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Comparison between different cosmological models

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Several cosmological models have been proposed in order to explain the current acceleration of the Universe. Recently, the normal branch of the DGP (after Dvali, Gabadadze, and Porrati) brane model with a generalized Chaplygin gas was studied as a model which can cross the phantom divide line avoiding the future singularity. In the present work, we wish to address the question of whether or not the aforementioned model has a better fit to supernovae data compared to cold dark matter with a cosmological constant, the (generalized) Chaplygin gas, and the DGP model with the self-accelerating branch without extra fluid for dark energy. We have found that the Chaplygin-DGP model has the worst fit, while the two-fluid model with Chaplygin gas and dust (baryons) has the best fit among the theoretical cosmological models considered here.

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According to our current understanding about the cosmos, we live in a flat universe which expands in an accelerating rate and it is dominated by a dark component. Identifying the origin and nature of dark energy is one of the biggest challenges for modern cosmology. Since we can only speculate about what dark energy could be, many cosmological models have been proposed and studied so far. The simplest candidate is a cosmological constant with state parameter $w = -1$, but models with a dynamical component with an evolving state parameter $w(z)$ also exist in the literature (for a review on dark energy models see e.g. [1]). In fact, observational data cannot rule out the possibility that $w < -1$ [2]. Among the various theoretical models, a scalar field with negative kinetic term (phantom) [3] behaves like a perfect fluid with $w < -1$, while a combination of a canonical and a phantom scalar field (quintom) [4] can give a state parameter that crosses the phantom divide line $w = -1$. However, scalar field models with a state parameter $w < -1$ face theoretical problems [5]. Furthermore, if the evolution of the Universe is dominated by a fluid with $w < -1$, then a future singularity (big rip) may be induced [6].

On the other hand, brane models [7] inspired by string theory have attracted a lot of interest as a new theoretical arena for addressing longstanding problems in particle physics and cosmology. A very interesting model was proposed several years ago [8] and its most appealing feature is that it can explain the cosmic acceleration at late times without a dark energy component. Unfortunately, the self-accelerating branch is problematic because of the appearance of ghosts and other pathologies [9]. However, the normal branch of this model with a generalized Chaplygin gas [10] was studied recently [11]. There it was shown that it is possible to cross the phantom divide line avoiding the singularity. The Chaplygin gas is characterized by the interesting property that it can unify the descriptions of dark energy and dark matter.

In an earlier work, it was found that in different dark energy parametrizations, most of the models with better fits cross the phantom divide line $w = -1$ [12]. It would be interesting to check whether or not the Chaplygin-Dvali, Gabadadze, and Porrati (DGP) model has a better fit to supernovae data compared to cold dark matter with a cosmological constant (LCDM), the four-dimensional (generalized) Chaplygin gas, and the DGP model with its self-accelerating branch. This is what we would like to address in this brief report.

The comparison of a theoretical model against supernovae data relies on the minimization of

$$\chi^2 = \sum_i \frac{(\mu_{\mathrm{obs}}(z_i) - \mu_{\mathrm{th}}(z_i))^2}{\sigma_i^2} \tag{1}$$

with μ the distance modulus, $\mu = m - M$, where M is the absolute magnitude and m is the apparent magnitude. The theoretical apparent magnitude μ_{th} is given by [1,12]

$$\mu_{\mathrm{th}} = 5\log_{10}\left(\frac{d_L(z)}{\mathrm{Mpc}}\right) + 25, \tag{2}$$

where the luminosity distance d_L for a flat universe can be expressed as (for distances in cosmology see e.g. [13])

$$d_L(z) = (1+z)c \int_0^z \frac{dx}{H(x)} \tag{3}$$

and $H(z)$ is the Hubble parameter as a function of redshift. The latter is the key object that one has to compute within a given theoretical cosmological model. We now summarize here the exact analytical form of $H(z)$ for the following five models: LCDM, four-dimensional (generalized) Chaplygin gas, DGP brane model with the self-accelerating branch, and the Chaplygin-DGP cosmology.

(a) LCDM: This is the model which is still in agreement with all observational data. The Hubble parameter

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$H(z)$ is given by the well-known expression

$$H(z) = H_0\sqrt{\Omega_\Lambda + \Omega_M(1+z)^3} \tag{4}$$

with the constraint $\Omega_M + \Omega_\Lambda = 1$.

(b) Generalized Chaplygin gas: It is the usual four-dimensional two-fluid model. The first fluid component is nonrelativistic matter (dust), while the equation of state for the second fluid component is given by

$$w(z) = \frac{p_{\rm gch}}{\rho_{\rm gch}} = -\frac{A}{\rho_{\rm gch}^{\alpha+1}}, \tag{5}$$

where A, α are the two parameters of the model. Integrating the conservation equation one obtains

$$\rho_{\rm gch} = (A + B(1+z)^{3(1+\alpha)})^{1/(1+\alpha)}, \tag{6}$$

where B is an integration constant. Note that the energy density behaves like dust for large redshifts and like a cosmological constant for small z. Therefore, the Chaplygin gas could unify the descriptions of dark matter and dark energy. The present value of the state parameter $w \equiv w(z=0)$ is given by

$$w = -\frac{A}{A+B}, \tag{7}$$

and the Hubble parameter as a function of redshift can be expressed as

$$\frac{H(z)^2}{H_0^2} = \Omega_b(1+z)^3 + (1-\Omega_b) \times ((1+w)(1+z)^{3(1+\alpha)} - w)^{1/(1+\alpha)}, \tag{8}$$

with Ω_b the baryon density. Because within this model dark matter and dark energy are unified, the nonrelativistic component is taken to be just baryons with $\Omega_b = 0.04$ as a prior.

(c) Chaplygin gas: This is a special case of the generalized Chaplygin gas in which $\alpha = 1$. It is a single parameter model, the only parameter being w.

(d) DGP with the self-accelerating branch: The cosmology of this model was first studied by Deffayet [14]. The model simply contains an Einstein-Hilbert term both in the bulk and on the brane

$$S = \frac{M^3}{2}\int d^5X\sqrt{-G}\hat{R} + \frac{m^2}{2}\int d^4x\sqrt{-g}R, \tag{9}$$

where M, G, $\hat{R}$ are the five-dimensional quantities, while m, g, R are the four-dimensional ones. Defining the distance scale $r_c = m^2/(2M^3)$ and the corresponding density $\Omega_{r_c} = 1/(4r_c^2H_0^2)$, one can obtain the following expression for $H(z)$

$$H(z) = H_0(\sqrt{\Omega_{r_c}} + \sqrt{\Omega_M(1+z)^3 + \Omega_{r_c}}), \tag{10}$$

with the constraint

$$\Omega_{r_c} = \left(\frac{1-\Omega_M}{2}\right)^2. \tag{11}$$

(e) Chaplygin-DGP: We make use of the function $H(z)$ found e.g. in [11]. It reads

$$\frac{H(z)}{H_0} = \sqrt{K} - \sqrt{\Omega_{r_c}}, \tag{12}$$

where

$$K \equiv \Omega_{r_c} + \Omega_M(1+z)^3 + \Omega_{ch}(A_s + (1-A_s) \times (1+z)^{3(1+\alpha)})^{1/(1+\alpha)}, \tag{13}$$

with the constraint $\Omega_{ch} = 1 - \Omega_M + 2\sqrt{\Omega_{r_c}}$.

We have compared the above theoretical models against the SNIa Gold dataset [15] and we have obtained for each model the values of the parameters that best fit the data. Our results are summarized in Table I. The $\chi^2_{\rm min}/(\text{d.o.f})$ is determined as follows. If N is the number of observational points (here $N = 157$) and n is the number of independent variables in a given model, (1 for LCDM, 1 for DGP, 1 for Chaplygin gas, 2 for generalized Chaplygin gas, and 4 for Chaplygin-DGP), then the number of degrees of freedom is given by $N - n$. Then the $\chi^2_{\rm min}/(d.o.f)$ is given by $\chi^2_{\rm min}/(N-n)$. We also report here the values of the parameters for each model that correspond to best fit.

$$\Omega_M = 0.3 \tag{14}$$

for LCDM,

$$\Omega_M = 0.2 \tag{15}$$

for DGP,

$$w = -0.94 \tag{16}$$

$$\alpha = 2.95 \tag{17}$$

for generalized Chaplygin gas,

$$w = -0.83 \tag{18}$$

for Chaplygin gas, and

$$A_s = 0.97 \tag{19}$$

$$\alpha = -0.29 \tag{20}$$

TABLE I. The $\chi^2_{\rm min}$ and the $\chi^2_{\rm min}$ per degrees of freedom for the five models considered here.

Model	$\chi^2_{\rm min}$	$\chi^2_{\rm min}/(\text{d.o.f})$
LCDM	177.127	1.135
DGP	178.078	1.142
Gen. Chaplygin gas	174.213	1.124
Chaplygin gas	175.007	1.122
Chaplygin-DGP	177.067	1.157

$$\Omega_M = 0.31 \tag{21}$$

$$\Omega_{r_c} = 0.01 \tag{22}$$

for the Chaplygin-DGP model. We see that the two-fluid model with Chaplygin gas and baryons has the best fit among the five models considered here, while the Chaplygin-DGP model has the worst fit. The reason for this is essentially the large number of parameters of the model. In an earlier work it was found that in different dark energy parametrizations, models that exhibit crossing of the phantom divide line $w = -1$ have better fits to supernovae data [12]. However, here we see that is not the case for the Chaplygin-DGP model.

In summary, in this brief report we have compared five theoretical models against supernovae data. The models under consideration are the standard LCDM model, and four more models that have been proposed for different theoretical reasons each. These are the DGP brane model, which explains the current cosmic acceleration without dark energy; the (generalized) Chaplygin gas, which unifies the description of dark matter and dark energy; and finally the Chaplygin-DGP model, which crosses the phantom divide line without resorting to a phantom fluid. Our results can be seen in Table I. We have found that among the above models, the two-fluid model with Chaplygin gas and baryons has the best fit, while the Chaplygin-DGP model has the worst fit due to the big number of its parameters.

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