Dynamical Lorentz and *CPT* symmetry breaking in a 4D four-fermion model

M. Gomes,[1,*] T. Mariz,[1,+] J. R. Nascimento,[1,2,‡] and A. J. da Silva[1,§]

[1]*Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, 05315-970, São Paulo, SP, Brazil*

[2]*Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, 58051-970, João Pessoa, Paraíba, Brazil*

(Received 5 October 2007; published 5 May 2008)

In a 4D chiral Thirring model we analyze the possibility that radiative corrections may produce spontaneous breaking of Lorentz and *CPT* symmetry. By studying the effective potential, we verified that the chiral current $\bar{\psi}\gamma^\mu\gamma_5\psi$ may assume a nonzero vacuum expectation value which triggers Lorentz and *CPT* violations. Furthermore, by making fluctuations on the minimum of the potential we dynamically induce a bumblebee-like model containing a Chern-Simons term.

DOI: 10.1103/PhysRevD.77.105002 PACS numbers: 11.30.Cp, 11.15.Ex, 14.70.−e

I. INTRODUCTION

The Lorentz invariance is one of the most well-established symmetries in physics having survived a variety of stringent tests. Nevertheless, recently there has been an active interest on the possibility that more fundamental theories may induce small violations of Lorentz invariance into the standard model, at levels accessible to high precision experiments [1]. The original motivation for this idea arose from the fact that the spontaneous breaking of Lorentz symmetry may appear in the context of string theory [2] (in field theory the breaking was first studied in [3]). To systematically investigate this possibility, a standard model extension (SME) including all possible terms which may violate Lorentz and/or *CPT* invariance, was constructed [4].

The breaking of the Lorentz symmetry in the SME was generated by a procedure analogous to the Higgs mechanism in which a scalar field gains a vacuum expectation value (VEV) to furnish masses for the standard model particles. Nonzero expectation values for tensor fields that contain Lorentz indices select specific directions in the spacetime, breaking Lorentz invariance spontaneously. As an example, let us consider a toy model whose Lagrangian describes a vector field B_μ in such way to induce spontaneous Lorentz and *CPT* violation [5–7],

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ - m - eB\!\!\!/\gamma_5)\psi - \tfrac{1}{4}\lambda(B_\mu B^\mu - \beta^2)^2, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. The Maxwell form of the kinetic part of B_μ can be justified by energy considerations [8] without recourse to a gauge invariance principle. The self-interaction in this "bumblebee" model triggers a Lorentz and *CPT*-violating VEV $\langle B_\mu\rangle = \beta_\mu$. Very interesting terms are obtained when we consider fluctuations about the vacuum through the redefinition $B_\mu = \beta_\mu + A_\mu$, where the shifted field is assumed to have a zero VEV, $\langle A_\mu\rangle = 0$. The Lagrangian (1) becomes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ - m - b\!\!\!/\gamma_5 - eA\!\!\!/\gamma_5)\psi - \frac{1}{4}\lambda\Big(A_\mu A^\mu - \frac{2}{e}A\cdot b\Big)^2, \tag{2}$$

with $b_\mu = e\beta_\mu$, presenting the term $b_\mu\bar{\psi}\gamma^\mu\gamma_5\psi$ which violates the Lorentz and *CPT* symmetry. This term can be used to produce through radiative corrections the Chern-Simons Lagrangian [9],

$$\mathcal{L}_{\rm CS} = \tfrac{1}{2}\kappa^\mu\epsilon_{\mu\nu\lambda\rho}A^\nu F^{\lambda\rho}, \tag{3}$$

with $\kappa_\mu \propto b_\mu$, since they have the same C, P, and T transformation properties. Both at zero [9–23] and at finite temperature [24–28], in the non-Abelian case [29], and in contexts which include gravity [30,31], this issue has been carefully investigated.

In the present work, we will analyze the spontaneous breaking of Lorentz and *CPT* symmetry [32] via the Coleman-Weinberg mechanism [33]. Our objective is to examine the possibility of causing a spontaneous Lorentz and *CPT* symmetry breaking through radiative corrections starting from the self-interacting fermionic theory given by the Lagrangian

$$\mathcal{L}_0 = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{G}{2}(\bar{\psi}\gamma_\mu\gamma_5\psi)(\bar{\psi}\gamma^\mu\gamma_5\psi), \tag{4}$$

and dynamically inducing a bumblebee model with a Chern-Simons term. A similar mechanism was proposed a long time ago [34] as a way to generate the quantum electrodynamics (QED) through radiative corrections without invoking local $U(1)$ gauge invariance [35–37]. For some recent developments, see [38–40].

The model given by (4) is nonrenormalizable and must be thought as a low energy effective theory arising from a more fundamental, yet unknown theory, in the same sense as the original proposal of Nambu and Jona-Lasinio (NJL) [41] for QCD. As in the NJL model an ultraviolet (UV)

*mgomes@fma.if.usp.br
+tmariz@fma.if.usp.br
‡jroberto@fma.if.usp.br
§ajsilva@fma.if.usp.br

cutoff will be present in the results, which will represent our lack of knowledge of the physics beyond that scale. In fact, we will use a variant of the dimensional regularization prescription and the parameter $\epsilon = 4 - D$ will be present (a correspondence between ϵ and a momentum cutoff Λ is discussed in many places in the literature [42,43]).

This paper is organized as follows. In the Sec. II we show that a Higgs-like potential may be induced through radiative corrections from the Lagrangian (4), instead of having been added from the start as in the bumblebee model (1), leading to the appearance of a Lorentz- and CPT-violating VEV $\langle\bar{\psi}\gamma_\mu\gamma_5\psi\rangle \neq 0$. After taking into account fluctuations about this vacuum, the radiative corrections at one loop are examined in Sec. III. Section IV contains some final comments.

II. EFFECTIVE POTENTIAL

In order to eliminate the self-interaction term of Eq. (4), it is convenient to introduce an auxiliary field B_μ, so that the above Lagrangian can be rewritten as

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_0 + \frac{g^2}{2}\left(B_\mu - \frac{e}{g^2}\bar{\psi}\gamma_\mu\gamma_5\psi\right)^2 \\ &= \frac{g^2}{2}B_\mu B^\mu + \bar{\psi}(i\not\partial - m - e\not B\gamma_5)\psi,\end{aligned} \tag{5}$$

where $G = e^2/g^2$. To verify the possibility that a bumblebee potential can be induced through radiative corrections from this Lagrangian, we consider the generating functional defined as

$$Z(\bar{\eta}, \eta) = \int DB_\mu D\psi D\bar{\psi} e^{i\int d^4x(\mathcal{L} + \bar{\eta}\psi + \bar{\psi}\eta)}. \tag{6}$$

By performing the fermionic integration we get

$$\begin{aligned}Z(\bar{\eta}, \eta) = \int DB_\mu \exp\Big[& iS_{\rm eff}[B] \\ & + i\int d^4x\left(\bar{\eta}\frac{1}{i\not\partial - m - e\not B\gamma_5}\eta\right)\Big],\end{aligned} \tag{7}$$

where the effective action is given by

$$S_{\rm eff}[B] = \frac{g^2}{2}\int d^4x B_\mu B^\mu - i\,{\rm Tr}\ln(i\not\partial - m - e\not B\gamma_5). \tag{8}$$

The "Tr" stands for the trace over Dirac matrices as well as the trace over the integration in momentum or coordinate spaces. Thus, the effective potential turns out to be

$$V_{\rm eff} = -\frac{g^2}{2}B_\mu B^\mu + i\,{\rm tr}\int\frac{d^4p}{(2\pi)^4}\ln(\not p - m - e\not B\gamma_5), \tag{9}$$

where the classical field is in a coordinate independent configuration. As we are interested in verifying the existence of a nontrivial minimum, we look for solutions of the expression

$$\frac{dV_{\rm eff}}{dB_\mu}\bigg|_{B=\beta} = -\frac{g^2}{e}b^\mu - i\Pi^\mu = 0, \tag{10}$$

where $b^\mu = e\beta^\mu \neq 0$ and Π^μ is the one-loop tadpole amplitude:

$$\Pi^\mu = {\rm tr}\int\frac{d^4p}{(2\pi)^4}\frac{i}{\not p - m - \not b\gamma_5}(-ie)\gamma^\mu\gamma_5. \tag{11}$$

To evaluate this integral we will follow the perturbative route where now the propagator is the usual $S(p) = i(\not p - m)^{-1}$ and $-i\not b\gamma_5$ is considered as insertions in this propagator. At this point a graphical representation may be helpful. With the conventions indicated in Fig. 1 the contributions to Π^μ are shown in Fig. 2. Our regularization procedure, the dimensional reduction scheme [44], consists in calculating the traces of the Dirac matrices in 4 dimensions and afterwards promoting the metric tensor $g^{\mu\nu}$ and the integrals to D dimensions. Proceeding in this way, we found that the first and third graphs as well as graphs with more than three insertions vanish [45]. The remaining contributions, i.e., the second and fourth graphs, give

$$\Pi^\mu = \left[-\frac{im^2e}{\pi^2\epsilon} + \frac{im^2e}{2\pi^2}\ln\left(\frac{m^2}{\mu'^2}\right) - \frac{ib^2e}{3\pi^2}\right]b^\mu, \tag{12}$$

with $\epsilon = 4 - D$, $\mu'^2 = 4\pi\mu^2e^{-\gamma}$, and μ having been the renormalization spot. Then, the expression (10) can be rewritten as

$$\left[-\frac{1}{G_R} + \frac{m^2}{2\pi^2}\ln\left(\frac{m^2}{\mu'^2}\right) - \frac{b^2}{3\pi^2}\right]eb_\mu = 0, \tag{13}$$

where we have introduced the renormalized coupling constant

$$\frac{1}{G_R} = \frac{1}{G} + \frac{m^2}{\pi^2\epsilon}. \tag{14}$$

Therefore, we see that a nontrivial solution of this gap equation is

$$b^2 = -3\pi^2\left[\frac{1}{G_R} - \frac{m^2}{2\pi^2}\ln\left(\frac{m^2}{\mu'^2}\right)\right]. \tag{15}$$

From this equation we see that a nontrivial minimum with a timelike b_μ is possible if

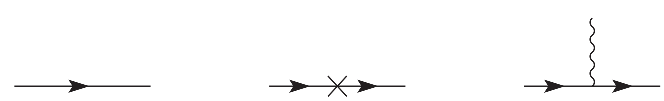

FIG. 1. Feynman rules. Continuous and wave lines represent the fermion propagator and the auxiliary field, respectively. The cross indicates the $-i\not b\gamma_5$ insertion in the fermion propagator and the trilinear vertex corresponds to $-ie\gamma^\mu\gamma^5$.

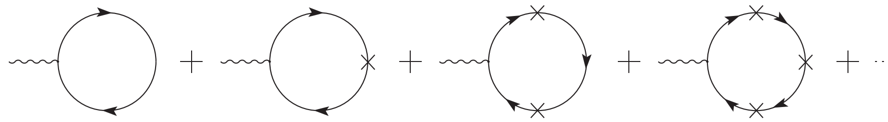

FIG. 2. Contributions to the tadpole Π^μ.

$$G_R > \frac{2\pi^2}{m^2 \ln(\frac{m^2}{\mu'^2})}, \tag{16}$$

whereas a nonzero spacelike b_μ requires

$$G_R < \frac{2\pi^2}{m^2 \ln(\frac{m^2}{\mu'^2})}. \tag{17}$$

The situation we are interested in is the case where the effective potential possesses a nonzero minimum given by Eq. (15), and therefore a VEV breaks the Lorentz invariance, i.e., $\langle B_\mu \rangle = \beta_\mu \neq 0$. This breaking of Lorentz invariance implies in a modification of the dispersion relation which may be useful in the study of ultrahigh energy cosmic rays [46,47].

III. ONE-LOOP CORRECTIONS AND THE INDUCED CHERN-SIMONS TERM

Let us now study the fluctuations, $B_\mu = \beta_\mu + A_\mu$, around the nontrivial minimum of the potential. We anticipate that, due to the breaking of the Lorentz and *CPT* symmetry, Chern-Simons terms will occur. The generating functional (7) expressed in terms of the shifted field is

$$Z(\bar\eta, \eta) = \int DA_\mu \exp\bigg[iS_{\rm eff}[A, b] + i\int d^4x\bigg(\bar\eta \frac{1}{i\not\partial - m - \not b\gamma_5 - e\not A\gamma_5}\eta\bigg)\bigg], \tag{18}$$

where the effective action is given by

$$S_{\rm eff}[A, b] = \int d^4x\bigg(\frac{g^2}{2}A_\mu A^\mu + \frac{g^2}{e}A_\mu b^\mu + \frac{g^2}{2e^2}b_\mu b^\mu\bigg) - i\,{\rm Tr}\ln(i\not\partial - m - \not b\gamma_5 - e\not A\gamma_5). \tag{19}$$

Up to a field independent factor which may be absorbed in the normalization of the generating functional, we get

$$S'_{\rm eff}[A, b] = \int d^4x\bigg(\frac{g^2}{2}A_\mu A^\mu + \frac{g^2}{e}A_\mu b^\mu\bigg) + S^{(n)}_{\rm eff}[A, b], \tag{20}$$

where

$$S^{(n)}_{\rm eff}[A, b] = i\,{\rm Tr}\sum_{n=1}^{\infty}\frac{1}{n}\bigg[\frac{i}{i\not\partial - m - \not b\gamma_5}(-ie)\not A\gamma_5\bigg]^n. \tag{21}$$

The formally divergent contributions in this formula are the tadpole, the self-energy, and the three and four point vertex functions of the field A_μ. The tadpole is given by

$$S^{(1)}_{\rm eff}[A, b] = i\,{\rm Tr}\frac{i}{i\not\partial - m - \not b\gamma_5}(-ie)\not A\gamma_5 = i\int d^4x\Pi^\mu A_\mu, \tag{22}$$

where Π^μ was given in (12) due to (10).

The self-energy term, which corresponds to $n = 2$, yields

$$S^{(2)}_{\rm eff}[A, b] = \frac{i}{2}\,{\rm Tr}\frac{i}{i\not\partial - m - \not b\gamma_5}(-ie)\not A\gamma_5\frac{i}{i\not\partial - m - \not b\gamma_5}\times(-ie)\not A\gamma_5 = \frac{i}{2}\int d^4x\Pi^{\mu\nu}A_\mu A_\nu, \tag{23}$$

where

$$\Pi^{\mu\nu} = {\rm tr}\int\frac{d^4p}{(2\pi)^4}\frac{i}{\not p - m - \not b\gamma_5}(-ie)\gamma^\mu\gamma_5 \times\frac{i}{\not p - i\not\partial - m - \not b\gamma_5}(-ie)\gamma^\nu\gamma_5. \tag{24}$$

By expanding in powers of $\not b\gamma_5$, the above result can be expressed graphically as in Fig. 3. The second and third graphs are separately finite and furnish a nonlocal Chern-Simons term. Similar to what happens in extended QED [48–50] the coefficient of this generated Chern-Simons term is ambiguous, i.e., different regularizations produce distinct results; for example, by using the 't Hooft-Veltman prescription [51,52] the coefficient vanishes. The divergent parts of the fourth, fifth, and sixth graphs cancel among themselves (we have also verified that graphs with three and four insertions of the vertex $-i\not b\gamma_5$ vanish); so only the first graph turns out to be divergent. We get

$$\Pi^{\mu\nu} = ie^2g^{\mu\nu}\bigg[-\frac{m^2}{\pi^2\epsilon} + \frac{m^2}{2\pi^2}\ln\bigg(\frac{m^2}{\mu'^2}\bigg) - \frac{b^2}{3\pi^2}\bigg] - \frac{ie^2}{6\pi^2\epsilon}(g^{\mu\nu}\Box - \partial^\mu\partial^\nu) + \frac{ie^2}{12\pi^2}\bigg[\ln\bigg(\frac{m^2}{\mu'^2}\bigg) + 1\bigg]\times(g^{\mu\nu}\Box - \partial^\mu\partial^\nu) - \frac{ie^2}{6\pi^2}\epsilon^{\mu\nu\lambda\rho}b_\lambda\partial_\rho - \frac{ie^2}{12\pi^2}\partial^\mu\partial^\nu - \frac{2ie^2}{3\pi^2}b^\mu b^\nu, \tag{25}$$

valid for $\Box/m^2 \ll 1$.

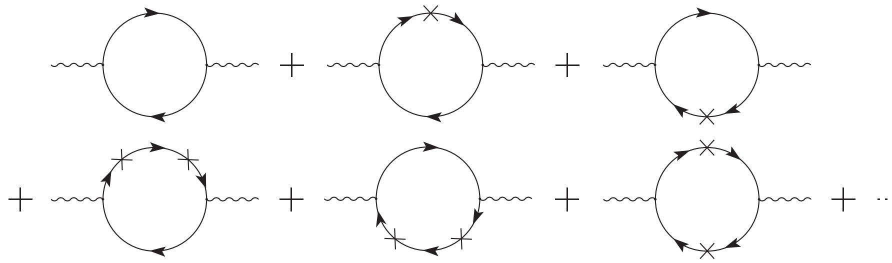

FIG. 3. Contributions to the vacuum polarization $\Pi^{\mu\nu}$.

Notice that UV divergences may also appear in the third term of the series in Eq. (21), as Furry theorem is not applicable. For $n = 3$ the expression (21) gives

$$S_{\rm eff}^{(3)}[A, b] = \frac{i}{3}\,\mathrm{Tr}\frac{i}{i\not\partial - m - \not b\gamma_5}(-ie)\not A\gamma_5\frac{i}{i\not\partial - m - \not b\gamma_5} \times(-ie)\not A\gamma_5\frac{i}{i\not\partial - m - \not b\gamma_5}(-ie)\not A\gamma_5 = \frac{i}{3}\int d^4x\Pi^{\mu\nu\rho}A_\mu A_\nu A_\rho, \tag{26}$$

where

$$\Pi^{\mu\nu\rho} = \mathrm{tr}\int\frac{d^4p}{(2\pi)^4}\frac{i}{\not p - m - \not b\gamma_5} \times(-ie)\gamma^\mu\gamma_5\frac{i}{\not p - i\not\partial - m - \not b\gamma_5} \times(-ie)\gamma^\nu\gamma_5\frac{i}{\not p - i\not\partial - i\not\partial' - m - \not b\gamma_5} \times(-ie)\gamma^\rho\gamma_5, \tag{27}$$

which, as a power series in $\not b\gamma_5$, is given by the graph expansion of Fig. 4. In the above formula the derivatives $\not\partial$ and $\not\partial'$ act on A_μ and A_ν, respectively. Because of properties of the trace of Dirac matrices the first graph results finite, whereas the divergent parts of the second, third, and fourth graphs cancel among themselves, in the same way as what happens with some one-loop contributions to Lorentz-violating QED [53]. The leading terms in the expansion in $\Box/m^2$ yield

$$\Pi^{\mu\nu\rho} = \frac{ie^3}{12\pi^2}(\epsilon^{\mu\nu\rho\lambda}\partial_\lambda - \epsilon^{\mu\nu\rho\lambda}\partial'_\lambda) + \frac{ie^3}{3\pi^2}(g^{\mu\nu}b^\rho + g^{\mu\rho}b^\nu + g^{\nu\rho}b^\mu). \tag{28}$$

In principle the fourth term of the series in (21) may be divergent but it results finite since the leading term is similar to the one in QED where, as it is known, it is finite. We obtain

$$S_{\rm eff}^{(4)} = \frac{e^4}{12\pi^2}\int d^4x(A_\mu A^\mu)^2 + \mathcal{O}\Big(\frac{\Box}{m^2}\Big). \tag{29}$$

The results obtained so far allow us to write the effective Lagrangian as

$$\mathcal{L} = -\frac{1}{4Z_3}F_{\mu\nu}F^{\mu\nu} + \frac{e^2}{24\pi^2}b^\mu\epsilon_{\mu\nu\lambda\rho}A^\nu F^{\lambda\rho} - \frac{e^2}{24\pi^2}(\partial_\mu A^\mu)^2 + \frac{e^4}{12\pi^2}\Big(A_\mu A^\mu - \frac{2}{e}A\cdot b\Big)^2 + \frac{e}{2b^2}A_\mu A^\mu\langle A_\nu\rangle b^\nu + \langle A_\mu\rangle A^\mu, \tag{30}$$

where

$$\frac{1}{Z_3} = \frac{e^2}{6\pi^2\epsilon} - \frac{e^2}{12\pi^2}\Big[\ln\Big(\frac{m^2}{\mu'^2}\Big) + 1\Big] \tag{31}$$

and

$$\langle A_\mu\rangle = \Big[\frac{1}{G_R} - \frac{m^2}{2\pi^2}\ln\Big(\frac{m^2}{\mu'^2}\Big) + \frac{b^2}{3\pi^2}\Big]eb_\mu. \tag{32}$$

The requirement that $\langle A_\mu\rangle = 0$, such that B_μ acquires a VEV $\langle B_\mu\rangle \neq 0$, was already studied in Eqs. (10)–(15), with the solutions (16) and (17). By defining a renormalized field $A_R^\mu = Z_3^{-1/2}A^\mu$ and a renormalized coupling constant $e_R = Z_3^{1/2}e$, we get

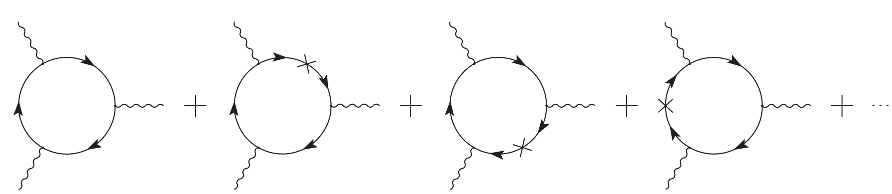

FIG. 4. Contributions to the three-point $\Pi^{\mu\nu\rho}$.

$$\mathcal{L} = -\frac{1}{4}F_{R\mu\nu}F_R^{\mu\nu} + \frac{e_R^2}{24\pi^2}b^\mu\epsilon_{\mu\nu\lambda\rho}A_R^\nu F_R^{\lambda\rho} - \frac{e_R^2}{24\pi^2} \times (\partial_\mu A_R^\mu)^2 + \frac{e_R^4}{12\pi^2}\left(A_{R\mu}A_R^\mu - \frac{2}{e_R}A_R\cdot b\right)^2. \quad (33)$$

This Lagrangian is exactly the extended QED by the Chern-Simons term, added of a gauge-fixing term and of a potential that do not trigger a Lorentz and *CPT* violation. We should stress that the (finite) Chern-Simons coefficient is ambiguous and depends on the particular regularization scheme used [48–50].

By substituting the expression (31) ($Z_3 \cong 6\pi^2\epsilon/e^2$) into the renormalized coupling constant, we obtain the result $e_R^2 \cong 6\pi^2\epsilon$ which is the same one for the induced QED [34,35,38,42,43]. In the limit $\epsilon \to 0$ we would have a trivial free theory with vanishing coupling constant. But as we remarked in the introduction we must keep ϵ at some small but nonvanishing value so that Eq. (33) has to be interpreted as an effective theory. Bumblebee models of this type have been discussed in flat and curved spacetime [54,55].

IV. CONCLUSIONS

We have shown that a bumblebee potential can be induced through radiative corrections from a 4D chiral Thirring model, as the conditions (16) and (17) hold for timelike and spacelike b_μ, respectively. By considering the fluctuations on the minimum of the potential, the QED extended by the Chern-Simons term is dynamically generated.

ACKNOWLEDGMENTS

Authors are grateful to Professor V. Alan Kostelecký for some enlightenments. This work was partially supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). The work by T. M. has been supported by FAPESP, Project No. 06/06531-4.

[1] *CPT and Lorentz Symmetry III*, edited by V. A. Kostelecký (World Scientific, Singapore, 2005).
[2] V. A. Kostelecký and S. Samuel, Phys. Rev. D **39**, 683 (1989); V. A. Kostelecký and R. Potting, Nucl. Phys. **B359**, 545 (1991).
[3] S. M. Carroll, G. B. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).
[4] D. Colladay and V. A. Kostelecký, Phys. Rev. D **55**, 6760 (1997); **58**, 116002 (1998).
[5] V. A. Kostelecký and R. Lehnert, Phys. Rev. D **63**, 065008 (2001).
[6] B. Altschul and V. A. Kostelecky, Phys. Lett. B **628**, 106 (2005).
[7] O. Bertolami and J. Paramos, Phys. Rev. D **72**, 044001 (2005).
[8] J. L. Chkareuli, C. D. Froggatt, and H. B. Nielsen, Nucl. Phys. **B609**, 46 (2001); Phys. Rev. Lett. **87**, 091601 (2001).
[9] R. Jackiw and V. A. Kostelecký, Phys. Rev. Lett. **82**, 3572 (1999).
[10] M. Pérez-Victoria, Phys. Rev. Lett. **83**, 2518 (1999).
[11] J. M. Chung and P. Oh, Phys. Rev. D **60**, 067702 (1999).
[12] J. M. Chung, Phys. Rev. D **60**, 127901 (1999).
[13] W. F. Chen, Phys. Rev. D **60**, 085007 (1999).
[14] J. M. Chung, Phys. Lett. B **461**, 138 (1999).
[15] C. Adam and F. R. Klinkhamer, Phys. Lett. B **513**, 245 (2001).
[16] G. Bonneau, Nucl. Phys. **B593**, 398 (2001).
[17] Yu. A. Sitenko, Phys. Lett. B **515**, 414 (2001).
[18] M. Chaichian, W. F. Chen, and R. González Felipe, Phys. Lett. B **503**, 215 (2001).
[19] J. M. Chung and B. K. Chung, Phys. Rev. D **63**, 105015 (2001).
[20] A. A. Andrianov, P. Giacconi, and R. Soldati, J. High Energy Phys. 02 (2002) 030.
[21] D. Bazeia, T. Mariz, J. R. Nascimento, E. Passos, and R. F. Ribeiro, J. Phys. A **36**, 4937 (2003).
[22] Y. L. Ma and Y. L. Wu, Phys. Lett. B **647**, 427 (2007).
[23] G. Bonneau, Nucl. Phys. **B764**, 83 (2007).
[24] J. R. Nascimento, R. F. Ribeiro, and N. F. Svaiter, arXiv: hep-th/0012039.
[25] L. Cervi, L. Griguolo, and D. Seminara, Phys. Rev. D **64**, 105003 (2001).
[26] D. Ebert, V. C. Zhukovsky, and A. S. Razumovsky, Phys. Rev. D **70**, 025003 (2004).
[27] T. Mariz, F. A. Brito, J. R. Nascimento, E. Passos, and R. F. Ribeiro, J. High Energy Phys. 10 (2005) 019.
[28] J. R. Nascimento, E. Passos, A. Y. Petrov, and F. A. Brito, J. High Energy Phys. 06 (2007) 016.
[29] M. Gomes, J. R. Nascimento, E. Passos, A. Y. Petrov, and A. J. da Silva, Phys. Rev. D **76**, 047701 (2007).
[30] T. Mariz, J. R. Nascimento, E. Passos, and R. F. Ribeiro, Phys. Rev. D **70**, 024014 (2004).
[31] T. Mariz, J. R. Nascimento, A. Y. Petrov, L. Y. Santos, and A. J. da Silva, arXiv:0708.3348.
[32] This possibility has been recently argued from an axion-Wess-Zumino model: A. A. Andrianov, R. Soldati, and L. Sorbo, Phys. Rev. D **59**, 025002 (1998).
[33] S. R. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).
[34] J. D. Bjorken, Ann. Phys. (N.Y.) **24**, 174 (1963).
[35] I. Bialynicki-Birula, Phys. Rev. **130**, 465 (1963).
[36] G. Guralnik, Phys. Rev. **136**, B1404 (1964).
[37] T. Eguchi, Phys. Rev. D **14**, 2755 (1976).

[38] J. Bjorken, arXiv:hep-th/0111196.
[39] P. Kraus and E. T. Tomboulis, Phys. Rev. D **66**, 045015 (2002).
[40] A. Jenkins, Phys. Rev. D **69**, 105007 (2004).
[41] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
[42] K. Akama and T. Hattori, Phys. Lett. B **392**, 383 (1997).
[43] K. Akama, arXiv:hep-ph/9706442.
[44] A. J. Buras, arXiv:hep-ph/9806471; G. Altarelli, G. Curci, G. Martinelli, and S. Petrarca, Nucl. Phys. B **187**, 461 (1981).
[45] By following a technique similar to the one in Ref. [11], using cutoff regularization we verified that higher powers than three in b_μ vanish.
[46] S. Coleman and S. L. Glashow, Phys. Rev. D **59**, 116008 (1999).
[47] S. R. Coleman and S. L. Glashow, Phys. Lett. B **405**, 249 (1997).
[48] R. Jackiw, Int. J. Mod. Phys. B **14**, 2011 (2000).
[49] M. Perez-Victoria, J. High Energy Phys. 04 (2001) 032.
[50] W. F. Chen, arXiv:hep-th/0106035.
[51] G. 't Hooft and M. J. G. Veltman, Nucl. Phys. **B44**, 189 (1972).
[52] P. Breitenlohner and D. Maison, Commun. Math. Phys. **52**, 11 (1977).
[53] V. A. Kostelecky, C. D. Lane, and A. G. M. Pickering, Phys. Rev. D **65**, 056006 (2002).
[54] V. A. Kostelecky, Phys. Rev. D **69**, 105009 (2004).
[55] R. Bluhm and V. A. Kostelecky, Phys. Rev. D **71**, 065008 (2005).