Vacuum energy, the cosmological constant, and compact extra dimensions: Constraints from Casimir effect experiments

L. Perivolaropoulos*

Department of Physics, University of Ioannina, Greece

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We consider a universe with a compact extra dimension and a cosmological constant emerging from a suitable ultraviolet cutoff on the zero-point energy of the vacuum. We derive the Casimir force between parallel conducting plates as a function of the following scales: plate separation, radius of the extra dimension and cutoff energy scale. We find that there are critical values of these scales where the Casimir force between the plates changes sign. For the cutoff energy scale required to reproduce the observed value of the cosmological constant, we find that the Casimir force changes sign and becomes repulsive for plate separations less than a critical separation $d_0 = 0.6$ mm, assuming a zero radius of the extra dimension (no extra dimension). This prediction contradicts Casimir experiments which indicate an attractive force down to plate separations of 100 nm. For a nonzero extra dimension radius, the critical separation d_0 gets even larger than 0.6 mm and remains inconsistent with Casimir force experiments. We conclude that with or without the presence of a compact extra dimension, vacuum energy with any suitable cutoff cannot play the role of the cosmological constant.

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Cosmological observations during the last decade have indicated that the universe expansion has been accelerating in contrast to expectations based on the attractive gravitational force of matter that would tend to decelerate the universe expansion. The observed accelerating expansion of the universe indicates that either the universe is dominated by an energy form with repulsive gravitational properties (dark energy) or general relativity needs to be modified on cosmological scales (modified gravity) (see [1] and references therein). Dark energy may appear in the form of a cosmological constant (constant energy density [2]), quintessence (variable energy density due to an evolving scalar field [3]), a perfect fluid filling throughout space [4], etc. The simplest among the above possibilities is the cosmological constant which is also favored by most cosmological data [5] compared to alternative more complicated models. It is therefore important to identify the possible origins of such a cosmological constant.

There have been several proposals concerning the origin of the cosmological constant even though none of them is completely satisfactory. Some of them include the zero-point energy of the vacuum [6], anthropic considerations [7], brane cosmology [8], degenerate vacua [9], etc. The simplest physical mechanism that could lead to a cosmological constant is the zero-point energy of the vacuum made finite by an ultraviolet cutoff. A natural value of such a cutoff is the Planck scale leading to an energy density of the vacuum $\rho_V = (2.44 \times 10^{27})^4$ eV^4. However, such a value of the energy density leads to a cosmological constant which is 123 orders of magnitude larger than the observed one.

If the accelerating expansion of the universe is due to a cosmological constant emerging due to zero-point fluctuations of the vacuum, the required energy density of the vacuum would be $\rho_\Lambda \simeq 10^{-11}$ eV^4 corresponding to a cutoff scale [see Eq. (1) below] of $\omega_c \simeq 10^{-3}$ eV ($l_c \simeq 0.1$ mm). Even though there is no apparent physical motivation for such a cutoff scale, it cannot be *a priori* excluded unless it is shown to be in conflict with specific experimental data.

There are various types of laboratory experiments which are able to probe directly quantities related to the zero-point energy of the vacuum. Such experiments include measurements of the Casimir force [10] and measurements of the current noise in Josephson junctions (see [11,12] for a debate on the effectiveness of Josephson junction experiments on probing the energy of the vacuum). Even though these nongravitational experiments can only probe changes of the zero-point energy under variations of system parameters or of external couplings, these changes can be modified in the presence of a cutoff of the absolute value of the zero-point energy. Therefore, these experiments can become indirect probes of the absolute value of the zero-point energy under the assumption of the presence of a cutoff.

According to the Casimir effect [13–15], the presence of macroscopic bodies (like a pair of conducting plates) leads to a modification of the zero-point vacuum fluctuations due to the introduction of nontrivial boundary conditions. This modification manifests itself as a force between the macroscopic bodies that distort the vacuum. When the role of the macroscopic bodies is played by a pair of conducting plates, the discreteness imposed on the fluctuation field modes, lowers the vacuum energy and leads to an attractive

*http://leandros.physics.uoi.gr

force between the plates (the *Casimir force*), which has been measured by several experiments [10]. In the presence of a finite cutoff length scale [16] l_c, electromagnetic field modes with wavelengths $\lambda < l_c$ are suppressed and the vacuum energy lowering due to the presence of the plates gets modified. This in turn leads to modifications of the Casimir force between the plates compared to the case where no cutoff has been imposed. In fact it may be shown [12,17,18] that when the plate separation d becomes significantly less than the cutoff scale l_c then the force between the plates changes sign and becomes repulsive! However, there is no experimental indication for change of sign of the Casimir force down to separations $d \simeq 100$ nm. This imposes a constraint on the vacuum energy cutoff as $l_c < 100$ nm corresponding to a vacuum energy $\rho_V > 1\ \mathrm{eV}^4$. Such a value of ρ_V is clearly inconsistent with the vacuum energy corresponding to the cosmological constant ($\rho_\Lambda \simeq 10^{-11}\ \mathrm{eV}^4$). This inconsistency increases further if the contribution of interactions other that electromagnetism is included since in that case the predicted ρ_V from the Casimir effect would get even larger to include the contribution of other fields. It is therefore clear that either vacuum energy is not responsible for the accelerating expansion of the universe or there is a missing ingredient in the calculation of the Casimir force as a function of the cutoff scale.

One such possible missing ingredient could be the presence of a universal compact extra dimension [19] with compactification scale R. Even though the current experimental bounds on R are quite stringent ($R \lesssim (300\ \mathrm{GeV})^{-1} \simeq 10^{-9}$ nm [20]) it is still instructive to consider this possibility in the context of the Casimir effect with a cutoff in the vacuum energy. The propagation of vacuum energy modes along this extra dimension modifies the field spectrum and affects accordingly the Casimir force.

The main goal of this brief report is to address the following question: "What is the effect of an extra compact dimension on the Casimir force between two parallel plates and how does this effect change as a function of the vacuum energy cutoff scale?"

We start by deriving the vacuum energy in the region between two parallel plates. Consider a 3 dimensional Euclidean space with an extra compact dimension of scale R. In this space consider a box of square parallel conducting plates of large surface L^2 parallel to the xy plane placed at a small distance d apart. The vacuum energy of the quantized electromagnetic field in the region between the plates [13,19,21] is

$$\mathcal{E} = \frac{1}{2}\sum_{\vec{k},\lambda} \hbar\omega_{\vec{k}}\, g\left(\frac{\omega_{\vec{k}}}{\omega_c}\right), \tag{1}$$

where $g(x)$ is a UV cutoff regulator ($\lim_{x\to 0} g(x) = 1$, $\lim_{x\to\infty} g(x) = 0$) and λ counts the polarization modes.

Also

$$\omega_{\vec{k}} = c\sqrt{k_x^2 + k_y^2 + k_d^2 + k_R^2}, \tag{2}$$

where

$$k_x = \frac{\pi m_x}{L} \qquad k_y = \frac{\pi m_y}{L} \qquad k_d = \frac{\pi n}{d} \qquad k_R = \frac{N}{R} \tag{3}$$

with $m_x.m_y$, n, $N = 0, 1, 2 \ldots$. In the large L limit Eq. (1) takes the form

$$\frac{\mathcal{E}}{L^2} = \frac{3\hbar c}{4\pi} \sum_{n,N=0}^{\infty\prime} \int_0^\infty dq q \omega_q g\left(\frac{\omega_{\vec{k}}}{\omega_c}\right), \tag{4}$$

where $\omega_q^2 = c^2\sqrt{q^2 + (\frac{\pi n}{d})^2 + (\frac{N}{R})^2}$ and the factor 3 is due to the three polarization degrees of freedom in the presence of the extra dimension.

The prime ($'$) on the sum implies that when $N = n = 0$ we should put a factor $\frac{1}{3}$ (only one degree of freedom from polarization) while if only one of the N, n is 0 we should put an extra factor $\frac{2}{3}$ (only two degrees of freedom from polarization). It is straightforward to show that the modification of the vacuum energy due to the presence of the plates is

$$\begin{aligned}\Delta u(d, R) &\equiv u^{\mathrm{vac}}(d, R) - u^{\mathrm{vac}}_{\gg}(d, R)\\ &= \frac{3\omega_c^3\hbar}{4\pi c^2} \sum_{N=0}^{\infty}\left(\sum_{n=0}^{\infty} F(n, N) - \int_0^\infty dq F(q, N)\right)\\ &\quad - h(n, N),\end{aligned} \tag{5}$$

where

$$\begin{aligned}h(n, N) = \frac{3\omega_c^3\hbar}{4\pi c^2}\Bigg[&\frac{2}{3}F(0, 0) + \frac{1}{3}\sum_{n=1}^{\infty} F(n, 0)\\ &+ \frac{1}{3}\sum_{N=1}^{\infty} F(0, N) - \frac{1}{3}\int_0^\infty dq F(q, 0)\Bigg]\end{aligned} \tag{6}$$

and

$$F(n, N) \equiv \int_{\sqrt{(n/\alpha)^2+(N/\beta)^2}}^{\infty} dv v^2 g(v) \tag{7}$$

with

$$\alpha \equiv \frac{\omega_c d}{c\pi} \qquad \beta \equiv \frac{\omega_c R}{c}. \tag{8}$$

In the limit $\beta \to 0$ corresponding to $R \to 0$ (no extra dimension), only the $N = 0$ mode contributes, and the above expressions reduce to the well-known [18,21] forms of the regularized vacuum energy.

We now specify an exponential form for the UV cutoff function $g(v)$. Other smooth forms of $g(v)$ also lead to similar results. For $g(v) = e^{-v}$ we have $F(n, N) = e^{-s}(2 + s(s + 2))$ with $s = \sqrt{(\frac{n}{\alpha})^2 + (\frac{N}{\beta})^2}$. In Fig. 1 we

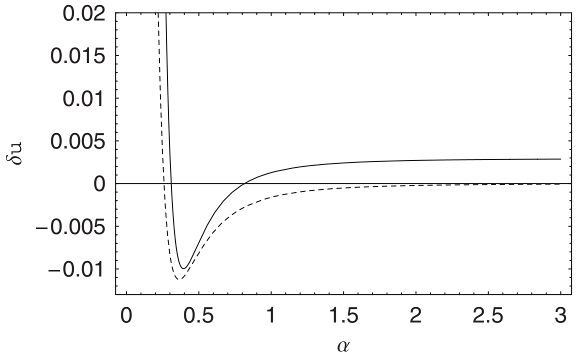

FIG. 1. The normalized vacuum energy $\delta u \equiv \frac{\Delta u}{(\frac{3\omega_c^3 \hbar}{4\pi c^2})}$ as a function α (dimensionless form of d) for $\beta = 0$ (dashed line) and $\beta = 1$ (continuous line). The curve corresponding to $\beta = 1$ has been shifted to lower values so that the locations of the minima can be compared more easily. Notice that the minimum shifts slightly to larger values as we increase the extra dimension size.

show the normalized vacuum energy $\delta u \equiv \frac{\Delta u}{(\frac{3\omega_c^3 \hbar}{4\pi c^2})}$ as a function of α for $\beta = 0$ and $\beta = 1$. Notice that for a small value of R ($R\omega_c \to 0$) we recover the result of Ref. [18] where the repulsive nature of the Casimir force was demonstrated for plate separations d much smaller than the cutoff scale $c\omega_c^{-1}$ without the presence of extra dimensions. For $\beta = 0$ (no extra dimension), the Casimir force becomes repulsive for $\alpha < \alpha_0 \simeq 0.36$. For a cutoff leading to the observed value of cosmological constant ($\omega_c \simeq 10^{-3}$ eV) we find the critical separation $d_0 \simeq 0.6$ mm such that for separations $d < d_0$ the Casimir force becomes repulsive. Since the Casimir force has been experimentally shown to be attractive down to plate separations $d \simeq 100$ nm [10] it becomes clear that the observed cosmological constant cannot be due to vacuum energy with appropriate cutoff and no extra dimensions.

The introduction of a compact extra dimension with finite size has two effects:

(i) The cutoff scale $\omega_c(\beta)$ required to match the observed value of the cosmological constant slowly decreases

(ii) The critical dimensionless separation $\alpha_0(\beta)$ for which the Casimir force changes sign increases (see Fig. 1).

In order to demonstrate the first effect we may evaluate the predicted vacuum energy density as a function of the dimensionless size β of the extra dimension. We have

$$\rho_V(\omega_c, \beta) = \frac{u_{\gg}^{\mathrm{vac}}(d)}{d} = \frac{3\hbar\omega_c^4}{4\pi^2 c^3} Q(\beta), \tag{9}$$

where

$$Q(\beta) = \sum_{N=0}^{\infty\prime} \int_0^\infty dq' \int_{\sqrt{q'^2+(N/\beta)^2}}^\infty v^2 e^{-v}. \tag{10}$$

Demanding

$$\rho_V(\omega_c, \beta) = \rho_\Lambda = 10^{-11}\ \mathrm{eV}^4 \tag{11}$$

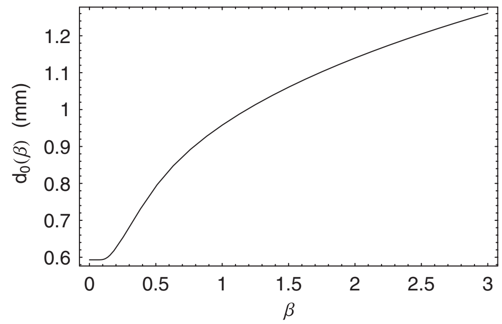

FIG. 2. The plate distance $d_0(\beta)$ in mm for which the Casimir force changes sign, as a function of the dimensionless scale of compatification β. In converting from the dimensionless distance $\alpha_0(\beta)$ to the dimensionful distance $d_0(\beta)$ in mm we have assumed that the cutoff is equal to $\omega_0^\Lambda(\beta)$ [Eq. (12)] required to reproduce the observed cosmological constant with vacuum energy.

we find

$$\omega_c^\Lambda(\beta) = \left(\frac{4\pi^2}{3} Q(\beta)^{-1} 10^{-11}\right)^{1/4}\ \mathrm{eV}. \tag{12}$$

In Fig. 2 we show a plot of the dimensionful plate distance $d_0(\beta)$ corresponding to a sign change of the Casimir force. This is found by converting the dimensionless minima α_0 of Fig. 1 to the corresponding dimensionful plate distances d_0 assuming a cutoff equal to $\omega_c^\Lambda(\beta)$ in Eq. (8) for α. As shown in Fig. 2 $d_0(\beta)$ slowly increases with β as expected by inspection of Fig. 1 which shows the minimum α_0 of $\Delta u(\alpha, \beta)$ shifts to larger values as we increase the dimensionless size β of the extra dimension.

It is easy to convert the dimensionless parameter β to the compactification scale R for a particular value of the cutoff ω_c. For example for the cosmological constant cutoff $\omega_c = \omega_c^\Lambda(\beta)$ we obtain using Eqs. (8) and (12)

$$R = \frac{3\beta}{(8.2 \times Q(\beta))^{-1/4}}. \tag{13}$$

Finally, it is straightforward to use Eqs. (5) and (8) to evaluate the Casimir force for $\omega_c = \omega_c^\Lambda(\beta)$. The resulting force is shown in Fig. 3 for $\beta = 0$ ($R = 0$) and $\beta = 1$

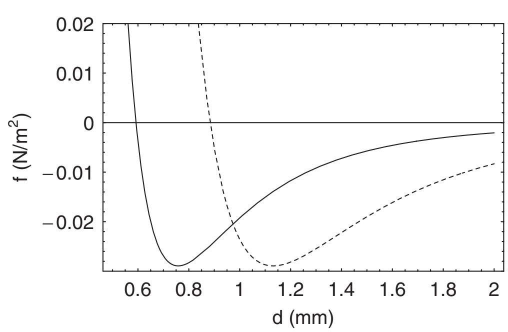

FIG. 3. The Casimir force per unit surface predicted for a vacuum cutoff corresponding to the cosmological constant as a function of the plate distance d in mm for $\beta = 0$ (no extra dimension, continuous line) and $\beta = 1$ (dashed line).

($R \simeq 0.7$ mm). Clearly, the plate separation where the force changes sign depends weakly on the size of the extra dimension and is always larger than 0.6 mm. We conclude that even in the presence of compact extra dimensions a cosmological constant induced by zero-point vacuum fluctuations with appropriate cutoff is in conflict with experimental measurements of the Casimir force which indicate an attractive force down to separations of $d \simeq 100$ nm.

Finally, we point out that our approach connecting the Casimir effect in the presence of a cutoff with the cosmological constant is distinct from previous global approaches [22] which view the universe as a system of large (cosmological size) and small compact dimensions (finite system) where the vacuum energy gets modified due to the finiteness of the system. In these models a cosmological constant is generated by the Casimir energy associated to some field propagating in the extra dimensions. That mechanism which does not implement a cutoff and does not address the issue of ultraviolet contributions (set to zero by an unknown mechanism), remains a valid candidate model for the generation of a cosmological constant but it cannot be tested with Casimir force laboratory experiments even though tight constraints are imposed by the predicted deviations from Newton's law.

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