Nonsupersymmetric Seiberg duality, orientifold QCD, and noncritical strings

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We propose an electric-magnetic duality and conjecture an exact conformal window for a class of nonsupersymmetric $U(N_c)$ gauge theories with fermions in the (anti)symmetric representation of the gauge group and N_f additional scalar and fermion flavors. The duality exchanges $N_c \to N_f - N_c \mp 4$ leaving N_f invariant, and has common features with Seiberg duality in $\mathcal{N} = 1$ super QCD (SQCD) with SU or SO/Sp gauge group. At large N the duality holds due to planar equivalence with $\mathcal{N} = 1$ SQCD. At finite N we embed these gauge theories in a setup with D-branes and orientifolds in a nonsupersymmetric, but tachyon-free, noncritical type 0B string theory and argue in favor of the duality in terms of boundary and crosscap state monodromies as in analogous supersymmetric situations. One can verify explicitly that the resulting duals have matching global anomalies. Finally, we comment on the moduli space of these gauge theories and discuss other potential nonsupersymmetric examples that could exhibit similar dualities.

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I. INTRODUCTION

A. Setting the stage

Seiberg duality [1] is an impressive statement about the infrared dynamics of strongly coupled gauge theories. It states that two gauge theories, which are distinct in the ultraviolet (UV), flow in the infrared (IR) to the same fixed point. In the prototypical example of $\mathcal{N} = 1$ super QCD (SQCD) with gauge group $SU(N_c)$ and N_f flavors, the low-energy dynamics in the conformal window ($3N_c/2 < N_f < 3N_c$) is described by a nontrivial scale-invariant theory of interacting quarks and gluons, which has a dual formulation in terms of a "magnetic" theory with gauge group $SU(N_f - N_c)$ and the same of number of flavors. Currently there is no proof of Seiberg duality, but in supersymmetric cases (e.g. $\mathcal{N} = 1$ SQCD) there is overwhelming evidence both from field theory [1] and string theory [2,3].

In field theory, besides 't Hooft anomaly matching, which is not based on supersymmetry, one can perform a number of nontrivial consistency checks that rely on the power of supersymmetry; in particular, holomorphy and the properties of the superconformal algebra (see [4] for a review).

In string theory, one embeds the gauge theory of interest in a D-brane setup in a ten-dimensional type II superstring vacuum (with fivebranes in flat space—for a review see [5] and references therein—or in near-singular Calabi-Yau compactifications [6]), or lifts to M-theory [7–11]. These situations include extra degrees of freedom, but by going to a convenient region of moduli space (alas, typically not the one of direct interest for the gauge theory) the description simplifies and one can draw interesting conclusions.

More recently, it has been understood how to obtain a useful, controllable embedding of interesting SQCD-like theories in the corner of parameter space most relevant for the gauge theory. For example, in the case of $\mathcal{N} = 1$ SQCD (with a quartic coupling as we will see in Sec. III), this description involves N_c D3- and N_f D5-branes in noncritical type IIB superstrings on

$$\mathbb{R}^{3,1} \times SL(2)_1/U(1), \tag{1.1}$$

where $SL(2)_1/U(1)$ is a supersymmetric coset of the $SL(2, \mathbb{R})$ Wess-Zumino-Witten (WZW) model at level 1 [12]. String theory in (1.1) arises within the ten-dimensional setup that realizes $\mathcal{N} = 1$ SQCD (with two orthogonal NS5-branes), in a suitable near-horizon decoupling limit that takes properly into account the backreaction of the NS5-branes [13]. Since the setup provided by (1.1) admits an exact world sheet conformal field theory (CFT) formulation, an explicit analysis of perturbative string theory is possible in this case.

It has been proposed in [14] that Seiberg duality for $\mathcal{N} = 1$ SQCD can be understood in the noncritical string context in terms of D-brane monodromies. We will review this argument in detail in Sec. III with some important new elements that modify some of the basic points in the analysis of [14]. We will find this implementation of the duality particularly useful in this paper, because in contrast with the noncritical string considered here, the critical ten-

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dimensional description suffers from a closed string tachyon instability and its M-theory lift is less understood.

It is an interesting and compelling question whether similar phenomena, e.g. the appearance of a conformal window or the existence of Seiberg dual theories, can arise and be properly understood in nonsupersymmetric gauge theories. Here are some known facts about this question.

The presence of a conformal window in QCD, more specifically $SU(3)$ Yang-Mills theory[1] with N_f Dirac fermions in the fundamental representation was argued many years ago in [15]. The conclusions of [15] are based on a perturbative analysis of the two-loop beta function, which predicts a conformal window for $N_f^* < N_f < 33/2$. Reference [16] estimated the lower bound of the conformal window in $SU(N_c)$ QCD with N_f flavors at $N_f^* \simeq 4N_c$ (i.e. $N_f \simeq 12$ for $N_c = 3$). Lattice Monte Carlo studies [17,18] of QCD with N_f flavors suggest $N_f^* > 8$. In search of a QCD Seiberg dual, 't Hooft anomaly matching was examined without definite conclusion in [19].

Seiberg duality in the context of nonsupersymmetric models has been discussed also in theories that are connected to $\mathcal{N} = 1$ SQCD by soft supersymmetry breaking [20–22] or by a discrete projection that breaks supersymmetry. The first attempt to construct a nonsupersymmetric Seiberg dual in the large N limit by discrete projection was made by Schmaltz in [23] using the orbifold projection of [24]. Schmaltz proposed a $U(N_c) \times U(N_c)$ "orbifold" theory as a candidate for the electric theory. Seiberg duality was expected to follow in the large N limit as a consequence of a "planar equivalence" between the parent and daughter theories of the projection. The validity of this proposal relies, however, on the presence of an unbroken global $\mathbb{Z}_2$ symmetry [25,26], which is not always guaranteed. In the string realization of the gauge theory, this requirement translates to a very simple condition: the condition that the closed string background is tachyon-free [27]. Hence, the string realization of the gauge theory in [23] with D-branes and NS5-branes in the *tachyonic* type 0B string theory [28] suggests that the necessary $\mathbb{Z}_2$ symmetry is in fact broken in this case. It was therefore proposed in [29] that a gauge theory that lives on branes of a *nontachyonic* type 0′B string theory [30,31] (the Sagnotti model) would be a better candidate for an electric theory.

This logic leads us naturally to the "orientifold" gauge theories of [32] (for a review see [33]). These are QCD-like, nonsupersymmetric $U(N_c)$ gauge theories with fermions in the (anti)symmetric representation of the gauge group and N_f scalar and fermion flavors. In fact, for $N_c = 3$ and $N_f = 0$, the theory with fermions in the antisymmetric representation *is* QCD with one flavor. Moreover, these theories have been argued, in the large N limit, to be planar equivalent to $\mathcal{N} = 1$ SQCD. So we know to leading order in $1/N$ that these theories have the same structure as $\mathcal{N} = 1$ SQCD—in particular, they have a conformal window at $\frac{3}{2}N_c < N_f < 3N_c$ and exhibit Seiberg duality. Our aim in this paper is to extend this picture beyond the large N regime, where planar equivalence is lost, and to make some exact predictions about the conformal window and Seiberg duality at finite N. A specific result is the prediction for a finite N conformal window at $\frac{3}{2}N_c \mp \frac{20}{3} \leq N_f \leq 3N_c \pm \frac{4}{3}$ with $N_c > 5$. The plus/minus signs refer to the specifics of the orientifold projection. The plan of the paper is as follows.

[1]Similar statements can be made also for any number of colors $N_c > 3$.

B. Overview of the paper

In the first part of Sec. II we review known facts about the orientifold field theories of interest, namely their definition and symmetries. We consider two models, one with the "gaugino" in the antisymmetric representation of the $U(N_c)$ gauge group and another with the gaugino in the symmetric representation of the gauge group. We will call the first model OQCD-AS and the second OQCD-S. We will refer to any of these theories collectively as OQCD. In both cases, there are N_f quark multiplets in the fundamental and antifundamental of the gauge group. The matter content of both theories is summarized in Table I.

In the second part of Sec. II we present the definition and symmetries of a corresponding set of theories, which we will claim are the magnetic duals of OQCD-AS and OQCD-S. The matter content of these models is summarized in Table II.

Our primary motivation for the proposal of this nonsupersymmetric electric-magnetic duality comes from string theory. In Sec. III we explain how we can embed both the electric and magnetic descriptions of OQCD in a highly curved, but exact type 0B noncritical string theory background and how we can motivate Seiberg duality as a statement about boundary and crosscap state monodromies. The setup involves N_c D3-branes, N_f D5-branes and an O′5-plane that projects out the tachyonic mode of type 0B string theory. The outcome of the string theory setup is a proposal for a duality between the electric description of OQCD-AS (respectively OQCD-S) with gauge group $U(N_c)$ and N_f flavors and the magnetic description with gauge group $U(N_f - N_c + 4)$ (respectively $U(N_f - N_c - 4)$) and the same number of flavors N_f. A similar analysis has been performed for type IIB string theory in (1.1) with D3- and D5-branes [12], giving Seiberg duality for $\mathcal{N} = 1$ SQCD with gauge group $SU(N_c)$ [14]. Adding O5-planes in the type IIB setup gives Seiberg duality for $SO(N_c)$ or $Sp(N_c/2)$ gauge group [34]. We note, however, that the analysis of the D-brane monodromies in the present paper (in both cases of SQCD and OQCD) departs significantly from those in the latter two references.

We want to emphasize that noncritical string theory on (1.1) with an O′5-plane is forced upon us in an almost unique way. From the analysis of the gauge invariant

TABLE I. The matter content of the electric description of OQCD. The left (right) four columns depict the matter content, representations and $U(1)_R$ quantum numbers of the OQCD theory in the antisymmetric (symmetric) projection.

	OQCD-AS $U(N_c)$	OQCD-AS $SU(N_f)$	OQCD-AS $SU(N_f)$	OQCD-AS $U(1)_R$	OQCD-S $U(N_c)$	OQCD-S $SU(N_f)$	OQCD-S $SU(N_f)$	OQCD-S $U(1)_R$
A_μ	Adjoint N_c^2	•	•	0	Adjoint N_c^2	•	•	0
λ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$ $\frac{N_c(N_c-1)}{2}$	•	•	1	$\square\square$ $\frac{N_c(N_c-1)}{2}$	•	•	1
$\tilde{\lambda}$	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$ $\frac{N_c(N_c-1)}{2}$	•	•	1	$\overline{\square\square}$ $\frac{N_c(N_c+1)}{2}$	•	•	1
Φ	$\overline{\square}$ $\bar{N}_c$	$\square$ N_f	•	$\frac{N_f-N_c+2}{N_f}$	$\overline{\square}$ $\bar{N}_c$	$\square$ N_f	•	$\frac{N_c(N_c+2)}{N_f}$
Ψ	$\square$ N_c	$\square$ N_f	•	$\frac{-N_c+2}{N_f}$	$\square$ N_c	$\square$ N_f	•	$\frac{-N_c-2}{N_f}$
$\tilde{\Phi}$	$\square$ N_c	•	$\overline{\square}$ $\bar{N}_j$	$\frac{N_f-N_c+2}{N_f}$	$\square$ N_c	•	$\overline{\square}$ $\bar{N}_j$	$\frac{N_f-N_c+2}{N_f}$
$\tilde{\Psi}$	$\overline{\square}$ $\bar{N}_c$	•	$\overline{\square}$ $\bar{N}_j$	$\frac{-N_c-2}{N_f}$	$\overline{\square}$ $\bar{N}_c$	•	$\overline{\square}$ $\bar{N}_f$	$\frac{-N_c-2}{N_f}$

TABLE II. The matter content of the proposed magnetic description of OQCD-AS (left) and OQCD-S (right), with number of colors $\tilde{N}_c = N_f - N_c + 4$ and $\tilde{N}_c = N_f - N_c - 4$, respectively.

	OQCD-AS $(\tilde{N}_c = N_f - N_c + 4)$ $U(\tilde{N}_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_R$	OQCD-S $(\hat{N}_c = N_f - N_c - 4)$ $U(\tilde{N}_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_R$
A_μ	Adjoint $\tilde{N}_c^2$	•	•	0	Adjoint $\hat{N}_c^2$	•	•	0
λ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$ $\frac{\bar{N}_c(\bar{N}_c-1)}{2}$	•	•	1	$\square\square$ $\frac{\hat{N}_c(\hat{N}_c+1)}{2}$	•	•	1
$\tilde{\lambda}$	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$ $\frac{\bar{N}_c(\bar{N}_c-1)}{2}$	•	•	1	$\overline{\square\square}$ $\frac{\hat{N}_c(\hat{N}_c+1)}{2}$	•	•	1
ϕ	$\square$ $\tilde{N}_c$	$\overline{\square}$ $\bar{N}_f$	•	$\frac{N_c-2}{N_f}$	$\square$ $\hat{N}_c$	$\overline{\square}$ $\bar{N}_f$	•	$\frac{N_c+2}{N_f}$
ψ	$\overline{\square}$ $\bar{\tilde{N}}_c$	$\overline{\square}$ $\bar{N}_f$	•	$\frac{N_c-N_f-2}{N_f}$	$\overline{\square}$ $\bar{\hat{N}}_c$	$\overline{\square}$ $\bar{N}_f$	•	$\frac{N_c-N_f+2}{N_f}$
$\tilde{\phi}$	$\overline{\square}$ $\bar{\tilde{N}}_c$	•	$\square$ N_f	$\frac{N_c-2}{N_f}$	$\overline{\square}$ $\bar{\hat{N}}_c$	•	$\square$ N_f	$\frac{N_c+2}{N_f}$
$\tilde{\psi}$	$\square$ $\tilde{N}_c$	•	$\square$ N_f	$\frac{N_c-N_f-2}{N_f}$	$\square$ $\hat{N}_c$	•	$\square$ N_f	$\frac{N_f-N_f+2}{N_f}$
M	•	$\square$ N_f	$\overline{\square}$ $\bar{N}_f$	$\frac{2N_f-2N_c+4}{N_f}$	•	$\square$ N_f	$\overline{\square}$ $\bar{N}_f$	$\frac{2N_f-2N_c-4}{N_f}$
χ	•	$\square\square$ $\frac{N_f(N_f+1)}{2}$	•	$\frac{N_f-2N_c+4}{N_f}$	•	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$ $\frac{N_f(N_f-1)}{2}$	•	$\frac{N_f-2N_c-4}{N_f}$
$\tilde{\chi}$	•	•	$\overline{\square\square}$ $\frac{N_f(N_f+1)}{2}$	$\frac{N_f-2N_c+4}{N_f}$	•	•	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$ $\frac{N_f(N_f-1)}{2}$	$\frac{N_f-2N_c-4}{N_f}$

operators of OQCD in the large N limit [33], we expect a purely bosonic spectrum. This implies, using general ideas from holography [35], that in order to engineer OQCD with a D-brane setup in string theory, we have to embed D-branes in a background with a closed string spectrum that is also purely bosonic in space-time. At the same time, the validity of planar equivalence with $\mathcal{N} = 1$ SQCD at large N requires [25,26] that a certain discrete symmetry (charge conjugation in the present case) is not spontaneously broken. This condition translates to a tachyon-free closed string background [27]. So gauge theory considerations alone imply that we are looking for a string theory that has a purely bosonic closed string spectrum without closed string tachyons. Besides two-dimensional examples (which are clearly irrelevant for our purposes) the only other examples that are known with these features are the noncritical string theories of [36], a close cousin of which is the theory we are discussing in Sec. III.[2]

In Sec. IV we collect the evidence for the proposed duality and discuss its limitations. The evidence in favor of our proposal includes:

(1) Seiberg duality is guaranteed to hold at infinite N because of planar equivalence with $\mathcal{N} = 1$ SQCD. This alone fixes the basic features of the magnetic duals, e.g. the matter content and the form of the dual Lagrangian.

(2) The string theory embedding motivates a definite proposal for the duality at finite N. The details of the string theory construction and the interpolation between the electric and magnetic descriptions are a hybrid of the corresponding setups in the context of $\mathcal{N} = 1$ SQCD with $U(N_c)$ or $SO(N_c)/Sp(N_c/2)$ gauge groups. Hence, certain arguments that can be used in favor of the validity of the duality in those cases suggest (however, do not rigorously prove) the validity of the duality in OQCD as well.

(3) Global anomalies can be computed explicitly and 't Hooft anomaly matching is found to hold for the proposed duals at any N.

[2]The Sagnotti model in ten dimensions is also a theory with a tachyon-free, purely bosonic closed string spectrum, but includes an additional open string sector with space-filling D9-branes.

In Sec. V we discuss the implications of our proposal for the IR dynamics of OQCD. One of them is a prediction for the precise range of the conformal window and an emerging picture of the phase structure of the theory as a function of the number of flavors N_f. Another interesting question concerns the quantum moduli space of OQCD. The parent $\mathcal{N} = 1$ SQCD has a whole space of vacua for $N_f \geq N_c$ parametrized by the vacuum expectation values (vevs) of meson and baryon fields. At finite N quantum corrections lift the classical moduli space of OQCD and one is left with a unique vacuum. To demonstrate this we compute the one-loop Coleman-Weinberg potential for the vev of the squark fields.

We conclude in Sec. VI with a brief discussion on possible extensions of this work.

II. DUALITY IN NONSUPERSYMMETRIC GAUGE THEORY: A PROPOSAL

In this section, we propose an IR duality between an electric and a magnetic version of the nonsupersymmetric OQCD gauge theories. We present the precise definition of these theories and summarize their most salient features.

A. The electric theory

The electric version of the gauge theory we will discuss here comes into two variants: OQCD-AS and OQCD-S. The matter content of these theories is given in Table I. In both cases, the boson representations are identical to the boson representations of the original $U(N_c)$ super-QCD. The difference occurs in the fermionic sector. The original gaugino is replaced by a gaugino in either the antisymmetric representation (in OQCD-AS) or in the symmetric representation (in OQCD-S). Similarly, the quarks (the "superpartners" of the "squarks"), although still in bifundamental representations, do not transform exactly as the quarks of the original supersymmetric theory. Their representations are fixed by their coupling to the gaugino and the quark, namely by the terms

$$\mathrm{OQCD-AS:}\ \lambda_{[ij]}\Phi^i_\alpha\bar{\Psi}^{j\alpha} + \tilde{\lambda}^{[ij]}\tilde{\Phi}^\alpha_i\bar{\tilde{\Psi}}_{j\alpha} \tag{2.1a}$$

$$\mathrm{OQCD-S:}\ \lambda_{\{ij\}}\Phi^i_\alpha\bar{\Psi}^{j\alpha} + \tilde{\lambda}^{\{ij\}}\tilde{\Phi}^\alpha_i\bar{\tilde{\Psi}}_{j\alpha}, \tag{2.1b}$$

where i, j are color indices and α is a flavor index. The tree-level Lagrangian of the theory is inherited from the supersymmetric theory, hence it contains the same fields and the same interaction terms as in the supersymmetric $U(N_c)$ gauge theory. Altogether, the Lagrangian looks like a hybrid of bosons from the $U(N_c)$ and fermions from the $SO(N_c)$ $[Sp(N_c/2)]$ theories.

Both theories exhibit an anomaly free $SU(N_f) \times SU(N_f) \times U(1)_R$ global symmetry. We call the anomaly free axial symmetry $U(1)_R$, although it is not an R-symmetry. Moreover, it will be important for our considerations below that the baryon $U(1)_B$ symmetry is gauged and that the gauge group is $U(N_c)$ instead of $SU(N_c)$. Consequently, there are no baryon operators in the theories we will consider.

At the classical level, the model admits a moduli space, parametrized by the vevs of scalars, exactly as in the supersymmetric $U(N_c)$ theory. At the quantum level, and finite N, this moduli space is lifted as no supersymmetry is present. This will be discussed further in Sec. V. Nevertheless, due to planar equivalence [33] in the large N limit, both OQCD-AS and OQCD-S become equivalent to the $U(N_c)$ electric SQCD theory in the common sector of C-parity even states [26]. Hence, in this limit the nonsupersymmetric effects are suppressed and OQCD exhibits the same moduli space as the SQCD theory.

B. The magnetic theory

The planar equivalence at infinite N raises the possibility of an electric-magnetic duality in OQCD even at finite N. Our purpose here is to make a definite proposal for this finite N duality. We will propose that the Seiberg dual of the $U(N_c)$ electric OQCD-AS theory is a $U(N_f - N_c + 4)$ magnetic OQCD-AS theory. Similarly, the dual of the electric OQCD-S is a $U(N_f - N_c - 4)$ magnetic OQCD-S theory. At infinite N, both magnetic duals become planar equivalent to the magnetic $U(N_f - N_c)$ SQCD theory.

The matter content of the proposed magnetic theories is summarized in Table II. Besides the gauge bosons and the "gluinos," this table contains, as in SQCD, additional fundamental degrees of freedom, which comprise a complex scalar field M (the magnetic meson) and Weyl fermions χ, $\tilde{\chi}$ (the magnetic "mesinos").

The representations of the fermions are fixed by the same rules as in the electric case. The representation of the gaugino is antisymmetric in OQCD-AS and symmetric in OQCD-S. The mesino representation is symmetric in OQCD-AS and antisymmetric in OQCD-S, otherwise the anomalies would not match. This is similar to the $SO(N)$ $[Sp(N/2)]$ SQCD case, where the mesino representation is symmetric when the gaugino is antisymmetric (and antisymmetric when the gaugino is symmetric).

The tree-level Lagrangians of the magnetic theories are again inherited, due to planar equivalence at infinite N, from the supersymmetric theory. An important feature of the magnetic Lagrangians are the meson, "mesino" couplings

$$\mathrm{OQCD-AS:}\ M^\alpha_\beta\psi_{\alpha k}\tilde{\psi}^{\beta k} + \chi_{\{\alpha\beta\}}\phi^\alpha_k\psi^{\beta k} + \tilde{\chi}^{\{\alpha\beta\}}\tilde{\phi}^k_\alpha\tilde{\psi}_{\beta k} \tag{2.2a}$$

$$\mathrm{OQCD-S:}\ M^\alpha_\beta\psi_{\alpha k}\tilde{\psi}^{\beta k} + \chi_{[\alpha\beta]}\phi^\alpha_k\psi^{\beta k} + \tilde{\chi}^{[\alpha\beta]}\tilde{\phi}^k_\alpha\tilde{\psi}_{\beta k}. \tag{2.2b}$$

III. EMBEDDING IN STRING THEORY

OQCD is a four-dimensional gauge theory with a purely bosonic spectrum of gauge invariant operators in the large N limit [33]. This is an indication, using general ideas from holography [35], that in order to engineer OQCD with a D-brane setup in string theory, we should embed D-branes in a nonsupersymmetric vacuum with a purely bosonic closed string spectrum. At the same time, the validity of planar equivalence with $\mathcal{N} = 1$ SQCD at large N requires [25–27] the absence of closed string tachyons. Higher dimensional backgrounds with these features are not common in string theory. The only such examples that we are aware of are the noncritical examples of [36], which are noncritical variants of the type 0′B string theory in ten dimensions [30,31]. Fortunately, a close cousin of the closed string theories presented in [36] provides the right setup for the string theory embedding of OQCD. Electric-magnetic duality in this context will be discussed using the techniques of [14], which were applied in an analogous type IIB construction to $\mathcal{N} = 1$ SQCD. Note, however, that we correct some statements made in [14], thus obtaining a partially modified picture of electric-magnetic duality in the context of noncritical string theory.

We would like to mention in passing that there is a close relation between the noncritical embedding of gauge theories that we consider here and the more familiar Hanany-Witten (HW) constructions [37] in critical string theory, with configurations of NS5-branes, D-branes, and orientifolds in flat space-time (see [5] for an extensive review). For $\mathcal{N} = 2$ gauge theories this relation has been discussed in detail in [38]. For $\mathcal{N} = 1$ SQCD it has been discussed in [12,14] (see also [39] for related issues). However, in this paper, we will not make explicit use of HW constructions (or their M-theory lifts for that matter), since they are embedded in tachyonic type 0 string theory and the required orientifold planes do not fully project out the closed string tachyon (more details are given at the end of Sec. III A). On the contrary, closed string tachyons are not an issue in the noncritical description. Moreover, as always, the noncritical description isolates and captures in HW constructions the region of moduli space that is most relevant for the gauge theory of interest (for a more detailed discussion of these aspects we refer the reader to [12,14,38] and references therein).

In this section we will focus mostly on the physics and the implications of the string theory setup at hand. Many explicit technical details are left for the interested reader in Appendices A and B.

A. OQCD in type 0′B noncritical strings

Our starting point is type 0B string theory on the exact perturbative string background

$$\mathbb{R}^{3,1} \times \frac{SL(2)_1}{U(1)}. \tag{3.1}$$

The flat $\mathbb{R}^{3,1}$ factor requires on the world sheet the usual four free bosons and their fermionic superpartners. The second factor is captured by an axial coset of the $SL(2,\mathbb{R})$ supersymmetric WZW model at level $k = 1$. In target space this superconformal field theory describes the Euclidean two-dimensional black hole [40–42] (commonly known as the "cigar" geometry). At $k = 1$ this geometry has curvatures at the string scale and is highly curved, hence the geometric intuition is not always a useful or accurate guide for the description of this setup. Here are some useful facts about the supersymmetric $SL(2)/U(1)$ coset.

It is a superconformal field theory with $\mathcal{N} = (2, 2)$ supersymmetry that has a mirror dual formulation as $\mathcal{N} = 2$ Liouville theory [43–45]. An important aspect of this duality is the presence of a nonperturbative winding condensate in the CFT. Far from the tip of the cigar, the coset CFT is well approximated by the free theory $\mathbb{R}_{\sqrt{2}} \times U(1)_1$, i.e. a linear dilaton field ρ with background charge $Q = \sqrt{2}$, a free boson X at the self-dual radius[3] $R = \sqrt{2}$, and their supersymmetric partners, a Dirac fermion with left- and right-moving components $\psi^{\pm} = \psi^{\rho} \pm i\psi^{x}$ and $\tilde{\psi}^{\pm} = \tilde{\psi}^{\rho} \pm i\tilde{\psi}^{x}$. In terms of this free field description, the $\mathcal{N} = 2$ Liouville potential reads

$$\delta \mathcal{S}_{\rm L}(\mu, \bar{\mu}) = \frac{i}{2\pi} \int d^2 z [\mu \psi^{+} \tilde{\psi}^{+} e^{-(\rho + i(X_{\rm L} - X_{\rm R})/\sqrt{2})} + \bar{\mu} \psi^{-} \tilde{\psi}^{-} e^{-(\rho - i(X_{\rm L} - X_{\rm R})/\sqrt{2})}]. \tag{3.2}$$

$X_{\rm L}$, $X_{\rm R}$ denote the left- and right-moving parts of $X(z, \bar{z})$.

In the case at hand $k = 1$, and the Liouville coupling constant μ is given by mirror symmetry in terms of the effective string coupling constant as follows:[4]

$$\mu^{{\rm ren};k=1} := \lim_{\epsilon \to 0^+} \epsilon^{-(1/2)} \mu^{k=1+\epsilon} = \frac{2}{g_{\rm eff}}. \tag{3.3}$$

According to this relation, when $\mu \to 0$, the background becomes strongly coupled. A more detailed description of the supersymmetric $SL(2,\mathbb{R})/U(1)$ coset and $\mathcal{N} = 2$ Liouville theory can be found, for example, in [48] and references therein.

The type 0B theory has a closed string tachyon. In order to project it out of the physical spectrum we can use, like in ten dimensions [30,31], a space-filling orientifold; in more technical terms, the orientifold should be spacefilling in $\mathbb{R}^{3,1}$ and B-type in the $SL(2)/U(1)$ supercoset [36]. We now proceed to discuss the physics of this orientifold and its implications in more detail.

[3]Hereafter we set $\alpha' = 2$.

[4]The relation found in [46] needs to be renormalized in the limit $k \to 1^+$. See also [47]. For convenience, we will denote $\mu^{{\rm ren};k=1}$ simply as μ in what follows.

1. The closed string sector

From the world sheet point of view, type 0B string theory on (3.1) is described by the tensor product of a free $\mathbb{R}^{3,1}$ super-CFT, the usual ghosts and superghosts and the supercoset $SL(2)/U(1)$, with left-right symmetric boundary conditions for the world sheet fermions. Physical states have delta-function normalizable wave functions in the coset (i.e. belong to the continuous representations, see Appendix A); their left- and right-moving conformal weights, for radial momentum P, read

$$\Delta = \frac{1}{2}p_\mu p^\mu + P^2 + \frac{(n+w)^2}{4} + N - \frac{1}{4} = 0, \quad (3.4a)$$

$$\bar{\Delta} = \frac{1}{2}p_\mu p^\mu + P^2 + \frac{(n-w)^2}{4} + \bar{N} - \frac{1}{4} = 0, \quad (3.4b)$$

where n (respectively w) is the momentum (respectively the winding) around the compact direction of the coset.[5] N, $\bar{N}$ are the left-, right-moving oscillator levels and p_μ is the flat space four-dimensional momentum. From the torus partition function (see Appendix B), one finds the following spectrum of lowest lying modes (see also [49]):

(i) A real tachyon with $m^2 = -1/2$ for $n = w = 0$ in the NS_-NS_- sector.

(ii) A pair of complex massless scalars, for ($n = \pm 1$, $w = 0$) and ($n = 0$, $w = \pm 1$), in the NS_-NS_- sector.

(iii) A real massless Ramond-Ramond (RR) scalar and a massless self-dual RR two-form in six dimensions from the R_+R_+ sector, plus another real scalar and an anti-self-dual two-form from the R_-R_- sector. From the four-dimensional perspective, one gets two complex scalars and two vectors.

The lowest modes in the NS_+NS_+ fields ($g_{\mu\nu}$, $B_{\mu\nu}$, Φ) are massive, with mass squared $m^2 = 1/2$. The positive mass shift is due to the presence of a linear dilaton.

In order to obtain a tachyon-free spectrum, we need to perform a specific orientifold projection [we refer the reader to [48] for a general analysis of orientifolds in $SL(2)/U(1)$]. The orientifold that turns out to be appropriate for our present purposes is very similar to the one utilized in [36], and is defined by the following action on primary states:[6]

$$\mathcal{P} = \Omega(-)^{Q_R}: |n, w\rangle \otimes |0\rangle_{NS} \rightarrow (-)^{n+w+1}|n, -w\rangle \otimes |0\rangle_{NS}, \quad (3.5)$$

where Ω is the standard world sheet parity. In this definition, Q_R is the charge of the left-moving $U(1)_R$ world sheet symmetry

$$Q_R = \oint \frac{dz}{2\pi i}(i\psi^\rho\psi^x + i\sqrt{2}\partial X) = F + n + w, \quad (3.6)$$

where F is the left-moving world sheet fermion number. As with the critical type 0′B theory defined in [30,31], the world sheet parity (3.5) acts on the GSO-invariant states of the corresponding type IIB model simply as Ω.

The closed string tachyon is odd under $\mathcal{P}$, hence it is projected out. The resulting theory has a tachyon-free spectrum with a Hagedorn density of states, but no space-time fermions. The invariant massless physical states are

(i) A complex massless scalar from the states $|\pm 1, 0\rangle \otimes |0\rangle_{NS}$ in the NS_-NS_- sector.

(ii) A real massless scalar from $(|0, +1\rangle + |0, -1\rangle) \otimes |0\rangle_{NS}$ in the NS_-NS_- sector.

(iii) A four-dimensional massless vector and a pair of massless real scalars from the R_+R_+ sector.

Compared to the type IIB case, instead of having two complex massless scalars in the NSNS sector, one has one complex and one real scalar. The missing state is the antisymmetric combination of winding one states.

Interestingly enough, this is not the whole story. It was found in [48] (see also [34]) that an orientifold described by the parity (3.5) alone does not give a consistent crosscap state[7] in the $SL(2)/U(1)$ CFT. The full crosscap state contains a second piece, with a wave function that is localized near the tip of the cigar. The parity associated with the localized piece of the crosscap state acts on closed string states as

$$\tilde{\mathcal{P}} = (-)^n\mathcal{P} = (-)^{F+w}\Omega. \quad (3.7)$$

The presence of this parity does not affect the closed string spectrum as determined with an asymptotic linear dilaton analysis above.[8]

The full consistency of the CFT requires that the $\mathcal{N} = 2$ Liouville potential (3.2) is invariant under any parity we want to consider. Indeed, one can check that under both parities $\mathcal{P}$ and $\tilde{\mathcal{P}}$ the $\mathcal{N} = 2$ Liouville potential transforms as follows:

$$\mathcal{P}, \tilde{\mathcal{P}}: \delta\mathcal{S}_L(\mu, \bar{\mu}) \rightarrow \delta\mathcal{S}_L(\bar{\mu}, \mu). \quad (3.8)$$

Consequently, these are symmetries of the CFT at the nonperturbative level if and only if $\mu \in \mathbb{R}$.

[5]Only the former is preserved by the interactions taking place near the tip of the cigar.

[6]This orientifold is B-type in $SL(2)/U(1)$, whereas the orientifold appearing in [36] was A-type. A-type boundary conditions are Dirichlet boundary conditions for the $U(1)$ boson X whereas B-type are Neumann.

[7]In this paper we will use heavily the boundary and crosscap state formalism in boundary conformal field theory. For an introduction to this formalism we refer the reader to the reviews [50–52] and references therein.

[8]For instance, one can check that the parity (3.7) does not contribute to the Klein bottle amplitude with a term proportional to the volume of the asymptotic linear dilaton cylinder. There is only a localized, finite contribution from states with zero radial momentum P. More details can be found in Appendix B, or in Ref. [48].

The fully consistent crosscap state, including both an extensive and a localized component that correspond, respectively, to the parities $\mathcal{P}$ and $\tilde{\mathcal{P}}$, can be found with modular boostrap methods, using the results of [48]. It turns out to be similar to the RR part of the supersymmetric type II orientifold found in [34], and reads

$$|\mathcal{C}\rangle = |\mathcal{C}; \text{ext}\rangle + |\mathcal{C}; \text{loc}\rangle = \int_0^\infty dP \sum_{\eta=\pm 1}\Big[\sum_{w\in 2\mathbb{Z}+1}\Psi_{\text{ext}}\Big(P, \frac{w}{2}, \eta\Big)|\mathcal{C}; P, \frac{w}{2}; \eta\rangle\rangle_R + \sum_{w\in 2\mathbb{Z}}\Psi_{\text{loc}}\Big(P, \frac{w}{2}, \eta\Big)|\mathcal{C}; P, \frac{w}{2}; \eta\rangle\rangle_R\Big], \quad (3.9)$$

with the wave functions[9]

$$\Psi_{\text{ext}}\Big(P, \frac{w}{2}, \eta\Big) = \frac{\sqrt{2}}{4\pi^2}\mu^{iP-(w/2)}\bar{\mu}^{iP+(w/2)} \times \frac{\Gamma(1-2iP)\Gamma(-2iP)}{\Gamma(1-iP+\frac{w}{2})\Gamma(-iP-\frac{w}{2})} \quad (3.10a)$$

$$\Psi_{\text{loc}}\Big(P, \frac{w}{2}, \eta\Big) = \frac{\sqrt{2}}{4\pi^2}\eta(-)^{w/2}\mu^{iP-(w/2)}\bar{\mu}^{iP+(w/2)} \times \frac{\Gamma(1-2iP)\Gamma(-2iP)}{\Gamma(1-iP+\frac{w}{2})\Gamma(-iP-\frac{w}{2})}\cosh\pi P. \quad (3.10b)$$

$|\mathcal{C}; P, w/2; \eta\rangle\rangle_R$ denote B-type crosscap Ishibashi states with superconformal gluing conditions $G^\pm = \eta\tilde{G}^\pm$ on the real axis. The flat space-time part of these Ishibashi states is the standard one associated with the parity $(-)^F\Omega$. Although $\mu = \bar{\mu}$ in our case, it will still be useful to keep the holomorphic/antiholomorphic dependence on μ explicit in the crosscap (and later boundary state) wave functions.

The orientifold we are discussing here is space filling, hence we will call it an O′5-plane. Notice that it sources only modes in the RR sector with zero momentum n. The extensive part of the orientifold sources odd winding modes, which are all massive, whereas the localized part sources even winding modes. The zero winding mode is a massless RR field. Since there are no massless, nondynamical RR tadpoles from the extensive part of the orientifold, there is no need to add extra D-branes for consistency [36]. Thus, we have a consistent theory of closed strings with a purely bosonic spectrum. This should be contrasted with the corresponding ten-dimensional case [30,31], where D-branes need to be added to cancel a nonvanishing RR tadpole.

[9]The wave functions in Eqs. (3.10a) and (3.10b) are identified with the coefficients of the one-point functions on the disc for closed string modes with radial momentum P, winding w and zero angular momentum n, in the RR sector.

2. A brief comment on the relation with NS5-brane configurations

There is a configuration of NS5-branes with an orientifold plane in ten-dimensional type 0A string theory, whose near-horizon region, in a suitable limit, will be described by the type 0B noncritical string theory on (3.1) in the presence of the O′5-plane. This configuration involves two orthogonal NS5-branes and an O′4-plane stretched along the following directions:

$$\begin{aligned} &\text{NS}\,5\text{: } 0\,1\,2\,3\,4\,5 && \text{at } x^6 = 0, x^7 = x^8 = x^9 = 0\\ &\text{NS}5'\text{: } 0\,1\,2\,3\,8\,9 && \text{at } x^6 = L, x^4 = x^5 = x^7 = 0\\ &\text{O}'4\text{: } 0\,1\,2\,3\,6 && \text{at } x^4 = x^5 = x^7 = x^8 = x^9 = 0. \end{aligned}$$

The O′4-plane is the standard O′ plane in ten dimensions associated with the orientifold projection $(-)^F I_5\Omega$ (I_5 is the reversal parity in the transverse space $x^i \to -x^i$, with $i = 4, 5, 7, 8, 9$.)[10]

One can argue, as in [13], that the tachyon-free type 0B noncritical string theory of this section describes the near-horizon geometry of the above configuration in a double-scaling limit, where the asymptotic string coupling $g_s \to 0$ and $L/\sqrt{\alpha'}g_s$ is kept fixed. Apparently, as we take this near-horizon limit, the bulk tachyon decouples and is left outside the near-horizon throat [36]. One can think of this situation as the exact opposite of localized closed string tachyons in nonsupersymmetric orbifolds [53].

Having said this, we could proceed with the above fivebrane configuration to construct a HW setup that realizes the electric description of OQCD [33]. The HW setup requires, besides the fivebranes and the O′4-plane, N_c D4-branes parallel to the O′4-plane suspended between the NS5-branes along the 6-direction and N_f D6-branes along the 0 1 2 3 7 8 9 plane. Then, keeping the bulk tachyon at its unstable maximum, we could further use what we know from similar supersymmetric configurations in type IIA to argue for Seiberg duality in OQCD and recover the results of Sec. II. We will not follow this route here, instead we will go to the noncritical description, which allows for more explicit control in a tachyon-free environment and argue for Seiberg duality there.

B. D-branes

$\mathcal{N} = 1$ SQCD can be realized in the type IIB noncritical background (3.1) with an appropriate combination of localized and extended B-type branes in $SL(2)/U(1)$ [12] (see also [14,47,54] for earlier work). The boundary states we will use are the same as those of the supersymmetric setup. These "dyonic" branes are not elementary in

[10]A mirror description of this setup is given by wrapped D-branes with an orientifold in the deformed conifold geometry as in [6].

oriented type 0B string theory, however they are elementary in the presence of the O′5 orientifold.

1. Color branes

The "color" branes, i.e. the branes that will provide the gauge degrees of freedom, are D3-branes in the type 0′B theory that are localized near the tip of the cigar and have Neumann boundary conditions in $\mathbb{R}^{3,1}$. Their open string spectrum is made only of the *identity representation* of the $\mathcal{N}=2$ superconformal algebra, see Appendix A for more details.

In the $SL(2)/U(1)$ boundary conformal field theory, the corresponding boundary states obey B-type boundary conditions and can be expressed as a linear combination of B-type boundary Ishibashi states [12]:

$$\left|\mathcal{B}; P, \frac{w}{2}; \eta\right\rangle\!\!\Big\rangle \mathrm{NSNS/RR}, \qquad P\in\mathbb{R}^+, \qquad w\in\mathbb{Z}, \qquad \eta=\pm 1. \tag{3.11}$$

These Ishibashi states have nonvanishing couplings to winding closed string modes only. They are associated to $\mathcal{N}=2$ continuous representations with $Q_R=-\tilde{Q}_R=w$ in $SL(2)/U(1)$ and obey in $\mathbb{R}^{3,1}$ the standard Neumann boundary conditions. As with the crosscap Ishibashi states above, the label $\eta=\pm1$ refers to the superconformal boundary conditions.[11]

Boundary states with $\eta=\pm1$ (called, respectively, electric or magnetic) are separately GSO invariant in type 0B string theory. The orientifold action, however, maps one to the other. More specifically, the action of the parities (3.5) and (3.7) on the Ishibashi states (3.11) is

$$\mathcal{P},\tilde{\mathcal{P}}\left|\mathcal{B};P,\frac{w}{2};\eta\right\rangle\!\!\Big\rangle_{\mathrm{NSNS}}=(-)^{w+1}\left|\mathcal{B};P,-\frac{w}{2};-\eta\right\rangle\!\!\Big\rangle_{\mathrm{NSNS}}, \tag{3.12a}$$

$$\mathcal{P},\tilde{\mathcal{P}}\left|\mathcal{B};P,\frac{w}{2};\eta\right\rangle\!\!\Big\rangle_{\mathrm{RR}}=(-)^{w+1}\left|\mathcal{B};P,-\frac{w}{2};-\eta\right\rangle\!\!\Big\rangle_{\mathrm{RR}} \tag{3.12b}$$

Then one can check that the D3-brane boundary state, which is invariant under the both parities $\mathcal{P}$, $\tilde{\mathcal{P}}$ is of the same form as the boundary state of the BPS D3-brane in type IIB string theory obtained in [12]

$$|D3\rangle=\sum_{a=\mathrm{NSNS,RR}}\int_0^\infty \mathrm{d}P\sum_{w\in\mathbb{Z}}\Phi_a\Big(P,\frac{w}{2}\Big)\Big[\left|\mathcal{B};P,\frac{w}{2};+\right\rangle\!\!\Big\rangle_a + (-)^{w+1}\left|\mathcal{B};P,\frac{w}{2};-\right\rangle\!\!\Big\rangle_a\Big], \tag{3.13}$$

where

$$\Phi_{\mathrm{NSNS}}(P,m)=\frac{\sqrt{2}}{32\pi^2}\mu^{iP-m}\bar{\mu}^{iP+m}\times\frac{\Gamma(\frac{1}{2}+m+iP)\Gamma(\frac{1}{2}-m+iP)}{\Gamma(2iP)\Gamma(1+2iP)}, \tag{3.14a}$$

$$\Phi_{\mathrm{RR}}(P,m)=\frac{\sqrt{2}}{32\pi^2}\mu^{iP-m}\bar{\mu}^{iP+m}\times\frac{\Gamma(1+m+iP)\Gamma(-m+iP)}{\Gamma(2iP)\Gamma(1+2iP)}. \tag{3.14b}$$

Notice that this boundary state does not carry any labels, i.e. it has no open string modulus.

The annulus and Möbius strip amplitudes for this brane are presented in Appendix B. The former is identical to the D3-brane annulus amplitude in type IIB found in [12], see Eq. (B4), and vanishes by supersymmetry. The latter contains a contribution from the Ramond sector only, see Eq. (B5), hence this amplitude breaks explicitly the Bose-Fermi degeneracy and is responsible for the supersymmetry breaking in our D-brane setup. Adding the two contributions, we can read off the spectrum of massless states

$$\mathcal{A}_{\text{D3-D3}}+\mathcal{M}_{\text{D3;}OO'5}=V_4\int_0^\infty\frac{dt}{2t}\frac{1}{(16\pi^2t)^2}\times\Big[\frac{N_c^2}{2}(2+\mathcal{O}(q))_{\mathrm{NS}_+}-\frac{N_c(N_c\pm1)}{2}(2+\mathcal{O}(q))_{\mathrm{R}_+}\Big]. \tag{3.15}$$

The Neveu-Schwarz (NS) sector contains a $U(N_c)$ gauge boson. The R sector contains a Dirac fermion transforming in the symmetric (upper sign) or antisymmetric (lower sign) representation of $U(N_c)$, depending on the sign of the orientifold charge.

2. Flavor branes

The "flavor" branes are space-filling D5-branes labeled by a continuous variable $s\in\mathbb{R}^+$ parametrizing the minimum distance of the brane from the tip of the cigar. A second continuous parameter $\theta\in[0,2\pi)$ parametrizes the value of a Wilson loop around the compact direction of the cigar. In the asymptotic cylinder part of the cigar, the D5-branes are double-sheeted and look like D-$\bar{\mathrm{D}}$ pairs (without an open string tachyon however). This geometry is responsible for the $U(N_f)\times U(N_f)$ global symmetry of the four-dimensional gauge theory that we will engineer. Moreover,

[11]Our definition, which follows standard practice, has the property

$$_{\mathrm{NSNS}}\langle\eta|e^{-\pi TH_c}|\eta'\rangle_{\mathrm{NSNS}}\sim\vartheta\Big[{0 \atop \frac{1-\eta\eta'}{2}}\Big](iT),$$

$$_{\mathrm{RR}}\langle\eta|e^{-\pi TH_c}|\eta'\rangle_{\mathrm{RR}}\sim\vartheta\Big[{1 \atop \frac{1-\eta\eta'}{2}}\Big](iT),$$

where $\vartheta[{a \atop b}](\tau)$ are standard theta functions.

although space filling, the flavor D5-branes are not charged under a nondynamical six-form RR potential in six dimensions.

The full D5 boundary states are given in terms of the same B-type Ishibashi states as the color branes (3.11)

$$|D5; s, \frac{\theta}{2\pi}\rangle = \sum_{a=\mathrm{NSNS,RR}} \int_0^\infty dP \sum_{w\in\mathbb{Z}} \Phi_a\Big(s, \theta; P, \frac{w}{2}\Big) \times \Big(|\mathcal{B}; P, \frac{w}{2}; +\rangle\rangle_a - (-)^w |\mathcal{B}; P, \frac{w}{2}; -\rangle\rangle_a\Big), \tag{3.16}$$

where now

$$\Phi_{\mathrm{NSNS}}\Big(s, \theta; P, \frac{w}{2}\Big) = \frac{\sqrt{2}}{16\pi^2}(-)^w e^{-iw\theta} \mu^{iP-(w/2)} \bar{\mu}^{iP+(w/2)} \times \frac{\Gamma(-2iP)\Gamma(1-2iP)}{\Gamma(\frac{1}{2}-\frac{w}{2}-iP)\Gamma(\frac{1}{2}+\frac{w}{2}-iP)} \times \cos(4\pi s P), \tag{3.17a}$$

$$\Phi_{\mathrm{RR}}\Big(s, \theta; P, \frac{w}{2}\Big) = \frac{\sqrt{2}}{16\pi^2}(-)^w e^{-iw\theta} \mu^{iP-(w/2)} \bar{\mu}^{iP+(w/2)} \times \frac{\Gamma(-2iP)\Gamma(1-2iP)}{\Gamma(1-\frac{w}{2}-iP)\Gamma(\frac{w}{2}-iP)} \times \cos(4\pi s P). \tag{3.17b}$$

The annulus amplitude for open strings stretched between identical flavor branes (*5-5 strings*) is given by Eq. (B6) in Appendix B. The massless spectrum comprises of an $\mathcal{N}=1$ chiral multiplet (with the quantum numbers of a massless meson), which is part of a continuous spectrum of modes. Vacuum expectation values of the scalar fields in this multiplet should be interpreted as parameters of the four-dimensional gauge theory [12]. The massive spectrum contains a vector multiplet, which accounts for the gauge degrees of freedom on the D5-brane. The positive mass shift in this multiplet is due to the linear dilaton.

The action of the orientifold parity on the open strings attached to the flavor branes can be read from the corresponding Möbius strip amplitudes, which appear in Appendix B. As before, the Möbius strip amplitudes are nonzero only in the Ramond sector, hence they leave the bosons unaffected but project the fermions. In particular, the fermionic superpartner of the massless chiral multiplet transforms, after the orientifold projection, in the (anti) symmetric representation of the diagonal $U(N_f)$ flavor group (the opposite projection compared to that of the gauginos).

The nonvanishing Möbius strip amplitude manifestly shows that space-time supersymmetry is broken on the flavor branes and leads to a net force between the flavor D5-branes and the orientifold plane, which we discuss in Appendix D. This force, which arises as a one-loop effect on the flavor branes, has no consequences on the dynamics of the four-dimensional OQCD theory that will be engineered in the next subsection. Indeed, we will consider the open string dynamics in a low-energy decoupling limit where the dynamics on the flavor branes are frozen. In this limit, only fields from the *3-3* and *3-5* sectors are allowed to run in loops.

3. Flavor open strings

Open string stretched between flavor branes and color branes (*3-5 strings*) transforms in the fundamental representation of $U(N_c)$. From the relevant annulus amplitude, see Eq. (B10), one gets a nontachyonic, supersymmetric open string spectrum either from a flavor brane with $\theta = 0$ ($|D5(s,0)\rangle$) or a flavor antibrane $\overline{|D5(s,1/2)\rangle}$ with $\theta=\pi$ [12]. For these branes and N_c color branes, the *3-5* sector includes the following light degrees of freedom:

(i) A flavor brane with $\theta = 0$ gives an $\mathcal{N}=1$ massive vector multiplet with four-dimensional mass $m = \sqrt{2s^2+1/2}$ in the fundamental of $U(N_c)$.

(ii) A flavor antibrane with $\theta=\pi$ gives two quark chiral multiplets with mass $m=\sqrt{2}s$, in the fundamental and antifundamental of $U(N_c)$.

Only the second case will be relevant for engineering the electric OQCD theory.

C. Realizing the electric theory

We are now in position to reveal the final picture. The electric version of OQCD can be obtained in noncritical type 0′B string theory as the low-energy description of the open string dynamics on N_c color branes $|D3\rangle$ and N_f flavor branes of the type $\overline{|D5(s,1/2)\rangle}$. In this configuration, the four-dimensional low-energy degrees of freedom are

(i) A $U(N_c)$ gauge field A_μ from *3-3 strings*.

(ii) A Dirac fermion in the symmetric or antisymmetric $U(N_c)$ representation from *3-3 strings*. Positive orientifold charge gives the symmetric representation and negative charge the antisymmetric.

(iii) N_f quark multiplets Φ in the $U(N_c)$ fundamental representation and N_f antifundamental multiplets $\tilde{\Phi}$ from *3-5 strings*. The mass of these multiplets is proportional to s, the parameter that appears in the labeling of the D5-brane boundary state.

In addition, from *5-5 strings* we get a massless chiral multiplet propagating in five dimensions. From the four-dimensional point of view, the dynamics of this multiplet are frozen; the vacuum expectation value of the scalar component of this multiplet gives a mass to the quarks, i.e. it is directly related to the parameter s above [12].

So far we have discussed the low-energy open string spectrum, but we have not determined unambiguously all the couplings in the low-energy string field theory Lagrangian.[12] The symmetries of the D-brane setup sug-

[12]From the gauge theory point of view, this is the UV Lagrangian defined at some scale below the string scale.

gest that the Lagrangian includes the usual minimal couplings of the OQCD theory, but, in principle, it is possible that additional couplings allowed by the symmetries are also present. One could check this by computing tree-level correlation functions in open string theory. This is a complicated exercise that requires explicit knowledge of open string correlation functions in the boundary conformal field theory of $SL(2)/U(1)$. This information goes beyond the currently available technology, so we will not pursue such a task any further here.

Nevertheless, it was pointed out in [14] (in a type IIB context, using the results of a computation in [47]) that the leading asymptotic backreaction of $N_c|\text{D3}\rangle$ and $N_f\overline{|\text{D5}(s,1/2)\rangle}$ boundary states on the profile of the dilaton and graviton fields in the bulk is proportional to $N_f - 2N_c$. One would expect the factor $N_f - 3N_c$ from a D-brane setup realizing just $\mathcal{N}=1$ SQCD without any extra couplings. On these grounds, Ref. [14] proposed that the gauge theory on the above D-brane setup is in fact $\mathcal{N}=1$ SQCD with an extra quartic superpotential coupling of the form

$$\int d^2\theta W = h\int d^2\theta(Q\tilde{Q})(Q\tilde{Q}), \tag{3.18}$$

where Q, $\tilde{Q}$ are the quark multiplets of $\mathcal{N}=1$ SQCD in the fundamental and antifundamental, respectively. This proposal is consistent with the one-loop beta function of this theory [55], which is also proportional to $N_f - 2N_c$. In the context of holography, this quartic coupling has been discussed recently in [56,57].

These observations carry over naturally to our nonsupersymmetric case. Hence, we propose that the above D-brane setup in type 0′B string theory realizes the electric OQCD-S theory (see Table I) if the orientifold charge is positive, or the OQCD-AS theory if the charge is negative, with an extra quartic coupling. We will find further evidence for the quartic coupling in a moment.

D. D-brane monodromies

Now we want to examine the behavior of the D3, D5 boundary states and the O′5 crosscap state under the $\mathbb{Z}_2$ transformation of the closed string modulus μ [the Liouville coupling constant, see Eq. (3.2)]

$$\mu \to -\mu. \tag{3.19}$$

Henceforth, we will refer to this transformation as the μ-transition following [58]. It has been argued in a similar type IIB setting [14,34] that one can use the μ-transition to interpolate between D-brane configurations that realize the electric and magnetic descriptions of SQCD with gauge groups SU, or SO/Sp. This fits very nicely with the HW description, where μ corresponds to the separation of the NS5 and NS5′ branes in the (x^6, x^7) plane.

Note that in the supersymmetric SU case, μ takes any value in the complex plane, so this interpolation can be made continuously without ever passing through the strong coupling point at $\mu = 0$. This should be contrasted with the SO/Sp cases [34] where μ is real, so a continuous interpolation from μ to $-\mu$ entails passing through the strong coupling point at the origin at which we lose perturbative control over the theory. In the present case of type 0′B string theory, we face a similar difficulty, since μ is again real. Our attitude towards this will be the same as in [34]: we will perform the μ transition (3.19) as a $\mathbb{Z}_2$ transformation and will account for any missing RR charge at the end of this process by adding the appropriate number of D-branes needed by charge conservation. Here we make the underlying assumption that nothing dramatic can happen along the μ real line that could affect the IR dynamics of the gauge theory on the branes. We discuss this point further in the next section.

To distinguish the boundary and crosscap states of the μ and $-\mu$ theories, we will explicitly label them by μ. To implement Seiberg duality, we supplement the μ-transition (3.19) with the following boundary and crosscap state transformations:[13]

$$|\text{D3};\mu\rangle \to |\text{D3};-\mu\rangle, \tag{3.20a}$$

$$\overline{|\text{D5};0,1/2;\mu\rangle} \to \overline{|\text{D5};i/2,0;-\mu\rangle} = |\text{D5};0,1/2;-\mu\rangle + \overline{|\text{D3};-\mu\rangle}, \tag{3.20b}$$

$$|\text{O}'5;\mu\rangle \to \overline{|\text{O}'5;-\mu\rangle}. \tag{3.20c}$$

These monodromies are different from the ones proposed in [14,34], so we will take a moment here to explain each one of them in detail.[14]

First of all, the $\mu \to -\mu$ transformation is equivalent, as a transformation of the $\mathcal{N}=2$ Liouville interaction (3.2), to a half-period shift $s_{\tilde{x}}$ along the angular direction in winding space, i.e. $X_L - X_R \to X_L - X_R + \sqrt{2}\pi$. Hence, the $\mathbb{Z}_2$ transformation (3.19) is equivalent to the multiplication of the D-brane wave functions with the phase $(-)^w$, which follows from the action

$$s_{\tilde{x}}\left|\mathcal{B};P,\frac{w}{2};\eta\right\rangle\!\!\right\rangle = (-)^w\left|\mathcal{B};P,\frac{w}{2};\eta\right\rangle\!\!\right\rangle, \quad s_{\tilde{x}}\left|\mathcal{C};P,\frac{w}{2};\eta\right\rangle\!\!\right\rangle = (-)^w\left|\mathcal{C};P,\frac{w}{2};\eta\right\rangle\!\!\right\rangle. \tag{3.21}$$

This is consistent with the μ dependence of the wave functions (3.14a), (3.14b), (3.17a), and (3.17b). The first

[13] Notice that we set the D5-brane parameter s to zero. In other words, we discuss what happens when the quark multiplets in the electric description are massless.

[14] Some of the monodromies proposed in [14,34] do not follow from the $\mu \to -\mu$ transformation and lead to D3-branes with negative tension. They also lead to a gauge theory spectrum with unexplained features that does not fit with the expected Seiberg duality. The monodromies we present here do not have these problems. Although we discuss a setup realizing OQCD, our analysis has analogous implications for the D-brane setups that realize $\mathcal{N}=1$ SQCD with SU or SO/Sp gauge groups, which were the subject of [14,34].

line (3.20a) exhibits the transformation of the D3-brane (i.e. the color brane) according to these rules.

The second line (3.20b) presents the transformation of the D5 boundary states. In order to obtain Seiberg duality, we want a process that affects only the physics near the tip of the cigar. In particular, we want to keep the value of the Wilson line on the D5-branes, as measured in the asymptotic region $\rho \to \infty$, fixed during the μ-transition. Otherwise, we simply rotate the whole configuration by half a period in the angular (winding) direction of the cigar to get back the electric description of the gauge theory.

In order to achieve this, we have to shift the D5 boundary state modulus $M = \frac{\theta}{2\pi} = \frac{1}{2}$ to $M = 0$ [notice that this offsets the effect of the $\mu \to -\mu$ transformation on the wave functions (3.17a) and (3.17b)]. At the same time, we want to keep the brane chiral, i.e. maintain the property $J = M$ of the D-brane labels, where, by definition, $J = \frac{1}{2} + is$. This is important for the following reason. The world sheet action for an open string ending on the D5 boundary state is captured by the boundary action [59]

$$\mathcal{S}_{\rm bdy} = \oint dx[\bar{\lambda}\partial_x\lambda - \mu_B\lambda\psi^+ e^{-((\rho_{\rm L}+iX_{\rm L})/\sqrt{2})} - \bar{\mu}_B e^{-((\rho_{\rm L}+iX_{\rm L})/\sqrt{2})}\psi^-\bar{\lambda} - \tilde{\mu}_B(\lambda\bar{\lambda} - \bar{\lambda}\lambda)(\psi^+\psi^- - i\sqrt{2}\partial X)e^{-\sqrt{2}\rho_{\rm L}}], \quad (3.22)$$

where λ, $\bar{\lambda}$ are boundary fermions and μ_B, $\bar{\mu}_B$, $\tilde{\mu}_B$ are boundary Liouville couplings that are related to the (J, M) labels of the branes and the bulk Liouville coupling μ by the following equations:[15]

$$\tilde{\mu}_B^{{\rm ren};k=1} = \frac{1}{2\pi}|\mu^{{\rm ren};k=1}|\cos\pi(2J-1), \quad (3.23a)$$

$$\mu_B^{{\rm ren};k=1}\bar{\mu}_B^{{\rm ren};k=1} = \frac{2}{\pi}|\mu^{{\rm ren};k=1}|\sin\pi(J-M)\sin\pi(J+M). \quad (3.23b)$$

For $J = M = \frac{1}{2}$ the boundary couplings μ_B and $\bar{\mu}_B$ vanish. Demanding that they still vanish after the transformation $M = \frac{1}{2} \to M = 0$ requires that we also set $J = 0$, i.e. that we perform an additional $\mathbb{Z}_2$ transformation on the D5 boundary state $\tilde{\mu}_B \to -\tilde{\mu}_B$.

This explains the transformation appearing in (3.20b). The equality that expresses the $J = M = 0$ boundary state as a linear combination of the fundamental D5-brane with $J = M = \frac{1}{2}$ and the D3-brane follows from the character identity (A12) in Appendix A. The importance of this relation for this setup was also pointed out in [14] (the character identity (A12) was observed earlier in [54]). In the HW context, it expresses the possibility of the flavor branes to break onto the NS5-brane after the rotation; a similar phenomenon was touched upon in [38] for $\mathcal{N} = 2$ setups.

[15]As in the bulk, see Eq. (3.3), these relations need to be renormalized in the limit $k \to 1^+$ [14].

Finally, in the last line (3.20c) the bar follows from the requirement that we get the same projection on the gauge theory fermions before and after the μ-transition [34]. From the μ-transition action on the Ishibashi states, Eq. (3.21), and the expression of the crosscap state (3.9), one observes that this transformation is such that the extensive part of the orientifold is invariant, while the localized part is reversed. As with the D5-brane, this is a consequence of the requirement to have a "local" operation, leaving the asymptotics invariant.

Consequently, if we start with the electric description of OQCD that arises in a setup that includes the states

$$N_c|{\rm D3}; \mu\rangle, \qquad N_f\overline{|{\rm D5}; 0, 1/2; \mu\rangle}, \qquad \pm|{\rm O}'5; \mu\rangle \quad (3.24)$$

we end up naively after the μ-transition and the annihilation of N_c D3 brane-antibrane pairs with a configuration that comprises of the boundary states $(N_f - N_c)\overline{|{\rm D3}; -\mu\rangle}$ and $N_f|{\rm D5}; 0, 1/2; -\mu\rangle$, together with the crosscap $\pm\overline{|{\rm O}'5; -\mu\rangle}$.

This, however, cannot be the final answer. Notice that in this process the C_0 RR scalar charge has changed. In appropriate normalizations where the $|{\rm D3}; \mu\rangle$ boundary state has unit RR charge, the total charge of the electric configuration is [14,34] (see also Appendix C for a detailed determination of the RR charges)

$$Q_e = N_c - \frac{N_f}{2} \pm 2. \quad (3.25)$$

The $\pm$ sign in this relation is the same as the $\pm$ sign appearing in front of the crosscap state in (3.24). We remind the reader that the $+$ sign corresponds to gauginos in the symmetric representation and the $-$ sign to gauginos in the antisymmetric.

The RR scalar charge of the $-\mu$ configuration is equal to

$$Q_m = N_c - \frac{N_f}{2} \mp 2. \quad (3.26)$$

Therefore, charge conservation requires the creation of new charge during the μ-transition. This is a familiar effect in HW setups with orientifolds [3]. Remember that massless RR charge is carried only by the $|\mathcal{C}; {\rm loc}\rangle$ part of the $|{\rm O}'5\rangle$ crosscap state [see Eq. (3.9)]. Hence, it is natural to proclaim that the new charge is carried by the only other localized object in the game, the D3-branes. Adding four D3 or $\overline{\rm D3}$ boundary states in the $-\mu$ configuration, we are canceling the deficiency (or surplus) of four units of scalar charge to obtain the following magnetic configuration

$$(N_f - N_c \mp 4)\overline{|{\rm D3}; -\mu\rangle}, \qquad N_f|D5; 0, 1/2; -\mu\rangle, \qquad \pm\overline{|{\rm O}'5; -\mu\rangle}. \quad (3.27)$$

The low-energy open string degrees of freedom of this configuration are string theory's "prediction" for the magnetic version of OQCD.

E. The magnetic theory

Low-energy open string degrees of freedom arise in three different sectors in the D-brane configuration (3.27): *3-3 strings*, *3-5 strings*, and *5-5 strings*. These can be determined from the annulus and Möbius strip amplitudes that appear in Appendix B. They are in fact the same as those appearing in Sec. III C for the electric setup of OQCD. The main difference is the all important change in the rank of the gauge group from N_c to $N_f - N_c \mp 4$. Besides that, the μ-transition exhibits a self-duality between the electric and magnetic setups. Indeed, both setups involve the same boundary and crosscap states (modulo a sign change in the RR sector which is naturally accounted for by the fact that we have rotated the theory in the bulk along the angular direction of the cigar).

The self-duality of the configuration matches very nicely with the proposal in Sec. III C that the low-energy theory on the D-branes is OQCD with a quartic coupling. In the large N limit, the theory is planar equivalent to $\mathcal{N} = 1$ SQCD with the quartic coupling (3.18). Seiberg duality for $\mathcal{N} = 1$ SQCD with a quartic coupling gives back the same theory with an inverse coupling constant for the quartic interaction and therefore exhibits the same self-duality property [55]. The dual Lagrangian can be obtained from the magnetic theory with a supersymmetric mass term for the magnetic meson if we integrate out the magnetic meson. For $N_f \neq 2N_c$, one recovers in the far infrared the usual duality between the electric and magnetic descriptions of SQCD without quartic couplings. The case $N_f = 2N_c$ is special. In this case, the quartic coupling becomes exactly marginal in the infrared.

The string theory picture of this section suggests a similar state of affairs also in OQCD. This is certainly true in the large N limit, because of planar equivalence, but we would like to propose here that this picture extends also at finite N. Furthermore, we consider this picture as evidence for the validity of the finite N Seiberg duality of Sec. II in the absence of quartic couplings.

IV. EVIDENCE FOR THE DUALITY

The string theory analysis of the previous section motivates an electric-magnetic duality for OQCD as postulated in Sec. II. This duality shares many common features with Seiberg duality in $\mathcal{N} = 1$ SQCD in the SU and SO/Sp cases. However, unlike the supersymmetric case, here we cannot use, in principle, the arsenal of supersymmetry to perform a set of nontrivial consistency checks. For instance, we cannot use holomorphy to fix the exact quantum form of potential terms, there is no known exact beta function formula [60–62] (except at large N [32]) and there is no superconformal symmetry or chiral ring structure that fixes the IR features of a class of special operators. What support can we then find in favor of our claim given that OQCD is a nonsupersymmetric gauge theory? In this section, we would like to summarize the evidence for the proposed duality and discuss its viability as a conjecture in nonsupersymmetric gauge theory.

A. Duality at large N

First of all, there is a sense in OQCD, in which we can expand around a supersymmetric point. At infinite N, namely, when both $N_c \to \infty$ and $N_f \to \infty$, with $g^2_{\rm YM} N_c$ and N_f/N_c kept fixed, the dynamics of the electric and magnetic theories is almost identical to the dynamics of $\mathcal{N} = 1$ SQCD. The reason is, as we mentioned above, "planar equivalence" [33]: the planar nonsupersymmetric electric (magnetic) theory is nonperturbatively equivalent to the supersymmetric electric (magnetic) theory in the common sector of C-parity even states [26]. This argument, by itself, is sufficient for the duality to hold.

A consistency check of the proposed duality at infinite N is a one-loop calculation of the Coleman-Weinberg potential for the "squark" field (this will be discussed in more detail in the next section). The potential remains flat. This is consistent with the existence of a moduli space in SQCD.

In addition, both the magnetic and the electric nonsupersymmetric theories admit a Novikov-Shifman-Vainshtein-Zakharov (NSVZ) beta function [32] at large N. The large-N NSVZ beta function supports the existence of a conformal window as in Seiberg's original paper [1].

We wish to add that the infinite-N equivalence with the supersymmetric theory does not involve any fine-tuning. Once the tree-level Lagrangian is given, quantum corrections will preserve the ratio between couplings as if the theory was supersymmetric.

We conclude that planar equivalence with SQCD is a nontrivial and nongeneric statement that proves the proposed duality at infinite N. Now the question about the fate of the duality becomes a question about the effects of $1/N$ corrections. Our evidence for a finite-N duality is weaker. One argument is that if a certain projection ("orientifolding") is made on both the electric and magnetic original theories, the IR-duality should still hold. Additional arguments can be made on the basis of anomaly matching and the string realization.

B. Anomaly matching

A nontrivial check of consistency that we can always perform independent of supersymmetry, is 't Hooft anomaly matching, i.e. we should verify that global anomalies are the same in the UV and IR descriptions of the theory. For concreteness, let us consider here the case of OQCD-AS (similar statements apply also to OQCD-S). The global $SU(N_f)^3$, $SU(N_f)^2 U(1)_R$, $U(1)_R$ and $U(1)_R^3$ anomalies are summarized for the electric and proposed magnetic de-

TABLE III. 't Hooft anomaly matching for OQCD-AS.

	Electric	Magnetic
$SU(N_f)^3$	$0 + N_c d^3(\square) = N_c d^3(\square)$	$0 + \tilde{N}_c(-d^3(\square)) + d^3(\square\square) = N_c d^3(\square)$
$SU(N_f)^2 U(1)_R$	$0 + N_c(\frac{-N_c+2}{N_f})d^2(\square) = \frac{-N_c^2+2N_c}{N_f} d^2(\square)$	$0 + \tilde{N}_c(\frac{N_c-N_f-2}{N_f})d^2(\square) + (\frac{N_f-2N_c+4}{N_f})d^2(\square\square) = \frac{-N_c^2+2N_c}{N_f} d^2(\square)$
$U(1)_R$	$(N_c^2 - N_c) + 2(N_c N_f \frac{-N_c+2}{N_f}) = -N_c^2 + 3N_c$	$(\tilde{N}_c^2 - \tilde{N}_c) + 2(\tilde{N}_c N_f \frac{N_c-N_f-2}{N_f}) + 2(\frac{1}{2}(N_f^2 + N_f)\frac{N_f-2N_c+4}{N_f}) = -N_c^2 + 3N_c$
$U(1)_R^3$	$(N_c^2 - N_c) + 2[(N_c N_f(\frac{-Nc+2}{N_f})^3] = N_c(N_c - 1 - 2\frac{(Nc-2)^3}{N_f^2})$	$(\tilde{N}_c^2 - \tilde{N}_c) + 2[\tilde{N}_c N_f(\frac{N_c-N_f-2}{N_f})^3] + 2[\frac{1}{2}(N_f^2 + N_f)(\frac{N_f-2N_c+4}{N_f})^3] = N_c(N_c - 1 - 2\frac{(N_c-2)^3}{N_f^2})$

scriptions of OQCD-AS in Table III. As before, we use the notation $\tilde{N}_c = N_f - N_c + 4$ and the terms in each box are ordered as ("gluino") + (quarks) in the electric theory and (gluino) + (quarks) + (mesino) in the magnetic theory. $d^2(R)\delta^{ab}$ and $d^3(R)d^{abc}$ for the representation R are, respectively, the traces $\mathrm{Tr}_R T^a T^b$, $\mathrm{Tr}_R T^a\{T^b, T^c\}$.[16] In Table III we make use of the following relations:

$$d^2(\square\square) = (N_f + 2)d^2(\square),$$
$$d^3(\square\square) = (N_f + 4)d^3(\square). \tag{4.1}$$

It is worth noticing that up to factors of 2 the matching works precisely as the matching of the anomalies in the supersymmetric $SO(N_c)$ case. This is not surprising, since the fermions in our model carry the same dimensions (up to factors of 2) as the fermions in the $SO(N_c)$ models.

The perfect matching of the above anomalies, including $1/N$ corrections, is our first independent consistency check for the duality directly in gauge theory. It suggests that the duality, as we formulated it in Sec. II, may hold even at finite N. There are known nonsupersymmetric cases, however, where 't Hooft anomaly matching is misleading as a basis for the proposal of an IR effective theory [19,63]. These are cases where the matching works in a nontrivial manner, but one can still argue that the so-proposed IR effective theory cannot be the correct one. Our case is different, however, because of planar equivalence at infinite N, which fixes the basic structure of the duality.

C. Hints from the string theory realization

The string theory realization of Sec. III cannot be taken as a proof for the proposed duality, but we believe it gives useful hints. Let us outline some of the pertinent issues.

What we presented in Sec. III is a procedure—the μ-transition—that relates two distinct configurations of D-branes and O-planes (exactly described in a nontrivial underlying two-dimensional world sheet CFT via the appropriate boundary and crosscap states). This procedure as a continuous process involves a number of nontrivial steps. Starting from the electric description with $\mu = \mu_0 \in \mathbb{R}^+$ we slide μ along the real axis until it goes through zero to $-\mu_0$. At zero μ the background string theory develops a linear dilaton singularity and becomes strongly coupled. This is the first nontrivial effect. Then as we emerge from zero μ four extra D-branes have been created and N_c D-$\bar{\mathrm{D}}$ pairs annihilate via open string tachyon condensation.

In order to argue for Seiberg duality in this setup we have to show that the IR dynamics on the branes are unaffected by the above procedure. For instance, one should show that any open string process that contributes in the extreme IR is independent of the sign of μ. This is plausible, given the fact that the boundary states are the same as in the corresponding SO/Sp setup in type IIB string theory [34] and the crosscap state is the RR part of its supersymmetric counterpart. We have not been able to show this explicitly however.

Another source of intuition that the μ-transition produces an actual Seiberg dual is based on the fact that the closed string modulus μ controls the bare coupling g_{YM} on the branes

$$g_{\mathrm{YM}}^2 \sim 1/|\mu|. \tag{4.2}$$

Hence, changing μ does not affect the IR dynamics. This naive picture could be spoiled if, for example, some extra nontrivial strong coupling IR dynamics take place at $\mu = 0$, so that when we emerge on the other side of the μ-line the IR dynamics on the branes are not anymore related with the IR dynamics on the brane setup of the electric description. In the supersymmetric case without orientifolds, type IIB string theory on $\mathbb{R}^{3,1} \times SL(2)_1/U(1)$ is related to type IIB string theory on the resolved conifold in a double-scaling limit [13,64]. In this limit, g_s and the volume of the blown-up S^2 cycle are taken to zero in such a way that D5-branes wrapping the S^2 have a finite tension [13]. The tension vanishes when μ on the $SL(2)/U(1)$ side becomes zero. In that case, the linear dilaton singularity is a signal of the corresponding conifold singularity. It is known [65–68] that the singularity of a slightly resolved conifold can be explained by taking into account the instanton contributions associated to D1-branes wrapping the S^2. These contributions are already present in the gauge theory on the D-brane setup of the electric description, so no abrupt change is anticipated as we go through the singularity.

[16] By definition $d^{abc} = \mathrm{Tr}_\square T^a\{T^b, T^c\}$..

The situation is more involved in the presence of an O4-plane. The NS5-branes act as domain walls for the orientifold charge, so when an O4-plane goes through an NS5-brane its charge changes [we can see this effect clearly in the noncritical description: the culprit for the extra charge is the localized part of the crosscap state $|\mathcal{C};\mathrm{loc}\rangle$—see Eq. (3.9)]. A consequence of this is a shift in RR charge as we go through the μ-transition, which has to be compensated by the creation of 4 (anti)D-branes. This a clear signal of a phase transition that occurs at $\mu = 0$. Still, what we know about Seiberg duality in SQCD with SO/Sp gauge groups suggests that there are no strong coupling dynamics that affect the IR physics during this phase transition. Given the similarities of the supersymmetric setup with the setup realizing OQCD, we are tempted to speculate that an analogous conclusion can be reached in the absence of supersymmetry, although, clearly, it would be desirable to substantiate this conclusion with a stronger argument.

V. IMPLICATIONS IN GAUGE THEORY

In the previous section we provided evidence for a Seiberg duality between two nonsupersymmetric (OQCD) gauge theories. For the moment let us accept this duality as a correct statement and discuss its implications.

At infinite N the dynamics of the electric and magnetic theories is supersymmetric in the sector of C-parity even states due to planar equivalence [33]. The immediate implications of this statement are: a quantum moduli space, a conformal window at $\frac{3}{2}N_c \leq N_f \leq 3N_c$ and confinement at $N_c \leq N_f < \frac{3}{2}N_c$. For $N_c + 1 < N_f < \frac{3}{2}N_c$ the magnetic description is IR free and provides a good effective description of the IR dynamics.

At finite N the nonsupersymmetric effects become more significant and determining exactly the IR dynamics becomes a complicated problem. As in ordinary QCD, a Banks-Zaks analysis of the two-loop beta function reveals that there should be a range of N_f where the IR dynamics are captured by a scale-invariant theory of interacting quarks and gluons. One of the most interesting implications of our proposal for a Seiberg duality in OQCD is a precise prediction for the exact range of the conformal window.

The one-loop beta function coefficient of the electric OQCD-AS theory is $\beta = 3N_c - N_f + \frac{4}{3}$. Similarly, the one-loop beta function coefficient of the magnetic theory is $\beta = 3(N_f - N_c + 4) - N_f + \frac{4}{3}$. Since the upper and lower parts of the conformal window are determined by the vanishing of the one-loop beta function coefficients of the electric and magnetic theories, we expect a conformal window for OQCD-AS when

$$\frac{3}{2}N_c - \frac{20}{3} \leq N_f \leq 3N_c + \frac{4}{3}, \qquad N_c > 5. \qquad (5.1)$$

When the gluino is in the symmetric representation (the OQCD-S theory), we expect a conformal window when

$$\frac{3}{2}N_c + \frac{20}{3} \leq N_f \leq 3N_c - \frac{4}{3}, \qquad N_c > 5. \qquad (5.2)$$

The restriction on the number of colors $N_c > 5$ is explained below.

Above the upper bound of the conformal window, the electric theories lose asymptotic freedom and become infrared free. Below the conformal window, i.e. when $N_f < \frac{3}{2}N_c \mp \frac{20}{3}$, the magnetic theories become infrared free and the electric theories are expected to confine. This is known as the free magnetic phase. However, we know from planar equivalence at infinite N that if we keep reducing the number of flavors, at some point the space of vacua of the theory will become empty. For $SU(N_c)$ $\mathcal{N} = 1$ SQCD this critical point occurs at $N_f = N_c$, precisely when we lose the magnetic description. It is natural to conjecture that a similar effect takes place for OQCD at finite N. The magnetic description is lost at the critical values $N_f = N_c - 4$ in OQCD-AS and $N_f = N_c + 4$ in OQCD-S. Below these values we expect the space of vacua of OQCD to be empty. The overall picture is summarized in Fig. 1. For OQCD-AS this picture makes sense only when $\frac{3}{2}N_c - \frac{20}{3} > N_c - 4$, whereas for OQCD-S it makes sense only when $3N_c - \frac{4}{3} > \frac{3}{2}N_c + \frac{20}{3}$. Both inequalities require $N_c > \frac{16}{3}$, or equivalently $N_c > 5$.

One of the nontrivial effects of the absence of supersymmetry at finite N is the lift of the classical moduli space. To see this, let us consider the Coleman-Weinberg potential for the vev of the squark field $\langle \tilde{\Phi}^i_\alpha \rangle = \langle \Phi^i_\alpha \rangle \equiv \delta^i_\alpha v^i$ of the electric OQCD-S theory. Keeping up to quadratic terms in the Lagrangian and integrating over all the fields, we obtain the effective potential (see Ref. [69])

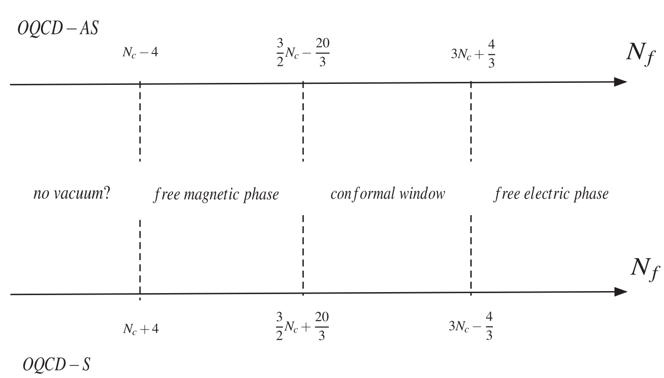

FIG. 1. The conjectured phase structure of OQCD as a function of the number of flavors N_f. At infinite N we recover the phase structure of $\mathcal{N} = 1$ SQCD with gauge group $U(N_c)$. This picture makes sense for $N_c > 5$.

$$E(v^1, v^2, \ldots, v^{N_c}) = N_c \int d^4p \log\Big(p^2 + \sum_{i=1}^{N_c} v^i v^i\Big) - (N_c+2) \int d^4p \log\Big(p^2 + \sum_{i=1}^{N_c} v^i v^i\Big), \tag{5.3}$$

namely,

$$E(v^2) = -2 \int d^4p \log(p^2 + v^2) \sim -\Lambda^4 - \Lambda^2 v^2 + (\log \Lambda^2/\nu^2) v^4 + \cdots, \tag{5.4}$$

where $v^2 \equiv \sum_i v^i v^i$, Λ is a UV cutoff and ν is an IR cutoff. The first term in the above equation (5.4) is a cosmological constant and thus has no implications on the gauge theory dynamics. The second term is a mass term for the scalars. It may be removed by a fine-tuned renormalization. The one-loop generated potential demonstrates that it is impossible to have a quantum moduli space at finite N. The minimum of the Coleman-Weinberg potential is at the origin $v^i = 0$, where the electric theory exhibits the full $SU(N_f)$ and $U(N_c)$ symmetries. It is worth noting that when the gaugino is in the antisymmetric representation (OQCD-AS), $v^i = 0$ is a maximum of the potential.

These observations do not invalidate our statement about Seiberg duality. It is still possible that there is a duality between the electric and magnetic theories in their true (unique) vacuum, rather than a duality in a large moduli space.

VI. OUTLOOK

In this paper we considered a nonsupersymmetric Seiberg duality between electric and magnetic "orientifold field theories." These theories are not generic nonsupersymmetric gauge theories. In the large N limit they exhibit planar equivalence with supersymmetric QCD. This nontrivial statement gives extra control and allows us to argue convincingly for Seiberg duality in this limit. Our discussion suggests that the duality may work also at finite N. An obvious question is whether we can generalize our results in other nonsupersymmetric cases. Namely, can we find other examples where we can argue for nonsupersymmetric Seiberg dualities in a similar fashion?

Another potential example relies on Seiberg duality between the $SO(N_c)$ electric SQCD and the $SO(N_f - N_c + 4)$ magnetic theory. To obtain a nonsupersymmetric descendant, one replaces the adjoint fermion in the electric theory by a symmetric fermion. Similarly, one replaces the adjoint fermion of the magnetic theory by a symmetric fermion and the symmetric fermion in the meson multiplet by an antisymmetric fermion. The result is a nonsupersymmetric electric theory and a nonsupersymmetric magnetic theory. We would like to propose that these theories form a pair of Seiberg duals. The evidence for the duality is identical to the one in the prime example of this paper. The global anomalies match and moreover, we may embed the theory in a noncritical string theory version of the Sugimoto model [70]. In addition, the electric and magnetic theories become supersymmetric at large N. It is interesting to explore this proposal and perhaps others in a future work.

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APPENDIX A: $\mathcal{N} = 2$ CHARACTERS AND USEFUL IDENTITIES

In the main text we determine the spectrum of open strings in various sectors by computing a corresponding set of annulus or Möbius strip amplitudes. The final expressions of these amplitudes are formulated in terms of the characters of the supersymmetric $SL(2)/U(1)$ theory. Such expressions appear in Appendix B. In this Appendix, we summarize the relevant notation and some useful facts about the $SL(2)/U(1)$ representations and their characters.

In view of the applications in this paper, we will concentrate on the characters of the $SL(2)/U(1)$ supercoset at level 1. These are characters of an $\mathcal{N} = 2$ superconformal algebra with central charge $c = 9$. They can be categorized in different classes corresponding to irreducible representations of the $SL(2)$ algebra in the parent WZW theory. In all cases the quadratic Casimir of the representations is $c_2 = -j(j-1)$. Here we summarize the basic representations.

One class of representations is the *continuous representations* with $j = 1/2 + ip$, $p \in \mathbb{R}^+$. The corresponding characters are denoted by $\mathrm{ch}_c(p, m)[^a_b]$, where the $N = 2$ superconformal $U(1)_R$ charge of the primary is $Q = 2m$, $m \in \mathbb{R}$.[17] The explicit form of the characters is

$$\mathrm{ch}_c(p, m; \tau, \nu)\begin{bmatrix} a \\ b \end{bmatrix} = q^{(p^2+m^2)/k} e^{4i\pi\nu(m/k)} \frac{\vartheta[^a_b](\tau, \nu)}{\eta^3(\tau)}, \tag{A1}$$

where $q = e^{2\pi i\tau}$. This character is similar to the character of a free theory comprising of two real bosons and a complex fermion. $\vartheta[^a_b](\tau, \nu)$ and $\eta(\tau)$ are, respectively, the standard theta and eta functions.

[17]The spectrum of R-charges is not necessarily continuous and depends on the model considered. For instance in the cigar CFT one has $m = (n + w)/2$ with $n, w \in \mathbb{Z}$.

Another important class of representations comprises *discrete representations* with j real. However, none of them are normalizable for $k = 1$.

While the closed string spectrum contains only continuous representations, the spectrum of open strings attached to localized D-branes is built upon the *identity representation* with $j = 0$. We denote the character of the identity representation by $\mathrm{ch}_{\mathbb{I}}(r)[{}^a_b]$. It has the form

$$\mathrm{ch}_{\mathbb{I}}(r;\tau,\nu)\begin{bmatrix}a\\b\end{bmatrix} = \frac{(1-q)q^{-(1/4+(r+a/2)^2)/k}e^{2i\pi\nu(2r+a/k))}}{(1+(-)^b e^{2i\pi\nu}q^{1/2+r+a/2})(1+(-)^b e^{-2i\pi\nu}q^{1/2-r-a/2})} \times \frac{\vartheta[{}^a_b](\tau,\nu)}{\eta^3(\tau)}. \tag{A2}$$

The primary states in the NS sector, for example, are, besides the identity, states of conformal dimension

$$\Delta = r^2 - |r| - \tfrac{1}{2}, \qquad r \neq 0. \tag{A3}$$

1. Extended characters

When the level k of $SL(2)/U(1)$ is rational it is often convenient to define *extended characters* [54], which serve as a useful repackaging of ordinary characters. For $k = 1$ the extended characters are defined by partially summing over integer units of spectral flow [71,72]. More explicitly, extended characters (denoted by capital letters) are defined as

$$\mathrm{Ch}_\star(\star,\star)\begin{bmatrix}a\\b\end{bmatrix}(\tau;\nu) = \sum_{\ell\in\mathbb{Z}}\mathrm{ch}_\star(\star,\star)\begin{bmatrix}a\\b\end{bmatrix}(\tau;\nu+\ell\tau). \tag{A4}$$

The stars stand for the appropriate, in each case, representation or quantum number. For example, the extended characters of the continuous representations are for k integer

$$\mathrm{Ch}_c(P,m)\begin{bmatrix}a\\b\end{bmatrix}(\tau;\nu) = \frac{q^{P^2/k}}{\eta^3(\tau)}\Theta_{2m,k}(\tau;2\nu)\vartheta\begin{bmatrix}a\\b\end{bmatrix}(\tau;\nu) \tag{A5}$$

with $2m \in \mathbb{Z}_{2k}$. $\Theta_{n,k}$ is a classical theta function.

2. Hatted characters

As usual, Möbius strip amplitudes are conveniently expressed in terms of hatted characters. These are defined as

$$\hat{\mathrm{ch}}_\star(\star;\tau)\begin{bmatrix}a\\b\end{bmatrix} = e^{-i\pi(\Delta-c/24)}\mathrm{ch}_\star(\star;\tau+1/2)\begin{bmatrix}a\\b\end{bmatrix}, \tag{A6}$$

where Δ is the scaling dimension of the primary state of the representation and c the central charge of the CFT. We refer the reader to [48] for a more detailed discussion of the properties of these characters in the $SL(2)/U(1)$ supercoset.

3. Modular transformation properties

In deriving the one-loop amplitudes in the direct or transverse channel, the following modular transformation properties are useful:[18]

$$\frac{\vartheta[{}^a_b](-1/\tau)}{\eta(-1/\tau)} = e^{-(i\pi ab/2)}\frac{\vartheta[{}^{-b}_{\ a}]}{\eta(\tau)}, \qquad \eta(-1/\tau) = (-i\tau)^{1/2}\eta(\tau), \tag{A7}$$

$$\frac{\vartheta[{}^a_b](-\frac{1}{4it}+\frac{1}{2})}{\eta^3(-\frac{1}{4it}+\frac{1}{2})} = \frac{1}{2t}e^{i\pi(1/4-b/2+3a/4)}\frac{\vartheta[{}^{\ \ a}_{a-b+1}](it+\frac{1}{2})}{\eta^3(it+\frac{1}{2})}, \tag{A8}$$

$$\mathrm{ch}_c\Big(P,m,-\frac{1}{4\tau}\Big)\begin{bmatrix}a\\b\end{bmatrix} = 4e^{i\pi ab/2}\int_0^\infty dP'\int_0^\infty dm' e^{-4\pi imm'} \times\cos(4\pi PP')\mathrm{ch}_c(P',m';\tau)\begin{bmatrix}-b\\a\end{bmatrix}, \tag{A9}$$

$$\hat{\mathrm{ch}}_c\Big(P,m,-\frac{1}{4\tau}\Big)\begin{bmatrix}a\\b\end{bmatrix} = 2e^{(i\pi/4)(1-a-2b)}\int_0^\infty dP'\int_0^\infty dm' \times e^{-2\pi imm'}\cos(2\pi PP') \times\hat{\mathrm{ch}}_c(P',m';\tau)\begin{bmatrix}a\\a-b+1\end{bmatrix}. \tag{A10}$$

4. Relations between identity and continuous characters

The definition of the extended identity characters is

$$\mathrm{Ch}_{\mathbb{I}}(\tau)\begin{bmatrix}a\\b\end{bmatrix} = \sum_{n\in\mathbb{Z}}\mathrm{ch}_{\mathbb{I}}\Big(n+\frac{a}{2};\tau\Big)\begin{bmatrix}a\\b\end{bmatrix}. \tag{A11}$$

Some useful identities between continuous and identity characters are

$$\mathrm{Ch}_{\mathbb{I}}(\tau)\begin{bmatrix}a\\b\end{bmatrix} = \mathrm{Ch}_c\Big(\frac{i}{2},\frac{a}{2};\tau\Big)\begin{bmatrix}a\\b\end{bmatrix} - (-)^b\mathrm{Ch}_c\Big(0,\frac{1+a}{2};\tau\Big)\begin{bmatrix}a\\b\end{bmatrix}, \tag{A12}$$

$$\sum_{n\in\mathbb{Z}}(-)^n\mathrm{ch}_{\mathbb{I}}(n;\tau)\begin{bmatrix}a\\b\end{bmatrix} = \sum_{n\in\mathbb{Z}}(-)^n\Big\{\mathrm{ch}_c\Big(\frac{i}{2};n+\frac{a}{2};\tau\Big)\begin{bmatrix}a\\b\end{bmatrix} + (-)^b\mathrm{ch}_c\Big(0,n+\frac{1+a}{2};\tau\Big)\begin{bmatrix}a\\b\end{bmatrix}\Big\}, \tag{A13}$$

[18]From now on we set $\nu = 0$.

$$\sum_{n\in\mathbb{Z}}(-)^n\widehat{\mathrm{ch}}_{\mathbb{I}}(n;\tau)\begin{bmatrix}1\\b\end{bmatrix}=\sum_{n\in 2\mathbb{Z}+1}\widehat{\mathrm{ch}}_c\Big(\frac{i}{2};\frac{n}{2};\tau\Big)\begin{bmatrix}1\\b\end{bmatrix}+\sum_{n\in 2\mathbb{Z}}(-)^b e^{\pi i n/2}\widehat{\mathrm{ch}}_c\Big(0,\frac{n}{2};\tau\Big)\begin{bmatrix}1\\b\end{bmatrix}, \tag{A14}$$

$$\sum_{n\in\mathbb{Z}}\widehat{\mathrm{ch}}_{\mathbb{I}}(n;\tau)\begin{bmatrix}1\\b\end{bmatrix}=-\sum_{n\in 2\mathbb{Z}+1}(-)^{(n-1)/2}\widehat{\mathrm{ch}}_c\Big(\frac{i}{2};\frac{n}{2};\tau\Big)\begin{bmatrix}1\\b\end{bmatrix}+\sum_{n\in 2\mathbb{Z}}(-)^b\widehat{\mathrm{ch}}_c\Big(0,\frac{n}{2};\tau\Big)\begin{bmatrix}1\\b\end{bmatrix}. \tag{A15}$$

5. Leading terms in character expansions

The identity characters have the following leading terms in an expansion in powers of $q=e^{2\pi i\tau}$ (here $\tau=it$):

$$\sum_{n\in\mathbb{Z}}\mathrm{ch}_{\mathbb{I}}(n;it)\begin{bmatrix}0\\0\end{bmatrix}\frac{\vartheta[^0_0](it)}{\eta^3(it)}=q^{-(1/2)}+2+\mathcal{O}(q), \tag{A16}$$

$$\sum_{n\in\mathbb{Z}}\mathrm{ch}_{\mathbb{I}}(n;it)\begin{bmatrix}0\\1\end{bmatrix}\frac{\vartheta[^0_1](it)}{\eta^3(it)}=q^{-(1/2)}-2+\mathcal{O}(q), \tag{A17}$$

$$\sum_{n\in\mathbb{Z}}\mathrm{ch}_{\mathbb{I}}(n;it)\begin{bmatrix}1\\0\end{bmatrix}\frac{\vartheta[^1_0](it)}{\eta^3(it)}=4+\mathcal{O}(q), \tag{A18}$$

$$\sum_{n\in\mathbb{Z}}(-)^n\widehat{\mathrm{ch}}_{\mathbb{I}}(n;it)\begin{bmatrix}1\\0\end{bmatrix}\frac{\vartheta[^1_0](it)}{\eta^3(it)}=4+\mathcal{O}(q). \tag{A19}$$

APPENDIX B: ONE-LOOP STRING AMPLITUDES

In this Appendix we provide the string theory one-loop amplitudes that are relevant for the discussion in the main text.

1. The closed string sector

The starting point of our construction in Sec. III is type 0B string theory on

$$\mathbb{R}^{3,1}\times\frac{SL(2)_1}{U(1)}. \tag{B1}$$

The closed string spectrum of this theory is summarized by the torus partition function:

$$\mathcal{T}_{0\mathrm{B}}=\frac{V}{2}\int\frac{\mathrm{d}\tau\mathrm{d}\bar\tau}{4\tau_2}\sum_{a,b\in\mathbb{Z}_2}\frac{\vartheta[^a_b](\tau)\vartheta[^a_b](\bar\tau)}{(8\pi^2\tau_2)^2\eta^3(\tau)\eta^3(\bar\tau)}\times\sum_{n,w\in\mathbb{Z}_2}\int_0^\infty\mathrm{d}p\,\mathrm{ch}_c\Big(p,\frac{n+w}{2};\tau\Big)\times\begin{bmatrix}a\\b\end{bmatrix}\mathrm{ch}_c\Big(p,\frac{n-w}{2};\bar\tau\Big)\begin{bmatrix}a\\b\end{bmatrix}. \tag{B2}$$

In this expression V is a factor that diverges proportionally to the overall volume of the space-time—this includes the volume of the 3 + 1 dimensional flat space-time and the volume of the cigar. The continuous representation characters were defined in the previous Appendix. As far as the fermions are concerned, (B2) is a typical, 0B diagonal, modular invariant sum.

The Klein bottle amplitude for the orientifold defined in Sec. III A, can be obtained in the transverse channel from the crosscap wave functions (3.10a) and (3.10b). It receives two contributions: one from the extended part of the crosscap state $|\mathcal{C};\mathrm{ext}\rangle$ and another from the localized part $|\mathcal{C};\mathrm{loc}\rangle$. In the direct channel (from which we can read off the orientifold action on closed string states), one finds by channel duality:

$$\mathcal{K}_{\mathrm{ext}}=\frac{1}{2}\int_0^\infty\frac{\mathrm{d}t}{4t^2}\frac{1}{(8\pi^2)^2}\langle\mathcal{C};\mathrm{ext}|e^{-(\pi/t)H_c}|\mathcal{C};\mathrm{ext}\rangle=-\frac{1}{2}\int_0^\infty\frac{\mathrm{d}t}{2t}\frac{1}{(8\pi^2t)^2}\sum_{a\in\mathbb{Z}_2}\sum_{n\in\mathbb{Z}}\int_0^\infty\mathrm{d}p\int_0^\infty\mathrm{d}p'\times\frac{\cos(4\pi pp')}{\sinh^2(\pi p)}(-)^{a+n+1}\mathrm{ch}_c\Big(p',\frac{n}{2};2it\Big)\times\begin{bmatrix}a\\1\end{bmatrix}\frac{\vartheta[^a_1](2it)}{\eta^3(2it)}, \tag{B3a}$$

$$\mathcal{K}_{\mathrm{loc}}=\frac{1}{2}\int_0^\infty\frac{\mathrm{d}t}{4t^2}\frac{1}{(8\pi^2)^2}\langle\mathcal{C};\mathrm{loc}|e^{-(\pi/t)H_c}|\mathcal{C};\mathrm{loc}\rangle=-\frac{1}{2}\int_0^\infty\frac{\mathrm{d}t}{4t^3}\frac{1}{(8\pi^2)^2}\sum_{a\in\mathbb{Z}_2}\sum_{n\in\mathbb{Z}}\int_0^\infty\mathrm{d}p\int_0^\infty\mathrm{d}p'\times\cos(4\pi pp')(-)^{a+1}\mathrm{ch}_c\Big(p',\frac{n}{2};2it\Big)\begin{bmatrix}a\\1\end{bmatrix}\frac{\vartheta[^a_1](2it)}{\eta^3(2it)}. \tag{B3b}$$

The extended contribution to the Klein bottle amplitude is, as expected, divergent and reproduces the expression anticipated from the known asymptotic form of the parity [48]. The localized contribution to the Klein bottle amplitude exhibits a delta-function density of states localized at $p'=0$.

2. The open string sectors

In this paper we consider D-brane setups with D3-branes (the color branes) and D5-branes (the flavor branes). There are three types of open strings: color-color strings, flavor-flavor strings and color-flavor strings.

3. Color-color strings

The annulus amplitude for N_c color D3-branes, characterized by the wave functions (3.14a) and (3.14b), reads in the open string channel

$$\mathcal{A}_{\text{D3-D3}} = V_4 N_c^2 \int_0^\infty \frac{dt}{4t} \frac{1}{(16\pi^2 t)^2} \sum_{a,b=0}^{1} \text{Ch}_{\mathbb{I}}(it)\begin{bmatrix} a \\ b \end{bmatrix} \times \frac{\vartheta[^a_b](it)}{\eta^3(it)}, \tag{B4}$$

where V_4 is the volume of $\mathbb{R}^{3,1}$. This expression involves only the characters of the identity representation, defined in Appendix A.

In the presence of the O′5 orientifold, the Möbius strip amplitude is

$$\mathcal{M}_{\text{D3;O}'5} = \mp V_4 N_c \int_0^\infty \frac{dt}{4t} \frac{1}{(16\pi^2 t)^2} \sum_{n\in\mathbb{Z}} (-)^n \sum_{b=0}^{1} \widehat{\text{ch}}_{\mathbb{I}}(n; it) \times \begin{bmatrix} 1 \\ b \end{bmatrix} \frac{\vartheta[^1_b]\left(it + \frac{1}{2}\right)}{\eta^3\left(it + \frac{1}{2}\right)}. \tag{B5}$$

The overall sign is fixed by the orientifold charge. For example, the − sign leads to a symmetric projection of the gauginos and arises from a positive orientifold charge. Note that the annulus amplitude (B4) is *twice* what it would be in type IIB, since the color brane is not a fundamental boundary state in type 0B. However, in type 0′B this brane is fundamental.

With the use of these amplitudes and the character expansions (A16)–(A19) one can deduce easily the low-energy degrees of freedom of *3-3* open strings.

4. Flavor-flavor strings

Next we consider the noncompact flavor D5-branes, whose wave functions are given by Eqs. (3.17a) and (3.17b). The annulus amplitude for two flavor branes with parameters (s_1, m_1) and (s_2, m_2) reads in the open string channel

$$\mathcal{A}_{\text{D5}(s_1,m_1)\text{-D5}(s_2,m_2)} = V_4 \int_0^\infty \frac{dt}{4t} \frac{1}{(16\pi^2 t)^2} \sum_{a,b=0}^{1} (-)^a \frac{\vartheta[^a_b]}{\eta^3} \times \int_0^\infty dP \Big\{ \rho_1(P; s_1|s_2) \text{Ch}_c\Big(P, m_1 - m_2 + \frac{1-a}{2}; it\Big)\begin{bmatrix} a \\ b \end{bmatrix} + (-)^b \rho_2(P; s_1|s_2) \times \text{Ch}_c\Big(P, m_1 - m_2 + \frac{a}{2}; it\Big)\begin{bmatrix} a \\ b \end{bmatrix}\Big\}. \tag{B6}$$

In our previous notation, $m_i = \theta_i/2\pi$. The spectral densities ρ_1, ρ_2 are given by the expressions

$$\rho_1(P; s_1|s_2) = 4 \int_0^\infty dP' \frac{\cos(4\pi s_1 P')\cos(4\pi s_2 P')\cos(4\pi P P')}{\sinh^2(2\pi P)}, \tag{B7a}$$

$$\rho_2(P; s_1|s_2) = 4 \int_0^\infty dP' \frac{\cos(4\pi s_1 P')\cos(4\pi s_2 P')\cos(4\pi P P')\cosh(2\pi P')}{\sinh^2(2\pi P)}. \tag{B7b}$$

Both densities have an infrared divergence at $P' \to 0$, which is related to the infinite volume of the cigar. This infinity can be regularized by computing *relative* annulus amplitudes, i.e. by substracting the annulus amplitude of a reference brane [12,73].

In Sec. III we consider D5-branes with $\theta = 0 \bmod \pi$. The setup of Sec. III C involves N_f D5 boundary states $|\overline{\text{D5}(s, 1/2)}\rangle$ for which the Möbius strip amplitude reads in the open string channel

$$\mathcal{M}_{\overline{\text{D5}(s,1/2)};\text{O}'5} = \mp N_f \int_0^\infty \frac{dt}{4t} \frac{1}{(16\pi^2 t)^2} \int_0^\infty dP \times \sum_{N\in\mathbb{Z}} \Big[(-)^N - \rho_3(P; s)\widehat{\text{ch}}_c(P, N; it)\begin{bmatrix} 1 \\ 0 \end{bmatrix} \times \frac{\vartheta[^1_0](it + 1/2)}{\eta 3(it + 1/2)} - \rho_4(P; s) \times \widehat{\text{ch}}_c\Big(P, N + \frac{1}{2}; it\Big)\begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{\vartheta[^1_0](it + 1/2)}{\eta 3(it + 1/2)} \Big] \tag{B8}$$

with spectral densities

$$\rho_3(P; s) = 2 \int_0^\infty dP' \frac{\cos(8\pi s P')\cos(4\pi P P')}{\sinh^2(2\pi P')} = \rho_1(P; s|s) - 2 \int_0^\infty dP' \frac{\cos(4\pi P P')}{\sinh^2(2\pi P')}, \tag{B9a}$$

$$\rho_4(P; s) = 2 \int_0^\infty dP' \frac{\cos(8\pi s P)\cos(4\pi P P')}{\cosh(2\pi P)}. \tag{B9b}$$

We observe that the density ρ_3, coming from the extensive part of the crosscap, is divergent, and coincides with the density ρ_1 in the annulus amplitude for two identical flavor branes, see Eq. (B7a), up to an s-independent term that cancels out when we compute a *relative amplitude* in order to remove the infrared divergence. The density ρ_4 is infrared finite, as it comes from the localized piece of the orientifold.

The low-energy spectrum of *5-5* open strings in Sec. III can be determined straightforwardly from the above amplitudes.

5. Color-flavor open strings

Flavor degrees of freedom in gauge theory arise from *3-5* strings. The open string spectrum of this sector can be determined from the annulus amplitude between a color D3-brane and a flavor D5-brane. By definition, there is no Möbius strip amplitude contributing to this sector. The flavor brane has parameters (s, θ). In order to cover the general case we will include here amplitudes involving flavor branes with both positive and negative RR-charge, i.e. both D5 and $\overline{\text{D5}}$ branes. We call $\varepsilon = \pm 1$ the sign of the flavor RR-charge. One finds in the open string channel:

$$\mathcal{A}_{\text{D3-D5}(s,\theta,\epsilon)} = \int_0^\infty \frac{\mathrm{d}t}{4t} \frac{1}{(16\pi^2 t)^2} \sum_{a,b=0}^{1} (-)^{a+b(1-\varepsilon)} \times \mathrm{Ch}_c\left(s, \frac{\theta}{2\pi} + \frac{a}{2}; it\right)\begin{bmatrix} a \\ b \end{bmatrix} \frac{\vartheta[{}^a_b](it)}{\eta^3(it)}. \quad \text{(B10)}$$

There are no nontrivial spectral densities in this case and one can read off the spectrum immediately by using the extended continuous character definition (A5).

APPENDIX C: RR CHARGES

In this Appendix we determine the RR scalar charge of the D3-, D5-branes and of the O′5-plane. The massless RR field C_0 has quantum numbers $P = 0$, $w = 0$, $n = 0$. The charge of the above objects is proportional to the one-point of C_0 on the disc. This quantity is provided by the wave-functions of the boundary/crosscap states in the main text.

Specifically, for D3-branes we have [see Eq. (3.14b)]

$$\lim_{P\to 0} \Phi_R(P, 0) = \mathcal{N}_{\text{D3}} \lim_{P\to 0} \frac{\Gamma(iP)}{\Gamma(2iP)} = 2\mathcal{N}_{\text{D3}}. \quad \text{(C1)}$$

For D5-branes of the type $\overline{|\text{D5}; 0, \frac{1}{2}\rangle}$ we have [see Eq. (3.17b)]:

$$\lim_{P\to 0} \Phi_R\left(0, \overline{\frac{1}{2}}; P, 0\right) = -\mathcal{N}_{\text{D5}} \lim_{P\to 0} \frac{\Gamma(-2iP)}{\Gamma(-iP)} = -\frac{\mathcal{N}_{\text{D5}}}{2} = -\mathcal{N}_{\text{D3}}. \quad \text{(C2)}$$

Finally, for the O′5-plane [see Eq. (3.10b)]

$$\lim_{P\to 0} (P, 0; +) = \mathcal{N}_{\text{O}'5} \lim_{P\to 0} \frac{\Gamma(-2iP)}{\Gamma(-iP)} = \frac{\mathcal{N}_{\text{O}'5}}{2} = 4\mathcal{N}_{\text{D3}}. \quad \text{(C3)}$$

In these expressions $\mathcal{N}_{\text{D3}}$, $\mathcal{N}_{\text{D5}} = 2\mathcal{N}_{\text{D3}}$, and $\mathcal{N}_{\text{O}'5} = 8\mathcal{N}_{\text{D3}}$ are, respectively, the normalizations of the D3, D5 boundary states and the O′5 crosscap state.

APPENDIX D: FORCES BETWEEN D-BRANES AND O-PLANES

Here we consider the forces that arise between the branes and/or orientifold planes at one string loop in our setups. By construction, the annulus amplitudes vanish, hence they do not give a net potential to the brane moduli. The Möbius strip amplitudes, however, get contributions from the RR sector only and break the Bose-Fermi degeneracy of the open string spectra. This breaking of supersymmetry leads to attractive or repulsive forces between the O′5 orientifold plane and the D-branes. Since the color D3-brane has no moduli, this contribution generates only a constant potential. However, the Möbius strip amplitude for the flavor D5-brane generates a potential for the brane modulus s, which characterizes the minimum distance of the brane from the tip of the cigar.

From the closed string point of view, the force between the orientifold and the flavor brane comes from the exchange of massless RR closed string modes. It is captured by the $t \to 0$ (or $\ell \equiv 1/t \to \infty$) asymptotics of the Möbius strip amplitude (B8):

$$\mathcal{M}_{\overline{\text{D5}(s,1/2)};\text{O}'5} \sim \pm N_f \int^\infty \mathrm{d}\ell \int_0^\infty \mathrm{d}P \frac{\cos(4\pi s P)}{\cosh(\pi P)} \times e^{-(\pi l/2)P^2}[1 + \mathcal{O}(e^{-(\pi l/2)})]_{\text{RR}}. \quad \text{(D1)}$$

Only the localized part of the orientifold sources massless fields, hence this is the only one that contributes in this regime. We are interested in the small s behavior of the function $F(s) \equiv -\partial_s \mathcal{M}_{\overline{\text{D5}(s,1/2)};\text{O}'5}$, where quarks from the *3-5* sector are nearly massless. $F(s)$ is the "force" felt by the flavor branes in the presence of the orientifold plane. Since we focus on the $\ell \to \infty$ asymptotics, the P-integral in (D1) is dominated by the region $P \to 0$. Hence, we get a linear expression in the modulus s at first order: $F(s) \propto \pm s$. This result shows that the force is attractive towards $s = 0$ for an orientifold with positive charge (OQCD-S setup). Similarly, for an orientifold with negative charge (the OQCD-AS setup) the force is repulsive.

It should be noted that this force, which arises as a one-loop effect on the flavor branes, has no consequences on the dynamics of the four-dimensional OQCD theory that was engineered in the intersection of D3- and D5-branes in Sec. III. In order to isolate the four-dimensional gauge theory dynamics from the rest of the open string degrees of freedom we need to take a low-energy decoupling limit where the dynamics on the flavor D5-branes are frozen. In this limit, only fields from the *3-3* and *3-5* sectors can run in loops to generate potentials. The force $F(s)$ appearing above is not one of these effects, because it arises from a loop in the *5-5* sector.

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