Impact of secondary non-Gaussianities on the search for primordial non-Gaussianity with CMB maps

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When constraining the primordial non-Gaussianity parameter $f_{\rm NL}$ with cosmic microwave background anisotropy maps, the bias resulting from the covariance between primordial non-Gaussianity and secondary non-Gaussianities to the estimator of $f_{\rm NL}$ is generally assumed to be negligible. We show that this assumption may not hold when attempting to measure the primordial non-Gaussianity out to angular scales below a few tens arcminutes with an experiment like Planck, especially if the primordial non-Gaussianity parameter is around the minimum detectability level with $f_{\rm NL}$ between 5 and 10. In the future, it will be necessary to jointly estimate the combined primordial and secondary contributions to the cosmic microwave background bispectrum and establish $f_{\rm NL}$ by properly accounting for the confusion from secondary non-Gaussianities.

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I. INTRODUCTION

The search for primordial non-Gaussianity with constraints on the non-Gaussianity parameter $f_{\rm NL}$ using cosmic microwave background (CMB) anisotropy maps is now an active topic in cosmology today [1,2]. The 3-year Wilkinson Microwave Anisotropy Probe (WMAP) has allowed the constraint that $-54 < f_{\rm NL} < 114$ at the 95% confidence level [3] for the local model of primordial non-Gaussianity, while the 5-year WMAP data improve this to $-9 < f_{\rm NL} < 111$ at the same 95% confidence level [4]. An independent study using WMAP 3-year data, however, claims a nonzero detection of primordial non-Gaussianity at the 95% confidence level with $26.9 < f_{\rm NL} < 146.7$ [5]. This result, if correct, has significant cosmological implications since the expected value under standard inflationary models is $f_{\rm NL} \lesssim 1$ [6–11], though alternative models of inflation, such as the ekpyrotic cosmology [12,13], generally predict a large primordial non-Gaussianity with $f_{\rm NL}$ at few tens.

Most studies that constrain $f_{\rm NL}$ with CMB anisotropy maps make use of an estimator for $f_{\rm NL}$ through $\hat{f}_{\rm NL} = \frac{\hat{S}_{\rm prim}}{N}$ [14–16], where

$$\hat{S}_{\rm prim} = \sum_{\mathbf{pq}} B^{\rm prim}_{l_1 l_2 l_3} \mathcal{C}^{-1}_{\mathbf{pq}} \hat{B}^{\rm obs}_{l'_1 l'_2 l'_3} \tag{1}$$

when $B^{\rm prim}_{l_1 l_2 k_3}$ is the primordial bispectrum with the assumption that $f_{\rm NL} = 1$ [1], and $\mathcal{C}_{\mathbf{pq}}$ is the covariance matrix for bispectrum measurements involving triplets of $\mathbf{p} \equiv (l_1 l_2 l_3)$ and $\mathbf{q} \equiv (l'_1 l'_2 l'_3)$. This estimator is the optimal estimator for non-Gaussianity measurements, but given complications associated with estimating the covariance, existing studies make use of a suboptimal estimator which approximates the covariance with variance only such that $\mathcal{C}^{-1}_{\alpha\alpha'} \approx \sigma^{-2}(l_1, l_2, l_3)\delta_{\alpha\alpha'}$ and introduces a linear term to Eq. (1) to minimize the variance of $\hat{f}_{\rm NL}$ [16]. Note that N is the overall normalization factor that can be calculated from Eq. (1) by replacing $\hat{B}^{\rm obs}$ with $B^{\rm prim}$.

In general $\hat{B}^{\rm obs}_{l_1 l_2 l_3} = f_{\rm NL} B^{\rm prim}_{l_1 l_2 l_3} + B^{\rm ps}_{l_1 l_2 l_3} + A_{\rm SZ} B^{\rm SZ-\kappa}_{l_1 l_2 l_3} + A_{\rm ISW} B^{\rm ISW-\kappa}_{l_1 l_2 l_3} + \ldots$, where $B^{\rm ps}_{l_1 l_2 l_3}$ is the shape of the non-Gaussianity of unresolved radio point sources, and $B^{\rm SZ-\kappa}_{l_1 l_2 l_3}$ and $B^{\rm ISW-\kappa}_{l_1 l_2 l_3}$ are additional foreground, secondary non-Gaussianities from the Sunyave-Zel'dovich (SZ) and integrated Sachs-Wolfe (ISW) effects correlating with CMB lensing [17,18], respectively. With the general bispectrum defined using the reduced bispectrum $b_{l_1 l_2 l_3}$ as

$$B_{l_1 l_2 l_3} = \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}} \times \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix} b_{l_1 l_2 l_3}, \tag{2}$$

the reduced bispectrum of unresolved point sources is mode-independent and $b_{\rm ps} \equiv b_{l_1 l_2 l_3} \propto \int_0^{S_c} S^3 dn/dS$ is determined by the number counts dn/dS below the cutoff flux S_c. The reduced bispectra associated with lensing correlation takes the form of

$$b^{s-\kappa}_{l_1 l_2 l_3} = \frac{l_1(l_1+1) - l_2(l_2+1) - l_3(l_3+1)}{2} C^{\rm CMB}_{l_1} C^{s-\kappa}_{l_3} + \text{Perm.}, \tag{3}$$

where $C^{\rm CMB}_l$ is the CMB angular power spectrum and $C^{s-\kappa}_l$ is the cross power spectrum of CMB lensing and a secondary effect. Since the latter is model dependent, especially for SZ [17], the normalization factors $A_{\rm SZ}$ and $A_{\rm ISW}$ account for an uncertainty in the amplitude with $C^{s-\kappa}_l \propto A_s$ for each secondary effect.

In addition to these, there are non-Gaussianities from ISW [19], kinetic SZ/Ostriker-Vishniac [20], and thermal SZ [21]. We ignore ISW and kinetic SZ/Ostriker-Vishniac related bispectra as they are small compared to SZ generated non-Gaussianities. The SZ-SZ-SZ bispectrum is significant at arcminute angular scales, but given the power-law shot-noise behavior of the SZ bispectrum when $l < 1500$, the SZ contribution to the bispectrum can be thought of as an additional correction to b_{ps}. The shot-noise behavior of the SZ effect is especially applicable for the SZ contribution during reionization associated with hot electrons in supernovae bubbles Compton-cooling off of the CMB [22]. Thus, we do not separately include the total SZ bispectrum as a separate non-Gaussianity here.

When estimating f_{NL}, it is usually assumed that $\hat{B}^{obs}_{l_1 l_2 l_3} \approx f_{NL} B^{prim}_{l_1 l_2 l_3}$ when estimating the primordial bispectrum. This allows an estimator for f_{NL} through

$$\hat{S}_{prim} = f_{NL} \sum_{l_1 l_2 l_3} \frac{(B^{prim}_{l_1 l_2 l_3})^2}{\sigma^2(l_1, l_2, l_3)}, \tag{4}$$

with $\hat{f}_{NL} = \hat{S}_{prim}/N$. The above assumption that only the primordial non-Gaussianity can be considered is generally motivated by the fact that the covariance term associated with the mode overlap between $B^{prim}_{l_1 l_2 l_3}$ and additional secondary contributions to $B^{obs}_{l_1 l_2 l_3}$ via

$$\hat{S}_{prim,cov} = \sum_i A_i \sum_{l_1 l_2 l_3} \frac{B^{prim}_{l_1 l_2 l_3} B^i_{l_1 l_2 l_3}}{\sigma^2(l_1, l_2, l_3)}, \tag{5}$$

when $A_i = (b_{ps}, A_{SZ}, A_{ISW}, \ldots)$ is expected to be smaller than the dominant term from Eq. (4) [1]. Nevertheless, an estimate of f_{NL} only from Eq. (4) leads to a biased estimate because of the contributions from secondary anisotropies through Eq. (5).

While the CMB map contains a large number of secondary non-Gaussian signals, in terms of the covariance related to the f_{NL} measurement, what is necessary is not to account for all of these non-Gaussianities, but to account for non-Gaussianities with bispectrum shapes $B_{l_1 l_2 l_3}$ in (l_1, l_2, l_3) moment space that align with the shape of the primary bispectrum. In this respect, previous calculations have suggested that the point-source bispectrum may be ignored [1], but the ISW-lensing bispectrum must be accounted for in the Planck analysis [23].

Including the SZ-lensing bispectrum, we find that while the assumption that the covariance from secondary anisotropies can be mostly ignored for an experiment like WMAP, it may be necessary to account for certain covariances when estimating f_{NL} from a high-resolution experiment like Planck, especially if the underlying primordial non-Gaussianity has a value around f_{NL} between 5 and 10 consistent with the minimum amplitude detectable with Planck. At the minimum detectability level of WMAP with $f_{NL} \sim 20$, the secondary anisotropies involving both residual points sources and lensing correlations will bias f_{NL} by a factor between 1.2 and 1.5 if a primordial non-Gaussianity estimate is performed out to angular scales corresponding to $\ell > 700$.

II. ESTIMATE OF BIAS

To reach these conclusions, we first calculated $B^{prim}_{l_1 l_2 l_3}$ following Ref. [1] with the full radiation transfer function using a modified code of CMBFAST [24] for the standard flat ΛCDM cosmological model consistent with WMAP with $\Omega_b = 0.042$, $\Omega_c = 0.238$, $h = 0.732$, $n = 0.958$, and $\tau = 0.089$. We verified our calculations are consistent with prior calculations in the literature. In Fig. 1, we show the absolute value of the signal-to-noise square ratio for the primordial bispectrum (thick lines) and for the covariances between primary and secondary bispectra. The plotted quantity here involving $d(S/N)^2/d\ln l_3$ resembles the estimator $\hat{S}$ above, except for the sum over l_3 while keeping the sign (ignoring the sign results in a higher bias as described in Ref. [23]). We take the variance to be $\sigma^2(l_1, l_2, l_3) = C^{tot}_{l_1} C^{tot}_{l_2} C^{tot}_{l_3}$ and with $C^{tot}_l = C^{CMB}_l + C^{sec}_l + C^N_l$, we take instrumental parameters for WMAP and Planck consistent with previous calculations of the bispectrum to calculate the noise spectrum C^N_l [17]. We also include secondary power spectra involving point sources and the SZ effect through C^{sec}_l; this results in a degradation of overall signal-to-noise ratios by about 10% to 20%.

While the primordial calculation of the square of the signal-to-noise ratio is $(S/N)^2 = \sum (B^{prim})^2/\sigma^2$, the "signal-to-noise" square of the covariance follows from $(S/N)^2 = \sum B^{prim} B^{ps}/\sigma^2$, for example, for the point-source confusion, and these confusions should not be interpreted simply as the signal-to-noise ratio square to detect any of these secondary bispectra directly from the CMB maps. That signal-to-noise ratio is simply, for example, $(S/N)^2 = \sum (B^{ps})^2/\sigma^2$, but we do not concentrate on this quantity since the estimator for f_{NL} relies on an assumption related to the expected mode structure of the primordial bispectrum.

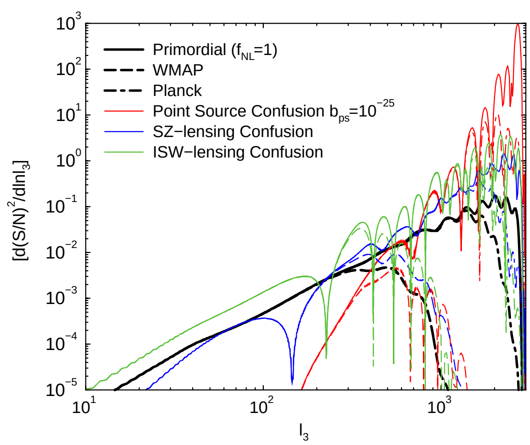

FIG. 1 (color online). Absolute values of signal-to-noise ratio squared for the detection of primordial bispectrum (black lines) assuming $f_{NL} = 1$ as a function of l_3. The signal-to-noise ratios squared for WMAP and Planck are shown with dashed and dot-dashed line, respectively. At $l = 5000$, the top, middle, and bottom lines show the confusion resulting from the covariance between primary and point-source, primary and SZ-lensing, and primary and ISW-lensing bispectra, respectively.

Instead of squared signal-to-noise ratios, to highlight the bias introduced to $f_{\rm NL}$ when the estimator ignores secondary non-Gaussianity covariances, we calculated $f_{\rm NL}^{\rm tot} = f_{\rm NL} + f_{\rm bias}$ where $f_{\rm bias}$ is the bias that is generated artificially by the correlation of modes between the primordial bispectrum and secondary bispectra. To properly normalize the relative contribution from secondary non-Gaussianities, we assume normalizations for the point-source bispectrum consistent with WMAP with $b_{\rm ps} = 3 \times 10^{-25}$, consistent with Q + V + W residual foreground [3], and Planck with $b_{\rm ps} = 5 \times 10^{-27}$. The $b_{\rm ps}$ value for Planck is higher by a factor of 2 to 3 than the values routinely quoted in the literature for unresolved radio sources in Planck high-resolution maps, but this is due to the fact that we believe $b_{\rm ps}$ includes a contribution from the SZ-SZ-SZ bispectrum from both clusters at low redshifts and supernovae halos during reionization with a power-law shot-noise spectrum when $l < 1500$. For the ISW-lensing and SZ-lensing bispectrum, we follow the calculation of Ref. [17] and generate the SZ contribution and the SZ correlation with dark matter halos responsible for lensing of the CMB using the halo model [21]. To account for an overall uncertainty and the variation in SZ and ISW amplitudes we have introduced an overall amplitude $A_{\rm SZ}$ and $A_{\rm ISW}$, respectively. Finally, to illustrate our results, we assume $f_{\rm NL}$ consistent with roughly the minimum detectable primordial non-Gaussianity with WMAP and Planck with $f_{\rm NL} = 20$ and 5, respectively. As we find later, the dominant confusion is from lensing bispectra and not from point sources.

III. RESULTS AND DISCUSSION

We summarize our results in Fig. 2, where we plot $f_{\rm NL}^{\rm tot}$ which can be thought of as the total primordial non-Gaussianity parameter that one will extract with the above estimator for $f_{\rm NL}$ when no attempt has been made to separate out the confusion from secondary anisotropies. For the most part, the bias is negligible and becomes only important when $l > 500$. For WMAP, shown with a dashed line in Fig. 2 with the assumption that $f_{\rm NL} = 20$ if non-Gaussianity measurements are attempted out to $l > 700$, capturing basically all information in WMAP maps, then one finds a bias between a factor of 1.5 to 2 if $f_{\rm NL} \sim 20$. If $f_{\rm NL} > 30$, then the relative contribution from secondary non-Gaussianities are subdominant compared to the primordial non-Gaussianity. Alternatively, if WMAP data are used to constrain that $f_{\rm NL} < 30$, then such a constraint must account for the covariances from secondary non-Gaussianities, especially those involving CMB lensing.

With Planck, non-Gaussianity estimates can be extended to $l_{\max} \sim 2000$, but at such small angular scales, one finds a bias higher by a factor of more than 2 relative to the lowest value of $f_{\rm NL}$ that can be reached with Planck (dot-dashed line). In return, if Planck data were to constrain $f_{\rm NL}$ to be below ~20, then such a constraint must account for the

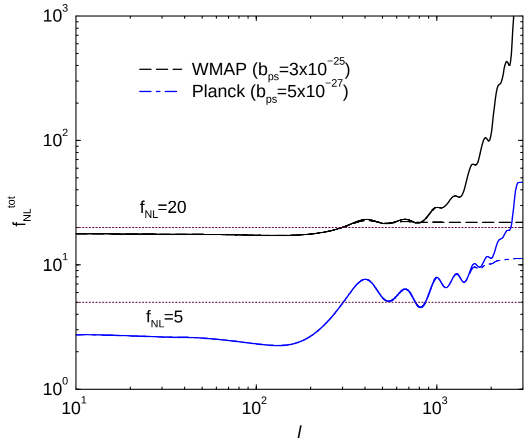

FIG. 2 (color online). The maximum non-Gaussianity measured with an optimal estimator for the primordial bispectrum $f_{\rm NL}^{\rm tot}$, which includes the true underlying primordial non-Gaussianity with $f_{\rm NL}$ as labeled on the figure and the bias correction coming from the unaccounted secondary anisotropies. The bias is generally small and nonexisting if primordial non-Gaussianity measurements are limited out to $l < 500$, but depending on the value of $f_{\rm NL}$ and the residual point-source contamination, the correction is generally a factor of 1.5 to 2. If $f_{\rm NL} \lesssim 10$, for Planck, it is necessary to account for secondary non-Gaussianities properly.

confusion from secondary anisotropies to the "optimal" estimator of $f_{\rm NL}$, since lensing non-Gaussianities produce a correction to $f_{\rm NL}$ with $f_{\rm bias} \sim 10$.

To account for secondary non-Gaussianities, one can modify existing "optimal estimators" for $f_{\rm NL}$ and jointly fit for both the primordial non-Gaussianity and the secondary non-Gaussianities through a series of estimators $\hat{S}_\alpha$ where α denotes the non-Gaussianity of interest with

$$\hat{S}_\alpha = N_{\alpha,\beta} K_\beta, \tag{6}$$

where

$$N_{\alpha,\beta} = \sum_{l_1 l_2 l_3} B^{\alpha}_{l_1 l_2 l_3} \mathcal{C}^{-1}_{\mathbf{pq}} B^{\beta}_{l'_1 l'_2 l'_3}, \tag{7}$$

with the covariance $\mathcal{C}_{\mathbf{pq}}$ between the triplets $\mathbf{p} = (l_1 l_2 l_3)$ and $\mathbf{q} = (l_1 l_2 l_3)$, and K_β refers to the set of non-Gaussianity parameters: ($f_{\rm NL}$, $b_{\rm ps}$, $A_{\rm SZ}$, $A_{\rm ISW}$). This method assumes that one has a good model for (l_1, l_2, l_3) dependence of secondary bispectra $B^{\beta}_{l_1 l_2 l_3}$. Even if the point-source covariance is small, the amplitude of the point-source confusion is generally unknown. Moreover, at $l < 1500$ many secondary bispectra such as the SZ effect have a power-law behavior similar to the bispectrum of point sources. Thus, it would be necessary to determine the amplitude $b_{\rm ps}$ from a joint fit.

Our suggestion that an accounting of secondary anisotropies is necessary for primordial non-Gaussian measurement is different from the general assumption in the literature that one can simply ignore the covariance between primordial and secondary non-Gaussianities. This partly comes through, for example, the suggestion that primordial and point-source bispectra are orthogonal following results from an exercise that involved jointly measuring non-Gaussian amplitudes f_{NL} and b_{ps} using a set of simulated maps in Ref. [2] to study if there are biases in the estimators. However, this study used simulated non-Gaussian maps that did not include any point sources with $b_{\mathrm{ps}} = 0$. This sets the covariance to be zero, and we believe this may have led to the wrong conclusion that there is no bias in the optimal estimator for f_{NL} from unresolved point sources, though such a bias is expected to be small, but non-negligible if $f_{\mathrm{NL}} \sim 1$. Our conclusions are consistent with some of the observations in Ref. [23].

Here we have considered the confusion from secondary non-Gaussianities such as point sources and those generated by CMB lensing. Additional contributions to the bispectrum exist with correlations between SZ, ISW and point sources as they all trace the same large-scale structure at low redshifts. Previous studies using the halo model to describe the nonlinear density field have shown correlations such as between SZ and radio sources to be small [21], but since the bispectra in these cases are of the form $\mathrm{SZ} - \mathrm{PS} - \mathrm{PS}^{\mathrm{nl}}$, these bispectra may have a multipolar dependence in (l_1, l_2, l_3) that is more aligned with the CMB primary bispectrum. Because of the lower amplitude of the non-Gaussianity, the overall bias in f_{NL} is expected to be less than 2 . While our discussion has concentrated on a momentum-independent non-Gaussianity parameter f_{NL}, or the so-called local type associated squeezed triangles, it is easy to generalize the calculation for more complex descriptions of $f_{\mathrm{NL}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ [25]. Because of differences in mode overlap, the exact momentum dependence will change the covariance contributions and the impact of secondary non-Gaussianities will be different between attempts to measure local f_{NL} and, for example, equilateral f_{NL}.

Based on our calculations on the covariance between lensing and primary bispectra we have suggested a potential confusion for f_{NL} measurement in Planck data. It is unlikely that our observation on the importance of secondary non-Gaussianities changes any of the current constraints on the non-Gaussianity parameter with WMAP data given that they mostly lead to $f_{\mathrm{NL}} \lesssim 100$ roughly. The secondary non-Gaussianities, however, could impact the significance of any detections of primordial non-Gaussianity, especially if the detection is marginally different from zero [5]. For such studies, the exact significance of the detection should include an accounting of the secondary non-Gaussianity and the overlap with primordial bispectrum in the optimal estimator used to establish f_{NL}.

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