Bloch-Nordsieck estimates of high-temperature QED

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(Received 15 March 2008; published 29 May 2008)

In anticipation of a subsequent application to QCD, we consider the case of QED at high temperature. We introduce a Fradkin representation into the exact, Schwingerian, functional expression of a fermion propagator, as well as a new and relevant version of the Bloch-Nordsieck model, which extracts the soft contributions of every perturbative graph, in contradistinction to the assumed separation of energy scales of previous semiperturbative treatments. Our results are applicable to the absorption of a fast particle which enters a heat bath, as well as to the propagation of a symmetric pulse within the thermal medium due to the appearance of an instantaneous, shockwave-like source acting in the medium. An exponentially decreasing time dependence of the incident particle's initial momentum combines with a stronger decrease in the particle's energy, estimated by a sum over all Matsubara frequencies, to model an initial "fireball," which subsequently decays in a Gaussian fashion. When extended to QCD, qualitative applications could be made to RHIC scattering, in which a fireball appears, expands, and is damped away.

DOI: 10.1103/PhysRevD.77.105027 PACS numbers: 11.10.Wx, 11.15.Tk, 12.20.Ds

I. INTRODUCTION

In a previous article [1], a hot quantum field toy model was used in order to test and appreciate the calculational efficiency of the functional methods long developed and used by one of us in a large variety of situations (for example, Refs. [2–4]). The result for the two-point function came out both nontrivial and remarkably simple, opening up some interesting physical interpretations. The scalar model, however, had little to connect it to the physical theories of QED and QCD. In the case under consideration of QED, and at the same level of approximation, the same two-point function exhibits a far richer structure of entwined contributions and associated mechanisms, which are the matter of the present article.

The mechanisms for depletion of a high-energy particle's energy E and momentum p when incident upon a medium at equilibrium temperature T, where $p \gg T$, suggested in the following sections are intended to be a small improvement to the seminal work of Weldon [5], Takashiba [6], and Blaizot and Iancu [7–9] of a decade ago. Our techniques and points of view are somewhat different from theirs, but it is essential to begin by acknowledging our debt to these authors, who first introduced and implemented the idea of a Bloch-Nordsieck (BN) approximation in order to estimate the physics of such energy and momentum depletion.

It may be useful to note the interpretations that we bring to this subject, and we here enumerate the special aspects of our approach which the interested reader will encounter below.

(1) At the very beginning, we separate and discard (the infinities of) those aspects of free-particle mass and wave-function renormalization from the specific effects of the medium on the particle. This is simple to perform in a functional approach, but rather complicated in the conventional, momentum-space expansion of the proper self-energy part of the inverse fermion propagator (of the particle which is entering the medium). Specific effects of the effective mass change in this model due to the motion of the particle in the medium may be found in [10].

(2) We introduce a modified BN approximation appropriate to the case when the particle's momentum is decreasing as it moves into the medium. Energy losses are calculated by the usual, Matsubara replacement of $E \to \omega_n$ and thermally averaged according to the Martin-Schwinger/Matsubara formalism we use, as described in detail in a previous publication [1]. But momentum loss is described by a separate, "Doppler" mechanism, which replaces the constant BN momentum p by $p(t) = p(0)\exp(-\Gamma t)$, where t is the duration of time the particle has been in the medium, and Γ is specified by a simple, semiclassical argument. We feel this choice of semiclassical BN momentum is more physical than the conventional procedure, appropriate to high-energy scattering, of retaining the constant value which the particle has upon entering the medium; but we place no particular emphasis on this Doppler mechanism for calculating $p(t)$. Other models may well be better for the description of this decreasing BN parameter, but this one is simple, and physically reasonable, and has the interesting consequence of modeling the appearance of a "fireball" at the initial stages of the particle's thermal history.

(3) We view the medium as an effective mechanism for the loss of a particle's energy and momentum, without requiring the particle to remain continuously on its mass shell (this is good quantum mechanics,

because the experiments we are describing do not measure this property). Only after thermalization, when $p(t)$ has decreased to the order of T, and its derived exponential decay law is no longer relevant, only when the particle joins its many identical twins in the equilibrium distribution at temperature T, can the particle be supposed to be on its mass shell.

(4) We rigorously maintain the nature of our BN approximation, with all real or virtual k_μ coupled to the incident particle required to satisfy $|\vec{p}| \gg |\vec{k}|$. As a result, all integrals are finite and easily approximated. For reasons stated in Sec. III [after Eqs. (3.11) and (3.16)], we do not employ the conventional hard thermal loop (HTL) analysis to describe pair-production generated by virtual photons emitted in the medium by the incident particle; rather, we estimate such pair-production using a straightforward functional representation and find it multiplying the ordinary Bremsstrahlung [a contribution to the decay exponent of $g^2(\vec{p}^2)^2$] by a factor of $g^2 \ln(\vec{p}^2/m^2)$. Were the coupling large, rather than that of QED, this term could be suppressed by the unitary denominator factor as described after Eq. (3.16).

(5) We are able to provide an explicit expression for the time dependence of the thermalization process, as the particle's |thermal average propagator|^2 initially increases—corresponding to the fireball—and then decreases rapidly, as given by a specific, Gaussian decay. Were we to restrict the final Matsubara sum to $n = 0$ only, that falloff would be exponential; but we are able to sum over all n, and the result is a stronger, Gaussian approach to thermalization.

(6) Our model calculation is able to distinguish longitudinal and transverse components of the fireball. We do not actually compute distributions which resemble a true fireball; rather, we use the word to represent a short-lived enhancement of probability as a function of time in the medium, corresponding to the incident particle's ability to generate a longitudinal burst of secondary particles and photons. By "transverse fireball" we mean a short-lived enhancement of probability as a function of the incident particle's time in the medium, which can serve to generate a symmetric pulse of secondaries in an arbitrary direction.

The paper is organized as follows. Section II describes the essential features of our BN derivation for the fermionic two-point function, in quenched approximation. The result turns out to be remarkably simple. For the sake of completeness, the theoretical steps which come before that treatment are deferred to Appendix A. Then, a Doppler model for the fast particle momentum damping is used to conclude Sec. II and the Doppler model itself is described in Appendix B. In Sec. III, the approximation of quenching is removed so as to take fermionic loop leading effects into account. The transverse fireball is discussed in Sec. IV. A summary and a discussion of our results are presented in Sec. V.

II. QUENCHING WITHIN THE BLOCH-NORDSIECK APPROXIMATION SCHEME

The main steps of the approach are as follows, as succinctly as possible.

Inherent to the BN approximation scheme, ordered exponentials which appear in the rigorous Fradkin representation of the fermion propagator [3,4],

$$(e^{g\int_0^s ds' \boldsymbol{\sigma}\cdot\mathbf{F}(y-u(s'))})_+, \qquad \boldsymbol{\sigma}_{\mu\nu} = \tfrac{1}{4}[\gamma_\mu, \gamma_\nu], \qquad \mathbf{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.1}$$

are suppressed, because, as in all eikonal/BN models, they generate terms proportional to soft photon momenta, which can be neglected in comparison to the particle's momentum. Because of our suppression of conventional mass renormalization, and the understood appearance of spinorial wave functions on either side of the final propagator, the fermionic propagator at zero temperature will have its $(m - i\gamma\cdot p)$ factor replaced by $2m$ and will read

$$\langle x_0, \vec{p}|\mathbf{S}_c|y_0, \vec{y}\rangle = i(2m)e^{-i\vec{p}\cdot\vec{y}}\int_0^\infty ds \times \int dp_0 e^{-is(\omega^2-p_0^2)}e^{-ip_0(x_0-y_0)}, \tag{2.2}$$

where $\omega^2 = \vec{p}^2 + m^2$. With the same approximations, the free-fermion thermal propagator is given by

$$\tilde{\mathbf{S}}_{\rm th}(\omega, z_0) = (2m)\frac{i}{\tau}\sum_{n=-\infty}^{+\infty}\int_0^\infty ds e^{-is(\omega^2-\omega_n^2)}e^{-i\omega_n z_0}, \qquad \omega_n = \frac{(2n+1)\pi}{\tau}, \qquad z_0 = x_0 - y_0, \tag{2.3}$$

and, by direct evaluation, is equal to

$$\tilde{\mathbf{S}}_{\rm th}(\omega, z_0) = (2m)\frac{i}{2\omega}\{[1-\tilde{n}(\omega)]e^{-i\omega z_0} - \tilde{n}(\omega)e^{+i\omega z_0}\}, \tag{2.4}$$

for $z_0 > 0$, where $\tilde{n}(\omega)$ is the Fermi-Dirac distribution function [11]. For the thermal propagator in the presence of a background A_μ field, the corresponding BN approximation gives

$$\langle x_0, \vec{p}|\mathbf{G}_{\rm th}^{\rm BN}[A]|y_0, \vec{y}\rangle = \frac{i}{\tau}\sum_{n=-\infty}^{+\infty}\int_0^\infty ds e^{-is(\omega^2-\omega_n^2)} \times e^{-i\omega_n z_0}e^{-ig\int_0^s ds' p\cdot A(y-2s'p)}, \tag{2.5}$$

where $p_\mu = (\omega_n, \vec{p})$. The thermal 2-point fermion function, in quenched BN approximation, is then given by

$$\langle \vec{p}, n|\mathbf{S}_{\rm th}^{\prime \rm BN}|\vec{y}, y_0\rangle = e^{-(i/2)\int(\delta/\delta A_\mu)\mathbf{D}_{\rm th}^{\mu\nu}(\delta/\delta A_\nu)} \cdot \langle \vec{p}, n|\mathbf{G}_{\rm th}^{\rm BN}[A]|\vec{y}, y_0\rangle|_{A\to 0}\cdot \mathbf{Z}_0[i\tau], \tag{2.6}$$

where $\mathbf{Z}_0[i\tau]$ is the free partition function, and in the real-time imaginary-temperature formalism being used for the linkage operator with Matsubara sum, one has $\mathbf{D}_{\rm th}^{\mu\nu} = \mathbf{D}_c^{\mu\nu} + \delta\mathbf{D}_{\rm th}^{\mu\nu}$, with $\mathbf{D}_c^{\mu\nu}$ the causal free-photon propagator, and $\delta\mathbf{D}_{\rm th}^{\mu\nu}$ the proper thermal part of $\mathbf{D}_{\rm th}^{\mu\nu}$,

$$\begin{aligned}\delta\mathbf{D}_{\mu\nu}^{\rm th}(u-v) &= \frac{i}{(2\pi)^3}\int d^4k\,\delta(\vec{k}^2-k_0^2)\frac{e^{ik\cdot(u-v)}}{e^{\beta|k_0|}-1}\hat{\mathbf{D}}_{\mu\nu} \\ &= \frac{i}{(2\pi)^3}\int\frac{d^3\vec{k}}{2k}\frac{e^{i\vec{k}\cdot(\vec{u}-\vec{v})}}{e^{\beta k}-1}(e^{-ik_0(u_0-v_0)} \\ &\quad + e^{+ik_0(u_0-v_0)})\hat{\mathbf{D}}_{\mu\nu}\end{aligned} \tag{2.7}$$

with $k = |\vec{k}|$. In the Coulomb gauge to be used, one has $A_0 = 0$, $\nabla\cdot\vec{A} = 0$, and

$$\hat{\mathbf{D}}_{\mu\nu} = \delta_{\mu i}\left(\delta_{ij} - \frac{k_ik_j}{\vec{k}^2}\right)\delta_{j\nu}. \tag{2.8}$$

In this first, quenched approximation, all complications related to conventional, $T = 0$, free-particle mass and wave-function renormalization are removed by suppressing the $\{-\frac{i}{2}\int\frac{\delta}{\delta A_\mu}\mathbf{D}_c^{\mu\nu}\frac{\delta}{\delta A_\nu}\}$ part of the complete linkage operator appearing in Eq. (2.6) and by retaining only the thermal part of it, that is,

$$e^{-(i/2)\int\frac{\delta}{\delta A_\mu}\delta\mathbf{D}_{\mu\nu}^{\rm th}(\delta/\delta A_\nu)}\frac{i}{\tau}\sum_{n=-\infty}^{+\infty}\int_0^\infty ds e^{-is(\omega^2-\omega_n^2)}e^{-i\omega_n z_0} \times e^{-ig\int_0^s ds'\vec{p}\cdot\vec{A}(y_0-2s'\omega_n,\vec{y}-2s'\vec{p})}\Big|_{A=0}, \tag{2.9}$$

which gives,

$$\frac{i}{\tau}\sum_{n=-\infty}^{+\infty}\int_0^\infty ds e^{-is(\omega^2-\omega_n^2)}e^{-i\omega_n z_0} \times e^{2ig^2\int_0^s ds_1\int_0^s ds_2 p_\mu\delta\mathbf{D}_{\mu\nu}^{\rm th}((s_1-s_2)p)p_\nu}. \tag{2.10}$$

In essence, this term's contribution corresponds to the particle's energy loss due to the bremsstrahlung produced under the enhancement of the heat bath's photons; for ease of presentation, that bremsstrahlung produced by the slowing particle inside the heat bath, will be reconsidered in Sec. III with the $\mathbf{D}_c^{\mu\nu}$ portion of the linkage operation.

The argument of the last exponential factor of Eq. (2.10) can be written as

$$2ig^2\int_0^s ds_1\int_0^s ds_2\frac{1}{2}\int\frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{k}\frac{e^{2i\vec{k}\cdot\vec{p}(s_1-s_2)}}{e^{\beta k}-1}2 \times\cos[2(s_1-s_2)k\omega_n]\left(\vec{p}^2-\frac{(\vec{k}\cdot\vec{p})^2}{\vec{k}^2}\right)e^{-(k/p)}, \tag{2.11}$$

where a factor $\exp(-|\vec{k}|/|\vec{p}|)$ has been inserted as a simple way of enforcing the BN approximation. Certainly, other such limitations are possible and the details of the calculation will be changed somewhat, but the physics will be essentially the same. The integrals in Eq. (2.11) are well defined, but cannot be carried out exactly. A sensible, approximate evaluation is proposed in Appendix C, leading to an overall form of

$$-\xi^2g^2f\left(\frac{T}{p}\right)(\vec{p}^2)^2s^2, \qquad f\left(\frac{T}{p}\right)\simeq\frac{(T/p)^2}{1+T/p}, \tag{2.12}$$

where ξ combines some numerical factors and where the approximation for $f(T/p)$ is valid in the regime $|\vec{p}|\gg T$. In Eq. (2.12), it is worth noting that power of s^2; had we obtained a power of s instead, then we would be lead back to the scalar field situation in which the whole set of BN approximations reduced to an exponential temporal damping of the original free-field result (cf. Eq. (3.35) in Ref. [1]). As might have been expected in the case of QED, the s^2 law leads to a more involved behavior that we now must evaluate.

One is then left with the expression

$$\begin{aligned}&\frac{i}{\tau}\sum_{n=-\infty}^{+\infty}\int_0^\infty ds e^{-is(\omega^2-\omega_n^2)}e^{-i\omega_n z_0}e^{-a^2s^2}, \\ &a^2 = \xi^2g^2(\vec{p}^2)^2f\left(\frac{T}{p}\right).\end{aligned} \tag{2.13}$$

The free-field result, $\tilde{\mathbf{S}}_{\rm th}$ of Eqs. (2.3) and (2.4), can still be used to rewrite Eq. (2.13) in the form

$$\begin{aligned}e^{-a^2(i(\partial/\partial\omega^2))^2}(2m)\frac{i}{\tau}\sum_{n=-\infty}^{+\infty}\int_0^\infty ds e^{-is(\omega^2-\omega_n^2)}e^{-i\omega_n z_0} \\ = e^{+a^2(\partial/\partial\omega^2)^2}\tilde{\mathbf{S}}_{\rm th}(\omega, z_0),\end{aligned} \tag{2.14}$$

and, by using the representation:

$$e^{+a^2(\partial/\partial\omega^2)^2} = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}db e^{-b^2+2ab(\partial/\partial\omega^2)},$$

it is possible to express our BN-approximated result in the remarkable and rather simple form of a Gaussian averaged, ω-translated free-field propagator,

$$\mathbf{S}_{\rm th}^{\prime\rm BN}(\omega, z_0) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}db e^{-b^2}\tilde{\mathbf{S}}_{\rm th}(\sqrt{\omega^2-2ab}, z_0). \tag{2.15}$$

However simple in principle, an exact integration of Eq. (2.15) remains out of reach. Fortunately, the following considerations concerning orders of magnitude are helpful in order to extract the essence of the result.

At a small enough coupling constant, one can expect to have $g\xi \leq 1$. In Appendix C, we found ξ itself is small and less than 1. Also, starting from $|\vec{p}|\gg T$, all the way down to thermalization, where $|\vec{p}|$ becomes on the order of T, one has clearly $\sqrt{f} \simeq (T/p)/\sqrt{1+T/p} < 1$. And finally, the essential part of Eq. (2.15) is given by the range of $|b| < 1$. It therefore appears sensible to replace Eq. (2.15) by

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} db e^{-b^2} \tilde{\mathbf{S}}_{\text{th}}(\omega(1 + g\xi\sqrt{f}b), z_0). \tag{2.16}$$

Inserting Eq. (2.4), (with the first, overall factor of $1/\omega$ left constant for simplicity), integration on the parameter b can be performed, with a convergent and damped result of

$$i\frac{2m}{\omega}\left\{\frac{1}{2}e^{-i\omega z_0} - e^{-\omega/T+(a/2\omega T)^2} \times \cos\left(\left[\omega - 2T\left(\frac{a}{2\omega T}\right)^2\right]z_0\right)\right\}e^{-a^2 z_0^2/4\omega^2}. \tag{2.17}$$

As noted in item 5 of the introduction, had we retained only the $n = 0$ term of Eq. (2.13), the result would have had the same structure, but with a slower falloff in z_0. Defining the quantity

$$q = e^{-\omega/T+(a/2\omega T)^2} \cos\left(\left[\omega - 2T\left(\frac{a}{2\omega T}\right)^2\right]z_0\right), \tag{2.18}$$

then, the squared modulus of Eq. (2.17) reads

$$\frac{4m^2}{\omega^2}\left[\frac{1}{4} + q^2 - q\cos(\omega z_0)\right]e^{-a^2 z_0^2/2\omega^2} \tag{2.19}$$

whose leading contribution, in view of Eq. (2.18), is the first term of Eq. (2.19),

$$\frac{m^2}{\omega^2}\exp\left(-\frac{z_0^2\alpha T^2}{1+T/p}\right) \to \frac{m^2}{\omega^2}\exp(-z_0^2\alpha T^2), \tag{2.20}$$

the meaning(s) of which will be discussed in detail in Sec. III. In order to represent the thermalization of such an incident particle, one may introduce the ratio of Eq. (2.20), taken at a given z_0 value, to its initial value at $z_0 = 0$.

Let $R(z_0 T, p(z_0)/T, \alpha) \equiv R(z_0)$ be this ratio. One has

$$R(z_0) = \left(\frac{p(0)}{p(z_0)}\right)^2 \exp\left(-\frac{z_0^2\alpha T^2}{1+T/p(z_0)}\right), \tag{2.21}$$

where we have resorted to the approximation of $\omega \simeq p$, which agrees with the ordering of scales: $p \gg T \gg m$. Restoring all of the conventional units, one can write

$$z_0 T \to \frac{z_0 k_B T}{\hbar} = \frac{c z_0}{\lambda_c}\frac{k_B T}{mc^2}, \tag{2.22}$$

where k_B is the Boltzmann constant and λ_c the Compton wavelength of the traveling particle. We assume a reasonable, semiclassical model for the decrease of $\vec{p} = \vec{p}(z_0)$, of form $\vec{p}(0)\exp(-\Gamma z_0)$ with $\Gamma = \Gamma_{\text{Doppler}} = \xi\alpha c/\lambda_c (kT/mc^2)^2$, where ξ is a numerical constant and $\lambda_c = \hbar/mc$, as derived in Appendix B. Then,

$$p(z_0) = p(0)e^{-\Gamma z_0}, \qquad \Gamma = \xi\frac{\alpha c}{\lambda_c}\left(\frac{k_B T}{mc^2}\right)^2, \tag{2.23}$$

and the rise and subsequent falloff of $R(z_0)$ can simply be read off from the expression

$$R(z_0) = \exp\left\{\left(\frac{k_B T}{mc^2}\right)^2 \frac{c z_0}{\lambda_c}\left[2\xi\alpha - \frac{c z_0}{\lambda_c}\frac{1}{1+T/p(z_0)}\right]\right\}. \tag{2.24}$$

If one assumes a specific form for the "linear" density of equilibrium heat bath photons in the Doppler computation, e.g., $\rho(\nu) = N(\nu) = [\exp(h\nu/k_B T) - 1]^{-1}$, that is, the conventional Planck distribution, then the constant ξ can be evaluated, and a specific value of z_0 predicted for when the exponential factor of Eq. (2.24) vanishes, and the fireball starts to decrease.

One finds, for example,

$$z_0 = 2\xi\alpha\frac{\lambda_c}{c} - \frac{1}{\Gamma}W_0\left(-2\Gamma\xi\alpha\frac{\lambda_c}{c}\frac{T}{p_0}e^{2(\xi\alpha k_B T/mc^2)^2}\right) \tag{2.25}$$

as the time after which a possible fireball starts to decrease. In Eq. (2.25), W_0 is the principal branch of the Lambert W function [12]. Provided its argument lies within the convergence radius of $1/e$, one has

$$2\left(\frac{T}{p_0}\right)\left(\frac{\xi\alpha k_B T}{mc^2}\right)^2 \exp\left[2\left(\frac{\xi\alpha k_B T}{mc^2}\right)^2\right] \le e^{-1}, \tag{2.26}$$

and then, W_0 can be replaced by its series expansion, and Eq. (2.25) may be approximated as

$$z_0 \simeq 2\xi\alpha\frac{\lambda_c}{c}\left(1 + \exp\left[2\left(\frac{\xi\alpha k_B T}{mc^2}\right)^2\right] + \cdots\right). \tag{2.27}$$

Note that $\mathcal{O}(\xi\alpha k_B T/mc^2) < 1$ is a necessary condition for Eq. (2.26) to be satisfied and the series expansion of W_0 to be reliable. In a system of natural units such as $\hbar = c = k_B = 1$, this is equivalent to $\mathcal{O}(\alpha T/m) < 1$, and this condition somewhat specifies, and restricts, the amount by which T is assumed to be much larger than m (remember the assumed ordering of $p \gg T \gg M$). One has then for Δz_0 an estimation on the order of α/m.

III. FERMIONIC LOOPS

In order to include pair production as a mechanism for the loss of the incident particle's energy, it is necessary to include at least the simplest closed-lepton loop, whose absorptive part corresponds to the probability of pair production. Let us approximate $\mathbf{L}[A]$, defined in Appendix A, in the simplest way, as $\mathbf{L}[A] = \frac{i}{2}\iint A_\mu(x)\mathbf{K}^{\mu\nu}(x-y)A_\nu(y)$, where the gauge invariant representation of $\mathbf{K}_{\mu\nu}$ reads [2]

$$\tilde{\mathbf{K}}_{\mu\nu}(k) = -k^2\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\Pi(k^2) = -k^2\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)[\Pi(0) + \Pi_R(k^2)] \quad (3.1)$$

with the renormalized part of Π_R,

$$\Pi_R(k^2) = -\frac{2\alpha}{\pi}\int_0^1 duu(1-u)\ln\left(1 + u(1-u)\frac{k^2}{m^2}\right). \quad (3.2)$$

Here, m is the mass of the looping fermions and renormalization has been performed so that $\Pi_R(0) = 0$. It is a real-valued function of k^2, as long as $k_0^2 < \vec{k}^2 + 4m^2$, but if $k_0^2 > \vec{k}^2 + 4m^2$, it develops an imaginary part given by the discontinuity of the logarithm across the cut, of value $2i\pi\Theta(k_0 - \sqrt{\vec{k}^2 + 4m^2})$. The real part of the logarithm, for large $\vec{k}^2$, is proportional to $\ln(\vec{k}^2/m^2)$, plus additive constants.

We have now, instead of Eq. (2.6), the expression

$$\langle\vec{p}, n|\mathbf{S}'^{\rm BN}_{\rm th}|\vec{y}, y_0\rangle = e^{-(i/2)\int(\delta/\delta A_\mu)D^{\mu\nu}_{\rm th}(\delta/\delta A_\nu)} \times \langle\vec{p}, n|\mathbf{G}_{\rm th}[A]|\vec{y}, y_0\rangle\frac{e^{\mathbf{L}[A]}}{\mathbf{Z}[i\tau]}\bigg|_{A=0}, \quad (3.3)$$

where $\mathbf{Z}[i\tau]$ is the normalization factor corresponding to the interacting partition function.

Using the BN approximation specified in Eq. (2.5), as well as the simplest approximation to $\mathbf{L}[A]$, the functional differentiation of Eq. (3.3) can be performed exactly with the help of the functional identity

$$\exp\left[-\frac{i}{2}\iint\frac{\delta}{\delta A}\cdot\mathbf{D}_{\rm th}\cdot\frac{\delta}{\delta A}\right] \cdot\exp\left[\frac{i}{2}\iint A\cdot\mathbf{K}\cdot A - i\int f\cdot A\right]\bigg|_{A=0} = \exp\left[\frac{i}{2}\iint f\cdot\mathbf{D}_{\rm th}\frac{1}{1-\mathbf{K}\cdot\mathbf{D}_{\rm th}}\cdot f\right] \cdot\exp\left[-\frac{1}{2}\,{\rm Tr}\ln(1-\mathbf{K}\cdot\mathbf{D}_{\rm th})\right], \quad (3.4)$$

where $f_\mu = gp_\mu\int_0^s ds'\delta(x-(y-s'p))$ and the Trace-Log determinantal factor of Eq. (3.4) has no relation to the traveling particle and is absorbed into the partition-function relation

$$\mathbf{Z}[\beta] = e^{-(1/2)\,{\rm Tr}\ln(1-\mathbf{K}\cdot\mathbf{D}_{\rm th})}|_{\tau\to -i\beta}\cdot\mathbf{Z}_0[\beta]. \quad (3.5)$$

The resulting thermal propagator has the same form as given in Eqs. (2.9) and (2.10), except that the term $\exp\{2ig^2\iint p\cdot\delta\mathbf{D}_{\rm th}\cdot p\}$ is now replaced by

$$\exp\left\{2ig^2\int_0^s ds_1\int_0^s ds_2\int\frac{d^4k}{(2\pi)^4} \times\left[p\cdot\mathbf{D}_{\rm th}\left(\frac{1}{1-\mathbf{K}\cdot\mathbf{D}_{\rm th}}\right)\cdot p\right]e^{-2i(s_1-s_2)(k_0\omega_n-\vec{k}\cdot\vec{p})}\right\}. \quad (3.6)$$

Recalling $\mathbf{D}^{\mu\nu}_{\rm th} = \mathbf{D}^{\mu\nu}_c + \delta\mathbf{D}^{\mu\nu}_{\rm th}$, Eq. (2.7), and $k^2\mathbf{D}_c(k) = \hat{\mathbf{D}}$, the denominator in the large parenthesis of Eq. (3.6), $1-\mathbf{K}\cdot\mathbf{D}_{\rm th}$, reduces to $1+\Pi(k^2)$, whereas the numerator, with $\mathbf{D}_{\rm th}(k)$, separates into two distinct parts, of which we consider first the contribution coming from $\mathbf{D}_c(k)$

$$\exp\left[\frac{i}{2}\int f\cdot\mathbf{D}_c\frac{1}{1+\Pi(k^2)}\cdot f\right] = \exp\left[-\frac{i}{2}\int f\cdot\mathbf{D}_c\frac{1}{1+\Pi(0)}\frac{1}{1+\frac{\Pi_R}{1+\Pi(0)}}\cdot f\right] \to \exp\left[\frac{i}{2}\int f\cdot\mathbf{D}_c\cdot f\right] \cdot\exp\left[-\frac{i}{2}\int f\cdot\mathbf{D}_c\frac{\Pi_R}{1+\Pi_R}\cdot f\right], \quad (3.7)$$

where the factor of $[1+\Pi(0)]^{-1}$ renormalizes all g^2 dependence in the last line, since $Z_3^{-1} = 1+\Pi(0)$ and $g_R^2 = Z_3 g^2$.

We first consider the first exponential factor on the right-hand side of Eq. (3.7): In a $T=0$, non-BN calculation where $k>p$ is permitted, this term generates the UV divergences associated with mass and wave-function renormalization, and those terms should properly be discarded as in Sec. II. In a $T>0$ context, this term describes the damping of the particle's energy due to ordinary bremsstrahlung (in contrast, the remaining part related to $\delta\mathbf{D}_{\rm th}$ in Eq. (3.6) describes the damping of the particle's energy as "enhanced" by the thermal photon heat bath in which the particle is slowing down).

The evaluation of this first term begins with

$$\exp\left\{4ig^2p_ip_j\int_0^s ds_1\int_0^{s_1}ds'\int\frac{d^4k}{(2\pi)^4}\frac{e^{-2is'(k_0\omega_n-\vec{k}\cdot\vec{p})}}{k^2-i\varepsilon} \times\left(\delta_{ij}-\frac{k_ik_j}{\vec{k}^2}\right)e^{-k/p}\right\}, \quad (3.8)$$

and the integration over k_0 can be carried out by contour integration:

$$-\int dk_0\frac{e^{-2is'k_0\omega_n}}{[k_0-(k-i\varepsilon)][k_0+(k-i\varepsilon)]} = +\frac{i\pi}{k}[\Theta(n)e^{-2is'k\omega_n}+\Theta(-n)e^{2is'k\omega_n}]. \quad (3.9)$$

Relying again on the approximations used in Sec. II, which amount basically to the neglect of the oscillating factors of Eqs. (3.8) and (3.9), Eq. (3.9) then becomes simply the quantity $i\pi/k$. Inserted into Eq. (3.8), one gets immediately a contribution of

$$\exp\left[-\frac{4}{3\pi}\alpha(\vec{p}^2)^2 s^2\right]. \tag{3.10}$$

That is, the first term in the right-hand side of Eq. (3.7) adds the amount $\frac{4}{3\pi}\alpha(\vec{p}^2)^2$ to the a^2 constant which appears in Eq. (2.13), an additional damping independent of the thermalized heat bath (at this level of approximation, of course).

The remaining factor of Eq. (3.7) displays a nice example of the basic unitarity of QED: even though we have used the lowest g^2-order approximation to $\mathbf{L}[A]$, in conjunction with our BN treatment, a very large Π_R cannot produce an overly large effect, for automatic damping (in the Hartree-Fock sense) is provided by its denominator. At first, we shall assume a weak effect, and accordingly replace that denominator by 1; and then, subsequently, the modifications will be noted when the complete denominator is used. But before we proceed with that very contribution, an interesting point must be made concerning the contributions attached to the $\delta\mathbf{D}_{\text{th}}^{\mu\nu}$ piece of the full $\mathbf{D}_{\text{th}}^{\mu\nu}$ propagator appearing in Eq. (3.6).

The contribution attached to $\delta\mathbf{D}_{\text{th}}^{\mu\nu}$ in Eq. (3.6) may be written in a way similar to Eq. (3.7)

$$\begin{aligned}\exp\left[\frac{i}{2}\int f\cdot\delta\mathbf{D}_{\text{th}}\frac{1}{1+\Pi(k^2)}\cdot f\right] &= \exp\left[\frac{i}{2}\int f\cdot\frac{\delta\mathbf{D}_{\text{th}}}{1+\Pi(0)}\cdot f\right]\\ &\quad\cdot\exp\left[-\frac{i}{2}\int f\cdot\frac{\delta\mathbf{D}_{\text{th}}}{1+\Pi(0)}\frac{\Pi_R}{1+\Pi(0)}\cdot f\right],\end{aligned} \tag{3.11}$$

where the first factor, in the right-hand side of Eq. (3.11), is that part already calculated in Sec. II which leads to the result of Eq. (2.13). The second term would correspond to a pair-production mechanism, enhanced by the thermal heat bath photons. However, since $\Pi_R(k^2)$ vanishes at $k^2 = 0$, this term vanishes because $\delta\mathbf{D}_{\text{th}}(k)$ is proportional to $\delta(k^2)$ [see Eq. (2.7)]. Over the relevant k-integration range, the leading thermal contribution to $\Pi(k)$, that is the so-called hard thermal loop polarization tensor, $\Pi^{\text{HTL}}(k_0, k)$, can be as large and even larger than the renormalized $T = 0$ part, $\Pi_R(k^2)$, [13,14]. However, this leading thermal piece of $\Pi^{\text{HTL}}(k_0, k)$ is also proportional to k^2 and therefore does not contribute either to the second term on the right-hand side of Eq. (3.11).

Returning to the second right-hand side factor of Eq. (3.7) in its lowest g^4 order, $\exp\{-(i/2)\int f\cdot\mathbf{D}_c\Pi_R\cdot f\}$, one needs to evaluate

$$\exp\left\{4ig^2\int_0^s ds_1\int_0^{s_1}ds'\int\frac{d^4k}{(2\pi)^4}\frac{e^{-2is'(k_0\omega_n-\vec{k}\cdot\vec{p})}}{k^2-i\varepsilon}\times\Pi_R(k^2)\left(\vec{p}^2-\frac{(\vec{p}\cdot\vec{k})^2}{\vec{k}^2}\right)e^{-k/p}\right\}, \tag{3.12}$$

and focus interest on the contribution coming from the imaginary part of $\Pi_R(k^2)$, given by $-(2i\alpha/3)\Theta(k^2 - 4m^2)$. Note that the renormalization prescription of $\Pi_R(0) = 0$ ensures that there is no singularity at $k^2 = 0$. And, therefore, the k_0 integral over the real part of $\Pi_R(k^2)$ receives no contribution [10].

To evaluate the absorptive part of Eq. (3.12), consider first

$$\int dk_0\frac{\Theta(k_0-\sqrt{\vec{k}^2+4m^2})}{[k_0-k+i\varepsilon][k_0+k-i\varepsilon]} \to \frac{1}{2k}\int_{\sqrt{\vec{k}^2+4m^2}}^{\Lambda}dk_0\left(\frac{1}{k_0-k}-\frac{1}{k_0+k}\right), \tag{3.13}$$

where the upper limit of Λ cannot be chosen larger than the particle's available energy (which it gives to the virtual photon, which then produces the pair). And even though one cannot make a mass-shell measurement of the particle as it passes through the medium, its energy surely cannot be too far from its mass-shell value, which is essentially cp (until thermalization occurs, $cp > k_BT$). Taking Λ on the order of p, the oscillating factors of Eq. (3.12) are again sufficiently small to be neglected, and what remains is the simple integral

$$-\frac{1}{2k}\int_{\sqrt{\vec{k}^2+4m^2}}^{p}dk_0\left(\frac{1}{k_0-k}-\frac{1}{k_0+k}\right) = \frac{1}{k}\left\{\ln\frac{p+k}{p-k}+\ln\frac{\sqrt{\vec{k}^2+4m^2}-k}{\sqrt{\vec{k}^2+4m^2}+k}\right\}. \tag{3.14}$$

With $k = xp$, this combination can be approximately reduced to

$$-\frac{1}{k}\left[\ln(1-x^2)+\ln\left(\frac{\vec{p}^2}{4m^2}\right)\right]. \tag{3.15}$$

Then, combining all factors and retaining only the most important $\ln(\vec{p}^2/m^2)$ dependence, one finds for the absorptive contribution of Eq. (3.12), the amount

$$-\frac{4}{3\pi}\left(\frac{g^2}{4\pi}\right)^2 s^2(\vec{p}^2)^2\ln\left(\frac{\vec{p}^2}{m^2}\right). \tag{3.16}$$

The remaining denominator factor of Eq. (3.7) can be taken into account by writing $1+\Pi_R(k^2) = 1 + g^2(u - iv)$ and identifying the new absorptive part of $\Pi_R/[1+\Pi_R]$ as $-ig^2v/[(1+g^2u)^2+g^4v^2]$. Here, one has $u = \mathcal{O}(\ln(\vec{k}^2/m^2))$, which, after integration, translates into a denominator factor of $\ln(\vec{p}^2/m^2)$, thereby removing the $\ln(\vec{p}^2/m^2)$ factor of Eq. (3.16), and effectively substituting a factor of $[\ln(\vec{p}^2/m^2)]^{-1}$. However, other, higher-order corrections from the photon polarization may change the result. From this simple photon bubble, in our BN approximation and for small coupling, there appears little pair-production enhancement of the bremsstrahlung damping of Eq. (3.10).

In our calculation, pair production is considered as one of the processes of energy depletion of the incident particle. The fermion-loop pairs are not considered thermalized at the instant of production and have no knowledge of the medium without subsequent interaction, which is irrelevant to the incident particle's energy loss. In contrast, previous calculations [7–9] have replaced internal photon lines with effective (resummed) photon propagators in the HTL approximation, in which the closed-fermion-loop momenta are assumed to be larger than those of soft thermal photons in the construction of the photon polarization tensor [15–18]. The loop fermions we use are not taken as thermalized; rather, we employ the conventional, renormalized, photon polarization tensor and extract its imaginary contribution as that piece of the calculation relevant to pair production.

IV. TRANSVERSE VS LONGITUDINAL FIREBALLS

This section develops the interpretation of the resulting fermionic two-point function which has been introduced in Ref. [1]. Collecting all three damping factors, with $p \gg T$, the a^2 constant in Eq. (2.13) now becomes

$$\tilde{a}^2 = 4\pi\xi^2\alpha T^2\vec{p}^2 + \frac{4}{3\pi}\alpha(\vec{p}^2)^2 - \frac{4}{3\pi}\alpha^2(\vec{p}^2)^2 \ln\left(\frac{\vec{p}^2}{m^2}\right)$$
$$= \frac{4}{3\pi}\alpha(\vec{p}^2)^2\left\{\left[1 + \left(\frac{2\pi T}{p}\right)^2\right] - \alpha \ln\left(\frac{\vec{p}^2}{m^2}\right)\right\}, \qquad (4.1)$$

where it is encouraging to recognize a term of $1 + (\frac{2\pi T}{p})^2$, peculiar to rigorous one-loop perturbative calculations [13], and Eq. (2.17) can be rewritten as

$$i\frac{2m}{\omega}\left\{\frac{1}{2}e^{-i\omega z_0-(\tilde{a}^2/4\omega^2)z_0^2} - e^{-(\omega/T)-(\tilde{a}^2/4\omega^2)(z_0^2-(1/T^2))}\cos\left(\left[\omega - 2T\left(\frac{\tilde{a}}{2\omega T}\right)^2\right]z_0\right)\right\}. \qquad (4.2)$$

The second term of Eq. (4.2) describes the disturbance inside the heat bath which is isotropic in the medium. Were one to calculate the spatial thermal propagator, by a Fourier transform over $\vec{p}$ at any given time (before thermalization) in the medium, the cosine factor of Eq. (4.2) would correspond to the appearance of a disturbance propagating with exponential phase factors of $[i(\vec{p}\cdot\vec{z} - Qz_0)]$ and $[i(\vec{p}\cdot\vec{z} + Qz_0)]$, with Q the square-bracket constant that appears in the cosine's argument of Eq. (4.2); and if the dummy variable $\vec{p}$ is changed to $-\vec{p}$ in the integration over the second exponential factor, the result suggests the propagation of symmetric, "transverse" back-to-back pulses in any arbitrary direction. This is true for the free propagator and interacting propagator and is to be expected of a thermal Green's function which not only describes the effects of an incident particle entering the medium, but also contains a description of any "tsunami-like" disturbance originating in the medium. (One may think here of the emission by the incident particle of a virtual photon with high energy and little momentum, which immediately decays into an electron-positron pair, which then comprise and sequentially generate the corresponding transverse fireballs.)

The factor, $e^{-\omega/T}$, in the second term of Eq. (4.2) comes from the Fermi-Dirac distribution and the combined factor, $\frac{1}{\omega}e^{-(\omega/T)+(\tilde{a}^2/4\omega^2T^2)}$, determines the initial relative magnitude of this second term compared to the first. The phase is also different from that of the incoming fermion by a shift of $2T(\frac{\tilde{a}}{2\omega T})^2$. The square modulus of this second term governs what we have called the transverse fireball, as defined in the item 6 of the introduction,

$$\frac{4m^2}{\omega^2}e^{-(2\omega/T)-(\tilde{a}^2/2\omega^2)(z_0^2-(1/T^2))} \times \cos^2\left(\left[\omega - 2T\left(\frac{\tilde{a}}{2\omega T}\right)^2\right]z_0\right). \qquad (4.3)$$

Similar to the analysis in Sec. II for the longitudinal counterpart which now reads

$$\frac{m^2}{\omega^2}\exp\left[-\frac{\tilde{a}^2z_0^2}{2\omega^2}\right]. \qquad (4.4)$$

One can see in Eq. (4.3) that the magnitude of the transverse disturbance rises as the incoming fermion starts to lose momentum/energy as $1/\omega^2$. However, the exponent decreases from its initial value which is Gaussian and much faster than $1/\omega^2$ as time goes on. Then the fireball shrinks in all directions once $z_0^2 > T^{-2}$ [or $z_0^2\hbar^{-2} > (k_BT)^{-2}$]. Hence, a very simple prediction arises from this BN-approximated QED calculation; which is a relative increase of the transverse disturbance as z_0 increases, over a duration extent of $\Delta z_0 = 1/T$.

As compared to Eqs. (2.26) and (2.27) for the longitudinal excitation (and neglecting the modification due to $a^2 \to \tilde{a}^2$), one would therefore have, because of $m > \alpha T$, the inequality $\Delta z_0^{(T)} > \Delta z_0^{(L)}$ for the durations after which transverse (T) and longitudinal (L) excitations quickly decay.

V. SUMMARY

Here, then, are all the damping factors estimated in a strict Bloch-Nordsieck framework, where there are no ultraviolet divergences and no infrared divergences. The evaluations involving the neglect of weakly oscillating integrands are of course approximate, but quite reasonable; and the results are physically correct in the sense that three sources of momentum and energy loss are included. It is worth noting that the results are also elegant in that they involve the complete thermal propagator and provide a continuous time dependence of the process, a fireball growth, followed by thermal decay which is Gaussian, rather than a simple exponential. We emphasize that, using straightforward functional methods, we sum over relevant contributions of the thermalized photons in the process of calculating the rapid thermalization of the incident particle. In contrast to the HTL approach, we do not consider fermion-loop lines which define pair production to be thermalized.

In this article, the case of a rapid massive fermion entering a thermalized QED plasma has been considered. Through the evaluation of the thermal fermion propagator, our focus has been to investigate depletion mechanisms that, in the ideal case of an infinite plasma, bring the incident fermion down to thermal equilibrium. The same formalism generates the probability requirement for the appearance of shock waves developing in the thermalized medium [1].

In order to go beyond the limitations of pure one-loop perturbative calculations, the present estimate is carried out in a nonperturbative way with a BN formalism associated with a realistically decreasing incident particle momentum. Not only do these approximations open the road to tractable calculations, but hopefully, they should also be physically relevant to the processes under consideration.

Within a convenient real-time/imaginary-temperature formalism, calculations are first carried out with the help of the quenching approximation, and we find a simple and elegant expression in terms of the free, noninteracting thermal fermion propagator. Then, the quenching approximation is relaxed by taking leading effects of fermion loops into account. And remarkably enough, up to the redefinition of a key parameter, the simple form of the "quenched result" is preserved.

For the incident particle, energy depletion mechanisms are taken to be induced by bremsstrahlung and pair production, whereas an intuitive, semiclassical Doppler model is formulated to account for momentum damping. In contrast to previous calculations a complete time-dependent description of the physical processes at play is obtained. In particular, the possibility of longitudinal and transverse shock waves is seen to develop in the plasma with different amplitudes and phases. Over two different scales of time duration, both excitations start to increase, and then quickly decay with a Gaussian law. This relatively simple QED analysis was motivated by the experimental runs at RHIC; and it will be interesting to learn, in a future investigation, if these relatively simple results also hold in QCD.

ACKNOWLEDGMENTS

H. M. Fried would like to thank the J. Schwinger Foundation for travel Grant No. JSF07080000 which contributed to the research and writing of this paper.

APPENDIX A: ON THE FUNCTIONAL FORMALISM

The fully dressed thermal Green's function is

$$\mathbf{S}'_{\mathrm{th}} = e^{\mathfrak{D}_A^{\mathrm{th}}} \cdot \left[\mathbf{G}_{\mathrm{th}}[A] \frac{e^{\mathbf{L}_{\mathrm{th}}[A]}}{\mathbf{Z}[i\tau]} \right] \Bigg|_{A\to 0}, \tag{A1}$$

where $\mathbf{L}_{\mathrm{th}}[A] = \mathrm{Tr}\ln[1 - ig(\gamma \cdot A)\mathbf{S}_{\mathrm{th}}]$ and the $\mathfrak{D}_A^{\mathrm{th}}$ operator of the linkage in the configuration space representation is

$$\mathfrak{D}_A^{\mathrm{th}} = -\frac{i}{2} \int dx \int dy \frac{\delta}{\delta A_\mu(x)} \cdot \mathbf{D}_{\mathrm{th}}^{\mu\nu}(x-y) \cdot \frac{\delta}{\delta A_\nu(y)}. \tag{A2}$$

The thermal fermion Green's function $\mathbf{G}_{\mathrm{th}}[A]$ is taken in the conventional Matsubara formalism but a Matsubara representation is not needed and is not used for the thermal photon propagator of the linkage operator. In the momentum representation of our formalism, the thermal photon propagator is separated into two parts,

$$\mathbf{D}_{\mathrm{th}}^{\mu\nu} = \mathbf{D}_c^{\mu\nu} + \delta\mathbf{D}_{\mathrm{th}}^{\mu\nu} \tag{A3}$$

with a corresponding splitting of the linkage operator Eq. (A2). The linkage operation can accordingly separate into two steps, the causal ($T = 0$) and thermal ($T \neq 0$) part, and the order of functional operation can be exchanged. For example,

$$\mathbf{S}'_{\mathrm{th}} = \left\{ e^{\Delta\mathfrak{D}_A^{(\mathrm{th})}} \left[e^{\mathfrak{D}_A^{(c)}} \left(\mathbf{G}_{\mathrm{th}}[A] \frac{e^{\mathbf{L}_{\mathrm{th}}[A]}}{\mathbf{Z}[i\tau]} \right) \right] \right\} \Bigg|_{A\to 0}. \tag{A4}$$

In addition to the Bremsstrahlung effect at $T > 0$, the first linkage operation, with $\exp[\mathfrak{D}_A^{(c)}]$, will produce factors of mass and wave-function renormalization, so that Eq. (A4) may be written approximately as

$$\mathbf{S}'_{\mathrm{th}} = \left\{ e^{\Delta\mathfrak{D}_A^{(\mathrm{th})}} \left(\mathbf{G}_{\mathrm{th},R}[A] \frac{e^{\mathbf{L}_{\mathrm{th},R}[A]}}{\mathbf{Z}_{,R}[i\tau]} \right) \right\} \Bigg|_{A\to 0}, \tag{A5}$$

where mass and wave-function renormalizations that have nothing to do with the medium are included in the fermion Green's functional, $\mathbf{G}_{\mathrm{th},R}[A]$, and closed-fermion-loop functional, $\mathbf{L}_{\mathrm{th},R}[A]$. For notational simplicity, the renormalization symbol, R, will be dropped in the following, and the mixed representation will hold

$$\langle \vec{p}, n | \mathbf{S}'_{\mathrm{th}} | \vec{y}, y_0 \rangle = e^{\Delta\mathfrak{D}_A^{(\mathrm{th})}} \left(\langle \vec{p}, n | \mathbf{G}_{\mathrm{th}}[A] | \vec{y}, y_0 \rangle \frac{e^{\mathbf{L}_{\mathrm{th}}[A]}}{\mathbf{Z}[i\tau]} \right) \Bigg|_{A\to 0}, \tag{A6}$$

where the "quenching" approximation is used; that is, the fermion determinant is suppressed, and Eq. (A6) is replaced (with the subscript Q for "quenched") by

$$\langle \vec{p}, n | \mathbf{S}'_{\mathrm{th}} | \vec{y}, y_0 \rangle_Q = \frac{1}{\mathbf{Z}_0[i\tau]} e^{\Delta\mathfrak{D}_A^{(\mathrm{th})}} \langle \vec{p}, n | \mathbf{G}_{\mathrm{th}}[A] | \vec{y}, y_0 \rangle |_{A\to 0}, \tag{A7}$$

where $\mathbf{Z}_0[i\tau]$ is the free partition function whose relation to $\mathbf{Z}[i\tau]$ is the following:

$$\mathbf{Z}[i\tau] = e^{\mathfrak{D}_A^{(\mathrm{th})}} e^{\mathbf{L}_{\mathrm{th}}[A]}|_{A\to 0} \cdot \mathbf{Z}_0[i\tau].$$

One finds [4,10]

$$\langle\vec{p}, n|\mathbf{S}'_{\rm th}|\vec{y}, y_0\rangle_Q = (\mathbf{Z}_0[i\tau])^{-1}[(2\pi)^3\tau]^{-1/2}e^{-i(\vec{p}\cdot\vec{y}-\omega_n y_0)}i\int_0^\infty ds e^{-is(m^2+p^2)}e^{-(1/2)\,{\rm Tr}\ln(2h)}\int d[w]\exp\left\{\frac{i}{4}\int_0^s ds_1 \times\int_0^s ds_2 w(s_1)\cdot h^{-1}(s_1,s_2)\cdot w(s_2)\right\}\left\{m - i\gamma\cdot\left[p + g^2\int\frac{d^4k}{(2\pi)^4}\int_0^s ds'\tilde{\mathbf{D}}_{\rm th}(k)\cdot[w'(s')-2p] \times e^{-ik\cdot[w(s)-w(s')]+2ik\cdot p(s-s')}\right]\right\}\exp\left\{\frac{i}{2}g^2\int\frac{d^4k}{(2\pi)^4}\int_0^s ds_1\int_0^s ds_2 e^{ik\cdot[w(s_2)-w(s_1)]-2ik\cdot p(s_2-s_1)} \times[w'(s_1)-2p]\cdot\tilde{\mathbf{D}}_{\rm th}(k)\cdot[w'(s_2)-2p]\right\}, \tag{A8}$$

where

$$h(s_1,s_2) = \int_0^s ds'\Theta(s_1-s')\Theta(s_2-s'), \quad h^{-1}(s_1,s_2) = \frac{\partial}{\partial s_2}\frac{\partial}{\partial s_1}\delta(s_1-s_2) \tag{A9}$$

and

$$e^{+(1/2)\,{\rm Tr}\ln(2h)} = \int d[w]\cdot\exp\left\{\frac{i}{4}\int_0^s ds_1\int_0^s ds_2 w(s_1)\cdot h^{-1}(s_1,s_2)\cdot w(s_2)\right\}. \tag{A10}$$

The Bloch-Nordseick set of approximations is completed by the replacements, which reflect the neglect of momentum fluctuation of magnitude less than p, $[w(s')-2s'p]\rightarrow -2s'p$ and $[w'(s')-2p]\rightarrow -2p$, so as to get eventually the expression

$$\langle\vec{p}, n|\mathbf{S}'^{\rm BN}_{\rm th}|\vec{y}, y_0\rangle_Q \simeq (\mathbf{Z}_0[i\tau])^{-1}[(2\pi)^3\tau]^{-1/2}e^{-i(\vec{p}\cdot\vec{y}-\omega_n y_0)}\{m - i\gamma\cdot p\}i\int_0^\infty ds e^{-is(m^2+p^2)} \cdot e^{2ig^2\int(d^4k/(2\pi)^4)\int_0^s ds_1\int_0^s ds_2[p\cdot\tilde{\mathbf{D}}_{\rm th}(k)\cdot p]e^{2ik\cdot p(s_1-s_2)}} \tag{A11}$$

which, in the main text, is at the level of Eqs. (2.10) and (2.11). Note that in passing from Eq. (A8) to (A11), the huge parenthesis of Eq. (A8),

$$\left\{m - i\gamma\cdot\left[p + g^2\int\frac{d^4k}{(2\pi)^4}\int_0^s ds'\tilde{\mathbf{D}}_{\rm th}(k)\cdot[w'(s') - 2p]e^{-ik\cdot[w(s)-w(s')]+2ik\cdot p(s-s')}\right]\right\}$$

has simply been replaced by $\{m - i\gamma\cdot p\}$. That is, the medium generated mass-shift, a long noticed phenomenon [19–21], has been withdrawn from our computation, because in this article, focus is placed on the depletion mechanisms acting on the incident fermion, and the subsequently generated shockwaves inside the thermalized medium.

APPENDIX B: ON THE DOPPLER MODEL

We here propose an elementary, one-dimensional derivation which avoids the problematic Lorentz transformation of the temperature T [22], of the $p(z_0)$ falloff. Let $\rho(\nu)$ be the linear density of photons per unit length, at thermal equilibrium. Then $\rho(\nu)\delta\nu$ is the number of such photons, of energy $h\nu$, per unit of time, in the frequency interval $\delta\nu$. The momentum carried by any photon in that distribution is $h\nu/c$. What the charged traveling particle sees is a Doppler shift of frequencies: for the photons "approaching head on," $\nu\rightarrow\nu_+$, and for the photons "approaching from the rear," $\nu\rightarrow\nu_-$, with

$$\nu_+ = \nu\sqrt{\frac{c+v}{c-v}}, \qquad \nu_- = \nu\sqrt{\frac{c-v}{c+v}}. \tag{B1}$$

From the elementary diagram of QED, let $\eta\alpha$ be the absorption probability of a photon by the fermionic line, where η stands for some numerical constant, α for the fine structure constant.

The number of photons absorbed per unit of time in the frequency interval $\delta\nu$ is thus $\alpha\eta\rho(\nu)\delta\nu$, the same in either front and rear directions. This allows the calculation of the momentum change induced by the process, assuming that all the other interactions, with the heat-bath thermalized photons, average out to zero. One gets

$$\frac{{\rm d}p}{{\rm d}t} = -\eta\alpha\int_0^\infty {\rm d}\nu\rho(\nu)\frac{h\nu}{c}\left(\sqrt{\frac{c+v}{c-v}} - \sqrt{\frac{c-v}{c+v}}\right), \tag{B2}$$

that is,

$$\frac{{\rm d}p}{{\rm d}t} = -p\Gamma_{\rm Doppler}, \qquad \Gamma_{\rm Doppler} = \left(\frac{\eta\pi^2}{3}\right)\frac{\alpha c}{\lambda_c}\left(\frac{k_B T}{mc^2}\right)^2, \tag{B3}$$

and this is the Γ constant that appears in Sec. II, Eq. (2.23).

APPENDIX C: APPROXIMATION SCHEME IN THE s INTEGRAL

The integral in the exponential factor of Eq. (2.11) becomes

$$-\frac{g^2}{2\pi^3}\vec{p}^2\int_0^s ds_1\int_0^{s_1}ds'\int d\Omega(1-\zeta^2)\times\int_0^\infty dkk\frac{\cos(2k\omega_n s')\cos(2kp\zeta s')}{e^{\beta k}-1}e^{-(k/p)}, \quad (C1)$$

where $\zeta=\cos\theta$. First carrying out both the s_1 and s' integrals, it reduces to

$$-\frac{g^2}{8\pi^3}\vec{p}^2\int d\Omega(1-\zeta^2)\int_0^\infty dk\frac{ke^{-k/p}}{(e^{\beta k}-1)}\times\left[\frac{1-\cos(2sQ^{(+)})}{(Q^{(+)})^2}+\frac{1-\cos(2sQ^{(-)})}{(Q^{(-)})^2}\right], \quad (C2)$$

where $Q^{(\pm)}=k(p\zeta\pm\omega_n)$. The integral over k is a bit complicated and cannot be carried out exactly. To continue evaluation, observe the oscillating factor $\exp[-is(\omega^2-\omega_n^2)]$ in the s integral of Eq. (2.10); when $s>s_{\max}=(\omega^2-\omega_n^2)^{-1}$, the oscillating factor effectively removes any contribution. The arguments of the cosine factors are

$$|sQ^{(\pm)}|<s_{\max}k|p\zeta\pm\omega_n|\ll s_{\max}p|p\zeta\pm\omega_n| < \frac{p|p\zeta\pm\omega_n|}{\omega^2-\omega_n^2}.$$

Since $p\gg m$ and $|\zeta|<1$,

$$|sQ^{(\pm)}|\ll\frac{p}{|p\mp\omega_n|}=\frac{1}{|1\mp\frac{\omega_n}{p}|}=\frac{1}{|1\pm i(2n+1)\pi\frac{T}{p}|},$$

or effectively $|sQ^{(\pm)}|<1$. Thus, the arguments of the cosine functions are small and can be approximated as

$$\frac{1-\cos(2sQ^{(\pm)})}{(Q^{(\pm)})^2}\simeq\frac{1}{2}\frac{(2sQ^{(\pm)})^2}{(Q^{(\pm)})^2}=2s^2. \quad (C3)$$

Set $x=k/p$ and the k integral becomes

$$\int_0^\infty dk\frac{ke^{-k/p}}{e^{\beta k}-1}=\vec{p}^2\int_0^\infty dx\frac{xe^{-x}}{e^{x(p/T)}-1}\equiv\vec{p}^2 f\left(\frac{T}{p}\right);$$

which yields

$$-s^2\frac{g^2}{2\pi^3}(\vec{p}^2)^2\int_0^1 d\zeta(1-\zeta^2)\int_0^\infty dx\frac{xe^{-x}}{e^{x(T/p)}-1} = -\xi^2 s^2 g^2 f\left(\frac{T}{p}\right)(\vec{p}^2)^2, \quad (C4)$$

where all numerical factors have been combined into $\xi^2=\frac{4}{3\pi}$. Similar approximations are made to derive Eqs. (3.10) and (3.16).

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