

Topological black holes in Lovelock-Born-Infeld gravity

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In this paper, we present topological black holes of third order Lovelock gravity in the presence of cosmological constant and nonlinear electromagnetic Born-Infeld field. Depending on the metric parameters, these solutions may be interpreted as black hole solutions with inner and outer event horizons, an extreme black hole or naked singularity. We investigate the thermodynamics of asymptotically flat solutions and show that the thermodynamic and conserved quantities of these black holes satisfy the first law of thermodynamic. We also endow the Ricci flat solutions with a global rotation and calculate the finite action and conserved quantities of these class of solutions by using the counterterm method. We compute the entropy through the use of the Gibbs-Duhem relation and find that the entropy obeys the area law. We obtain a Smarr-type formula for the mass as a function of the entropy, the angular momenta, and the charge, and compute temperature, angular velocities, and electric potential and show that these thermodynamic quantities coincide with their values which are computed through the use of geometry. Finally, we perform a stability analysis for this class of solutions in both the canonical and the grand-canonical ensemble and show that the presence of a nonlinear electromagnetic field and higher curvature terms has no effect on the stability of the black branes, and they are stable in the whole phase space.

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I. INTRODUCTION

Over the last few years, several extra-dimensional models have been introduced in an attempt to deal with the hierarchy problem. These models can lead to rather unique and spectacular signatures at Terascale colliders such as the LHC and ILC. In higher dimensions, it is known that the Einstein-Hilbert (EH) Lagrangian, R , can only be regarded as the first order term in an effective action, so one may on general grounds expect that as one probes energies approaching the fundamental scale, significant deviations from EH expectations are likely to appear. This motivates one to consider the more general class of gravitational action:

$$I_G = \frac{1}{16\pi} \int d^{n+1}x \sqrt{-g} \mathcal{F}(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}).$$

The presence of higher curvature terms can also be seen in the renormalization of quantum field theory in curved spacetime [1], or in the construction of low energy effective action of string theory [2]. Among the higher curvature gravity theories, the so-called Lovelock gravity is quite special, whose Lagrangian consist of the dimensionally extended Euler densities. This Lagrangian is obtained by Lovelock as he tried to calculate the most general tensor that satisfies properties of Einstein's tensor in higher dimensions [3]. Since the Lovelock tensor contains derivatives of metrics of order not higher than two, the

quantization of linearized Lovelock theory is free of ghosts [4]. Thus, it is natural to study the effects of higher curvature terms on the properties and thermodynamics of black holes.

Accepting the nonlinear terms of the invariants constructed by Riemann tensor on the gravity side of the action, it seems natural to add the nonlinear terms in the matter action too. Thus, in the presence of an electromagnetic field, it is worthwhile to apply the action of Born-Infeld [5] instead of the Maxwell action. In this paper, we generalize static and rotating black hole solutions of third order Lovelock gravity in the presence of Maxwell field [6,7] to the case of these solutions in the presence of nonlinear electromagnetic fields. Indeed, it is interesting to explore new black hole solutions in higher curvature gravity and investigate which properties of black holes are peculiar to Einstein gravity, and which are robust features of all generally covariant theories of gravity. The first aim to relate the nonlinear electrodynamics and gravity has been done by Hoffmann [8]. He obtained a solution of the Einstein equations for a pointlike Born-Infeld charge, which is devoid of the divergence of the metric at the origin that characterizes the Reissner-Nordström solution. However, a conical singularity remained there, as it was later objected by Einstein and Rosen. The spherically symmetric solutions in Einstein-Born-Infeld gravity with or without a cosmological constant have been considered by many authors [9,10], while the rotating solutions of this theory is investigated in [11]. Also, these kinds of solutions in the presence of a dilaton field have been introduced in [12]. The static black hole solutions of Gauss-Bonnet-

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Born-Infeld gravity have been constructed in Ref. [13], and the rotating solution of this theory has been considered in [14].

The outline of our paper is as follows. We present the topological black holes of third order Lovelock gravity in the presence of Born-Infeld field in Sec. II. In Sec. III, we calculate the thermodynamic quantities of asymptotically flat solutions and investigate the first law of thermodynamics. In Sec. IV we introduce the rotating solutions with flat horizon and compute the thermodynamic and conserved quantities of them. We also perform a stability analysis of the solutions both in canonical and grand canonical ensemble. We finish our paper with some concluding remarks.

II. TOPOLOGICAL BLACK HOLES

The action of third order Lovelock gravity in the presence of nonlinear Born-Infeld electromagnetic field is

$$I_G = \frac{1}{16\pi} \int d^{n+1}x \sqrt{-g} (-2\Lambda + \mathcal{L}_1 + \alpha_2 \mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + L(F)), \quad (1)$$

where Λ is the cosmological constant, α_2 and α_3 are the second and third order Lovelock coefficients, $\mathcal{L}_1 = R$ is just the Einstein-Hilbert Lagrangian, $\mathcal{L}_2 = R_{\mu\nu\gamma\delta} R^{\mu\nu\gamma\delta} - 4R_{\mu\nu} R^{\mu\nu} + R^2$ is the Gauss-Bonnet Lagrangian,

$$\begin{aligned} \mathcal{L}_3 = & 2R^{\mu\nu\sigma\kappa} R_{\sigma\kappa\rho\tau} R^{\rho\tau}_{\mu\nu} + 8R^{\mu\nu}_{\sigma\rho} R^{\sigma\kappa}_{\nu\tau} R^{\rho\tau}_{\mu\kappa} \\ & + 24R^{\mu\nu\sigma\kappa} R_{\sigma\kappa\nu\rho} R^\rho_{\mu} + 3RR^{\mu\nu\sigma\kappa} R_{\sigma\kappa\mu\nu} \\ & + 24R^{\mu\nu\sigma\kappa} R_{\sigma\mu} R_{\kappa\nu} + 16R^{\mu\nu} R_{\nu\sigma} R^\sigma_{\mu} \\ & - 12RR^{\mu\nu} R_{\mu\nu} + R^3 \end{aligned} \quad (2)$$

is the third order Lovelock Lagrangian, and $L(F)$ is the Born-Infeld Lagrangian given as

$$L(F) = 4\beta^2 \left(1 - \sqrt{1 + \frac{F^2}{2\beta^2}} \right). \quad (3)$$

In the limit $\beta \rightarrow \infty$, $L(F)$ reduces to the standard Maxwell form $L(F) = -F^2$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Varying the action (1) with respect to the metric tensor $g_{\mu\nu}$ and electromagnetic vector field A_μ the equations of gravitation and electromagnetic fields are obtained as:

$$\begin{aligned} G_{\mu\nu}^{(1)} + \Lambda g_{\mu\nu} + \alpha_2 G_{\mu\nu}^{(2)} + \alpha_3 G_{\mu\nu}^{(3)} \\ = \frac{1}{2} g_{\mu\nu} L(F) + \frac{2F_{\mu\lambda} F^\lambda_{\nu}}{\sqrt{1 + \frac{F^2}{2\beta^2}}}, \end{aligned} \quad (4)$$

$$\partial_\mu \left(\frac{\sqrt{-g} F^{\mu\nu}}{\sqrt{1 + \frac{F^2}{2\beta^2}}} \right) = 0, \quad (5)$$

where $G_{\mu\nu}^{(1)}$ is the Einstein tensor, and $G_{\mu\nu}^{(2)}$ and $G_{\mu\nu}^{(3)}$ are the second and third order Lovelock tensors given as [15]:

$$\begin{aligned} G_{\mu\nu}^{(2)} = & 2(R_{\mu\sigma\kappa\tau} R_{\nu}^{\sigma\kappa\tau} - 2R_{\mu\rho\nu\sigma} R^{\rho\sigma} - 2R_{\mu\sigma} R^{\sigma}_{\nu} \\ & + RR_{\mu\nu}) - \frac{1}{2} \mathcal{L}_2 g_{\mu\nu}, \end{aligned} \quad (6)$$

$$\begin{aligned} G_{\mu\nu}^{(3)} = & -3(4R^{\tau\rho\sigma\kappa} R_{\sigma\kappa\lambda\rho} R^\lambda_{\nu\tau\mu} - 8R^{\tau\rho}_{\lambda\sigma} R^{\sigma\kappa}_{\tau\mu} R^\lambda_{\nu\rho\kappa} + 2R_{\nu}^{\tau\sigma\kappa} R_{\sigma\kappa\lambda\rho} R^\lambda_{\tau\mu} - R^{\tau\rho\sigma\kappa} R_{\sigma\kappa\tau\rho} R_{\nu\mu} + 8R^{\tau}_{\nu\sigma\rho} R^{\sigma\kappa}_{\tau\mu} R^\rho_{\mu\kappa} \\ & + 8R^{\sigma}_{\nu\tau\kappa} R^{\tau\rho}_{\sigma\mu} R^\kappa_{\rho} + 4R_{\nu}^{\tau\sigma\kappa} R_{\sigma\kappa\mu\rho} R^\rho_{\tau} - 4R_{\nu}^{\tau\sigma\kappa} R_{\sigma\kappa\tau\rho} R^\rho_{\mu} + 4R^{\tau\rho\sigma\kappa} R_{\sigma\kappa\tau\mu} R_{\nu\rho} + 2RR_{\nu}^{\kappa\tau\rho} R_{\tau\rho\kappa\mu} \\ & + 8R^{\tau}_{\nu\mu\rho} R^\rho_{\sigma} R^{\sigma\tau}_{\mu} - 8R^{\sigma}_{\nu\tau\rho} R^{\tau\rho}_{\sigma\mu} - 8R^{\tau\rho}_{\sigma\mu} R^{\sigma\tau}_{\nu\rho} - 4RR^{\tau\rho}_{\nu\mu\rho} R^\rho_{\tau} + 4R^{\tau\rho} R_{\rho\tau} R_{\nu\mu} - 8R^{\tau}_{\nu} R_{\tau\rho} R^\rho_{\mu} \\ & + 4RR_{\nu\rho} R^\rho_{\mu} - R^2 R_{\nu\mu}) - \frac{1}{2} \mathcal{L}_3 g_{\mu\nu}. \end{aligned} \quad (7)$$

Here we want to obtain the $(n + 1)$ -dimensional static solutions of Eqs. (4) and (5). We assume that the metric has the following form:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \quad (8)$$

where

$$d\Omega^2 = \begin{cases} d\theta_1^2 + \sum_{i=2}^{n-1} \prod_{j=1}^{i-1} \sin^2 \theta_j d\theta_i^2 & k = 1 \\ d\theta_1^2 + \sinh^2 \theta_1 d\theta_2^2 + \sinh^2 \theta_1 \sum_{i=3}^{n-1} \prod_{j=2}^{i-1} \sin^2 \theta_j d\theta_i^2 & k = -1 \\ \sum_{i=1}^{n-1} d\phi_i^2 & k = 0 \end{cases}$$

represents the line element of an $(n - 1)$ -dimensional hypersurface with constant curvature $(n - 1)(n - 2)k$ and volume V_{n-1} .

Using Eq. (5), one can show that the vector potential can be written as

$$A_\mu = -\sqrt{\frac{(n-1)}{2n-4}} \frac{q}{r^{n-2}} F(\eta) \delta_\mu^0, \quad (9)$$

where q is an integration constant which is related to the charge parameter and

$$\eta = \frac{(n-1)(n-2)q^2}{2\beta^2 r^{2n-2}}.$$

In Eq. (9) and throughout the paper, we use the following abbreviation for the hypergeometric function

$${}_2F_1\left(\left[\frac{1}{2}, \frac{n-2}{2n-2}\right], \left[\frac{3n-4}{2n-2}\right], -z\right) = F(z). \quad (10)$$

The hypergeometric function $F(\eta) \rightarrow 1$ as $\eta \rightarrow 0$ ($\beta \rightarrow \infty$) and therefore A_μ of Eq. (9) reduces to the gauge potential of Maxwell field. One may show that the metric function

$$f(r) = k + \frac{r^2}{\alpha} (1 - g(r)^{1/3}), \quad (11)$$

$$g(r) = 1 + \frac{3\alpha m}{r^n} - \frac{12\alpha\beta^2}{n(n-1)} \left[1 - \sqrt{1 + \eta} - \frac{\Lambda}{2\beta^2} + \frac{(n-1)\eta}{(n-2)} F(\eta) \right] \quad (12)$$

satisfies the field equations (4) in the special case

$$\alpha_2 = \frac{\alpha}{(n-2)(n-3)}, \quad \alpha_3 = \frac{\alpha^2}{72\binom{n-2}{4}},$$

where m is the mass parameter. Solutions of Gauss-Bonnet gravity are not real in the whole range $0 \leq r < \infty$ and one needs a transformation to make them real [14,16]. But, here the metric function $f(r)$ is real in the whole range $0 \leq r < \infty$.

$$T_+ = \frac{f'(r_+)}{4\pi} = \frac{(n-1)k[3(n-2)r_+^4 + 3(n-4)k\alpha r_+^2 + (n-6)k^2\alpha^2] + 12r_+^6\beta^2(1 - \sqrt{1 + \eta_+}) - 6\Lambda r_+^6}{12\pi(n-1)r_+(r_+^2 + k\alpha)^2}. \quad (16)$$

It is worthwhile to note that T_+ is zero for $m = m_{\text{ext}}$.

III. THERMODYNAMICS OF ASYMPTOTICALLY FLAT BLACK HOLES FOR $k = 1$

In this section, we consider the thermodynamics of spherically symmetric black holes which are asymptotically flat. This is due to the fact that only the entropy of asymptotically black holes of Lovelock gravity is well

In order to consider the asymptotic behavior of the solution, we put $m = q = 0$ where the metric function reduces to

$$f(r) = k + \frac{r^2}{\alpha} \left[1 - \left(1 + \frac{6\Lambda\alpha}{n(n-1)} \right)^{1/3} \right]. \quad (13)$$

Equation (13) shows that the asymptotic behavior of the solution is AdS or dS provided $\Lambda < 0$ or $\Lambda > 0$. The case of asymptotic flat solutions ($\Lambda = 0$) is permitted only for $k = 1$.

As in the case of black holes of Gauss-Bonnet-Born-Infeld gravity [13,14], the above metric given by Eqs. (8), (11), and (12) has an essential timelike singularity at $r = 0$. Seeking possible black hole solutions, we turn to looking for the existence of horizons. The event horizon(s), if there exists any, is (are) located at the root(s) of $g^{rr} = f(r) = 0$. Denoting the largest real root of $f(r)$ by r_+ , we consider first the case that $f(r)$ has only one real root. In this case $f(r)$ is minimum at r_+ and therefore $f'(r_+) = 0$. That is,

$$(n-1)k[3(n-2)r_+^4 + 3(n-4)k\alpha r_+^2 + (n-6)k^2\alpha^2] + 12r_+^6\beta^2(1 - \sqrt{1 + \eta_+}) - 6\Lambda r_+^6 = 0. \quad (14)$$

One can find the extremal value of mass, m_{ext} , in terms of parameters of metric function by finding r_+ from Eq. (14) and inserting it into equation $f(r_+) = 0$. Then, the metric of Eqs. (8), (11), and (12) presents a black hole solution with inner and outer event horizons provided $m > m_{\text{ext}}$, an extreme black hole for $m = m_{\text{ext}}$ [temperature is zero since it is proportional to $f'(r_+)$] and a naked singularity otherwise. It is a matter of calculation to show that m_{ext} for $k = 0$ becomes

$$m_{\text{ext}} = \frac{2(n-1)q_{\text{ext}}^2}{n} \left(\frac{\Lambda(\Lambda - 4\beta^2)}{2(n-1)(n-2)\beta^2 q_{\text{ext}}^2} \right)^{(n-2)/(2n-2)} \times F\left(\frac{\Lambda(\Lambda - 4\beta^2)}{4\beta^4}\right). \quad (15)$$

The Hawking temperature of the black holes can be easily obtained by requiring the absence of conical singularity at the horizon in the Euclidean sector of the black hole solutions. One obtains

known [17]. Usually entropy of black holes satisfies the so-called area law of entropy which states that the black hole entropy equals one-quarter of the horizon area [18]. One of the surprising and impressive features of this area law of entropy is its universality. It applies to all kinds of black holes and black strings of Einstein gravity [19]. However, in higher derivative gravity the area law of entropy is not satisfied in general [20]. It is known that the entropy of asymptotically flat black holes of Lovelock

gravity is [17]

$$S = \frac{1}{4} \sum_{k=1}^{[(n-2)/2]} k \alpha_k \int d^{n-1} x \sqrt{\tilde{g}} \tilde{\mathcal{L}}_k, \quad (17)$$

where the integration is done on the $(n-1)$ -dimensional spacelike hypersurface of the Killing horizon, $\tilde{g}_{\mu\nu}$ is the induced metric on it, \tilde{g} is the determinant of $\tilde{g}_{\mu\nu}$, and $\tilde{\mathcal{L}}_k$ is the k th order Lovelock Lagrangian of $\tilde{g}_{\mu\nu}$. Thus, the entropy for asymptotically flat black holes in third order Lovelock gravity is

$$S = \frac{1}{4} \int d^{n-1} x \sqrt{\tilde{g}} (1 + 2\alpha_2 \tilde{R} + 3\alpha_3 (\tilde{R}_{\mu\nu\sigma\kappa} \tilde{R}^{\mu\nu\sigma\kappa} - 4\tilde{R}_{\mu\nu} \tilde{R}^{\mu\nu} + \tilde{R}^2)), \quad (18)$$

where $\tilde{R}_{\mu\nu\rho\sigma}$ and $\tilde{R}_{\mu\nu}$ are Riemann and Ricci tensors and \tilde{R} is the Ricci scalar for the induced metric \tilde{g}_{ab} on the $(n-1)$ -dimensional horizon. It is a matter of calculation to show that the entropy of black holes is

$$S = \frac{V_{n-1}}{4} \left(r_+^4 + \frac{2(n-1)}{n-3} \alpha r_+^2 + \frac{n-1}{n-5} \alpha^2 \right) r_+^{n-5}. \quad (19)$$

The charge of the black hole can be found by calculating the flux of the electric field at infinity, yielding

$$Q = \frac{V_{n-1}}{4\pi} \sqrt{\frac{(n-1)(n-2)}{2}} q. \quad (20)$$

The electric potential Φ , measured at infinity with respect to the horizon, is defined by

$$\Phi = A_\mu \chi^\mu|_{r \rightarrow \infty} - A_\mu \chi^\mu|_{r=r_+}, \quad (21)$$

where $\chi = \partial/\partial t$ is the null generator of the horizon. One finds

$$\Phi = \sqrt{\frac{(n-1)}{2(n-2)}} \frac{q}{r_+^{n-2}} F(\eta_+). \quad (22)$$

The Arnowitt-Deser-Misner (ADM) mass of black hole can be obtained by using the behavior of the metric at large r . It is easy to show that the mass of the black hole is

$$M = \frac{V_{n-1}}{16\pi} (n-1)m. \quad (23)$$

We now investigate the first law of thermodynamics. Using the expression for the entropy, the charge, and the mass given in Eqs. (19), (20), and (23), and the fact that $f(r_+) = 0$, one obtains

$$M(S, Q) = \frac{(n-1)}{16\pi} \left\{ \frac{2r_+^n}{n(n-1)} \left(2\beta^2 \left[1 - \sqrt{1 + \mathfrak{S}} \right] + \frac{(n-1)\mathfrak{S}}{(n-2)} F(\mathfrak{S}) \right) - \Lambda \right\} - r_+^{n-2} + \alpha r_+^{n-4} - \frac{\alpha^2 r_+^{n-6}}{3}, \quad (24)$$

where

$$\mathfrak{S} = \frac{16\pi^2 Q^2}{\beta^2 r_+^{2n-2}}.$$

In Eq. (24) r_+ is the real root of Eq. (19) which is a function of S . One may then regard the parameters S and Q as a complete set of extensive parameters for the mass $M(S, Q)$ and define the intensive parameters conjugate to them. These quantities are the temperature and the electric potential

$$T = \left(\frac{\partial M}{\partial S} \right)_Q, \quad \Phi = \left(\frac{\partial M}{\partial Q} \right)_S. \quad (25)$$

Computing $\partial M/\partial r_+$ and $\partial S/\partial r_+$ and using the chain rule, it is easy to show that the intensive quantities calculated by Eq. (25) coincide with Eqs. (16) and (22), respectively. Thus, the thermodynamic quantities calculated in Eqs. (16) and (22) satisfy the first law of thermodynamics,

$$dM = TdS + \Phi dQ. \quad (26)$$

IV. THERMODYNAMICS OF ASYMPTOTICALLY ADS ROTATING BLACK BRANES WITH FLAT HORIZON

Now, we want to endow our spacetime solution (8) for $k=0$ with a global rotation. These kinds of rotating solutions in Einstein gravity have been introduced in [21]. In order to add angular momentum to the spacetime, we perform the following rotation boost in the $t - \phi_i$ planes

$$t \mapsto \Xi t - a_i \phi_i, \quad \phi_i \mapsto \Xi \phi_i - \frac{a_i}{l^2} t \quad (27)$$

for $i = 1 \dots [n/2]$, where $[x]$ is the integer part of x . The maximum number of rotation parameters is due to the fact that the rotation group in $n+1$ dimensions is $SO(n)$ and therefore the number of independent rotation parameters is $[n/2]$. Thus the metric of an asymptotically AdS rotating solution with $p \leq [n/2]$ rotation parameters for flat horizon can be written as

$$ds^2 = -f(r) \left(\Xi dt - \sum_{i=1}^p a_i d\phi_i \right)^2 + \frac{r^2}{l^4} \sum_{i=1}^p (a_i dt - \Xi l^2 d\phi_i)^2 + \frac{dr^2}{f(r)} - \frac{r^2}{l^2} \sum_{i < j}^p (a_i d\phi_j - a_j d\phi_i)^2 + r^2 \sum_{i=p+1}^{n-1} d\phi_i^2, \quad (28)$$

where $\Xi = \sqrt{1 + \sum_i^k a_i^2/l^2}$. Using Eq. (5), one can show that the vector potential can be written as

$$A_\mu = -\sqrt{\frac{(n-1)}{2n-4}} \frac{q}{r^{n-2}} F(\eta) (\Xi \delta_\mu^0 - \delta_\mu^i a_i) \quad (29)$$

(no sum on i).

One can obtain the temperature and angular momentum of the event horizon by analytic continuation of the metric. One obtains

$$T_+ = \frac{f'(r_+)}{4\pi\Xi} = \frac{r_+}{2(n-1)\pi\Xi} (2\beta^2(1 - \sqrt{1 + \eta_+}) - \Lambda), \quad (30)$$

$$\Omega_i = \frac{a_i}{\Xi l^2}, \quad (31)$$

where $\eta_+ = \eta(r = r_+)$. Next, we calculate the electric charge and potential of the solutions. The electric charge per unit volume V_{n-1} can be found by calculating the flux of the electric field at infinity, yielding

$$Q = \frac{1}{4\pi} \sqrt{\frac{(n-1)(n-2)}{2}} \Xi q. \quad (32)$$

Using Eq. (21) and the fact that $\chi = \partial_t + \sum_i^k \Omega_i \partial_{\phi_i}$ is the null generator of the horizon, the electric potential Φ is obtained as

$$\Phi = \sqrt{\frac{(n-1)}{2(n-2)}} \frac{q}{\Xi r_+^{n-2}} F(\eta_+). \quad (33)$$

A. Conserved quantities of the solutions

Here, we calculate the action and conserved quantities of the black brane solutions. In general the action and conserved quantities of the spacetime are divergent when evaluated on the solutions. A systematic method of dealing with this divergence for asymptotically AdS solutions of Einstein gravity is through the use of the counterterms method inspired by the anti-de Sitter conformal field theory (AdS/CFT) correspondence [22]. For asymptotically AdS solutions of Lovelock gravity with flat boundary, $\hat{R}_{abcd}(\gamma) = 0$, the finite action is [7,23]

$$I = I_G + \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^n x \sqrt{-\gamma} \{L_{1b} + \alpha_2 L_{2b} + \alpha_3 L_{3b}\} + \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^n x \sqrt{-\gamma} \left(\frac{n-1}{L} \right), \quad (34)$$

where L is

$$L = \frac{15l^2 \sqrt{\alpha(1-\lambda)}}{5l^2 + 9\alpha - l^2 \lambda^2 - 4l^2 \lambda}, \quad (35)$$

$$\lambda = \left(1 - \frac{3\alpha}{l^2} \right)^{1/3}. \quad (36)$$

One may note that L reduces to l as α goes to zero. The first

integral in Eq. (34) is a boundary term which is chosen such that the variational principle is well defined. In this integral $L_{1b} = K$, $L_{2b} = 2(J - 2\hat{G}_{ab}^{(1)} K^{ab})$ and

$$L_{3b} = 3(P - 2\hat{G}_{ab}^{(2)} K^{ab} - 12\hat{R}_{ab} J^{ab} + 2\hat{R} J - 4K\hat{R}_{abcd} K^{ac} K^{bd} - 8\hat{R}_{abcd} K^{ac} K_e^b K^{ed}),$$

where $\gamma_{\mu\nu}$ and K are induced metric and trace of extrinsic curvature of boundary, $\hat{G}_{ab}^{(1)}$ and $\hat{G}_{ab}^{(2)}$ are the n -dimensional Einstein and second order Lovelock tensors [Eq. (6)] of the metric γ_{ab} and J and P are the trace of

$$J_{ab} = \frac{1}{3} (2KK_{ac} K_b^c + K_{cd} K^{cd} K_{ab} - 2K_{ac} K^{cd} K_{db} - K^2 K_{ab}), \quad (37)$$

and

$$P_{ab} = \frac{1}{5} \{ [K^4 - 6K^2 K^{cd} K_{cd} + 8KK_{cd} K_e^d K^{ec} - 6K_{cd} K^{de} K_{ef} K^{fc} + 3(K_{cd} K^{cd})^2] K_{ab} - (4K^3 - 12KK_{ed} K^{ed} + 8K_{de} K_f^e K^{fd}) K_{ac} K_b^c - 24KK_{ac} K^{cd} K_{de} K_b^e + (12K^2 - 12K_{ef} K^{ef}) \times K_{ac} K^{cd} K_{db} + 24K_{ac} K^{cd} K_{de} K_{bf} K^{ef} \}. \quad (38)$$

Using Eqs. (1) and (34), the finite action per unit volume V_{n-1} can be calculated as

$$I = -\frac{1}{T_+} \left\{ \frac{r_+^n}{16\pi l^2} - \frac{r_+^n \beta^2 (\sqrt{1 + \eta_+} - 1)}{4n(n-1)\pi} + \frac{(n-1)q^2}{8n\pi r_+^{(n-2)}} F(\eta_+) \right\}. \quad (39)$$

Using the Brown-York method [24], the finite energy-momentum tensor is

$$T^{ab} = \frac{1}{8\pi} \left\{ (K^{ab} - K\gamma^{ab}) + 2\alpha_2 (3J^{ab} - J\gamma^{ab}) + 3\alpha_3 (5P^{ab} - P\gamma^{ab}) + \frac{n-1}{L} \gamma^{ab} \right\}, \quad (40)$$

and the conserved quantities associated with the Killing vectors $\partial/\partial t$ and $\partial/\partial \phi^i$ are

$$M = \frac{1}{16\pi} m(n\Xi^2 - 1), \quad (41)$$

$$J_i = \frac{1}{16\pi} n\Xi m a_i, \quad (42)$$

which are the mass and angular momentum of the solution. Now using Gibbs-Duhem relation

$$S = \frac{1}{T} \left(M - Q\Phi - \sum_{i=1}^k \Omega_i J_i \right) - I, \quad (43)$$

and Eqs. (33), (39), (41), and (42) one obtains

$$S = \frac{\Xi}{4} r_+^{n-1} \quad (44)$$

for the entropy per unit volume V_{n-1} . This shows that the entropy obeys the area law for our case where the horizon is flat.

B. Stability of the solutions

Calculating all the thermodynamic and conserved quantities of the black brane solutions, we now check the first law of thermodynamics for our solutions with flat horizon. We obtain the mass as a function of the extensive quantities S , \mathbf{J} , and Q . Using the expression for charge mass, angular momenta, and entropy given in Eqs. (32), (41), (42), and (44) and the fact that $f(r_+) = 0$, one can obtain a Smarr-type formula as

$$M(S, \mathbf{J}, Q) = \frac{(nZ - 1)J}{nl\sqrt{Z(Z - 1)}}, \quad (45)$$

where $J = |\mathbf{J}| = \sqrt{\sum_i J_i^2}$ and $Z = \Xi^2$ is the positive real root of the following equation:

$$\frac{Z^{1/2(n-1)}}{\sqrt{Z - 1}} = \frac{[4l^2\beta^2(1 - \sqrt{1 + \eta_+}) + n(n - 1)]S^{n/(n-1)}}{(n - 1)\pi l J} + \frac{4\pi l Q^2 F\left(\frac{\pi^2 Q^2}{\beta^2 S^2}\right)}{(n - 2)J(4S)^{(n-2)/(n-1)}}. \quad (46)$$

One may then regard the parameters S , J_i 's, and Q as a complete set of extensive parameters for the mass

$M(S, \mathbf{J}, Q)$ and define the intensive parameters conjugate to them. These quantities are the temperature, the angular velocities, and the electric potential

$$T = \left(\frac{\partial M}{\partial S}\right)_{J, Q}, \quad \Omega_i = \left(\frac{\partial M}{\partial J_i}\right)_{S, Q}, \quad \Phi = \left(\frac{\partial M}{\partial Q}\right)_{S, J}. \quad (47)$$

Straightforward calculations show that the intensive quantities calculated by Eq. (47) coincide with Eqs. (30), (31), and (33). Thus, these quantities satisfy the first law of thermodynamics:

$$dM = TdS + \sum_{i=1}^k \Omega_i dJ_i + \Phi dQ.$$

Finally, we investigate the local stability of charged rotating black brane solutions of third order Lovelock gravity in the presence of nonlinear electrodynamic Born-Infeld field in the canonical and grand canonical ensembles. In the canonical ensemble, the positivity of the heat capacity $C_{\mathbf{J}, Q} = T_+ / (\partial^2 M / \partial S^2)_{\mathbf{J}, Q}$ and therefore the positivity of $(\partial^2 M / \partial S^2)_{\mathbf{J}, Q}$ is sufficient to ensure the local stability. Using the fact that

$$\begin{aligned} {}_2F_1\left(\left[\frac{3}{2}, \frac{3n-4}{2n-2}\right], \left[\frac{5n-6}{2n-2}\right], -z\right) \\ = \frac{(3n-4)}{(n-1)z} \left\{ F\left(\frac{\pi^2 Q^2}{\beta^2 S^2}\right) - \frac{1}{\sqrt{1+z}} \right\}, \end{aligned}$$

it is easy to show that

$$\begin{aligned} \frac{\partial^2 M}{\partial S^2} = & \frac{2[(n-1)(n-2)^2 q^2 - \Lambda r_+^{(2n-2)} \sqrt{1 + \eta_+} + 2\beta^2 r_+^{(2n-2)} (\sqrt{1 + \eta_+} - 1)]}{(n-1)^2 \pi \Xi^2 r_+^{(3n-4)} \sqrt{1 + \eta_+}} \\ & - \frac{8(\Xi^2 - 1)\Lambda(-r_+^{2n-2} \sqrt{1 + \eta_+} \frac{(n-1)}{2} + 2\beta^2 l^2) + (n-1)(n-2)\beta l^2 q^2 + 2\beta^3 l^2 r_+^{(2n-2)} l^2}{\pi m l^2 \beta^2 \Xi^2 (n-1)^4 (4\Xi^2 + 1)(1 + \eta_+) r_+^{(4n-6)}}. \quad (48) \end{aligned}$$

Both of the two terms of Eq. (48) are positive, and therefore the condition for thermal equilibrium in the canonical ensemble is satisfied.

In the grand canonical ensemble, the positivity of the determinant of the Hessian matrix of $M(S, Q, \mathbf{J})$ with respect to its extensive variables X_i , $\mathbf{H}_{X_i X_j}^M = (\partial^2 M / \partial X_i \partial X_j)$, is sufficient to ensure the local stability. It is a matter of calculation to show that the determinant of $\mathbf{H}_{S, Q, \mathbf{J}}^M$ is:

$$|\mathbf{H}_{S, Q, \mathbf{J}}^M| = \frac{64\pi(2(n-2)\beta^2 \eta_+ - \Lambda \sqrt{1 + \eta_+} + 2\beta^2(\sqrt{1 + \eta_+} - 1))}{(n-2)(n-1)^3 m l^2 \Xi^6 r_+^{2(n-2)} [(n-2)\Xi^2 + 1] \sqrt{1 + \eta_+}} F(\eta_+) + \frac{8(n-1)\Xi}{r_+} T_+. \quad (49)$$

Equation (49) shows that the determinant of the Hessian matrix is positive, and therefore the solution is stable in the grand canonical ensemble too. The stability analysis given here shows that the higher curvature and nonlinear Maxwell terms in the action have no effect on the stability of black holes with flat horizon, and these kinds of black

holes are thermodynamically stable as in the case of toroidal black holes of Einstein-Maxwell gravity [25]. This phase behavior is also commensurate with the fact that there is no Hawking-Page transition for a black object whose horizon is diffeomorphic to \mathbb{R}^p and therefore the system is always in the high temperature phase [26].

V. CLOSING REMARKS

In this paper we considered both the nonlinear scalar terms constructing from the curvature tensor and electromagnetic field tensor in gravitational action, which are on similar footing with regard to the string corrections on gravity and electrodynamic sides. We presented static topological black hole solutions of third order Lovelock gravity in the presence of Born-Infeld gravity, which are asymptotically AdS for negative cosmological constant, dS for positive Λ . For the case of solutions with positive curvature horizon ($k = 1$), one can also have asymptotically flat solutions, provided $\Lambda = 0$. The topological solutions obtained in this paper may be interpreted as black holes with two inner and outer event horizons for $m > m_{\text{ext}}$, extreme black holes for $m = m_{\text{ext}}$ or naked singularity otherwise. We found that these solutions reduce to the solutions of Einstein-Born-Infeld gravity as the Lovelock coefficients vanish, and reduce to the solutions of third order Lovelock gravity in the presence of Maxwell field as β goes to infinity [6]. We consider thermodynamics of asymptotically flat solutions and found that the first law of thermodynamics is satisfied by the conserved and thermodynamic quantities of the black hole. We also consider

the rotating solution with flat horizon and computed the action and conserved quantities of it through the use of counterterm method. We found that the entropy obeys the area law for black branes with flat horizon. We obtained a Smarr-type formula for the mass of the black brane as a function of the entropy, the charge, and the angular momenta, and found that the conserved and thermodynamics quantities satisfy the first law of thermodynamics. We also studied the phase behavior of the $(n + 1)$ -dimensional rotating black branes in third order Lovelock gravity and showed that there is no Hawking-Page phase transition in spite of the angular momenta of the branes and the presence of a nonlinear electromagnetic field. Indeed, we calculated the heat capacity and the determinant of the Hessian matrix of the mass with respect to S , \mathbf{J} , and Q of the black branes and found that they are positive for all the phase space, which means that the brane is locally stable for all the allowed values of the metric parameters.

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