution in comparison with that on the solar mixing angle, and we neglect it in the following discussion.

If CP invariance holds, we have $\alpha_{32} = 0, \pi$, and Eq. (3.5) simplifies to

$$\sigma(|\langle m\rangle|_\pm) \simeq \sqrt{\Delta m^2_\odot\sin^4\theta_\odot\delta^2(\sin^2\theta_\odot) + \frac{\sin^4\theta_{13}\Delta m^2_{\rm A}}{4}\delta^2(\Delta m^2_{\rm A}) + \Delta m^2_{\rm A}\sigma^2(\sin^2\theta_{13})}, \quad (3.6)$$

where we have neglected $\sqrt{\Delta m^2_\odot}\sin^2\theta_\odot$ with respect to $\sqrt{\Delta m^2_{\rm A}}$. In Eq. (3.6), $|\langle m\rangle|_\pm$ refers to $\eta_{32} = \pm 1$. The contribution of the error on $\Delta m^2_{\rm A}$ in $\sigma(|\langle m\rangle|_\pm)$ is suppressed by the factor $\sin^2\theta_{13}$ and can also be neglected, while the errors on $\sin^2\theta_{13}$ and on $\sin^2\theta_\odot$ can give sizable contributions to $\sigma(|\langle m\rangle|_\pm)$, and both should be taken into account. For the current best fit values of the oscillation parameters, $\sigma(|\langle m\rangle|_\pm)$ is given to a good approximation by $\sigma(|\langle m\rangle|_\pm) \cong \sqrt{\Delta m^2_{\rm A}}\sqrt{(0.057\delta^2(\sin^2\theta_\odot)) + \sigma^2(\sin^2\theta_{13})}$. It is clear from this expression that, for an error on $\sin^2\theta_\odot$ of 4%–8%, the two terms in $\sigma(|\langle m\rangle|_\pm)$ are of the same order if $\sigma(\sin^2\theta_{13}) = 0.004$, while for $\sigma(\sin^2\theta_{13}) \gtrsim 0.006$ the error on $\sin^2\theta_{13}$ typically gives the dominant contribution in $\sigma(|\langle m\rangle|_\pm)$.

For neutrinos of equal CP parities, i.e., $\alpha_{32} = 0$, the mean value of $|\langle m\rangle|$ is predicted to be in the few meV range, and the expected relative error $\sigma(|\langle m\rangle|_\pm)$ varies between 7% and 15%, depending on the specific values of errors and best fit values of the parameters. If the neutrinos ν_2 and ν_3 have opposite CP parities, i.e., $\alpha_{32} = \pi$, the mean value of $|\langle m\rangle|$ is smaller as a partial cancellation between their contributions to $|\langle m\rangle|$ can take place. In this case the error on $|\langle m\rangle|$ can become as large as 30%–40%.

If CP symmetry is broken, the full expression for $\sigma(|\langle m\rangle|)$ [Eq. (3.5)] should be used. It can be shown, however, that $\sigma(|\langle m\rangle|) < \max(\sigma(|\langle m\rangle|_+), \sigma(|\langle m\rangle|_-))$.

By using Eq. (3.6) in the case of $\eta_{32} = -1$, we can study analytically the condition on $\sin^2\theta_{13}$ which guarantees that the predicted value of $|\langle m\rangle|$ is larger than 1 meV. By neglecting the dependence on $\sin^2\theta_{13}$ in $\sigma(|\langle m\rangle|)$, we find an approximate solution for $\sin^2\theta_{13}$:

$$\sin^2\theta_{13} < \frac{\sqrt{\Delta m^2_\odot}\sin^2\theta_\odot - 1\ {\rm meV} - n\sqrt{\Delta m^2_\odot\sin^4\theta_\odot\delta^2(\sin^2\theta_\odot) + \Delta m^2_{\rm A}\sigma^2(\sin^2\theta_{13})}}{\sqrt{\Delta m^2_{\rm A}}}. \quad (3.7)$$

In Fig. 3, we show the values of $\sin^2\theta_{13}$ versus $\Delta m^2_{\rm A}$ for which $|\langle m\rangle|_{\rm min} = 1$ meV is satisfied for $n = 1, 2, 3$ (dashed-dotted, dashed, and dashed–double-dotted lines, respectively). We use the best fit value of $\sin^2\theta_\odot$ and two values of the error on $\sin^2\theta_{13}$. If $\sin^2\theta_{13}$ is larger than the shown values, a strong cancellation between the two contributions to $|\langle m\rangle|$ can take place, and $|\langle m\rangle|_{\rm min} < 1$ meV. This would imply that, depending on the value of α_{32}, there are predicted values of $|\langle m\rangle|$ both smaller and larger than the future reference sensitivity used in this analysis. The

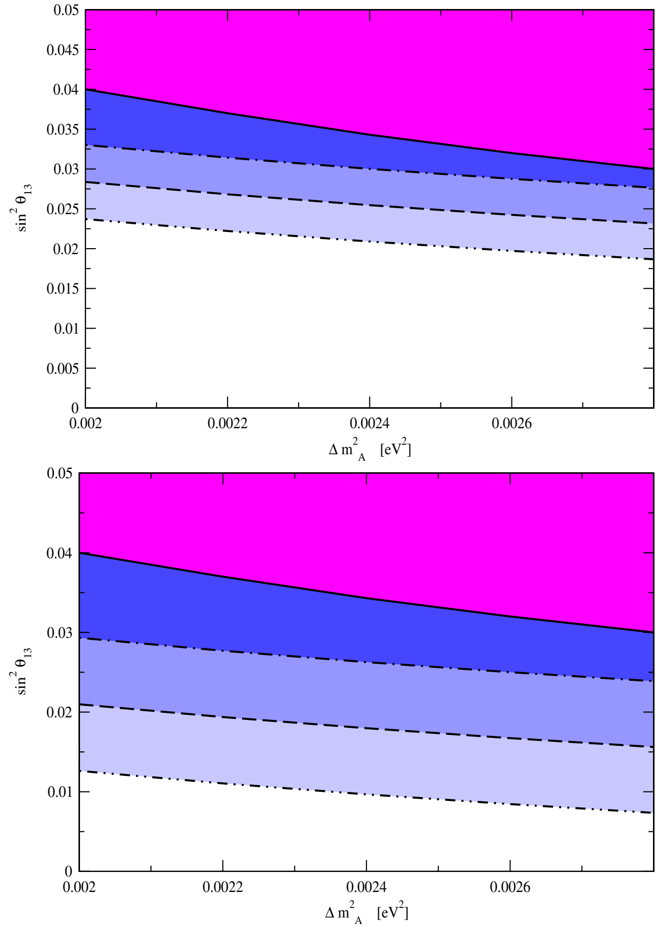

FIG. 3 (color online). The gray (blue online) regions show the values of $\sin^2\theta_{13}$ versus $\Delta m^2_{\rm A}$ for which $|\langle m\rangle|_{\rm min} < 1$ meV at 1σ (2σ) $[3\sigma]$ (region bounded from below by the dashed-dotted (dashed) [dashed–double-dotted] line for $\sin^2\theta_\odot = 0.32$. The error on $\sin^2\theta_{13}$ is taken to be 0.004 (0.008) in the upper (lower) plot. The medium-gray (magenta) region is excluded by the present bound on $\sin^2\theta_{13}$ [39].

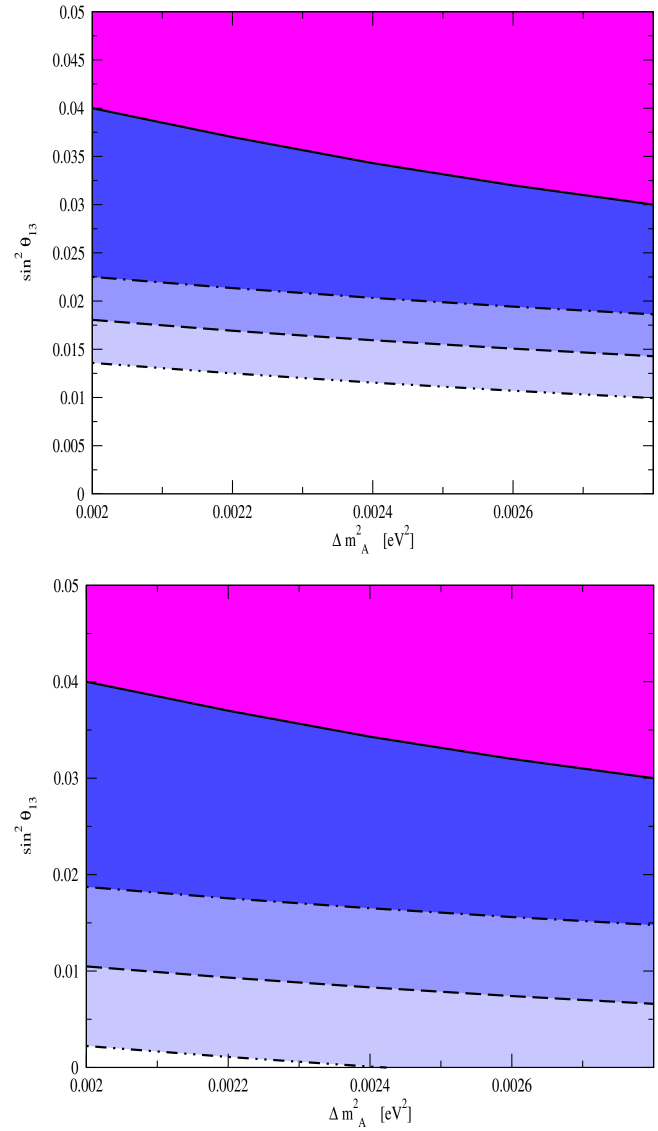

FIG. 4 (color online). The same as in Fig. 3 but for $\sin^2\theta_\odot = 0.26$.

possibility for a future experiment to find a positive signal of $(\beta\beta)_{0\nu}$-decay would depend on the unknown value of α_{32}.

The limiting value of $\sin^2\theta_{13}$ is in the 0.01–0.03 range. The precise value depends critically on the error on $\sin^2\theta_{13}$: For $\sigma(\sin^2\theta_{13}) \simeq 0.004$ (0.008), we have $\sin^2\theta_{13} < 0.02$ (0.01). The limit on $\sin^2\theta_{13}$ depends also on $\Delta m^2_{\rm A}$, as can be easily understood from Eq. (3.7): The larger $\Delta m^2_{\rm A}$, the smaller the bound on $\sin^2\theta_{13}$. The value of $\sin^2\theta_\odot$ controls the magnitude of the first term in $|\langle m\rangle|_-$ and therefore plays an important role in Eq. (3.7). We show the dependence on $\sin^2\theta_\odot$ in Figs. 4 and 5. The smaller the value of $\sin^2\theta_\odot$, the smaller $\sin^2\theta_{13}$ for which one can have $|\langle m\rangle|_{\rm min} < 1$ meV. If $\sin^2\theta_\odot = 0.26$ and $\sigma(\sin^2\theta_{13}) \simeq 0.004$, we have $|\langle m\rangle| > 0.001$ eV for values of $\sin^2\theta_{13} < 0.01$. If, however, $\sigma(\sin^2\theta_{13}) \simeq 0.008$, one can have $|\langle m\rangle|_{\rm min} < 1$ meV even if the (mean) value of $\sin^2\theta_{13} = 0$. On the contrary, for $\sin^2\theta_\odot = 0.40$, a large part of the relevant parameter space is already excluded by the present data [39], and we get $|\langle m\rangle| > 0.001$ eV for $\sin^2\theta_{13} < 0.03$ (0.02) in the case of $\sigma(\sin^2\theta_{13}) = 0.004$ (0.008).

The preceding rather detailed analysis shows that $|\langle m\rangle| \gtrsim 0.001$ eV typically for $\sin^2\theta_{13} \lesssim (0.01\text{–}0.02)$. Values of $\sin^2\theta_{13} \gtrsim (0.01\text{–}0.02)$ are within the sensitivity of the two reactor experiments Double-CHOOZ [69] and Daya Bay [70], which are under preparation, and of the currently operating and future long baseline neutrino oscillation experiments MINOS [8], OPERA [71], T2K, and NOνA [48]. The results of these experiments will be crucial for establishing whether the effective Majorana mass $|\langle m\rangle|$ in the case of a NH neutrino mass spectrum is limited from below and for determining its lower limit.

In the case of a NH spectrum, for $\sin^2\theta_{13} = 0$, only one contribution in $|\langle m\rangle|$ is relevant, the other two being sup-

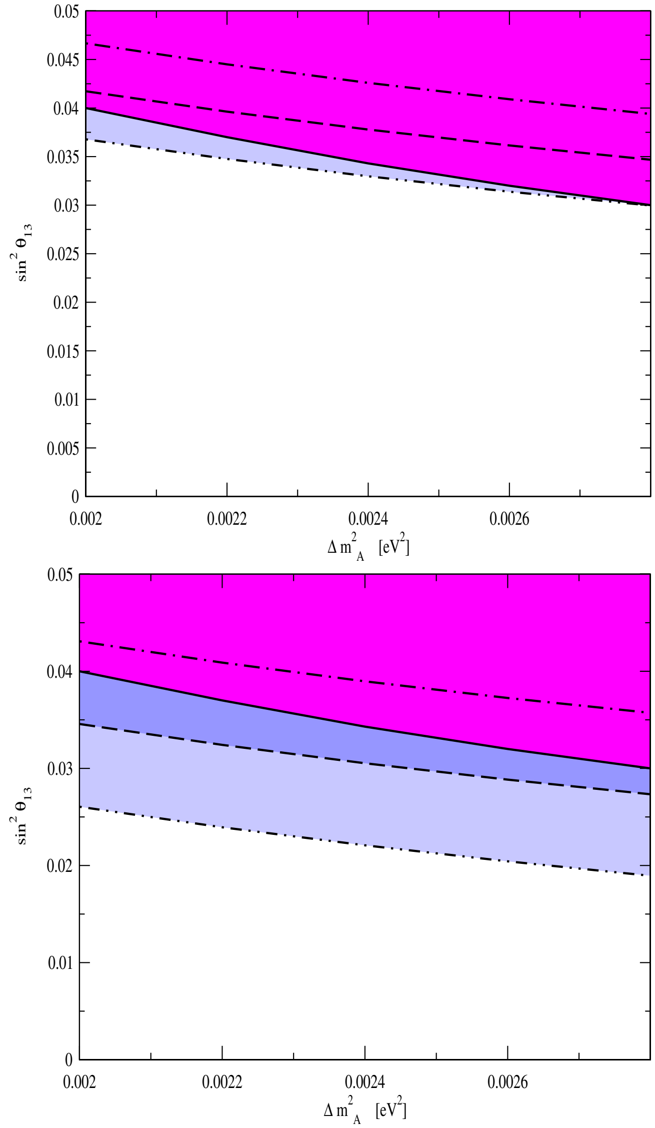

FIG. 5 (color online). The same as in Fig. 3 but for $\sin^2\theta_\odot = 0.40$.

pressed by the negligible values of m_1 and $\sin^2\theta_{13}$. In this case there is no dependence of $|\langle m\rangle|$ on α_{32}. If $\sin^2\theta_{13}$ has a value close to the existing upper limit, a sufficiently accurate measurement of $|\langle m\rangle|$ could allow one to distinguish the two possible CP-parity patterns or establish CP violation. Here we study what would be the requirements in order to have sensitivity to CP violation. We perform a simplified analysis in which we retain for both CP-parity patterns only the dominant term in the theoretical error on $|\langle m\rangle|$:

$$\sigma(|\langle m\rangle|) \simeq \sqrt{\Delta m_A^2}\,\sigma(\sin^2\theta_{13}). \tag{3.8}$$

The existence of a " just-CP-violating" region [18], signaling the possibility to search for CP violation, requires the allowed regions for the CP-conserving cases $\eta_{32} = 1$ and $\eta_{32} = -1$ not to overlap. This condition is satisfied provided

$$\sin^2\theta_{13} > n\sigma(\sin^2\theta_{13}), \tag{3.9}$$

where n is the number of $\sigma(\sin^2\theta_{13})$ considered. For example, for $\sigma(\sin^2\theta_{13}) = 0.004, 0.008$ and $n = 2$, we have $\sin^2\theta_{13} > 0.008, 0.016$. In this case, in principle, it would be possible to distinguish the two CP-parity patterns or find CP violation due to a Majorana CP-violating phase. CP violation would be established if the experimentally allowed value of $|\langle m\rangle|$ is within the just-CP-violating region, once the experimental error on $|\langle m\rangle|$, Δ, and the nuclear matrix element uncertainties are taken into account. Even if Eq. (3.9) is satisfied, this is a formidably challenging task. In the most optimistic case of $\sin^2\theta_{13}$ having a value close to the present 3σ bound, $\sin^2\theta_{13} \simeq 0.05$, for a nuclear matrix element uncertainty $\zeta = 1.5$ on $|\langle m\rangle|$, an error not larger than $\Delta = 0.5$ meV would be required. The width of the just-CP-violating region decreases rapidly with θ_{13}, and for smaller values of $\sin^2\theta_{13}$ the error required on $|\langle m\rangle|$ would be even smaller.

B. The case of small $\sin^2\theta_{13}$

Consider next the possibility of $\sin^2\theta_{13}$ having a rather small value, such that $\sqrt{m_1^2 + \Delta m_A^2}\sin^2\theta_{13} \lesssim 2 \times 10^{-4}$ eV $\ll 10^{-3}$ eV. For $m_1^2 \ll \Delta m_A^2$ this condition is fulfilled if $\sin^2\theta_{13} \lesssim 4 \times 10^{-3}$, while if, e.g., $m_1 \cong 0.05$ eV, it is satisfied provided $\sin^2\theta_{13} \lesssim 3 \times 10^{-3}$. These values of $\sin^2\theta_{13}$ can be tested, e.g., in future long baseline neutrino experiments with superbeams and beta beams and at neutrino factories [43,45].

We set $\sin^2\theta_{13} = 0$ for simplicity in the following discussion. The expression for $|\langle m\rangle|$ simplifies to

$$|\langle m\rangle| = |m_1\cos^2\theta_\odot + \sqrt{m_1^2 + \Delta m_\odot^2}\sin^2\theta_\odot e^{i\alpha_{21}}|. \tag{3.10}$$

For a NH neutrino mass spectrum, i.e., for $m_1 \ll \sqrt{\Delta m_\odot^2}\sin^2\theta_\odot/\cos^2\theta_\odot \cong 4 \times 10^{-3}$ eV, we always have $|\langle m\rangle| \cong 3 \times 10^{-3}$ eV. If, however, $m_1^2 \gtrsim \Delta m_\odot^2$, the neutrino mass spectrum will not be hierarchical. There are two possibilities.

(i) For $m_1^2 \gg \Delta m_\odot^2 \cong 7.6 \times 10^{-5}$ eV^2, we get

$$|\langle m\rangle| \cong m_1\sqrt{1 - \sin^2 2\theta_\odot \sin^2\frac{\alpha_{21}}{2}} \gtrsim m_1\cos 2\theta_\odot. \tag{3.11}$$

By taking $m_1 \gtrsim 2 \times 10^{-2}$ eV and the 2σ (3σ) lower limit on $\cos 2\theta_\odot$, $\cos 2\theta_\odot \gtrsim 0.26$ (0.20), we find $|\langle m\rangle| \gtrsim 5.2(4.0) \times 10^{-3}$ eV. In this case $m_2 = \sqrt{m_1^2 + \Delta m_\odot^2} \gtrsim 2.2 \times 10^{-2}$ eV, and the sum of neutrino masses satisfies

$$m_1 + m_2 + m_3 \gtrsim 9.5 \times 10^{-2} \text{ eV}. \quad (3.12)$$

(ii) If, however, $m_1^2 \sim \Delta m_\odot^2$ and $\alpha_{21} \sim \pi$, a cancellation between the two terms in Eq. (3.10) is possible, and $|\langle m \rangle|$ can be strongly suppressed, $|\langle m \rangle| \ll 10^{-3}$ eV. Consider the extreme case of $|\langle m \rangle| = 0$ [for a more detailed discussion of the $(\beta\beta)_{0\nu}$-decay in the case of $|\langle m \rangle| = 0$, see Sec. IV]. For $\alpha_{21} = \pi$, it is realized if [19,27]

$$|\langle m \rangle| = 0: m_1 = m_2 \tan^2\theta_\odot. \quad (3.13)$$

By using the relation $m_2 = (m_1^2 + \Delta m_\odot^2)^{1/2}$, we find that $|\langle m \rangle| = 0$ can hold in the case being studied if $m_1^2 = \Delta m_\odot^2 \sin^4\theta_\odot / \cos 2\theta_\odot \cong 2.2 \times 10^{-5}$ eV^2, where we have used the best fit values of $\Delta m_\odot^2$ and $\sin^2\theta_\odot$. This implies that $m_1 \cong 4.6 \times 10^{-3}$ eV, $m_2 \cong 10^{-2}$ eV, and, correspondingly,

$$m_1 + m_2 + m_3 \cong 6.4 \times 10^{-2} \text{ eV}. \quad (3.14)$$

It is not difficult to convince oneself, however, that, if $\alpha_{21} = \pi$, one obtains $|\langle m \rangle| \gtrsim \mu$ for

$$m_1 \gtrsim \frac{\mu}{\cos 2\theta_\odot}\Big[\cos^2\theta_\odot + \sin^2\theta_\odot \sqrt{1 + \mu^{-2}\Delta m_\odot^2 \cos 2\theta_\odot}\Big], \quad (3.15)$$

where the reference value $\mu = 10^{-3}$ eV in the case of interest. By using the best fit values of $\Delta m_\odot^2$ and $\sin^2\theta_\odot$ we get $m_1 \gtrsim 6.6 \times 10^{-3}$ eV. For the sum of neutrino masses we obtain $m_1 + m_2 + m_3 \gtrsim 6.7 \times 10^{-2}$ eV.

This qualitative analysis shows that, if $\sin^2\theta_{13} \lesssim 3 \times 10^{-3}$ and the sum of neutrino masses satisfies $m_1 + m_2 + m_3 \gtrsim 7 \times 10^{-2}$ eV, we will have $|\langle m \rangle| \geq 10^{-3}$ eV for any α_{21}.

C. Spectrum with partial hierarchy

The neutrino mass spectrum with a partial hierarchy interpolates between the normal hierarchical one ($m_1 \ll m_2 \ll m_3$) and the quasidegenerate case ($m_1 \simeq m_2 \simeq m_3$) and is characterized by values of m_1 which give a contribution to $|\langle m \rangle|$ comparable to the one of $\sqrt{\Delta m_\odot^2}$ and $\sqrt{\Delta m_A^2}$. Depending on the value of $\sin^2\theta_{13}$, the effects of m_1 become relevant for m_1 as small as a few $\times 10^{-4}$ eV (see Figs. 1 and 2), while for $m_1 \gtrsim 0.05$ eV the role of $\Delta m_\odot^2$ and Δm_A^2 is subdominant in $|\langle m \rangle|$, and the normal and inverted ordering give the same predictions for $|\langle m \rangle|$. In the following, we will consider the above-quoted values as conventional boundaries for m_1 in the spectrum with a partial hierarchy for the study of $|\langle m \rangle|$. As is well known [19], in the case of such a mass spectrum we can have $|\langle m \rangle| \ll 1$ meV and even $|\langle m \rangle| = 0$. However, this requires that the lightest neutrino mass m_1 has a value in the rather narrow interval $m_1 \sim$ (a few $\times 10^{-3}$–10^{-2}) eV. As a consequence, the sum of neutrino masses should also lie within a specific interval. Here we analyze the values of m_1 and $\sin^2\theta_{13}$ for which the indicated strong cancellation in $|\langle m \rangle|$ would not take place and we would have $|\langle m \rangle| \geq 1$ meV.

For the neutrino mass spectrum under discussion, all three contributions to $|\langle m \rangle|$ in Eq. (2.7) are relevant. We consider the effect of cancellations between the three terms in the case of *CP* invariance, in which there are four different neutrino *CP*-parity patterns. We will denote them as $+++$ $(+--)$ if $\alpha_{21,31} = 0$ (π), and $++-$ $(+-+)$ when $\alpha_{21} = 0$ (π) while $\alpha_{31} = \pi$ (0). The prediction in the case of *CP* violation will lie within the ones obtained for *CP* conservation. Obviously, if both $0 \lesssim \alpha_{21} \lesssim \pi/2$ and $0 \lesssim \alpha_{31} \lesssim \pi/2$, there will be no mutual compensation between the three terms in Eq. (2.7), and we would have $|\langle m \rangle| \gtrsim 3 \times 10^{-3}$ eV.

For each *CP*-parity pattern, we analyze what are the values of m_1 and $\sin^2\theta_{13}$ which would guarantee $|\langle m \rangle| \geq 1$ meV or, conversely, which would be implied by a negative result for a search of a neutrinoless double beta decay with a sensitivity of 1 meV, in the hypothesis of Majorana neutrinos. The effective Majorana mass parameter would be predicted to be smaller than 1 meV, if a sufficient cancellation between the three terms in the right-hand side of Eq. (2.7) takes place.

Here we use $\mu = 1$ meV as a reference value for $|\langle m \rangle|$, but similar results can be obtained for other values of μ in the few meV range.[6] The central value of m_1 can be found by solving Eq. (2.7) with $|\langle m \rangle| = 1$ meV, while the error on m_1 is obtained by propagating the errors on the oscillation parameters:

$$\sigma(m_1) = \left(\frac{\partial |\langle m \rangle|}{\partial m_1}\right)^{-1} \sigma(|\langle m \rangle|) \simeq \frac{\sigma(|\langle m \rangle|)}{\cos^2\theta_\odot \pm \frac{m_1 \sin^2\theta_\odot}{\sqrt{m_1^2 + \Delta m_\odot^2}}}. \quad (3.16)$$

The degree of cancellation between the three terms in $|\langle m \rangle|$ depends on the neutrino *CP*-parity pattern. The results for m_1 for the different *CP*-parity patterns are presented in Fig. 6 for three values of $\theta_\odot$, $\sin^2\theta_\odot = 0.26, 0.32$, and 0.40, using the prospective relative errors of 2%, 2%, and 4% for Δm_A^2, $\Delta m_\odot^2$, and $\sin^2\theta_\odot$, respectively, and the absolute error of 0.006 on $\sin^2\theta_{13}$.

We can understand the results in Fig. 6 by performing a simplified analysis neglecting $\sigma(m_1)$. We study each *CP*-parity pattern separately. In the following, we will use the present best fit values of $\Delta m_\odot^2$, Δm_A^2, and $\sin^2\theta_\odot$, unless otherwise indicated.

- For the *CP*-parity pattern $(+++)$, no cancellation takes place, and we will have $|\langle m \rangle| \gtrsim 2.5$ meV for

[6] Let us note that a similar analysis for $\mu = 0$ was performed in Ref. [19].

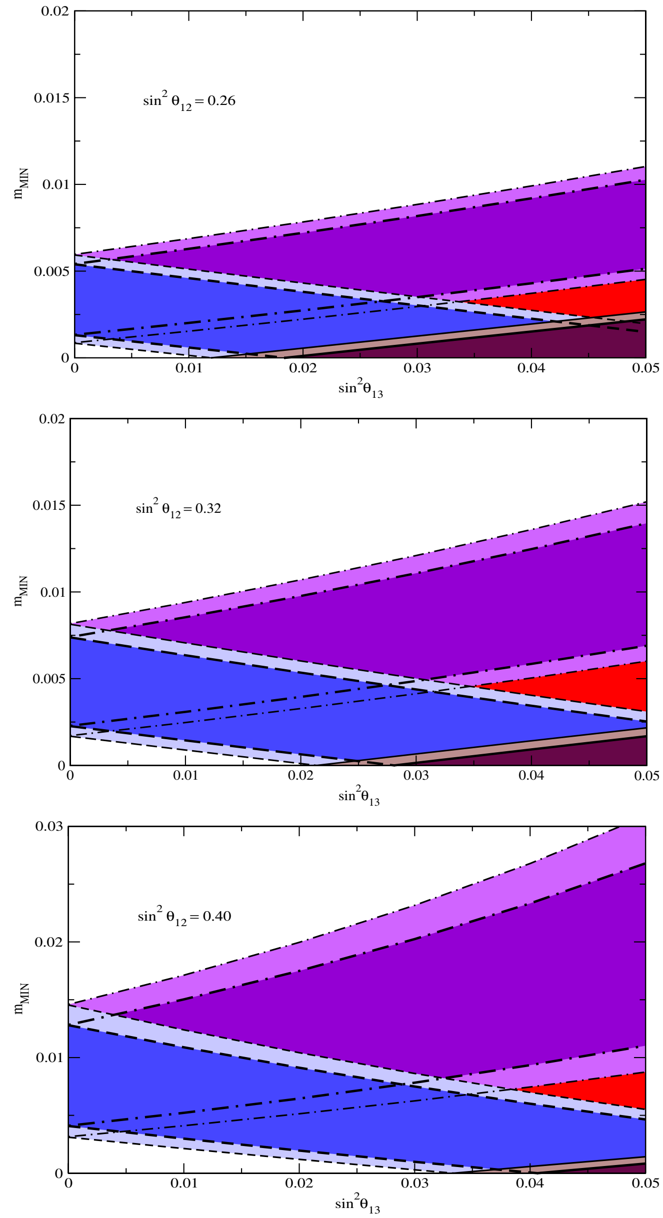

FIG. 6 (color online). The gray (color online) regions denote the ranges of $m_{\min}$ for which $|\langle m\rangle| < 1$ meV and are delimited by thick (thin) lines at 1σ (2σ). The CP-conserving patterns are indicated by (i) solid lines for the case $++-$, (ii) dashed lines for the $+-+$ one, and (iii) dashed-dotted lines for $+--$. The red triangular region requires CP violation. The present best fit values for $\Delta m^2_{\odot}$ and $\Delta m^2_{\rm A}$ are used.

any allowed value of θ_{13} and $\theta_{\odot}$. A negative search for a neutrinoless double beta decay with a sensitivity of a few meV, such that $|\langle m\rangle|_0 < |\langle m\rangle|_+ - n\sigma(|\langle m\rangle|)$, where $|\langle m\rangle|_0$ is the experimentally determined value of $|\langle m\rangle|$ and $|\langle m\rangle|_+$ corresponds to a NH spectrum and $\eta_{32} = +1$ (see Sec. III A), would strongly disfavor (if not rule out) this possibility.

- In the $(++-)$ case, a significant cancellation can take place only if the atmospheric term $\sqrt{\Delta m^2_{\rm A}}\sin^2\theta_{13}$ is of the same order as the sum of the first two terms in the right-hand side of Eq. (2.7). We can have $|\langle m\rangle| \geq \mu = 1$ meV for a given m_1 provided $\sin^2\theta_{13}$ satisfies

$$\sin^2\theta_{13} \leq \frac{m_1\cos^2\theta_{\odot} + \sqrt{m_1^2 + \Delta m^2_{\odot}}\sin^2\theta_{\odot} - \mu}{\sqrt{m_1^2 + \Delta m^2_{\rm A}}}. \tag{3.17}$$

The above inequality is always fulfilled for $m_1 \gtrsim \sqrt{\Delta m^2_{\odot}}$. For $m_1 \ll \sqrt{\Delta m^2_{\odot}}$, this condition becomes $\sin^2\theta_{13} \leq (\sin^2\theta_{13})_0$, with

$$(\sin^2\theta_{13})_0 = \sqrt{\frac{\Delta m^2_{\odot}}{\Delta m^2_{\rm A}}}\sin^2\theta_{\odot} - \frac{\mu}{\sqrt{\Delta m^2_{\rm A}}}. \tag{3.18}$$

It is satisfied for the best fit values of $\Delta m^2_{\odot}$, $\sin^2\theta_{\odot}$, and $\Delta m^2_{\rm A}$, while if one uses the 3σ allowed ranges of these parameters, the inequality implies $\sin^2\theta_{13} \lesssim 0.026$, 0.036, and 0.051 for $\sin^2\theta_{\odot} = 0.26$, 0.32, and 0.40, respectively. These values of $\sin^2\theta_{13}$ are close to the present 3σ upper bound. In summary, for values of $\sin^2\theta_{13} \leq (\sin^2\theta_{13})_0$, $|\langle m\rangle|$ is guaranteed to be larger than 1 meV for any m_1. For a given $\sin^2\theta_{13} > (\sin^2\theta_{13})_0$, we will have $|\langle m\rangle| \geq \mu = 1$ meV if m_1 satisfies $m_1 \geq (m_1^A)_-$, where

$$(m_1^A)_{\pm} = \frac{1}{\cos 2\theta_{\odot}}\Big[(\sqrt{\Delta m^2_{\rm A}}\sin^2\theta_{13} + \mu)\cos^2\theta_{\odot} \pm \sin^2\theta_{\odot} \times \sqrt{(\sqrt{\Delta m^2_{\rm A}}\sin^2\theta_{13} + \mu)^2 + \Delta m^2_{\odot}\cos 2\theta_{\odot}}\Big]. \tag{3.19}$$

In deriving Eq. (3.19) we have neglected m_1^2 with respect to $\Delta m^2_{\rm A}$ and have taken $\cos^2\theta_{13} \simeq 1$. The lower bound $(m_1^A)_-$ of m_1 in Eq. (3.19) increases with $\sin^2\theta_{13}$ but is rather small: For $\mu = 10^{-3}$ eV, $\sin^2\theta_{13} = 0.05$, and best fit values of the other relevant oscillation parameters, we get $(m_1^A)_- \cong 0.9 \times 10^{-3}$ eV.

- For the CP-parity pattern $(+-+)$, a partial cancellation can take place between the first and the second terms in Eq. (2.7); the cancellation would be significant only if $m_1 \sim$ a few meV. The second term in Eq. (2.7) would dominate, and we would have $|\langle m\rangle| \geq \mu = 10^{-3}$ eV only if m_1 and $\sin^2\theta_{13}$ are sufficiently small, more precisely, if $0 \leq m_1 \leq (-(m_1^A)_-)$ and $\sin^2\theta_{13} \leq (\sin^2\theta_{13})_0$, where $(m_1^A)_-$ and $(\sin^2\theta_{13})_0$ are given in Eqs. (3.18) and (3.19), respectively.
The sum of the first and third terms in Eq. (2.7) will dominate and will lead to $|\langle m\rangle| \geq \mu = 10^{-3}$ eV for $m_1 \geq (m_1^B)_+$, where

$$(m_1^B)_\pm = \frac{1}{\cos 2\theta_\odot}\Big[(\mu - \sqrt{\Delta m_A^2}\sin^2\theta_{13})\cos^2\theta_\odot \pm \sin^2\theta_\odot \times \sqrt{(\mu - \sqrt{\Delta m_A^2}\sin^2\theta_{13})^2 + \Delta m_\odot^2 \cos 2\theta_\odot}\Big], \tag{3.20}$$

provided

$$\sin^2\theta_{13} \leq \sqrt{\frac{\Delta m_\odot^2}{\Delta m_A^2}}\sin^2\theta_\odot + \frac{\mu}{\sqrt{\Delta m_A^2}}. \tag{3.21}$$

Given the experimental 3σ upper bound $\sin^2\theta_{13} < 0.05$, the second inequality is always satisfied for $\mu = 10^{-3}$ eV. For $\sin^2\theta_{13} = 0\ (0.02)$ we get from Eq. (3.20): $m_1 \gtrsim 6.6\ (4.7) \times 10^{-3}$ eV.

- Finally, consider the case $(+--)$. As the second and third terms in the right-hand side of Eq. (2.7) are summed constructively, a strong cancellation in $|\langle m\rangle|$ can happen only for sufficiently large values of m_1. We get $|\langle m\rangle| \geq \mu = 10^{-3}$ eV for $0 \leq m_1 \leq (-(m_1^B)_-)$ and for $m_1 \geq (m_1^A)_+$, where $(m_1^A)_+$ is given in Eq. (3.19). The maximal value of m_1 determined by Eq. (3.20) can be rather large. More specifically, we have $-(m_1^B)_- = 2.8\ (5.0)\ [7.6] \times 10^{-3}$ eV for $\sin^2\theta_{13} = 0\ (0.025)\ [0.05]$. For the minimal value of m_1 determined by the inequality $m_1 \geq (m_1^A)_+$, we get for $\sin^2\theta_{13} = 0\ (0.025)\ [0.05]$: $(m_1^A)_+ = 6.6(9.3)[12.1]\times 10^{-3}$ eV. In the latter case the sum of the neutrino masses is limited from below by $(m_1 + m_2 + m_3) \gtrsim 6.8\ (7.2)\ [7.9] \times 10^{-2}$ eV. Both $(-(m_1^B)_-)$ and $(m_1^A)_+$ increase with θ_{13} and $\sin^2\theta_\odot$.

It follows from the preceding discussion that, if a future highly sensitive $(\beta\beta)_{0\nu}$-decay experiment does not find a positive signal down to $|\langle m\rangle| \sim 1$ meV, Majorana neutrinos would still be allowed, but the spectrum would be constrained to be with normal ordering and m_1 would be bound to be smaller than $\sim 10^{-2}$ eV. The CP-parity pattern $(+++)$ will be strongly disfavored (if not ruled out) as well. If, in addition, it is found that $\sin^2\theta_{13} \lesssim 0.01$, (i) the CP-parity pattern $(++-)$ will also be disfavored, and (ii) m_1 would be constrained to lie in the interval $m_1 \sim (10^{-3}$–$10^{-2})$ eV. No other future neutrino experiment will have the capability of constraining the lightest neutrino mass (and the absolute neutrino mass scale) in the meV range. Obviously, the above limits would hold only if massive neutrinos are Majorana particles. If the lightest neutrino has a mass in the interval $m_1 \sim (10^{-3}$–$10^{-2})$ eV, this can have important effects on the generation of the baryon asymmetry of the Universe in the flavored leptogenesis scenario of matter-antimatter asymmetry generation [66].

IV. $(\beta\beta)_{0\nu}$ DECAY IN THE CASE OF $|\langle m\rangle| = 0$

In the present section we shall discuss briefly the possible implications of having $|\langle m\rangle| = 0$ for the process of $(\beta\beta)_{0\nu}$-decay. If $|\langle m\rangle| = 0$ as a consequence of conservation of a certain lepton charge, which could be, e.g., L_e, L, or $L' = L_e - L_\mu - L_\tau$, the $(\beta\beta)_{0\nu}$-decay will be strictly forbidden. However, in the case of a neutrino mass spectrum with normal ordering, one can have $|\langle m\rangle| = 0$, as we have seen, as a consequence of an "accidental" relation involving the neutrino masses, the solar neutrino, and CHOOZ mixing angles and the Majorana phase(s) in $U_{\rm PMNS}$. For the spectrum of the normal hierarchical type, the relation of interest is given by Eq. (3.3), while if $\sin^2\theta_{13}$ is negligibly small, it is shown in Eq. (3.13). Neither of the two relations can be directly associated with a symmetry which forbids $(\beta\beta)_{0\nu}$-decay. Thus, if $|\langle m\rangle| = 0$ is a consequence of Eq. (3.3) or (3.13), $(\beta\beta)_{0\nu}$-decay will still be allowed. In what follows we will estimate the nonzero contribution to the $(\beta\beta)_{0\nu}$-decay amplitude $A(\beta\beta)_{0\nu}$ due to the exchange of the light massive Majorana neutrinos ν_j in the case when $|\langle m\rangle| = 0$ and there is no symmetry forbidding the decay.

Suppose that neutrino masses and mixing arise due to the Majorana mass term of the three flavor neutrinos:

$$\mathcal{L}^M(x) = -\tfrac{1}{2} m_{ll'} \bar{\nu}^c{}_{lR} \nu_{l'L} + \text{H.c.}, \tag{4.1}$$

where $\nu_{lR}^c = C(\bar{\nu}_{lL})^{\rm T}$, $l = e, \mu, \tau$, C being the charge-conjugated matrix. We have $m_{ll'} = m_{l'l}$, $l, l' = e, \mu, \tau$ (see, e.g., [13]). The mass term in Eq. (4.1) is diagonalized by using the congruent transformation: $m = U^* m^d U^\dagger$, where $m^d = {\rm diag}(m_1, m_2, m_3)$ is a diagonal matrix formed by the masses of the Majorana neutrinos ν_j and U is the PMNS matrix.[7] The effective Majorana mass $\langle m\rangle$ arises in $A(\beta\beta)_{0\nu}$ from the virtual neutrino propagator (see, e.g., [13]):

$$\mathcal{P} = \sum_j U_{ej}^2 \frac{m_j}{q^2 - m_j^2} = P_1 + P_3 + P_5 + \cdots, \tag{4.2}$$

where

$$P_1 = \frac{1}{q^2}\sum_j U_{ej}^2 m_j = \frac{1}{q^2}\langle m\rangle, \tag{4.3}$$

$$P_3 = \frac{1}{q^2}\sum_j U_{ej}^2 m_j \frac{m_j^2}{q^2}, \text{ etc.} \tag{4.4}$$

Here q is the momentum of the virtual neutrino, and we have used the fact that $m_j^2 \ll |q^2|$. Typically, one has for the average momentum of the virtual neutrino in $(\beta\beta)_{0\nu}$-decay (see, e.g., [72,73]) $|q^2| \sim (100\ \text{MeV})^2$. As a consequence, the following inequalities hold:

[7] We work in the basis in which the charged lepton mass matrix is diagonal.

$|P_{2n+1}| \ll |P_1|$, $n = 1, 2, \ldots$. Usually, the terms P_3, P_5, etc., are neglected in the expression for $\mathcal{P}$. The dominant term $P_1 \propto \langle m \rangle$, which leads to $A(\beta\beta)_{0\nu} \propto \langle m \rangle$. The q^{-2} factor in P_1 gives rise to a Coulomb-like potential of interaction between the nucleons exchanging the virtual neutrino in the nucleus undergoing $(\beta\beta)_{0\nu}$-decay.

Assume now that $|\langle m \rangle| = 0$. In this case $P_1 = 0$, and the dominant term in the expression for $\mathcal{P}$ [Eq. (4.2)] will be P_3. If $|\langle m \rangle| = 0$ is not a consequence of a conservation of some lepton charge, we will have $P_3 \neq 0$ and $A(\beta\beta)_{0\nu} \neq 0$, in general. However, unless the $(\beta\beta)_{0\nu}$-decay amplitude receives contributions from mechanisms other than the exchange of the light Majorana neutrinos ν_j, the $(\beta\beta)_{0\nu}$-decay rate will be extremely strongly suppressed due to the fact that [72,73] $m_j^2/|q^2| < 10^{-16}$, where we have used $m_j < 1$ eV. Although allowed, $(\beta\beta)_{0\nu}$-decay will be practically unobservable if the P_3 term in $\mathcal{P}$ gives the dominant contribution in $A(\beta\beta)_{0\nu}$.

It is well known (see, e.g., [13]) that $|\langle m \rangle| = |m_{ee}|$, where m_{ee} is the ee-element of the Majorana mass matrix m of neutrinos [Eq. (4.1)]. If $|\langle m \rangle| = |m_{ee}| = 0$, the term $\bar{\nu}^c{}_{eR}\nu_{eL}$ will effectively be "regenerated" at higher orders from the other terms in $\mathcal{L}^M(x)$ [Eq. (4.1)]. The exchange of virtual ν_e mediated by this term will lead to $(\beta\beta)_{0\nu}$-decay. If we treat $\mathcal{L}^M(x)$ as an "interaction" term[8] and use perturbation theory, the virtual neutrino propagator in the $(\beta\beta)_{0\nu}$-decay amplitude will have, to leading order in the parameters $m_{ll'}$, the following form:

$$\mathcal{P} = \frac{1}{q^2}\frac{\tilde{m}^*}{q^2} + \cdots, \tag{4.5}$$

where

$$\tilde{m} = m_{e\mu}m^*_{\mu\tau}m_{\tau e} + m_{e\mu}m^*_{\mu\mu}m_{\mu e} + m_{e\tau}m^*_{\tau\mu}m_{\mu e} + m_{e\tau}m^*_{\tau\tau}m_{\tau e}. \tag{4.6}$$

It follows from the expression for the mass parameter $\tilde{m}$ that, if $m_{ee} = 0$, we will have $\tilde{m} = 0$ in the following cases [74–76]: (i) $m_{e\mu} = m_{e\tau} = 0$, (ii) $m_{e\mu} = m_{\tau\tau} = 0$, (iii) $m_{e\tau} = m_{\mu\mu} = 0$, and (iv) $m_{\tau\mu} = m_{\mu\mu} = m_{\tau\tau} = 0$. It is easy to see that the four cases in which $\tilde{m} = 0$ correspond to the conservation of the following lepton charges [75]: (i) L_e, (ii) $L_e - L_\tau$, (iii) $L_e - L_\mu$, and (iv) $L_e - L_\mu - L_\tau$. In all four cases the $(\beta\beta)_{0\nu}$-decay is strictly forbidden. However, all four cases are ruled out by the existing neutrino oscillation data (see, e.g., [76,77]). Thus, we can conclude that $\tilde{m} \neq 0$ and therefore $A(\beta\beta)_{0\nu} \neq 0$.

How large can the mass parameter $\tilde{m}$ be? By using the relation $m = U^* m^d U^\dagger$ and assuming that $m_{ee} = \langle m \rangle = 0$, it is not difficult to show that

$$\tilde{m}^* = \sum_j U_{ej}^2 m_j^3. \tag{4.7}$$

Thus, we recover the result obtained earlier by expanding the massive Majorana neutrino propagators in a power series of m_j^2/q^2:

$$\mathcal{P} = \frac{1}{q^2}\frac{\tilde{m}^*}{q^2} + \cdots = P_3 + \cdots, \tag{4.8}$$

where P_3 is given in Eq. (4.4). Therefore the $(\beta\beta)_{0\nu}$-decay will be extremely strongly suppressed if $m_{ee} = 0$ and $A(\beta\beta)_{0\nu} \neq 0$ is generated at higher order by the Majorana mass term [Eq. (4.1)].

V. CONCLUSIONS

Present and future searches for neutrinoless double beta decay aim at probing lepton number violation and the Majorana nature of neutrinos with remarkable precision. A wide experimental program is currently under discussion. Experiments with a sensitivity to the effective Majorana mass parameter $|\langle m \rangle|$ down to ~(50–10) meV are in a stage of preparation or planning and will take place in the future. These experiments will provide valuable information on the neutrino masses and the nature of massive neutrinos.

If future $(\beta\beta)_{0\nu}$-decay experiments show that $|\langle m \rangle| < 0.01$ eV, both the IH and the QD spectra will be ruled out for massive Majorana neutrinos. If, in addition, it is established in neutrino oscillation experiments that the neutrino mass spectrum is with *inverted ordering*, i.e., that $\Delta m^2_{\rm A} < 0$, the absence of a signal in neutrinoless double beta decay experiments sensitive to $|\langle m \rangle| \sim 10$ meV would be a strong indication that the massive neutrinos ν_j are Dirac fermions. At the same time, the alternative explanation based on the assumptions that the massive neutrinos ν_j are Majorana particles but there are additional contributions to the $(\beta\beta)_{0\nu}$-decay amplitude which interfere destructively with that due to the exchange of light massive Majorana neutrinos would also be possible. However, if $\Delta m^2_{\rm A}$ is determined to be positive in neutrino oscillation experiments, the upper limit $|\langle m \rangle| < 0.01$ eV would be perfectly compatible with massive Majorana neutrinos possessing a *normal hierarchical* mass spectrum or a mass spectrum with *normal ordering but a partial hierarchy*, and the quest for $|\langle m \rangle|$ would still be open. Under such circumstances, the next frontier in the searches for $(\beta\beta)_{0\nu}$-decay would most probably correspond to values of $|\langle m \rangle| \sim 0.001$ eV.

By taking $|\langle m \rangle| = 0.001$ eV as a reference value, we have investigated in the present article the conditions under which $|\langle m \rangle|$ in the case of a neutrino mass spectrum with normal ordering would satisfy $|\langle m \rangle| \geq 0.001$ eV. We have considered the specific cases of (i) a normal hierarchical neutrino mass spectrum, (ii) a relatively small value of the CHOOZ angle θ_{13}, as well as (iii) the general case of a

[8] In this case $\nu_{lL}(x)$ should be considered as zero mass fermion fields having the standard zero mass fermion propagator.

spectrum with normal ordering, a partial hierarchy, and a value of θ_{13} close to the existing upper limit. We have derived the ranges of the lightest neutrino mass m_1 and/or of $\sin^2\theta_{13}$ for which $|\langle m\rangle| \geq 0.001$ eV and have discussed some related phenomenological implications. We took into account the uncertainties in the predicted value of $|\langle m\rangle|$ due to the uncertainties in the measured values of the input neutrino oscillation parameters $\Delta m^2_{\odot}$, $\Delta m^2_{\rm A}$, and $\sin^2\theta_{\odot}$. For the latter, we have used the following prospective 1σ errors: 2%, 2%, and 4%, respectively.

In the present analysis we did not include the possible effects of the uncertainty related to the imprecise knowledge of the $(\beta\beta)_{0\nu}$-decay nuclear matrix elements. We hope (perhaps optimistically) that, by the time it will become clear whether the searches for $(\beta\beta)_{0\nu}$-decay will require a sensitivity to values of $|\langle m\rangle| < 0.01$ eV, the problem of sufficiently precise calculation of the $(\beta\beta)_{0\nu}$-decay nuclear matrix elements will be resolved.

We have found that in the case of a NH neutrino mass spectrum we get $|\langle m\rangle| \gtrsim 0.001$ eV for $\sin^2\theta_{13} \lesssim (0.01\text{–}0.02)$ and any value of the relevant Majorana phase (difference) α_{32}, provided the currently determined best fit values of the solar and atmospheric neutrino oscillation parameters $\Delta m^2_{\odot}$, $\Delta m^2_{\rm A}$, and especially $\sin^2\theta_{\odot}$ will not change considerably in the future high precision measurements (Fig. 3). For $0 \leq \alpha_{32} \leq \pi/2$, one has $|\langle m\rangle| \gtrsim 2.0 \times 10^{-3}$ eV for any $\sin^2\theta_{13}$, while if $\pi/2 < \alpha_{32} \leq 5\pi/6$, we get $|\langle m\rangle| \gtrsim 10^{-3}$ eV for any $\sin^2\theta_{13}$ allowed at 3σ by the existing data. Values of $\alpha_{32} \neq 0$ in the indicated ranges are required for the generation of the baryon asymmetry of the Universe in the flavored leptogenesis scenario in which the requisite CP violation is provided exclusively by the Majorana phase (difference) α_{32} [65].

We have investigated also the case when $\sin^2\theta_{13}$ has a rather small value $\sin^2\theta_{13} \lesssim 3 \times 10^{-3}$, but the neutrino mass spectrum is not hierarchical. We have found that in this case one has $|\langle m\rangle| \geq 10^{-3}$ eV for any value of the relevant Majorana phase α_{21} if the sum of neutrino masses satisfies $m_1 + m_2 + m_3 \gtrsim 7 \times 10^{-2}$ eV.

In the general case of a neutrino mass spectrum with a partial hierarchy (i.e., non-negligible lightest neutrino mass m_1) and sufficiently large $\sin^2\theta_{13}$, one finds $|\langle m\rangle| \geq 10^{-3}$ eV typically for $m_1 \lesssim$ a few $\times 10^{-3}$ eV and $m_1 \gtrsim 10^{-2}$ eV (Fig. 6). In the second case the sum of neutrino masses satisfies $m_1 + m_2 + m_3 \gtrsim 7.4 \times 10^{-2}$ eV. If a future highly sensitive $(\beta\beta)_{0\nu}$-decay experiment does not find a positive signal corresponding to $|\langle m\rangle| \geq 1$ meV, Majorana neutrinos would still be allowed, but the spectrum would be constrained to be with normal ordering and m_1 to be smaller than $\sim 10^{-2}$ eV. The CP-parity pattern $(+++)$ will be strongly disfavored (if not ruled out) as well. If, in addition, it is found that $\sin^2 \lesssim 0.01$, m_1 would be constrained to lie in the interval $m_1 \sim (10^{-2}\text{–}10^{-3})$ eV (for $\sin^2\theta_{\odot} \sim 0.32$), and the CP-parity pattern $(++-)$ will also be disfavored. No other future neutrino experiment, foreseeable at present, will have the capability of constraining the lightest neutrino mass (and the absolute neutrino mass scale) in the meV range. Obviously, the above constraints would hold only if massive neutrinos are Majorana particles. If the lightest neutrino has a mass in the interval $m_1 \sim (10^{-3}\text{–}10^{-2})$ eV, this can have important effects on the generation of the baryon asymmetry of the Universe in the flavored leptogenesis scenario of matter-antimatter asymmetry generation [66].

We have provided also an estimate of $|\langle m\rangle|$ when the three-neutrino masses and the neutrino mixing originate from neutrino mass term of Majorana type for the (left-handed) flavor neutrinos and $\sum_{j=1}^{3} m_j U_{ej}^2 = 0$, but there does not exist a symmetry which forbids the $(\beta\beta)_{0\nu}$-decay. Our results show that, although in this case the $(\beta\beta)_{0\nu}$-decay will be allowed, the corresponding effective Majorana mass parameter is determined by $\sum_{j=1}^{3} m_j^3 U_{ej}^2/q^2$, where q is the momentum of the virtual Majorana neutrino. For the average momentum of the virtual neutrino in $(\beta\beta)_{0\nu}$-decay, one typically has (see, e.g., [72,73]) $|q^2| \sim (100\ \text{MeV})^2$. As a consequence, the contribution to the $(\beta\beta)_{0\nu}$-decay amplitude $A(\beta\beta)_{0\nu}$ due to the light Majorana neutrino exchange will be strongly suppressed: $|\langle m\rangle| \ll 10^{-3}$ eV. Thus, if $\sum_{j=1}^{3} m_j U_{ej}^2 = 0$ and $(\beta\beta)_{0\nu}$-decay is observed in experiments with sensitivity to $|\langle m\rangle| \sim 10^{-3}$ eV, it would imply the existence of contributions to $A(\beta\beta)_{0\nu}$ due to mechanism(s) other than the three light Majorana neutrino exchange.

ACKNOWLEDGMENTS

Part of this work was done at the Aspen Center for Physics during the 2007 Summer Program on Neutrino Physics. We acknowledge support by the European Network of Theoretical Astroparticle Physics ILIAS/N6 under Contract No. RII3-CT-2004-506222. This work was also supported in part by the INFN program on ''Astroparticle Physics'' as well as by the Italian MIUR (PRIN and Internazionalizzazione Programs) and the Yukawa Institute of Theoretical Physics (YITP), Kyoto, Japan, within the joint SISSA-YITP research project on ''Fundamental Interactions and the Early Universe'' (S. T. P.).

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