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Hawking radiation of charged Dirac particles from a Kerr-Newman black hole

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Charged Dirac particles' Hawking radiation from a Kerr-Newman black hole is calculated using Damour-Ruffini's method. When energy conservation and the backreaction of particles to the space-time are considered, the emission spectrum is not purely thermal anymore. The leading term is exactly the Boltzman factor, and the deviation from the purely thermal spectrum can bring some information out, which can be treated as an explanation to the information loss paradox. The result can also be treated as a quantum-corrected radiation temperature, which is dependent on the black hole background and the radiation particle's energy, angular momentum, and charge.

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I. INTRODUCTION

Hawking's astonishing discovery that a black hole radiates black body spectrum, which takes nothing out of the black hole, has been a great development in the research of black hole thermodynamics [1,2]. However, it also sets up a disturbing and difficult problem about information conservation during black hole evaporation, which leads to the so-called "information loss paradox" and the violation of the underlying quantum unitary theory [3–6]. Since Hawking's significant discovery was published in 1970s, there have been many works to solve the two problems. From 2000 to now, at least three kinds of methods have been proposed about the issue.

(1) In 2000, Parikh and Wilczek proposed a semiclassical approach [7–10] to calculate the emission rate by treating Hawking radiation as a tunneling process and using WKB approximation. The barrier is created by the outgoing particles themselves. When self-gravitation of particles is considered, a corrected spectrum is given. After that, Zhang and Zhao extended this method to more general circumstances [11–15]. All of them can obtain the conclusion that the spectrum is no longer precisely thermal and some information can be taken out of the black hole. A possible explanation for information loss paradox and the loss of quantum unitary theory can be obtained.

(2) Marco Angheben *et al* proposed another method to investigate Hawking radiation [16–18]. By calculating the classical action I of emitting particles, which satisfies the relativistic Hamilton-Jacobi equation, the emission rate can also be obtained. In this method, the same conclusion as the first method can be drawn.

(3) Recently, Liu has proposed a new method about this topic [19]. Using the Damour-Ruffini method [20], Liu has investigated Hawking radiation of massive Klein-Gorden particles from a Reissner-Nordstrom black hole. When energy conservation and the particles' backreaction are taken into account, the same conclusion as the previous works can be obtained.

We will extend Liu's work to charged Dirac particles' Hawking radiation from a Kerr-Newman black hole. The original Damour-Ruffini method can give a proof to the fact that black holes have thermal radiation only using relativistic quantum mechanics in curved space-time. Neither the thermal balance between the black hole inside and outside, nor the collapse of the black hole is considered there. The Kerr-Newman black hole is a more general background to be investigated. Moreover, the massive, charged Dirac radiation particle with any angular momentum will be calculated. This is more complex than before. According to some recent papers [21–23], the result can also be treated as Hawking radiation at a quantum-corrected temperature, which is dependent on not only the black hole background, but also the radiation particle's energy, angular momentum, and charge.

II. DIRAC EQUATIONS IN A KERR-NEWMAN SPACE-TIME

The line element of the Kerr-Newman black hole can be written as

$$ds^2 = -\left(1 - \frac{2Mr - Q^2}{\Sigma^2}\right)dt^2 + \frac{\Sigma^2}{\Delta}dr^2 + \Sigma^2 d\theta^2 + \left[(r^2 + a^2) + \frac{(2Mr - Q^2)a^2\sin^2\theta}{\Sigma^2}\right]\sin^2\theta d\varphi^2 - \frac{2(2Mr - Q^2)a\sin^2\theta}{\Sigma^2}dtd\varphi, \tag{1}$$

where $\Delta \equiv r^2 - 2Mr + a^2 + Q^2$, $\Sigma^2 \equiv r^2 + a^2\cos^2\theta$, in which M, Q, a are the total mass, total charge, and angular momentum per unit mass of the black hole, respectively.

The event horizon $r_\pm$ is given by

$$r_\pm = M \pm \sqrt{M^2 - a^2 - Q^2}. \tag{2}$$

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In a curved space-time, Dirac equations of a charged particle's dynamics in Newman-Penrose formalism are given as [24,25]

$$\begin{aligned}
&(D+\varepsilon-\rho+ie\vec{A}\cdot\vec{l})F_1+(\bar{\delta}+\pi-\alpha+ie\vec{A}\cdot\vec{\bar{m}})F_2=i\frac{\mu_0}{\sqrt{2}}G_1,\\
&(\tilde{\Delta}+\mu-\gamma+ie\vec{A}\cdot\vec{n})F_2+(\delta+\beta-\tau+ie\vec{A}\cdot\vec{m})F_1=i\frac{\mu_0}{\sqrt{2}}G_2,\\
&(D+\bar{\varepsilon}-\bar{\rho}+ie\vec{A}\cdot\vec{l})G_2-(\delta+\bar{\pi}-\bar{\alpha}+ie\vec{A}\cdot\vec{m})G_1=i\frac{\mu_0}{\sqrt{2}}F_2,\\
&(\tilde{\Delta}+\bar{\mu}-\bar{\gamma}+ie\vec{A}\cdot\vec{n})G_1-(\bar{\delta}+\bar{\beta}-\bar{\tau}+ie\vec{A}\cdot\vec{\bar{m}})G_2=i\frac{\mu_0}{\sqrt{2}}F_1,
\end{aligned} \tag{3}$$

where μ_0 and e are rest mass and charge of the particle, respectively, and F_1, F_2, G_1, and G_2 are the four components of the wave functions. D, $\tilde{\Delta}$, δ, $\bar{\delta}$ are usual differential operators; α, β, γ, ε, ρ, π, μ, τ are spin coefficients. They are given as following

$$\begin{aligned}
&\alpha=\frac{1}{2}(l_{\mu;\nu}n^\mu\bar{m}^\nu-m_{\mu;\nu}\bar{m}^\mu\bar{m}^\nu),\qquad \rho=l_{\mu;\nu}m^\mu\bar{m}^\nu,\qquad \beta=\frac{1}{2}(l_{\mu;\nu}n^\mu m^\nu-m_{\mu;\nu}\bar{m}^\mu m^\nu),\\
&\pi=-n_{\mu;\nu}\bar{m}^\mu l^\nu,\qquad \gamma=\frac{1}{2}(l_{\mu;\nu}n^\mu n^\nu-m_{\mu;\nu}\bar{m}^\mu n^\nu),\qquad \mu=-n_{\mu;\nu}\bar{m}^\mu m^\nu,\qquad \varepsilon=\frac{1}{2}(l_{\mu;\nu}n^\mu l^\nu-m_{\mu;\nu}\bar{m}^\mu l^\nu),\\
&\tau=l_{\mu;\nu}m^\mu n^\nu,\qquad D=l^\mu\partial_\mu,\qquad \tilde{\Delta}=n^\mu\partial_\mu,\qquad \delta=m^\mu\partial_\mu,\qquad \bar{\delta}=\bar{m}^\mu\partial_\mu;
\end{aligned} \tag{4}$$

A_μ is the four-dimensional electromagnetic potential. Since the electromagnetic field of the Kerr-Newman black hole is axially symmetric, the coordinate components of A_μ are irrelevant to the coordinates t and φ. They are

$$A_0=-\frac{Qr}{\Sigma^2},\qquad A_1=A_2=0,\qquad A_3=\frac{Qra\sin^2\theta}{\Sigma^2}. \tag{5}$$

For a description of Kerr-Newman space-time in a Newman-Penrose formalism, we first need to choose a null tetrad frame. We can choose it as

$$\begin{aligned}
l_\mu&=\frac{1}{\Delta}(\Delta,-\Sigma^2,0,-a\Delta\sin^2\theta),\\
n_\mu&=\frac{1}{2\Sigma^2}(\Delta,\Sigma^2,0,-a\Delta\sin^2\theta),\\
m_\mu&=\frac{1}{\sqrt{2}\bar{\Sigma}}(ia\sin\theta,0,-\Sigma^2,-i(r^2+a^2)\sin\theta),\\
\bar{m}_\mu&=\frac{1}{\sqrt{2}\bar{\Sigma}^*}(-ia\sin\theta,0,-\Sigma^2,i(r^2+a^2)\sin\theta).
\end{aligned} \tag{6}$$

The contravariant forms of the basis vectors are

$$\begin{aligned}
l^\mu&=\frac{1}{\Delta}(r^2+a^2,\Delta,0,a),\\
n^\mu&=\frac{1}{2\Sigma^2}(r^2+a^2,-\Delta,0,a),\\
m^\mu&=\frac{1}{\sqrt{2}\bar{\Sigma}}\left(ia\sin\theta,0,1,\frac{i}{\sin\theta}\right),\\
\bar{m}^\mu&=\frac{1}{\sqrt{2}\bar{\Sigma}^*}\left(-ia\sin\theta,0,1,-\frac{i}{\sin\theta}\right),
\end{aligned} \tag{7}$$

where $\bar{\Sigma}=r+ia\cos\theta$ and $\bar{\Sigma}^*=r-ia\cos\theta$.

Because $(\frac{\partial}{\partial t})^a$ and $(\frac{\partial}{\partial\varphi})^a$ are Killing vector fields in Kerr-Newman space-time, we can put the four components of the wave function as following

$$\begin{aligned}
F_1&=e^{-i(\omega t-m\varphi)}(r-ia\cos\theta)^{-1}f_1(r,\theta),\\
F_2&=e^{-i(\omega t-m\varphi)}f_2(r,\theta),\\
G_1&=e^{-i(\omega t-m\varphi)}g_1(r,\theta),\\
G_2&=e^{-i(\omega t-m\varphi)}(r+ia\cos\theta)^{-1}g_2(r,\theta).
\end{aligned} \tag{8}$$

After calculating differential operators and spin coefficients in Eqs. (4) by using Eqs. (6) and (7), we can obtain the following equations by putting Eqs. (4)–(8) into Eq. (3):

$$\begin{aligned}
&\left(\partial_r - \frac{iK}{\Delta}\right)f_1 + \frac{1}{\sqrt{2}}\left(\partial_\theta - q + \frac{1}{2}\cot\theta\right)f_2 = \frac{1}{\sqrt{2}}(i\mu_0 r + a\mu_0\cos\theta)g_1,\\
&\Delta\left(\partial_r + \frac{iK}{\Delta} + \frac{r-M}{\Delta}\right)f_2 - \sqrt{2}\left(\partial_\theta + q + \frac{1}{2}\cot\theta\right)f_1 = -\sqrt{2}(i\mu_0 r + a\mu_0\cos\theta)g_2,\\
&\left(\partial_r - \frac{iK}{\Delta}\right)g_2 - \frac{1}{\sqrt{2}}\left(\partial_\theta + q + \frac{1}{2}\cot\theta\right)g_1 = \frac{1}{\sqrt{2}}(i\mu_0 r - a\mu_0\cos\theta)f_2,\\
&\Delta\left(\partial_r + \frac{iK}{\Delta} + \frac{r-M}{\Delta}\right)g_1 + \sqrt{2}\left(\partial_\theta - q + \frac{1}{2}\cot\theta\right)g_2 = -\sqrt{2}(i\mu_0 r - a\mu_0\cos\theta)f_1,
\end{aligned} \tag{9}$$

where $K = (r^2 + a^2)\omega - am - eQr$, $q = a\omega\sin\theta - \frac{m}{\sin\theta}$.

By separating variables as

$$\begin{aligned}
f_1(r,\theta) &= R_{-(1/2)}(r)S_{-(1/2)}(\theta) = R(r)S(\theta),\\
f_2(r,\theta) &= R_{+(1/2)}(r)S_{+(1/2)}(\theta),\\
g_1(r,\theta) &= R_{+(1/2)}(r)S_{-(1/2)}(\theta),\\
g_2(r,\theta) &= R_{-(1/2)}(r)S_{+(1/2)}(\theta),
\end{aligned} \tag{10}$$

we can get the decoupled Dirac equations. $R_{-(1/2)}(r)$, i.e., $R(r)$ and $R_{+(1/2)}(r)$, represent outgoing and ingoing waves, respectively. We are only interested in the radial outgoing wave equation, that is

$$\sqrt{\Delta}\frac{d}{dr}\left(\sqrt{\Delta}\frac{dR}{dr}\right) - \frac{i\mu\Delta}{\lambda + i\mu r}\frac{dR}{dr} + \left[\frac{K^2 + i(r-M)K}{\Delta} - 2i\omega r + ieQ - \frac{\mu K}{\lambda + i\mu r} - \mu^2 r^2 - \lambda^2\right]R = 0. \tag{11}$$

III. TORTOISE COORDINATE TRANSFORMATION AND ANALYTIC EXTENSION

Tortoise coordinate transformation can be given as

$$\begin{aligned}
r_* &= r + \frac{1}{\sqrt{M^2 - a^2 - Q^2}}\left[\left(Mr_+ - \frac{1}{2}Q^2\right)\ln\frac{r - r_+}{r_+} - \left(Mr_- - \frac{1}{2}Q^2\right)\ln\frac{r - r_-}{r_-}\right]\\
&= r + \frac{1}{2\kappa_+}\ln\frac{r - r_+}{r_+} - \frac{1}{2\kappa_-}\ln\frac{r - r_-}{r_-},
\end{aligned} \tag{12}$$

where $\kappa_\pm = \frac{r_+ - r_-}{2(r_\pm^2 + a^2)}$, then we have

$$dr_* = \frac{r^2 + a^2}{\Delta}dr. \tag{13}$$

Then, the radial function Eq. (11) in the tortoise coordinate system can be written as

$$\begin{aligned}
&(r^2 + a^2)^2\frac{d^2R}{dr_*^2} + \left[2r\Delta - (r-M)(r^2 + a^2) - (r^2 + a^2)\mu_0\Delta\frac{\mu_0 r + i\lambda}{\lambda^2 + \mu_0^2 r^2}\right]\frac{dR}{dr_*} + \Delta\left\{\frac{K^2}{\Delta} - \lambda^2 - \mu_0^2 r^2\right.\\
&\left. - \frac{\mu_0\lambda K - i\mu_0^2 K r}{\lambda^2 + \mu_0^2 r^2} - i\left[-eQ + 2\omega r - \frac{K(r-M)}{\Delta}\right]\right\}R = 0.
\end{aligned} \tag{14}$$

At the horizon $r = r_+$, Eq. (14) becomes

$$(r_+^2 + a^2)^2\frac{d^2R}{dr_*^2} - (r_+ - M)(r_+^2 + a^2)\frac{dR}{dr_*} + [K^2 + iK(r_+ - M)]R = 0, \tag{15}$$

which is a wave equation. Its solution is

$$R = e^{i(\omega - j\Omega - eV_0)r_*} = e^{i(\omega - \omega_0)r_*}, \tag{16}$$

where $\omega_0 = j\Omega + eV_0$, $\Omega = \frac{a}{r_+^2 + a^2}$, $V_0 = \frac{Qr_+}{r_+^2 + a^2}$, in which Ω is the angular velocity of the horizon and V_0 is the static electropotential of the horizon where θ is equal to 0 or π. Therefore, the radial solution can be written as

$$\Psi_\omega = e^{-i\omega t \pm i(\omega - \omega_0)r_*},$$

where " $+$" corresponds to the outgoing wave and " $-$" represents the ingoing wave.

Letting $\hat{r} = \frac{\omega - \omega_0}{\omega}r_*$, the radial solution becomes $\Psi_\omega = e^{-i\omega(t \mp \hat{r})}$. Using the advanced Eddington coordinate $v = t + \hat{r}$, in which the metric is well behaved and analytic over the whole coordinate range $0 < r < +\infty$, $-\infty < v < +\infty$ including r_+, the ingoing and outgoing waves are separately

$$\Psi_\omega^{\rm in} = e^{-i\omega v}, \tag{17}$$

$$\Psi_\omega^{\rm out} = e^{-i\omega v + i2(\omega - \omega_0)r_*}. \tag{18}$$

While Eq. (17) corresponds to a wave purely ingoing on r_+ and can be extended inside $r < r_+$, Eq. (18) represents an outgoing wave and has an infinite number of oscillations as $r \to r_+$ and therefore cannot be straightforwardly extended to the region inside r_+. We will in the following use and generalize to analytic curved spaces the well-known result of flat-space relativistic wave theories: The wave function $\Phi(x)$ describing a particle state (positive frequencies) can be analytically extended to complex points of the form $z = x + iy$ if y lies in the past cone; similarly, for an antiparticle state (negative frequencies) y has to lie in the future cone.

Since in advanced Eddington coordinates the vector $\frac{\partial}{\partial r}$ is everywhere null and past-directed, the prescription $r \to r - i0$ will yield the unique continuation of Eq. (17) describing an antiparticle state. According to quantum field theory, the ingoing negative frequency antiparticle is just the outgoing positive frequency particle. Although Eq. (18) has singularity on the horizon and therefore cannot be extended straightforwardly to the region inside the horizon, we can extend the outgoing wave Eq. (18) into the horizon and yield the ingoing negative frequency antiparticle by turning the $(-\pi)$ angle through the negative half complex plane. Let $(r - r_+) \to | r - r_+ | e^{-i\pi} = (r_+ - r)e^{-i\pi}$; the outgoing wave function inside and outside of the horizon are respectively [20]

$$\Psi_\omega^{\rm out}(r < r_+) = e^{-i\omega v}(r_+ - r)^{(i/\kappa_+)(\omega-\omega_0)} e^{(\pi(\omega-\omega_0))/\kappa_+)},$$

and

$$\Psi_\omega^{\rm out}(r > r_+) = e^{-i\omega v}(r - r_+)^{(i/\kappa_+)(\omega-\omega_0)}.$$

Therefore, thinking of Sannan's work in Ref. [26], the emission rate at the horizon is given by

$$\Gamma = \left| \frac{\Psi_\omega^{\rm out}(r > r_+)}{\Psi_\omega^{\rm out}(r < r_+)} \right|^2 = e^{(-2\pi(\omega-\omega_0)/\kappa_+)}. \tag{19}$$

IV. BACKREACTION OF THE RADIATION

Now we consider that the emitting particles have backreaction on the space-time. When a particle with energy ω_i, charge e_i, and angular momentum j_i comes out of the black hole, M should be substituted by $(M - \omega_i)$, Q should be substituted by $(Q - e_i)$, and a should be substituted by $a' = \frac{Ma - j_i}{M - \omega_i}$, then

$$\Gamma_i = e^{(-2\pi(\omega_i - \omega_{i0}))/\kappa_{i+}} \tag{20}$$

where

$$\omega_{i0} = j_i\Omega_i + e_iV_{i0} = \frac{j_ia' + e_i(Q - e_i)r_{i+}}{r_{i+}^2 + a'^2}, \qquad r_{i\pm} = (M - \omega_i) \pm \sqrt{(M - \omega_i)^2 - (Q - e_i)^2 - \left(\frac{Ma - j_i}{M - \omega_i}\right)^2},$$

$$\kappa_{i+} = \frac{r_{i+} - r_{i-}}{2(r_{i+}^2 + a'^2)} = \frac{\sqrt{(M - \omega_i)^2 - (Q - e_i)^2 - (\frac{Ma - j_i}{M - \omega_i})^2}}{((M - \omega_i) + \sqrt{(M - \omega_i)^2 - (Q - e_i)^2 - (\frac{Ma - j_i}{M - \omega_i})^2})^2 + (\frac{Ma - j_i}{M - \omega_i})^2}.$$

For many particles' emission, thinking that they radiate one by one, we have

$$\Gamma = \prod_i \Gamma_i = e^{\sum_i (-2\pi(\omega_i - \omega_{i0}))/\kappa_{i+})}. \tag{21}$$

If the emission is regarded as a continuous procession, the sum in Eq. (21) should be substituted by integration

$$\Gamma = e^{-2\pi \int (d\omega' - \Omega' dj' - V_0' de')/\kappa_+')} = e^{-2\pi\Lambda}, \tag{22}$$

where

$$\begin{aligned}\Lambda &= \int \frac{d\omega' - \Omega' dj' - V_0' de'}{\kappa_+'} \\ &= \int_{(0,0,0)}^{(\omega,j,e)} \frac{((M - \omega') + \sqrt{(M - \omega')^2 - (Q - e')^2 - (\frac{Ma - j'}{M - \omega'})^2})^2 + (\frac{Ma - j'}{M - \omega'})^2}{\sqrt{(M - \omega')^2 - (Q - e')^2 - (\frac{Ma - j'}{M - \omega'})^2}} d\omega' \\ &\quad - \frac{\frac{Ma - j'}{M - \omega'}}{\sqrt{(M - \omega')^2 - (Q - e')^2 - (\frac{Ma - j'}{M - \omega'})^2}} dj' - \frac{(Q - e')((M - \omega') + \sqrt{(M - \omega')^2 - (Q - e')^2 - (\frac{Ma - j'}{M - \omega'})^2})}{\sqrt{(M - \omega')^2 - (Q - e')^2 - (\frac{Ma - j'}{M - \omega'})^2}} de'.\end{aligned} \tag{23}$$

To make the calculation more simple, we do not need to do the integration directly. Instead we work on it in the following way: making use of the entropy S of the black hole satisfying

$$S = \frac{1}{4}A = \pi(r_+^2 + a^2), \tag{24}$$

where A is the area of the black hole horizon, and we can easily obtain that

$$\triangle S = \pi[(r_+'^2 + a'^2) - (r_+^2 + a^2)] = \pi[2(M-\omega)^2 - (Q-e)^2 + 2(M-\omega)\sqrt{(M-\omega)^2 - (Q-e)^2 - a'^2} - 2M^2 + Q^2 - 2M\sqrt{M^2 - Q^2 - a^2}], \tag{25}$$

in which $\triangle S = S(M-\omega, Q-e, a') - S(M, Q, a)$ is the difference between the entropies of the black hole before and after the emission. Then we have

$$\frac{\partial(\triangle S)}{\partial \omega} = -2\pi \frac{((M-\omega) + \sqrt{(M-\omega)^2 - (Q-e)^2 - (\frac{Ma-j}{M-\omega})^2})^2 + (\frac{Ma-j}{M-\omega})^2}{\sqrt{(M-\omega)^2 - (Q-e)^2 - (\frac{Ma-j}{M-\omega})^2}},$$
$$\frac{\partial(\triangle S)}{\partial j} = 2\pi \frac{(\frac{Ma-j}{M-\omega})^2}{\sqrt{(M-\omega)^2 - (Q-e)^2 - (\frac{Ma-j}{M-\omega})^2}}, \tag{26}$$
$$\frac{\partial(\triangle S)}{\partial e} = 2\pi \frac{(Q-e)[(M-\omega) + \sqrt{(M-\omega)^2 - (Q-e)^2 - (\frac{Ma-j}{M-\omega})^2}]}{\sqrt{(M-\omega)^2 - (Q-e)^2 - (\frac{Ma-j}{M-\omega})^2}}.$$

Comparing Eq. (23) with Eq. (26), we find that the integration in Eq. (23) satisfies the total differential condition. So Eq. (23) can be calculated out as following

$$\Lambda = -\frac{1}{2\pi}\int_{(0,0,0)}^{(\omega,j,e)} \frac{\partial(\triangle S)}{\partial \omega'} d\omega' + \frac{\partial(\triangle S)}{\partial j'} dj' + \frac{\partial(\triangle S)}{\partial e'} de' = -\frac{1}{2\pi}\int d(\triangle S) = -\frac{1}{2\pi}\triangle S, \tag{27}$$

so the emitting rate Γ is given by $\Gamma = e^{\triangle S}$.

V. CONCLUSIONS AND DISCUSSIONS

Following Liu's work [19], we calculated charged Dirac particles' Hawking radiation from a Kerr-Newman black hole using the improved Damour-Ruffini method. In this method, using the relativistic quantum mechanics in curved space-time, we can study not only the static and stationary black holes, but also dynamical ones. Furthermore, this method can be used for both bosons and fermions. In the 1980s, Zhao, Gui and Liu [27] proved that Dirac particles radiate thermally in Kerr-Newman space-time. In this paper, when energy conservation, charge conservation, angular momentum conservation, and the backreaction of emitting particles to the space-time are taken into account, we have concluded that the spectrum is not accurately thermal. To compare with the purely thermal spectrum, by expanding the emission rate Γ in ω, e, and j, we have

$$\Gamma = e^{\triangle S} = e^{-\beta(\omega-\omega_0)+o(\omega,e,j)^2}. \tag{28}$$

It is easy to find that the leading-order term gives the Boltzman factor, and the higher-order terms of ω, e, and j are a deviation from a purely thermal spectrum. Some information can be taken out of the black hole with the corrected spectrum and an explanation to information loss paradox can be obtained. The underlying unitary theory will possibly be satisfied too.

Actually, we can treat Eq. (28) in another way. In Refs. [21–23], the modified surface gravity and temperature due to one-loop backreaction effects are given for the most simple Schwarzschild black hole. The results for a Schwarzschild black hole are $\kappa(M) = \kappa_0(M)(1 + \frac{\alpha}{M^2})$ and $T(M) = T_0(M)(1 + \frac{\alpha}{M^2})$. Thinking of this modification idea, we can change Eq. (28) into the following:

$$\Gamma = e^{-\beta(\omega-\omega_0)+o(\omega,e,j)^2} = e^{-[\beta-(o(\omega,e,j)^2)/(\omega-\omega_0)](\omega-\omega_0)} = e^{-\beta[1-(o(\omega,e,j)^2)/(\beta(\omega-\omega_0))](\omega-\omega_0)}. \tag{29}$$

So, we can treat

$$\beta' = \beta\left[1 - \frac{o(\omega,e,j)^2}{\beta(\omega-\omega_0)}\right] \tag{30}$$

as an inverse quantum-corrected temperature. After some calculation, we find that this expression is not consistent with Refs. [21–23] in detail in a simple way because the corrections in prior references are not dependent on the radiation mass, charge and angular momentum, but we can see they are all something about corrected temperature. Maybe this should be the clue for people to extend the perfect black hole thermodynamics to general black hole thermodynamics. This is just what we will investigate in depth in the future.

Although we obtained the same conclusion as Zhang's work in [11], there are some differences between the two methods. First, Zhang's work is the development of Parikh's tunneling method. The dragging coordinate system is used because the event horizon and the infinite redshift surface coincide with each other in this coordinate system, which is necessary so that the WKB approximation can be used. In our work, we do not need to use the dragging coordinate system. Second, in Zhang's method, to conserve the symmetry of the space-time, the particle should still be an ellipsoid shell during the tunneling process, which means a should be taken as a constant. However, we do not need to do the assumption and substitute a with $a' = \frac{Ma-j}{M-\omega}$.

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