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$f(R)$ gravity: From the Pioneer anomaly to cosmic acceleration

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We use metric formalism in $f(R)$ modified gravity to study the dynamics of various systems from the solar system to the cosmological scale. We assume an ansatz for the derivative of action as a function of distance and describe the Pioneer anomaly and the flat rotation curve of the spiral galaxies. Having the asymptotic behavior of action, we propose the action of $f(R) = (R + \Lambda)(1 + \ln(R/R_c)/(R/R_0 + 2/\alpha))$ where in galactic and solar system scales it can recover our desired form. The vacuum solution of this action also results in a positive late time acceleration for the Universe. We fix the parameters of this model, comparing with the Pioneer anomaly, rotation curve of spiral galaxies, and supernova type Ia gold sample data.

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I. INTRODUCTION

Recent observations of the supernova type Ia and cosmic microwave background (CMB) radiation indicate that the Universe is under positive accelerating expansion [1]. This accelerating expansion is one of the important puzzles of the contemporary physics. A nonzero vacuum energy can drive the Universe to accelerate however one can ask why it is nonzero and why it is so small [2]. Adding a simple cosmological constant term to the Einstein equations can also accelerate the Universe. However, the problem with the cosmological constant is explaining why the energy density of matter and the cosmological constant are in the same order at the present time?

A varying dark energy model can partially solve this problem in which the density of dark energy traces the density of matter from the early Universe to the present time. Modified gravity can also provide an effective time varying equation of state. In these models the Einstein-Hilbert action is replaced with a generic form of $f(R)$ gravity [3]. In addition to the late time cosmic expansion, early inflationary era also can be achieved by an extra term to the action, as adding a cubic term to the $1/R$ gravity model [4]. Modifying action not only affects the dynamics of the Universe, it can also alter the dynamics at the galactic or solar system scales.

There are two main approaches of metric and Palatini formalism to extract the field equations from the action. Considering the non-Levi-Civita connection associated with the manifold, we can take the connection and the metric as the independence geometrical quantities. Varying the action with respect to these two parameters (so-called Palatini formalism) results in the field equations [5,6]. On the other hand in the metric formalism the connection is the Levi-Civita connection and we do variation of action with respect to the metric to derive modified gravity field equations. The advantage of the Palatini formalism is that the field equations are second order differential equations similar to the other parts of the physics.

One of the interesting issues in $f(R)$ gravity is studying the spherically symmetric solutions. In the case of Palatini formalism the solution is Schwarzschild–de Sitter metric with an effective cosmological constant. However in the metric formalism the solution of non-Einstein-Hilbert action suffers from a low-mass equivalent scalar field that is incompatible with solar system tests of general relativity, as long as the scalar field propagates over solar system scales [7,8]. One of the solutions to avoid the solar system tests is using action in such a way that reduces to the Einstein-Hilbert action in the low curvature regime at the solar system and, at the cosmological regime, acts as an effective cosmological constant [9,10]. The other problem in this issue is the consistency of the spherically symmetric solutions in $f(R)$ gravity [11] that will be addressed in this paper.

In this work, we try to extract an appropriate action for the modified gravity through the inverse solution. This method has been applied in the previous works both in the galactic [12] and cosmological scales [13]. Here we extend the previous works to the solar system scale, studying anomalies in the Pioneer acceleration, and obtain the appropriate action in the solar system scale. On the other hand following the method proposed by Capozziello *et al.* [14] we extract an appropriate action to provide a flat rotation curve in the spiral galaxies. Both the solar system and the galactic scale solutions are consistent with the modified gravity field equations in the first order of approximation. Finally we propose a generic function for the action to cover all the mentioned scales and in addition to provide a late time acceleration for the Universe. At the end we use the observational data of the Pioneer anomalies in solar system, rotation curve of the galaxies, and supernova type Ia in the cosmological scales to put constraint on the parameters of the model.

The organization of the paper is as follows: In Sec. II we introduce the modified gravity in metric formalism. Using an ansatz for the derivative of the action, we solve the field equation for the spherically symmetric metric, and derive the dynamics in solar system and galactic scales. We use the observational data in the solar system as well as the flat rotation curve of the spiral galaxies to constrain the parameters of the model. In Sec. III we propose a generic action where in the small scales it reduces to the appropriate actions in the galactic and solar system scales. Supernova type Ia gold sample data provide compatible results with the other observations. The conclusion is presented in Sec. IV.

II. SPHERICALLY SYMMETRIC SPACE

Let us take an action for the gravity that depends only on the Ricci scalar as $f(R)$ where in the simple case of $f(R) = R$, it is the so-called Einstein-Hilbert action. For a generic $f(R)$, there are two main approaches to extract the field equations. The first one is so-called "metric formalism" in which the variation of action is performed with respect to the metric. In the second approach, "Palatini formalism," the connection and metric are considered independent of each other and we do the variation for those two parameters independently. In this work we will follow the metric formalism.

A generic form of the action depending on the Ricci scalar can be written as follows:

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R) + S_m. \tag{1}$$

Varying the action with respect to the metric results in the field equations as

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)F(R) = \kappa T_{\mu\nu}, \tag{2}$$

where $F = df/dR$ and $\Box \equiv \nabla_\alpha\nabla^\alpha$. From Eq. (2), we take the trace and obtain the action in terms of F and Ricci scalar as

$$f(R) = \frac{1}{2}(3\Box F + FR - \kappa T). \tag{3}$$

By taking derivative from Eq. (3) with respect to r (radial coordinate of the metric) we rewrite this equation in terms of F and R as follows:

$$RF' - FR' + 3(\Box F)' = \kappa T', \tag{4}$$

where $' \equiv d/dr$ and $f'(R) = F(R)R'$.

Replacing $f(R)$ in favor of $F(R)$, we obtain the field equation in terms of $F(R)$

$$R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R = \frac{\kappa}{F}\left(T_{\mu\nu} - \frac{1}{4}g_{\mu\nu}T\right) + \frac{1}{F}\left(\nabla_\mu\nabla_\nu F - \frac{1}{4}g_{\mu\nu}\Box F\right). \tag{5}$$

Following the method introduced in [15], we solve the time-independent spherical symmetric field equation in the vacuum. Let us take a generic spherically symmetric metric as

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2. \tag{6}$$

Since the metric depends only on r, one can view Eq. (5) as a set of differential equations for $F(r)$, $B(r)$, and $A(r)$. For the spherically symmetric space both sides of Eq. (5) are diagonal and we have two independent equations. We rewrite Eq. (5) as

$$K_{[\mu]} = \frac{FR_{\mu\mu} - \nabla_\mu\nabla_\mu F - \kappa T_{\mu\mu}}{g_{\mu\mu}}, \tag{7}$$

where $K_{[\mu]}$ is an index independent parameter and $K_{[\mu]} - K_{[\nu]} = 0$ for all μ and ν. For the vacuum space $T_{\mu\nu} = 0$, $K_{[t]} - K_{[r]} = 0$ results in

$$2F\frac{X'}{X} + rF'\frac{X'}{X} - 2rF'' = 0, \tag{8}$$

where $X(r) = B(r)A(r)$. For $K_{[t]} - K_{[\theta]} = 0$,

$$B'' + \left(\frac{F'}{F} - \frac{1}{2}\frac{X'}{X}\right)B' - \frac{2}{r}\left(\frac{F'}{F} - \frac{1}{2}\frac{X'}{X}\right)B - \frac{2}{r^2}B + \frac{2}{r^2}X = 0. \tag{9}$$

In the case of Einstein-Hilbert action ($F = 1$) Eq. (8) reduces to $X = 1$ and Eq. (9) reduces to the Schwarzschild solution. We note that for this case Eq. (4) also reduces to $0 = 0$ identity. In the generic case having a F as a function of distance or as a function of Ricci scalar, we can obtain the metric elements from Eqs. (8) and (9).

Here we take an ansatz of $F(r) = (1 + r/d)^{-\alpha}$ for the derivative of action as a function of distance from the center, where α is a small dimensionless constant ($\alpha \ll 1$) and d is a characteristic length scale in the order of galactic size. Similar to the case of $F = 1$ we use Eqs. (8) and (9) to derive X and A. We start with the Eq. (8), the solution results in

$$X(r) = X_0\left(1 + \frac{r}{d}\right)^{-2(1+\alpha)}\left(1 + \frac{2-\alpha}{2}\frac{r}{d}\right)^{[4(1+\alpha)]/(2-\alpha)}, \tag{10}$$

where X_0 is a constant of integration and for $\alpha = 0$ we recover Schwarzschild metric, which implies $X_0 = 1$.

In what follows we obtain the metric element $B(r)$ by solving the differential equation of (9) for the solar system scales ($r \ll d$) and galactic scales ($r > d$). Once we derive the metric, the Ricci scalar and the corresponding action can be obtained. Finally we will use Eq. (4) to check the consistency of the solution.

A. Solar system scale ($r \ll d$)

In 1998 Anderson *et al.* [16] reported an unmodeled constant acceleration towards the Sun of about $a_P = 8.5 \times 10^{-10}$ m/s^2 for the spacecrafts Pioneer 10 (launched 2 March 1972), Pioneer 11 (launched 4 December 1973), Galileo (launched 18 October 1989) and Ulysses (launched 6 October 1990). In a subsequent report [17] they discussed in detail many suggested explanations for the effect and gave the value $a_P = (8.74 \pm 1.33) \times 10^{-10}$ m/s^2 directed towards the Sun. The data covered many years starting in 1980 when due to the large distance (20 AU) of Pioneer 10 from the Sun the solar radiation pressure became sufficiently small. The data were collected up to 1990 for Pioneer 11 (30 AU) and up to 1998 (70 AU) for Pioneer 10. In this section our aim is to explain this extra acceleration by the modified gravity model.

We assume the range of $r \ll d$ in our concern and neglect all the higher terms of r/d and $\alpha r/d$. $F(r)$ in this regime reduces to

$$F(r) = 1 - \frac{\alpha}{d} r. \tag{11}$$

This expression is similar to adding the first order of the perturbation of the action around the Einstein-Hilbert action. From Eq. (10), expanding X up to the first order results in $X = X_0 = 1$.

Using (11) in the differential Eq. (9) we obtain $B(r)$ as follows:

$$B(r) = \left[1 + \frac{\alpha}{d} r + \left(\frac{3}{2} + \ln\left|\frac{\alpha}{d} - \frac{1}{r}\right|\right)\frac{\alpha^2}{d^2} r^2\right] + c_1 r^2 + c_2\left[\frac{1}{3r} + \frac{\alpha}{2d} + \frac{\alpha^2}{d^2} r + \frac{\alpha^3}{d^3} r^2 \ln\left|\frac{\alpha}{d} - \frac{1}{r}\right|\right], \tag{12}$$

where c_1 and c_2 are the constants of the integration. For the case of $\alpha = 0$ (Einstein-Hilbert action) we use the Schwarzschild metric as the zero order which implies $c_1 = 0$ and $c_2 = -6m$. Using Eq. (12) we can obtain the Ricci scalar of this metric. We keep up to the first order of perturbation in the metric as

$$B(r) = 1 - \frac{2m}{r} + \frac{\alpha}{d} r. \tag{13}$$

The Ricci scalar in the spherically symmetric space for a generic case of X is:

$$R = -\frac{1}{X}\left[B'' + \frac{4}{r} B' + \frac{2}{r^2} B - \frac{X'}{X}\left(\frac{1}{2} B' + \frac{2}{r} B\right)\right] + \frac{2}{r^2}, \tag{14}$$

where substituting the metric elements, the corresponding Ricci scalar up to the first order, obtains as

$$R(r) = -\frac{6\alpha}{rd}. \tag{15}$$

Now to check the consistency condition, we apply the metric element as well as the Ricci scalar in Eq. (4). Here $\Box F$ up to the first order reduce to

$$\Box F = \frac{B}{X}\left(F'' + \frac{2}{r} F' - \frac{1}{2}\frac{X'}{X} F' + \frac{B'}{B} F'\right), \tag{16}$$

$$= -\frac{2\alpha}{rd}. \tag{17}$$

Doing simple algebra shows the consistency of the equation for the trace equation up to the first order of perturbation.

Now we replace r in favor of $R(r)$ in Eq. (11) and obtain $F(R)$ in terms of the Ricci scalar as

$$F(R) = 1 + \frac{6\alpha^2}{Rd^2}. \tag{18}$$

Finally integrating (18) yields action as follows:

$$f(R) = R + R_0 \ln\frac{R}{R_c}, \tag{19}$$

where $R_0 = 6\alpha^2/d^2$ and R_c is the constant of integration.

The equation of motion for a test particle from the metric can be obtained. Using the weak field regime, we define an effective potential as

$$\phi_N = -\frac{m}{r} + \frac{\alpha}{2d} r, \tag{20}$$

where the acceleration of the particles from this potential is

$$a = -\frac{m}{r^2} - \frac{\alpha}{2d}. \tag{21}$$

The first term at the right-hand side of this equation is the standard Newtonian gravity; however the second term is a constant acceleration, independent of the mass. We may correspond this extra term to the Pioneer anomalies and constrain it with the observed value of $a_P = (8.74 \pm 1.33) \times 10^{-10}$ m/s^2 which results in $\alpha/d \simeq 10^{-26} m^{-1}$.

B. Galactic scale ($r > d$)

Recently some of the authors have tried to explain the dynamics of galaxies by the modified gravity instead of assuming a dark matter halo for the galaxy [14]. We follow the same method to extract the action with the ansatz of $F(r) = (1 + r/d)^{-\alpha}$ proposed in this work. We assume r to be larger than the characteristic length scale of the model d and write $F(r) = (1 + r/d)^{-\alpha}$ for $\alpha \ll 1$ as follows:

$$F(r) \simeq (r/d)^{-\alpha} \simeq 1 - \alpha \ln(r/d). \tag{22}$$

This action can be considered as perturbation around the Einstein-Hilbert action. From Eq. (10)

$$X(r) = \left(\frac{r}{d}\right)^{\alpha}. \quad (23)$$

We follow the same procedure as we did in the case of solar system to extract the metric. Using Eqs. (22) and (23) in (9) we obtain $B(r)$ as

$$B(r) = \left(\frac{r}{d}\right)^{\alpha}\left[\frac{1}{1-\alpha} + e_1 r^{-(1-\alpha/2)} + e_2 r^{2(1-\alpha/2)}\right] = \frac{1}{1-\alpha}\left(\frac{r}{d}\right)^{\alpha}[1 + e'_1 r^{-(1-\alpha/2)} + e'_2 r^{2(1-\alpha/2)}], \quad (24)$$

where e_i's are the constants of integration and $e'_i = e_i(1-\alpha)$ for $i = 1, 2$. For $\alpha = 0$ Eqs. (22) and (23) reduce to $F = 1$ and $X = 1$ and we expect to recover Schwarzschild–de Sitter metric which yields $e'_1 = -2m$ and $e'_2 = \frac{1}{12}\Lambda$, where Λ is the cosmological constant. For generic case when $\alpha \neq 0$, from the dimensional analysis, the constants of the integration obtain as $e'_1 = -(2m)^{1-\alpha/2}$ and $e'_2 = (\Lambda/12)^{1-\alpha/2}$. We rewrite the metric elements after fixing e'_i's,

$$B(r) = \frac{1}{1-\alpha}\left[1 - \left(\frac{2m}{r}\right)^{1-\alpha/2} + \left(\frac{\Lambda r^2}{12}\right)^{(1-\alpha/2)}\right]\left(\frac{r}{d}\right)^{\alpha},$$
$$A(r) = (1-\alpha)\left[1 - \left(\frac{2m}{r}\right)^{1-\alpha/2} + \left(\frac{\Lambda r^2}{12}\right)^{(1-\alpha/2)}\right]^{-1}. \quad (25)$$

From the metric elements we get the following Ricci scalar

$$R(r) = -\frac{1}{(1-\alpha)r^2}\left[3\alpha + (12 - 3\alpha)\left(\frac{\Lambda r^2}{12}\right)^{1-\alpha/2} + \frac{\alpha^2}{2}\left(1 - 3\left(\frac{2m}{r}\right)^{1-\alpha/2}\right)\right]. \quad (26)$$

We keep the Ricci scalar up to the first order term in α and Λ, then (26) reduces to

$$R(r) = -\frac{3\alpha}{r^2} - \Lambda. \quad (27)$$

Again to check the consistency of the solution in this space, we substitute (22) and (27) in Eq. (4). The solution of this space satisfies the trace equation up to the first order of perturbation in terms of α. We note that in general, for the nonperturbed case the solutions might be inconsistent. Here the solution is valid only up to the first order of perturbation.

Eliminating r in favor of R from Eq. (27) and using (22), the derivative of action we obtain as

$$F(R) = \left(\frac{d^2}{3\alpha}|R + \Lambda|\right)^{\alpha/2}, \quad (28)$$

then

$$f(R) = \frac{1}{1+\alpha/2}\left(\frac{d^2}{3\alpha}\right)^{\alpha/2}|R + \Lambda|^{1+\alpha/2}. \quad (29)$$

For simplicity let us write action as

$$f(R) = f_0|R + \Lambda|^{1+\alpha/2}. \quad (30)$$

The dynamics of a test particle around this metric follow the geodesic equation in weak field regime,

$$\ddot{r} + \Gamma^r{}_{tt} = 0, \quad (31)$$

where substituting the corresponding metric elements we get the following velocity for a particle rotating around the center of a galaxy

$$v = \frac{c}{\sqrt{2}}\left(\frac{r}{d}\right)^{\alpha/2}\left[\left(\frac{2m}{r}\right)^{1-\alpha/2} + \alpha\right]^{1/2}, \quad (32)$$

where we ignored the Λr^2 term as it is 5 orders of magnitude smaller than $2m/r$. For $\alpha = 0$ we recover the standard Newtonian law for the rotation velocity of a test particle, in which $m = GM/c^2$ and M is the mass of galaxy. The extra term in Eq. (32) may provide contribution to the flat rotation curve. Figure 1 compares the rotation curve of a test particle around the center of a galaxy with an arbitrary unit in the modified and standard Newtonian gravity. Here we model the mass of the galaxy spherically distributed up to 3.3 kpc and obtain the rotation curve up to 66 kpc. For a typical spiral galaxy with the mass of $M = 10^{11}M_\odot$ and at the large distances (e.g. $r > d$) from the center, $v \sim 200$ km s^{-1} which roughly constrains $\alpha \simeq 10^{-6}$. In the previous Section we had an estimation for $\alpha/d \simeq 10^{-26}m^{-1}$ which provides the characteristic length scale of the model, $d \simeq 10$ kpc. We note that while Eq. (32) provides a flat rotation curve for the galaxy it does not support the Tully-Fisher relation.

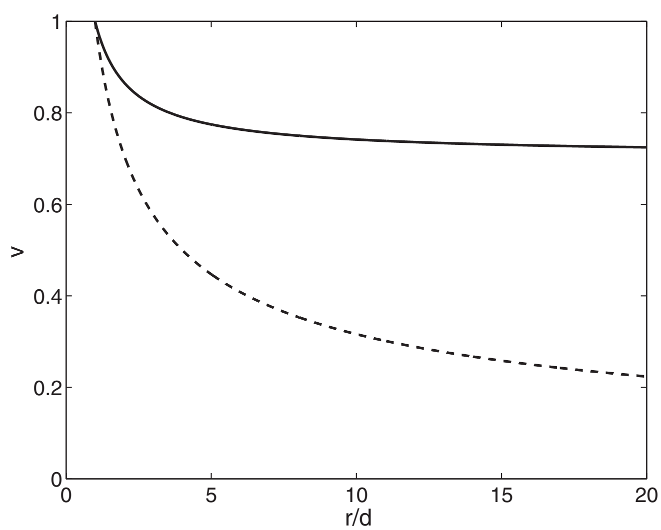

FIG. 1. Comparing the rotation curve of a galaxy in Newtonian gravity, $v \propto 1/r$ (dashed line) with the rotation curve from the modified gravity in the weak field regime (solid line), see Eq. (32). The parameters of the galaxy are taken as $M = 10^{11}M_\odot$ (mass of galaxy), $\alpha \simeq 10^{-6}$, and $d \simeq 10$ kpc.

III. PROPOSING A GENERIC ACTION

In the previous Section we obtained the asymptotic behavior of an action in the galactic and solar system scales. Those two actions could describe the observations in the corresponding length scales without need of dark matter. The action for the small scale varies with a logarithmic function and for the galactic scales with a power-law function. This asymptotic behavior of the actions guides us to guess a generic action which can cover also those two scales. Here we propose the action of

$$f(R) = R + \Lambda + \frac{R+\Lambda}{R/R_0 + 2/\alpha}\ln\frac{R+\Lambda}{R_c}. \qquad (33)$$

For the range of $R \gg \Lambda$ and $R/R_0 \gg 2/\alpha$, the action reduces to

$$f(R) = R + R_0 \ln\left(\frac{R}{R_c}\right). \qquad (34)$$

Comparing with (19) provides $R_0 = 6\alpha^2/d^2$. Using $|R(r)| = 6\alpha/rd$ and $R/R_0 \gg 1/\alpha$ satisfies the solar system range of $r \ll d$.

On the other hand for $\alpha \ll 1$ and $R \simeq R_0 \simeq \Lambda$ the action (33) can be written as

$$f(R) = (R+\Lambda)\left[1 + \frac{\alpha}{2}\ln\left(\frac{R+\Lambda}{R_c}\right)\right], \qquad (35)$$

where for small α, we write the action as

$$f(R) = \frac{(R+\Lambda)^{1+\alpha/2}}{R_c^{\alpha/2}}. \qquad (36)$$

For $\alpha \ll 1$, the action reduces to $f(R) = R + \Lambda$. We expect the best parameters of the model from supernova type Ia and CMB experiments should be around the ΛCDM model. To see the consistency of this action with the matter dominant epoch, we let $R \gg \Lambda$ and $R \gg R_0$. In this case the action reduces to $f(R) \to R$ (i.e. Einstein-Hilbert action) and the scale factor changes as $a \propto t^{2/3}$ with time.

In what follows we put constraint on the parameters of the model in (35). The generic Friedmann-Robertson-Walker (FRW) equation in modified gravity is

$$3H\dot{F} + 3H^2F - \frac{1}{2}(f - RF) = \kappa\rho_m. \qquad (37)$$

We use supernova type Ia gold sample with considering flat universe to constrain Ω_m, Ω_Λ, and α. Using action of (35), Eq. (37) is written as follows:

$$H^2 - \frac{\Lambda}{6} + \frac{\alpha}{2}\left\{H^2\left[\frac{R}{R+\Lambda} + \ln\frac{R+\Lambda}{R_c}\right] + \frac{R^2}{R+\Lambda} + \frac{R+2\Lambda}{(R+\Lambda)^2}H\dot{R}\right\} = H_0^2\Omega_m a^{-3}, \qquad (38)$$

where $3H_0^2\Omega_m = \kappa\rho_m$. For $\alpha = 0$ we recover the standard FRW equation,

$$H = H_0(\Omega_m a^{-3} + \Omega_\Lambda)^{1/2}. \qquad (39)$$

On the other hand, variation of action with respect to the metric preserves the conservation of energy momentum, so the matter density changes as $\rho = \rho_0 a^{-3}$.

From the constraint of the rotation curve of the spiral galaxies in the previous Section, $\alpha \simeq 10^{-6}$, we assume $\alpha \ll 1$ and this term is considered as a perturbation parameter in Eq. (38). We solve Eq. (38) by perturbing the Hubble parameter around ΛCDM solution, $H = H^{(0)} + \alpha H^{(1)}$, in which $H^{(0)}$ obtains from Eq. (39) and $H^{(1)}$ is calculated from

$$H^{(1)} = -\frac{1}{4}\frac{H^{(0)}}{R^{(0)}+\Lambda}\left\{H^{(0)}R^{(0)} + \frac{R^{(0)}+2\Lambda}{R^{(0)}+\Lambda}\dot{R}^{(0)} + \frac{R^{(0)2}}{6H^{(0)}} + H^{(0)}(R^{(0)}+\Lambda)^3\ln\frac{R^{(0)}+\Lambda}{R_c}\right\}, \qquad (40)$$

where $R^{(0)}$ and $\dot{R}^{(0)}$ are the zero order terms obtained from $H^{(0)}$ and $\dot{H}^{(0)}$. The relevant parameter for comparing the theoretical model with supernova type Ia data is the luminosity distance $D_L = D_L(z; \Omega_m, \alpha, R_c, \Lambda, h)$ and is related to the distance modulus of the supernovae as follows:

$$\mu = m - M = 5\log_{10}\left[\frac{D_L}{10pc}\right],$$
$$D_L = c(1+z)\int_0^z \frac{dz}{H(z; \Omega_m, \alpha, R_c, \Lambda, h)}, \qquad (41)$$

where the K correction is included in the distance modulus of the supernovae.

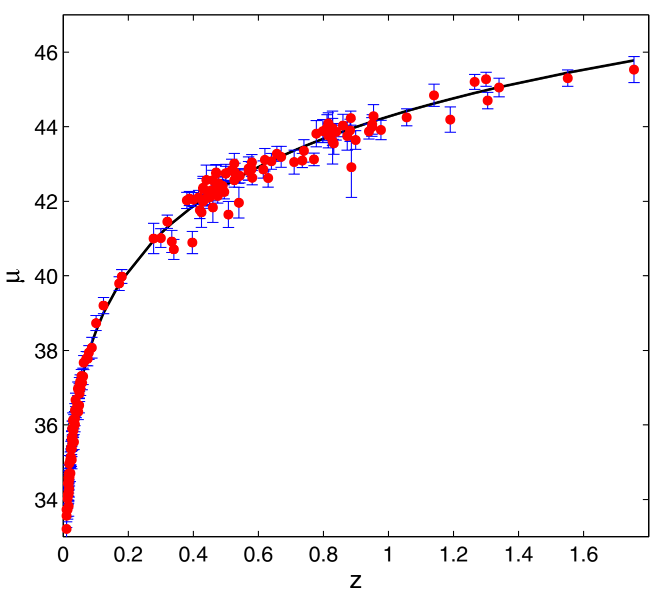

FIG. 2 (color online). Distance modulus of the supernova type Ia new gold sample in terms of redshift. The solid line shows the best fit values with the corresponding parameters of $h = 0.64$, $\Omega_m = 0.31$, $\Omega_\Lambda = 0.69$, and $\alpha \ll 10^{-3}$ with the corresponding $\chi^2_{\rm min}/N_{\rm dof} = 1.14$.

We do likelihood analysis using the Hubble parameter h, the cosmological parameters Ω_m and Ω_Λ, and α as the free parameters to find the best values. The comparison between the observed and theoretical distance moduli is done by χ^2 fitting as follows:

$$\chi^2 = \sum_i \frac{[\mu_{\rm obs}(z_i) - \mu_{\rm th}(z_i; \Omega_m, \alpha, R_c, \Lambda, h)]^2}{\sigma_i^2}. \quad (42)$$

The best value for χ^2, normalized to the number of degrees of freedom is $\chi^2/N_{\rm dof} = 1.14$ (see Fig. 2). The corresponding best values for the parameters of the model are: $\Omega_m = 0.31$, $\Omega_\Lambda = 0.69$, $h = 0.64$, and $\alpha \ll 10^{-3}$. The constraint on α is consistent with the results from the rotational velocity of spiral galaxies, $\alpha \simeq 10^{-6}$. Finally we should point out that R_c is not sensitive to the supernova type Ia data.

IV. SUMMARY AND DISCUSSION

In this work we tried to explain the anomalies in the acceleration of the Pioneer spacecraft and flat rotation curve of spiral galaxies in the framework of the modification of the gravity. We started by assuming an ansatz for the derivative of action in terms of distance from the center and did the inverse procedure to derive the metric and action of the space. In the solar system scale we extract a logarithmic extra term to the Einstein-Hilbert action and in the galactic scale we follow the same procedure and found a power-law action. The solution in both two regimes obtained as perturbation around the Einstein-Hilbert action and we showed that within this approximation the solutions are consistent with the modified gravity equations. We note that in the generic case we may not find a consistent solution in the spherical space [11]. Finally we proposed a generic action where in the asymptotic regimes reduce to our desired metric and actions in the solar system and galactic scales. For the cosmological scales this action provides a late time acceleration for the Universe. Finally we used the Pioneer data, flat rotation curve of galaxies, and the CMB and supernova type Ia gold sample to put constraint on the parameters of the model.

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[1] G. Hinshaw *et al.*, Astrophys. J. Suppl. Ser. **170**, 288 (2007); D. N. Spergel *et al.*, Astrophys. J. Suppl. Ser. **170**, 377 (2007); C. L. Bennett *et al.*, Astrophys. J. Suppl. Ser. **148**, 1 (2003); A. G. Riess *et al.* (Supernova Search Team Collaboration), Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.* (Supernova Cosmology Project Collaboration), Astrophys. J. **517**, 565 (1999); J. L. Tonry *et al.*, Astrophys. J. **594**, 1 (2003); C. B. Netterfield *et al.* (Boomerang Collaboration), Astrophys. J. **571**, 604 (2002); N. W. Halverson *et al.*, Astrophys. J. **568**, 38 (2002).

[2] V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000).

[3] S. M. Carroll, V. Duvvuri, M. Trodden, and M. S. Turner, Phys. Rev. D **70**, 043528 (2004); S. Nojiri and S. D. Odintsov, Phys. Rev. D **68**, 123512 (2003); Gen. Relativ. Gravit. **36**, 1765 (2004); Mod. Phys. Lett. A **19**, 627 (2004); G. J. Olmo, Phys. Rev. D **72**, 083505 (2005); S. Baghram, M. Farhang, and S. Rahvar, Phys. Rev. D **75**, 044024 (2007); M. S. Movahed, S. Baghram, and S. Rahvar, Phys. Rev. D **76**, 044008 (2007).

[4] T. P. Sotiriou, Phys. Rev. D **73**, 063515 (2006).

[5] D. N. Vollick, Phys. Rev. D **68**, 063510 (2003); G. Allemandi, A. Borowiec, and M. Francaviglia, Phys. Rev. D **70**, 043524 (2004); G. J. Olmo, W. Komp, X. H. Meng, and P. Wang, Classical Quantum Gravity **20**, 4949 (2003); X. H. Meng and P. Wang, Classical Quantum Gravity **21**, 951 (2004); G. M. Kremer and D. S. M. Alves, Phys. Rev. D **70**, 023503 (2004); G. Allemandi, A. Borowiec, M. Francaviglia, and S. D. Odintsov, Phys. Rev. D **72**, 063505 (2005).

[6] G. J. Olmo, Phys. Rev. Lett. **95**, 261102 (2005).

[7] A. L. Erickcek, T. L. Smith, and M. Kamionkowski, Phys. Rev. D **74**, 121501 (2006).

[8] T. Chiba, T. L. Smith, and A. L. Erickcek, Phys. Rev. D **75**, 124014 (2007).

[9] A. A. Starobinsky, JETP Lett. **86**, 157 (2007).

[10] W. Hu and I. Sawicki, Phys. Rev. D **76**, 064004 (2007).

[11] R. Saffari and S. Rahvar, arXiv:0710.5635.

[12] Y. Sobouti, Astron. Astrophys. **464**, 921 (2007); R. Saffari and Y. Sobouti, Astron. Astrophys. **472**, 833 (2007).

[13] S. Rahvar and Y. Sobouti, arXiv:astro-ph/0704.0680 [Mod. Phys. Lett. A (to be published)].

[14] S. Capozziello, V. F. Cardone, and A. Troisi, Mon. Not. R. Astron. Soc. **375**, 1423 (2007).

[15] T. Multamaki and I. Vilja, Phys. Rev. D **74**, 064022 (2006).

[16] J. D. Anderson *et al.*, Phys. Rev. Lett. **81**, 2858 (1998).

[17] J. D. Anderson *et al.*, Phys. Rev. D **65**, 082004 (2002).