

Time delay of light signals in an energy-dependent spacetime metric

A. F. Grillo and E. Luzio

INFN - Laboratori Nazionali del Gran Sasso, SS. 17bis, 67010 Assergi (L'Aquila), Italy

F. Méndez

Departamento de Física, Universidad de Santiago de Chile, Av. Ecuador 3493, Casilla 307 Stgo-2, Chile
(Received 17 January 2008; published 28 May 2008)

In this paper we review the problem of time delay of photons propagating in a spacetime with a metric that explicitly depends on the energy of the particles (gravity-rainbow approach). We show that corrections due to this approach—which is closely related to the double special relativity proposal—produce for small redshifts ($z \ll 1$) smaller time delays than in the generic Lorentz invariance violating case.

DOI: [10.1103/PhysRevD.77.104033](https://doi.org/10.1103/PhysRevD.77.104033)

PACS numbers: 04.50.Kd, 11.30.Cp

I. INTRODUCTION

The idea that relativistic symmetry might not be preserved at all energy scales has been a subject of an intense debate and study during last years. Proposals of how to modify the Lorentz symmetry and models implementing this idea, from which measurable consequences can be obtained, have been investigated by large [1].

In very general grounds, these proposals can be divided in two types: (a) those where Lorentz symmetry is broken by choosing a preferred reference frame and (b) those where Lorentz symmetry is deformed and the relativistic principle is preserved. In the present paper we will focus on the consequences for the time of flight of photons in case (b).

Double special relativity (DSR) [2] models fall in case (b). Generically, they are nonlinear realizations of the Lorentz group that incorporate a second invariant scale (momentum or energy scale) in order to solve the following problem: if Lorentz symmetry is valid only up to certain energy (or momentum) scale, then this scale must be invariant for all observers on inertial reference frames.

Even if this idea has concrete realizations for the case of one particle in the momentum space, a consistent approach in spacetime and multiparticle sector is still a matter of intense debate [3].

A possible solution to spacetime problem is the so-called rainbow gravity [4,5]. Here, a spacetime is introduced that is dual to the momentum space where Lorentz group has a nonlinear realization: as a result, the metric of this spacetime is energy dependent. This approach admits also curved spacetimes which are solutions of (modified) Einstein equations. We will refer to this space (curved and energy dependent) as *rainbow spacetime* [4].

The problem we address here is related with this modifications of spacetime structure and the possibility of testing it by redshift and/or time-of-flight measurements. Specifically, we are interested in the modification of photon redshifts generated by DSR-like changes in the dispersion relation.

Let us briefly review the standard case. In the cosmological standard model, the metric of the universe is given by the Friedman-Robertson-Walker line element

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + d\Omega^2 \right), \quad (1)$$

where t, r, θ, ϕ are the usual cosmological coordinates and k is the three-dimensional space curvature (which we will take equal to zero from here on).

Redshift z is a wavelength (or frequency) shift due to the fact that light signals propagate in background (1) and relates the wavelength of the photon at emission (λ) with the wavelength λ_0 of the photon today (in cosmological terms), namely

$$z \equiv \frac{\lambda_0 - \lambda}{\lambda} = \frac{\lambda_0}{\lambda} - 1 \quad (2)$$

or in terms of energy

$$E = E_0(z + 1). \quad (3)$$

All these definitions are still valid in the deformed case since they do not depend on details of propagation. The relation between z and the scale factor $a(t)$, however, depends indeed of those details. In concrete, from the fact that metric has the shape (1),—see for example [6]—since the space is a maximally symmetric one and then spatial coordinates can be chosen as a comoving reference system, one has

$$\frac{a_0}{a} = \frac{\lambda_0}{\lambda} = z + 1, \quad (4)$$

with $a_0 = a|_{z=0}$. In the next section we will discuss how this property changes and explore the consequences for the calculation of proper distances and time delay of photons.

II. RAINBOW REDSHIFT

Consider now an energy-dependent metric¹

$$ds^2 = -f^{-2}(E)dt^2 + g^{-2}(E)a^2(t)d\chi^2, \quad (5)$$

with $d\chi^2 = \gamma_{ij}dx^i dx^j$ is the spatial line element. Even if it is possible to perform calculations for general functions f and g , in the present paper we will focus on the propagation on this spacetime up to first order in M_{Pl}^{-1} , that is, we will take functions with the shape

$$f(E) \sim 1 + \phi \frac{E}{M_{\text{Pl}}}, \quad g(E) \sim 1 + \gamma \frac{E}{M_{\text{Pl}}} \quad (6)$$

where ϕ , γ are numerical constants of order 1. For example, for a DSR1 deformation (following the classification in [7]) we have $\phi = 0$, $\gamma = 1/2$, while for DSR2 we have $f = g$ to all orders in E/M_{Pl} and therefore $\phi = \gamma$.

Note that the definition of redshift (2) does not change, however its relation with a [Eq. (4) in the standard case] does. Indeed, if the relation between wavelength and momentum remains unchanged one has, considering the modified dispersion relation

$$E^2 f^2(E) - p^2 g^2(E) = 0 = E^2 f^2(E) - \frac{1}{\lambda^2(E)} g^2(E)$$

and

$$\frac{f(E_0)}{f(E)} \frac{g(E)}{g(E_0)} \frac{a(t_0)}{a(t)} = \frac{E}{E_0}.$$

The last equality is an assumption, depending on the (unknown) QM in rainbow space-time. It is however natural, since it produces wavelengths approaching the Planck length when energies approach the Planck mass.

Using the definition of z we can write previous expression as

$$\frac{a(z)}{a_0} = \frac{f(E_0)}{g(E_0)} \frac{g(E_0(z+1))}{f(E_0(z+1))} \frac{1}{z+1}. \quad (7)$$

To summarize, the definition of redshift in this rainbow spaces is still (2), and this just says that to a measured energy E_0 corresponds an emitted energy E at redshift z . Equation (7), on the other hand, says that to this redshift z corresponds a given value of a , which is determined by equations of motion.

Notice that in this framework [4] photons of different energies see a different expansion.

In the following section we will use both relations to calculate the proper distance traveled by a photon that is received, at present, with energy E_0 .

A. Photons proper distances

The comoving (proper) distance is the integral of equations of motion and for simplicity we will consider only radial trajectories, that is

$$r(z, E_0) = \int_t^{t_0} \frac{g(E)}{f(E)} \frac{dt'}{a(t')}.$$

Here, the speed of photons is energy dependent through f and g ; the constant factor c has been chosen unity. The previous equation can be rewritten as an integral in z as in the standard case. From (7) we have

$$r(z, E_0) = \frac{g_0}{f_0 a_0} \int_0^z \frac{g(E_0(z'+1))}{f(E_0(z'+1))} \frac{dz'}{H(z')} \times \left[(z'+1) \frac{f(E_0(z'+1))}{g(E_0(z'+1))} \right] \frac{dz'}{H(z')},$$

where $f_0 = f(E_0)$, $g_0 = g(E_0)$, $a_0 = a(t_0)$ and $H(z) = \dot{a}(t)/a(t)$ and it is given by [4]

$$H = \left[\frac{8\pi}{3} G(E) \frac{\rho}{f^2} + \frac{\Lambda(E)}{3} \right]^{1/2}. \quad (8)$$

G and Λ can be, in principle, functions of the energy.

It is possible to rewrite $r(z, E_0)$ in order to show explicitly the modifications due to the energy dependence of the metric. A direct calculation allows us to write

$$r(z, E_0) = \frac{g_0}{f_0 a_0} \int_0^z \left[1 - (z'+1) \frac{d}{dz'} \ln \left(\frac{g}{f} \right) \right] \frac{dz'}{H(z')}, \quad (9)$$

where f , g , and H are evaluated in $E_0(z'+1)$.

This last expression is valid for any symmetry deformation, however in this general form is not useful to extract information about possible physical consequences. Since we are interested in linear corrections, we can circumvent this problem by considering deformations of the type (6), namely,

$$G(E) \sim G \left(1 + \Gamma \frac{E}{M_{\text{Pl}}} \right), \quad \Lambda(E) \sim \Lambda \left(1 + \lambda \frac{E}{M_{\text{Pl}}} \right), \quad (10)$$

with G , Λ the Newton and cosmological constants (in the limit $M_{\text{Pl}} \rightarrow \infty$) and Γ , λ , numerical constants of order 1.

With (6) and (10) we can calculate explicitly first order corrections to proper distances in (9), namely

$$r(z, E_0) = \frac{1}{a_0} \int_0^z \frac{dz'}{H_{\text{st}}(z')} - \frac{E_0}{M_{\text{Pl}} a_0} \int_0^z \frac{dz'}{H_{\text{st}}(z')} \times \left(z'(\gamma - \phi) + (z'+1) \frac{\delta H^2(z')}{2H_{\text{st}}^2(z')} \right), \quad (11)$$

where H_{st} is the standard Hubble parameter [that is $f = 1$, $G(E) = G$, $\Lambda(E) = \Lambda$ in (8)] while δH^2 is the first order correction due to the dependences on E

¹Here we follow the notation of Magueijo and Smolin in [4]

$$\delta H^2 = \frac{8\pi}{3} G\rho(\Gamma - 2\phi) + \frac{\Lambda\lambda}{3}. \quad (12)$$

Let us consider the example of DSR1. We have

$$r_{\text{DSR1}}(z, E_0) = \frac{1}{a_0} \int_0^z \frac{dz'}{H_{\text{st}}(z')} - \frac{E_0}{2M_{\text{Pl}}a_0} \int_0^z \frac{dz'}{H_{\text{st}}(z')} \times \left(z' + (z' + 1) \frac{\delta H^2(z')}{H_{\text{st}}^2(z')} \right). \quad (13)$$

Note that the behavior of this function depends on functions G , Λ . In fact, if we consider (a very conservative approach) $\Gamma = 0 = \lambda$, we have

$$r_{\text{DSR1}}(z, E_0) = \frac{1}{a_0} \int_0^z \left[1 - z' \frac{E_0}{2M_{\text{Pl}}} \right] \frac{dz'}{H_{\text{st}}(z')}. \quad (14)$$

which is different from one obtained recently by Jacob and Piran [8] and the reason, in this particular case, is that they use the standard relation between a and z while for us, it is given by (7). In our case, this introduces an extra factor g_0 , which cancels the 1 in $z' + 1$. For this particular example our result coincides with that in [8] only for $z \gg 1$. However, a rather different behavior appears if we consider nonvanishing Γ and λ . In fact, since they are of the order 1, then the last term in (13) is of order one, that is $\delta H^2(z')/2H_{\text{st}}^2(z') \sim 1$ and then a correction similar to the one obtained in [8] is obtained. An illustrative case is $\Gamma = \Lambda \equiv \sigma$ and it gives

$$r_{\text{DSR1}}(z, E_0) = \frac{1}{a_0} \int_0^z \left[1 - \frac{E_0}{2M_{\text{Pl}}} (z' + (z' + 1)\sigma) \right] \frac{dz'}{H_{\text{st}}(z')}. \quad (15)$$

We will return to that in the discussion section.

For DSR2, instead, the only possible corrections, in the present approach, come from functions $G(E)$, $\Lambda(E)$. In fact, we have in (11)

$$r(z, E_0) = \frac{1}{a_0} \int_0^z \frac{dz'}{H_{\text{st}}(z')} - \frac{E_0}{M_{\text{Pl}}a_0} \int_0^z \frac{dz'}{H_{\text{st}}(z')} (z' + 1) \times \frac{\delta H^2(z')}{2H_{\text{st}}^2(z')}. \quad (16)$$

and for $\Gamma = 0 = \lambda$ no corrections are obtained. Instead, for the other case discussed before $\Gamma = \lambda \equiv \sigma$ we have a correction of the type obtained in [8].

B. Photons time delay

The time of flight of a photon that travels between two points labeled by t and t_0 is

$$\delta t = \int_{t_0}^t dt' = \int_0^z \frac{da}{H(z')a},$$

where we have chosen t_0 as the present time. Using (7) the lookback time is

$$\delta t = \int_0^z \left[-\frac{1}{z+1} + \frac{d}{dz'} \ln\left(\frac{g}{f}\right) \right] \frac{dz'}{H(z')}, \quad (17)$$

where f and g are evaluated in $E_0(z' + 1)$. This expression gives the time that a photon takes to travel from a source at a given z to the present (with $z = 0$) if the energy measured now is E_0 [what means of course that the energy at the emission time were $E_0(z + 1)$].

Consider now two photons produced at z with different energies there, arriving at present time ($z = 0$) (of course, with different energies). In the standard case, only the first term in the RHS of (17) is present and it does not depend on energy, therefore the difference on time of flight between these two photons will be zero, as is well known. However, in the present case, due to the dependence on the final energy [second term in RHS of (17)] we will have the following difference for the lookback time

$$\Delta t = \int_0^z \Delta \left[\frac{1}{H(z')} \frac{d}{dz'} \ln\left(\frac{g}{f}\right) \right] dz', \quad (18)$$

with

$$\Delta \left[\frac{1}{H(z')} \frac{d}{dz'} \ln\left(\frac{g}{f}\right) \right] \equiv \frac{1}{H^{(1)}} \frac{d}{dz'} \ln\left(\frac{g(E_0^{(1)}(z' + 1))}{f(E_0^{(1)}(z' + 1))}\right) - \frac{1}{H^{(2)}} \frac{d}{dz'} \ln\left(\frac{g(E_0^{(2)}(z' + 1))}{f(E_0^{(2)}(z' + 1))}\right),$$

where $E_0^{(1)}$ and $E_0^{(2)}$ are the energies of the two photons, measured at $z = 0$ and $H^{(i)}$ is H defined in (8) evaluated in $E_0^{(i)}$, for $i = 1, 2$.

The previous expression is valid for general functions f , g , G , Λ . In order to analyze the behavior of it we will consider again only first order contributions, namely, (6) and (10). Note that, since the derivative of $\ln(g/f)$ is of the order E/M_{Pl} , then contributions due to Γ and λ will not be present. In fact, a straightforward calculation shows that, in the linear approach, (18) becomes

$$\Delta t = \frac{\gamma - \phi}{M_{\text{Pl}}} \int_0^z \Delta \left[\frac{1}{H_{\text{st}}(z')} E_0 \right] dz',$$

but H_{st} does not depend on the energy of particles, then

$$\Delta t = \frac{\Delta E_0(\gamma - \phi)}{M_{\text{Pl}}} \int_0^z \frac{dz'}{H_{\text{st}}(z')}, \quad (19)$$

We see again that, for DSR2 there will be no (energy dependent) delay while for DSR1 we have

$$\Delta t = \frac{\Delta E_0}{2M_{\text{Pl}}} \int_0^z \frac{dz'}{H(z')}, \quad (20)$$

which coincides with the result of [9]. Corrections due to the dependence of G and Λ on E turn out to be second order in E/M_{Pl} .

A similar question can be formulated about differences on proper distances. Namely, two photons produced by a

source at z , but with different energies there—and therefore with different energies at $z = 0$ —will they have a shift on their proper distances? The answer for this rainbow spacetime can be obtained directly from (9), but it is not so illuminating and therefore we will consider again the linear approach, that is (11). It is straightforward to see that in this case

$$\Delta r(z, E_0) = \frac{\Delta E_0}{M_{\text{Pl}} a_0} \int_0^z \frac{dz'}{H_{\text{st}}(z')} \times \left(z'(\gamma - \phi) + (z' + 1) \frac{\delta H^2(z')}{2H_{\text{st}}^2(z')} \right).$$

Clearly we can define a time delay $\bar{\Delta}t = a_0 \Delta r(z, E_0)$, and then

$$\bar{\Delta}t = \frac{\Delta E_0}{M_{\text{Pl}}} \int_0^z \frac{dz'}{H_{\text{st}}(z')} \left(z'(\gamma - \phi) + (z' + 1) \frac{\delta H^2(z')}{2H_{\text{st}}^2(z')} \right). \quad (21)$$

Some comments are in order here. First, the fact that a time delay can be defined proportional to the proper distance depends also on the relation between a and z , which in our case is not the standard one; however this gives rise to second order corrections in $1/M_{\text{Pl}}$, which have been discarded in the examples considered in the present paper. Second, we would like to call the attention on the fact that Δt in (19) and $\bar{\Delta}t$ in (21) are different because they measure different physical properties. The first is related to actual measurements of times while the second is based on measurements of proper distances.

To finalize the present discussion, let us point out that our results are strongly dependent on the choice of $G(E)$, $\Lambda(E)$.

III. DISCUSSION AND CONCLUSIONS

In this paper we have explored the consequences on the determination of (energy dependent) proper distances and arrival times of photons produced by sources at redshift z in doubly special relativity models with modified dispersion relations.

The major problem in performing this kind of calculation is the still uncertain knowledge of the spacetime structure compatible with this symmetry deformation. As we pointed out in the introduction, this is an open problem, even if much work (and progress) has been done.

On the other hand there exist (partial) proposals (for instance [4]) that allow us to treat, at least in a self-

consistent way, the problem of the propagation of particles in cosmological spacetimes.

For the discussion carried out in the present paper, we have used the proposal of Magueijo and Smolin where the spacetime metric depends on the energy of the particle that probes this spacetime (the so called “rainbow metric”) and satisfies (modified) Einstein equations for some matter distribution. In this approach photons move on a modified geodesics as a consequence of the modification of the dispersion relation, and experience a modified and energy-dependent cosmological expansion.

In this context we find that (at least for photons emitted at small z) the effects of the modifications on the photon geodesics and of the scale factor do compensate and one obtains for the time delay of photons of different energies emitted at same z the result reported by Ellis *et al.* in [9], in the particular case of DSR1 photons propagating in a (deformed) FRW universe with constant G and Λ .

As was pointed out in previous section, our results depend strongly on the functional form of $G(E)$ and $\Lambda(E)$ [as well as that of $f(E)$ and $g(E)$]; this just reflects the strong dependence of the time delay on the structure of spacetime.

In conclusion, we have shown that, in this model, for G and Λ constant, the DSR1 approach and Ref. [8], give different results for small z , while they coincide for large values of redshift [see relation (7)]. On the other hand, with linear corrections to G and Λ , the modifications can compensate and then a similar result to [8] is obtained. For DSR2, instead, possible corrections arise only from this last effect.

For the lookback time, corrections from the dependence of G , λ on the energy of photons, are second order effects in M_{Pl}^{-1} and then only relation (7) is relevant. We have shown that time delays calculated in this way coincide with results in [9].

Finally, we would like to emphasize that our result is different from that recently reported by Jacob and Piran [8], but there different hypotheses on the spacetime structure are made. This in principle leaves open the possibility of experimentally distinguishing among different phenomenological consequences of quantum gravity.

ACKNOWLEDGMENTS

One of us (F.M) would like to thank L.N.G.S. (INFN) for the kind hospitality during the preparation of this manuscript and for the grant Fondecyt-Chile 1060079.

[1] The following references are not intended to be a complete list of works on this subject: V. Alan Kostelecky and S. Samuel, Phys. Rev. D **39**, 683 (1989); V. Alan Kostelecky

and Robertus Potting, Nucl. Phys. **B359**, 545 (1991); G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, and S. Sarkar, Nature (London) **393**, 763

- (1998); D. Colladay and V. A. Kostelecký, *Phys. Rev. D* **58**, 116002 (1998); R. Gambini and J. Pullin, *Phys. Rev. D* **59**, 124021 (1999); T. Jacobson, S. Liberati, and D. Mattingly, *Phys. Rev. D* **66**, 081302 (2002); V. A. Kostelecký and Charles D. Lane, *Phys. Rev. D* **60**, 116010 (1999); R. Jackiw and V. A. Kostelecký, *Phys. Rev. Lett.* **82**, 3572 (1999); D. V. Ahluwalia, *Nature (London)* **398**, 199 (1999); Tadashi Kifune, *Astrophys. J.* **518**, L21 (1999); G. Z. Adunas, E. Rodriguez-Milla, and D. V. Ahluwalia, *Phys. Lett. B* **485**, 215 (2000); O. Bertolami and C. S. Carvalho, *Phys. Rev. D* **61**, 103002 (2000); J. Alfaro, H. A. Morales-Tecotl, and L. F. Urrutia, *Phys. Rev. Lett.* **84**, 2318 (2000); Y. Jack Ng, D. S. Lee, M. C. Oh, and H. van Dam, *Phys. Lett. B* **507**, 236 (2001); S. M. Carroll, J. A. Harvey, V. A. Kostelecky, C. D. Lane, and T. Okamoto, *Phys. Rev. Lett.* **87**, 141601 (2001); J. Alfaro and G. Palma, *Phys. Rev. D* **65**, 103516 (2002); J. Alfaro, H. A. Morales-Tecotl, and L. F. Urrutia, *Phys. Rev. D* **65**, 103509 (2002); V. A. Kostelecky and M. Mewes, *Phys. Rev. D* **66**, 056005 (2002); H. Vankov and T. Stanev, *Phys. Lett. B* **538**, 251 (2002); J. Alfaro and G. Palma, *Phys. Rev. D* **65**, 103516 (2002); R. Aloisio, P. Blasi, P. L. Ghia, and Aurelio F. Grillo, *Phys. Rev. D* **62**, 053010 (2000); R. Aloisio, P. Blasi, A. Galante, P. L. Ghia, and A. F. Grillo, *Astropart. Phys.* **19**, 127 (2003); R. Aloisio, P. Blasi, A. Galante, and A. F. Grillo, *Astropart. Phys.* **20**, 369 (2003); J. M. Carmona, J. L. Cortes, J. Gamboa, and F. Mendez, *Phys. Lett. B* **565**, 222 (2003); *J. High Energy Phys.* 03 (2003) 058; A. Chakrabarti, *J. Math. Phys. (N.Y.)* **44**, 3800 (2003); R. C. Myers and M. Pospelov, *Phys. Rev. Lett.* **90**, 211601 (2003); Abel Camacho, *Gravitation Cosmol.* **1**, 89 (2004); Y. Jack Ng, arXiv:gr-qc/0401015; Joao Magueijo and Lee Smolin, *Phys. Rev. D* **71**, 026010 (2005); F. W. Stecker and S. T. Scully, *Astropart. Phys.* **23**, 203 (2005); M. Bojowald, H. A. Morales-Tecotl, and H. Sahlmann, *Phys. Rev. D* **71**, 084012 (2005); O. Bertolami and J. G. Rosa, *Phys. Rev. D* **71**, 097901 (2005); P. A. Bolokhov, S. G. Nibbelink, and M. Pospelov, *Phys. Rev. D* **72**, 015013 (2005); A. Corichi and D. Sudarsky, *Int. J. Mod. Phys. D* **14**, 1685 (2005); J. Alfaro, *Phys. Rev. Lett.* **94**, 221302 (2005); A. Camacho, *Classical Quantum Gravity* **23**, 7355 (2006); T. Jacobson, S. Liberati, and D. Mattingly, *Ann. Phys. (N.Y.)* **321**, 150 (2006); S. Kanno and J. Soda, *Phys. Rev. D* **74**, 063505 (2006); J. Alfaro, A. A. Andrianov, M. Cambiaso, P. Giacconi, and R. Soldati, *Phys. Lett. B* **639** (2006); P. Gaete and C. Wotzasek, *Phys. Rev. D* **75**, 057902 (2007); P. M. Crichigno and H. Vucetich, *Phys. Lett. B* **651**, 313 (2007); U. Jacob and T. Piran, *Nature Phys.* **3**, 87 (2007); R. Lehnert, arXiv:0711.4851; M. Gomes, J. R. Nascimento, E. Passos, A. Yu. Petrov, and A. J. da Silva, *Phys. Rev. D* **76**, 047701 (2007); Arianto, F. P. Zen, B. E. Gunara, Triyanta, and Supardi, *J. High Energy Phys.* 09 (2007) 048; J. M. Carmona, J. L. Cortes, and J. Indurain (Zaragoza U.) arXiv:0709.2267; N. E. Mavromatos, arXiv:0708.2250.
- [2] G. Amelino-Camelia, *Int. J. Mod. Phys. D* **11**, 35 (2002); G. Amelino-Camelia, arXiv:0106004; G. Amelino-Camelia, *Int. J. Mod. Phys. D* **11**, 1643 (2002); J. Magueijo and L. Smolin, *Phys. Rev. Lett.* **88**, 190403 (2002); *Phys. Rev. D* **67**, 044017 (2003).
- [3] Jerzy Kowalski-Glikman, *Phys. Lett. B* **547**, 291 (2002); A. A. Deriglazov, *Phys. Lett. B* **603**, 124 (2004); S. Mignemi, *Int. J. Mod. Phys. D* **15**, 925 (2006); *Phys. Rev. D* **72**, 087703 (2005); S. Gao and Xiao-ning Wu, arXiv:gr-qc/0311009; D. Kimberly, J. Magueijo, and J. Medeiros, *Phys. Rev. D* **70**, 084007 (2004); S. Hossenfelder, *Phys. Rev. D* **75**, 105005 (2007); *Phys. Lett. B* **649**, 310 (2007).
- [4] J. Magueijo and L. Smolin, *Classical Quantum Gravity* **21**, 1725 (2004).
- [5] P. Galan and G. A. Mena Marugan, *Int. J. Mod. Phys. D* **16**, 1133 (2007).
- [6] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (John Wiley & Sons, New York, 1972).
- [7] J. Lukierski and A. Nowicki, in *Physical and Mathematical Aspects of Symmetries*, edited by J-P Gazeau *et al.*, (Institute of Physics, Bristol, England, 2002), p. 287.
- [8] U. Jacob and T. Piran, *J. Cosmol. Astropart. Phys.* 01 (2008) 031.
- [9] J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos, and A. S. Sakharov, *Astron. Astrophys.* **402**, 409 (2003).