Weak gravity conjecture for effective field theories with N species

Qing-Guo Huang
School of Physics, Korea Institute for Advanced Study, 207-43, Cheongryangri-Dong, Dongdaemun-Gu, Seoul 130-722, Korea
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We conjecture an intrinsic UV cutoff for the validity of the effective field theory with a large number of species coupled to gravity. In four dimensions such an UV cutoff takes the form $\Lambda = \sqrt{\lambda/N} M_p$ for N scalar fields with the same potential $\lambda \phi_i^4$, $i = 1, \ldots, N$. This conjecture implies that the assisted chaotic inflation or N-flation might be in the swampland, not in the landscape. Similarly an UV cutoff $\Lambda = gM_p/\sqrt{N}$ is conjectured for the U(1) gauge theory with N species.

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String theory is proposed as a candidate of quantum gravity. The consistent perturbative superstring theory can only live in ten-dimensional spacetime. A key problem in string theory is how it eventually connects with experiments. To understand the effective low-energy physics in four dimensions, string theory must be compactified on some six-dimensional manifold. Different choices of the extra dimensional manifold leads to different low-energy effective theory. Recent developments on the flux compactification [1–3] can help us to stabilize some moduli. However, there are so many choices of flux. Supposing that each flux can take of order 10 values, we find of order 10^K distinct solutions, where K is the number of distinct topological types of flux. Calabi-Yau manifolds typically have $K \sim \mathcal{O}(10^2)$. It is quite unclear for us to get some testable predictions for string theory because there are so many different low-energy effective field theories.

In [4] Vafa suggested that the vast series of the metastable vacua is only semiclassically self-consistent, not really self-consistent. We say that they are in the *swampland*. The really self-consistent *landscape* is surrounded by the swampland. Even though we cannot pick out a unique compactification or a unique low-energy effective field theory in four dimensions, we still can ask which low-energy effective field theory can/cannot be compatible with gravity.

Usually we believe that the effective field theory breaks down at the Planck scale because the gravity becomes important and the full theory of quantum gravity is called for. However, this estimation seems to be naive. The gauge force strength is characterized by g^2 and the gravitational strength is roughly $G\Lambda^2 = \Lambda^2/M_p^2$ at energy scale Λ. Gravity becomes dominant when the energy scale goes to gM_p which is much lower than the Planck scale in the perturbative region. Arkani-Hamed *et al.* [5] conjecture that a U(1) gauge theory with gauge coupling g has an intrinsic UV cutoff

$$\Lambda \leq gM_p \tag{1}$$

for the validity. In cosmology, the scalar field plays a crucial role. Both the inflaton and quintessence are scalar fields. As a simple example, we consider the $\lambda\phi^4$ theory coupled to gravity. The interaction strength of the scalar field is the dimensionless scalar coupling λ, and the gravitational strength is still $G\Lambda^2$. Similarly, in [6] we proposed a weak gravity conjecture for the $\lambda\phi^4$ theory as follows:

$$\Lambda \leq \lambda^{1/2} M_p. \tag{2}$$

This conjecture infers that the single-field chaotic inflation cannot be achieved in string landscape. Recently the authors constructed a D-term chaotic inflation model in supergravity [7]. The potential of the scalar field takes the form $\lambda\phi^4$ which comes from the D-term of a U(1) gauge field in the same supermultiplet. The supersymmetry implies a relationship between the gauge coupling g and the scalar coupling λ as $\lambda = g^2$. We see that our weak gravity conjecture for $\lambda\phi^4$ theory is equivalent to that for the U(1) gauge theory. For other related works on weak gravity conjecture see [8–18].

We can imagine that there should be a lot of gauge and scalar fields in string landscape. A loop of a light field generates a quadratically divergent correction to the Planck scale [19–22]. A large number of light fields can enhance radiative corrections to the Planck mass. Demanding that the total correction $N\Lambda^2$ be smaller than M_p^2 leads to an upper bound on the UV cutoff Λ

$$\Lambda \leq \Lambda_G = \frac{M_p}{\sqrt{N}}, \tag{3}$$

where N is the number of light fields. In [23] Dvali used the black hole physics to give a strong argument to support this new energy scale for the U(1) gauge theory with N species. A large number of species of the quantum fields implies an inevitable hierarchy between the energy scale of the field theory and the Planck scale.

It is curious for us to ask what is the weak gravity conjecture with a large number of field species. In this paper we will propose some new conjectures on the U(1) gauge theory and $\lambda\phi^4$ scalar field theory with N species coupled to gravity, respectively. Our conjecture implies

*huangqg@kias.re.kr

that even the assisted slow-roll chaotic inflation might not be realized in the landscape. Here we only focus on the conjectures in four dimensions, even though some of our conjectures can be easily generalized to other dimensions.

Let us consider a scalar field theory containing N scalar fields ϕ_i with independent potential $V_i(\phi_i)$ for $i = 1, \ldots, N$. For simplicity, we assume that the potential for different ϕ_i takes the same form

$$V_i(\phi_i) = \lambda\phi_i^4. \tag{4}$$

We also assume that there are no cross-couplings between the different scalar fields. Our results are also valid as long as the cross-couplings can be taken as perturbations. A realization of this field theory is that the different scalar field ϕ_i are stuck to living on different points in the extra dimensions. The cross-coupling between two fields is exponentially suppressed by the distance between them in the extra dimensions. Let us consider the nonsupersymmetric fieldtheory landscape in [21]. Suppose the UV cutoff for the scalar field theory is Λ. The distance between two fields in the extra dimensions is required to be larger than Λ^{-1} in order to suppress the cross-coupling between them. This requirement leads to a lower bound on the size of the extra dimensions R, namely,

$$R \geq \Lambda^{-1}N^{1/(d-4)}, \tag{5}$$

where $d - 4$ is the number of the extra dimensions. On the other hand, the four-dimensional Planck scale M_p is related to the fundamental Planck scale $M_d = G_d^{-(1/(d-2))}$ in d dimensions by

$$M_p^2 = R^{d-4}M_d^{d-2}. \tag{6}$$

Combining (6) with (5), we find

$$M_p^2 \geq \frac{NM_d^{d-2}}{\Lambda^{d-4}}. \tag{7}$$

Naturally, the UV cutoff for the effective field theory is lower than the fundamental Planck scale and thus (7) reads $\Lambda^2 \leq M_p^2/N$ which is just the same as (3).

In the previous scenario the size of the extra dimensions is much larger than the fundamental Planck length M_d^{-1} and the length scale Λ^{-1} for the field theory. So we should take the contribution of the KK modes into account. Or equivalently, for convenience, we consider that the gravity propagates in the whole d-dimensional spacetime. The action for the field theory coupled to d-dimensional gravity is given by

$$S = \frac{1}{2\kappa_d^2}\int d^dx\sqrt{-g_d}R + \int d^4x\sqrt{-g} \times \sum_{i=1}^{N}\left(\frac{1}{2}(\partial\phi_i)^2 + \frac{1}{2}m^2\phi_i^2 + \lambda\phi_i^4\right), \tag{8}$$

where $\kappa_d^2 \sim G_d$ is the Newton coupling constant in d dimensions. The self-interaction strength of the scalar fields is still characterized by the dimensionless coupling λ and the gravitational strength is proportional to the d-dimensional Newton coupling constant $G_d\Lambda^{d-2} = (\Lambda/M_d)^{d-2}$. Requiring that the self-interaction of the scalar fields is larger than gravitational interaction leads to

$$\Lambda \leq \lambda^{1/(d-2)}M_d. \tag{9}$$

Substituting (9) into (7), we obtain

$$\Lambda \leq \sqrt{\frac{\lambda}{N}}M_p = \lambda^{1/2}\Lambda_G. \tag{10}$$

This is the weak gravity conjecture for the effective field theory with N scalar fields and independent potential $\lambda\phi_i^4$ for $i = 1, \ldots, N$. For $N = 1$ our conjecture is just the same as that in the case with single scalar field [6].

We can easily generalize the previous arguments to U(1) gauge theory with N species in four dimensions. Similarly, we also require that these N U(1) gauge fields live on the different points in the extra dimensions and these points are separated far more than Λ^{-1} away from each other. Now the N copies of the U(1) gauge theory are obtained in four dimensions. The bound (5) on the size of extra dimensions and (7) are still valid. The strength of the gauge force is proportional to g^2. The gravity also propagates in the whole d-dimensional spacetime. Requiring that the gauge force be larger than the gravitational interaction $G_d\Lambda^{d-2} = (\Lambda/M_d)^{d-2}$ yields

$$\Lambda \leq g^{2/(d-2)}M_d. \tag{11}$$

This is the weak gravity conjecture for the U(1) gauge theory with large extra dimensions. For details see [15] where we gave more evidence to support this conjecture. Combining the above inequality with (7), we get the weak gravity conjecture for the U(1) gauge theory with N species

$$\Lambda \leq \frac{g}{\sqrt{N}}M_p = g\Lambda_G. \tag{12}$$

This result is much more stringent than (3) in the perturbative region. Our conjecture is also consistent with the case for the single U(1) gauge field in four dimensions [5].

We see that $\Lambda_G = M_p/\sqrt{N}$ plays a role as the effective gravity scale in the weak gravity conjecture for a large number of field species. In [24] the authors suggest that the scale Λ_G should be taken as the quantum gravity scale and new gravitational dynamics must appear beyond this energy scale. In [20] Veneziano argues that the dimensionless gravity coupling $\alpha_G = G\Lambda^2$ has an upper bound $1/N$ at one-loop level. Here we also give an argument to show that Λ_G is the limiting temperature for a thermal gas containing N species.

Let us consider a thermal gas with N species in a box of size L. This thermal gas is heated to a temperature T which is assumed to be much higher than their mass and the particles in the thermal gas are treated as relativistic particles. The total energy of the thermal gas is given by

$$E \simeq NT^4L^3, \tag{13}$$

and entropy takes the form

$$S \simeq NT^3L^3. \tag{14}$$

If the size of the thermal system is smaller than the Schwarzchild radius of the black hole with mass E, this thermal system will eventually collapse into a black hole. Requiring that the thermal gas does not collapse into a black hole leads to a maximum size of the box

$$L < L_{\max} = \frac{M_p}{\sqrt{N}T^2} = \frac{\Lambda_G}{T^2}. \tag{15}$$

In [24] the authors suggested that the entropy of the thermal gas is bounded by $M_p^2L^2$ and then we get another upper bound on the size of the thermal system

$$L < \frac{M_p^2}{NT^3} = \frac{\Lambda_G^2}{T^3}. \tag{16}$$

But we cannot get any conclusive results by combining (15) with (16). Here we suggest that one take the Bekenstein entropy bound into account. In [25,26] Bekenstein argued that the total entropy of a system is not larger than the product of the energy and the linear size of the system, namely.

$$S < S_B = EL. \tag{17}$$

In our case the Bekenstein entropy bound implies a lower bound on the size of the thermal system

$$L > 1/T. \tag{18}$$

From the viewpoint of quantum mechanics, the temperature can be taken as the typical energy of a quanta in the thermal system and $1/T$ is its wavelength which should be shorter than the size of the system. Using (15) and (18), we find the temperature of the thermal gas with N species is bounded by the gravity scale Λ_G.

From now on we switch to inflationary cosmology. The requirement that the gravity be the weakest force leads to a stringent constraint on inflation. In [6] we argued that the single-field chaotic inflation cannot be achieved self-consistently if we take into account the weak gravity conjecture for $\lambda\phi^4$ theory. But the assisted chaotic inflation still survives.

First, we give a general discussion on the assisted inflation which is governed by N scalar fields with the same potentials. The spacetime geometry is quasi de Sitter space during inflation and the size of the maximum causal patch for the field theory is bounded by the Hubble size H^{-1}. The Hubble parameter can be taken as the IR cutoff for the field theory and it should be smaller than the UV cutoff $\Lambda_G = M_p/\sqrt{N}$, namely,

$$N \le \frac{M_p^2}{H^2} \sim S_{ds}. \tag{19}$$

The number of scalar fields is bounded by the de Sitter entropy. If the Hubble parameter exceeds the gravity scale Λ_G, the holographic entropy bound is violated. There is another argument for this result in [27]. The amplitude of the quantum fluctuations for the inflaton is $H/2\pi$ on the Hubble scale H^{-1} and thus each inflaton field fluctuation provides a gradient energy density $(\partial_\mu\delta\phi)^2 \sim (\delta\phi/H^{-1})^2 \sim H^4$. The total gradient energy density of the fluctuations of the inflatons is given by

$$N(\partial_\mu\delta\phi)^2 \sim NH^4. \tag{20}$$

A more explicit result in [27] is that the quantum fluctuations of scalar fields give a contribution to the average value of the energy-momentum tensor

$$\langle T_{\mu\nu}\rangle = \frac{3NH^4}{32\pi^2}g_{\mu\nu}. \tag{21}$$

Requiring that $3NH^4/(32\pi^2)$ is no greater than the total energy density of the inflatons $3M_p^2H^2$ yields

$$N \le \frac{32\pi^2M_p^2}{H^2} = \frac{4\pi H^{-2}}{G} = S_{dS}; \tag{22}$$

otherwise, inflation is shut off. The authors in [28] found that the assisted chaotic inflation cannot be eternal if we combine the weak gravity conjecture (2) with the entropy bound on the number of inflaton fields (19).

However, the conjecture for the scalar field theory with N species in this paper is much more stringent than the combination of the weak gravity conjecture for the single scalar field and the entropy bound on the number of the fields. Here we consider the assisted chaotic inflation. The equations of motion are given by

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{1}{3M_p^2}\sum_{i=1}^{N}\left(\frac{1}{2}\dot\phi_i^2 + \frac{1}{2}m^2\phi_i^2 + \lambda\phi_i^4\right), \tag{23}$$

$$\ddot\phi_i + 3H\dot\phi_i = -(m^2\phi_i + 4\lambda\phi_i^3), \qquad i = 1,\ldots N. \tag{24}$$

There is a unique attractor solution with $\phi_1 = \phi_2 = \ldots = \phi_N \equiv \phi$. Slow-roll assisted inflation happens if the Hubble parameter evolves very slowly. It is convenient for us to define a new slow-roll parameter for the assisted inflation

$$\epsilon_H \equiv -\frac{\dot H}{H^2}. \tag{25}$$

If $\epsilon_H \ll 1$, the slow-roll assisted inflation is achieved and the equations of motion are simplified to be

$$H^2 = \frac{N}{3M_p^2}\left(\frac{1}{2}m^2\phi^2 + \lambda\phi^4\right), \tag{26}$$

$$3H\dot{\phi} = -(m^2\phi + 4\lambda\phi^3), \qquad i = 1, \ldots N, \tag{27}$$

and the slow-roll parameter is given by

$$\epsilon_H = \frac{M_p^2}{2N}\left(\frac{dV(\phi)/d\phi}{V(\phi)}\right)^2. \tag{28}$$

Considering that the UV cutoff Λ in (10) be larger than the IR cutoff H, we have

$$\frac{N\lambda\phi^4}{3M_p^2} \leq H^2 \leq \Lambda^2 \leq \frac{\lambda}{N}M_p^2, \quad \text{or} \quad \phi \leq \frac{M_p}{\sqrt{N}} = \Lambda_G. \tag{29}$$

The vacuum expectation value (VEV) of the scalar field is less than the gravity scale Λ_G. Now the slow-roll parameter becomes

$$\epsilon_H \sim \frac{M_p^2}{N\phi^2} \geq 1, \tag{30}$$

which says that the slow-roll condition cannot be achieved and the assisted chaotic inflation is in the swampland.

Even though the previous discussions are restricted to chaotic inflation, we expect that the VEV of the canonical scalar field is bounded by Λ_G for the case with a large number of species. As a concrete example in string theory, the authors of [29] found the maximal variation of the canonical inflaton field for a D3 brane in the warped background as

$$\Delta\phi = \sqrt{T_3}R \leq \frac{2}{\sqrt{N_B}}M_p, \tag{31}$$

where R is the size of the throat and N_B is the number of the background D3 brane charge. If we introduce N mobile D3 branes in this scenario, the number of scalar fields is N which must be bounded by the background D3 charge N_B ($N < N_B$) for the validity of the background geometry in order to get the backreaction of the mobile D3 branes under control. Therefore, the bound in Eq. (31) can be written as

$$\Delta\phi \leq \frac{2}{\sqrt{N}}M_p = 2\Lambda_G. \tag{32}$$

Ignoring the coefficient, we find that the VEV of the canonical scalar field is bounded by the gravity scale Λ_G.

To summarize, we propose the weak gravity conjectures for the effective field theories with N species coupled to gravity. The weak gravity conjecture for the field theory with a large number of species is more stringent than the entropy bound. If the weak gravity conjecture is correct, it must indicate a new intrinsic property of quantum gravity.

The conjecture for the $\lambda\phi_i^4$ theory implies that even the assisted chaotic inflation might be in the swampland, not in the landscape. However, a possible realization of the assisted chaotic inflation in string theory called N-flation was proposed in [30]. In fact, they worked in a semiclassical way. If the conjectures in this paper are correct, N-flation is just semiclassically self-consistent, not really self-consistent. N-flation has the same predictions as single-field chaotic inflation. The accuracy of the Planck satellite which will be launched in 2008 [31] is much better than that of the WMAP (Wilkinson Microwave Anisotropy Probe), and its measurements will set more restricted constraints on the dynamics of inflation. We predict that the chaotic inflation/N-flation will be ruled out by Planck at a high confidence level, even though this model is nicely compatible with the present WMAP data [32]. The weak gravity conjecture is still a conjecture and its validity is uncertain, but it is a testable conjecture. If the N-flation or chaotic inflation is confirmed by the Planck satellite, it will falsify our conjecture.

In general we believe that the low-energy effective field theory is not applicable in the over-Planckian field space. In [11] Ooguri and Vafa also gave some arguments in string theory to support this statement. Our conjecture provides a concrete example for it. We also propose some possible observational consequences for the inflation models with sub-Planckian field values in [33]: a lower bound on the spectral index is obtained and the tensor perturbations can be neglected. It also brings a constraint on the equation of state parameter of quintessence [34].

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