Thermodynamics of the Schwarzschild-de Sitter black hole: Thermal stability of the Nariai black hole

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We study the thermodynamics of the Schwarzschild-de Sitter black hole in five dimensions by introducing two temperatures based on the standard and Bousso-Hawking normalizations. We use the first-law of thermodynamics to derive thermodynamic quantities. The two temperatures indicate that the Nariai black hole is thermodynamically unstable. However, it seems that black hole thermodynamics favors the standard normalization and does not favor the Bousso-Hawking normalization.

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I. INTRODUCTION

The Schwarzschild black hole with negative specific heat is in an unstable equilibrium with the heat reservoir of the temperature T [1]. Its fate under small fluctuations will be either to decay to hot flat space by Hawking radiation or to grow without limit by absorbing thermal radiation in the heat reservoir [2]. This means that an isolated black hole is never in thermal equilibrium. There exists a way to achieve a stable black hole in an equilibrium with the heat reservoir. A black hole could be rendered thermodynamically stable by placing it in AdS space. An important point to understand is how a black hole with positive specific heat could emerge from thermal radiation through a phase transition. To this end, the Hawking-Page phase transition between thermal AdS space and the Schwarzschild-AdS black hole was introduced [3–5].

Furthermore, a thermodynamic similarity between the event horizon of a black hole and the cosmological horizon of de Sitter space has been established since the work of Gibbons-Hawking [6]. The key point is that a cosmological horizon possesses temperature and entropy. Ginsparg and Perry have studied the thermal properties of the Schwarzschild-de Sitter black hole (SdS) [7]. However, an issue of the negative mass to the cosmological horizon has appeared when using the first law of thermodynamics to derive thermodynamic quantities [6,8]. This problem arises because the surface gravity κ_C of the cosmological horizon is negative. Using the first law of $dM_C = [\kappa_C/8\pi]dA$ leads to the mentioned result. A way to resolve this issue is to calculate the mass of the cosmological horizon using the Brown-York approach in the asymptotic future [9].

It is known that the SdS is intriguing, but it is difficult to analyze its thermodynamic properties because a black hole is inside the fixed cosmological horizon. The cosmological horizon may play a role of the heat reservoir for a black hole like the AdS space. In order to investigate the SdS, one introduces two kinds of temperatures based on the standard and Bousso-Hawking normalizations. The standard normalization provides the Hawking temperature T_H^E and Gibbons-Hawking temperature T_H^C for event and cosmological horizons, respectively [10]. They behave differently but have the zero temperature at the Nariai case which corresponds to the maximum black hole and minimum de Sitter space. These temperatures were derived by the analogy with asymptotically flat and AdS space. In the case of asymptotically flat spacetimes, a standard method to obtain the surface gravity is to choose the Killing field that goes to a unit time-translation at infinity. An observer staying there does not feel any acceleration. However, there is no asymptotic region and thus no preferred observer in de Sitter spacetimes. Hence one has to introduce another normalization to define appropriate temperatures. This is the Bousso-Hawking normalization [11]. At the point $r = r_0$ where the metric function satisfies $h'(r_0) = 0$, the black hole attraction and the cosmological repulsion exactly cancel out, and thus one may achieve the zero acceleration inside the cosmological horizon. Including this normalization into the expression of the surface gravity, one finds the Bousso-Hawking temperatures $T_{\rm BH}^{E/C}$. These do not vanish in the Nariai limit but approach a constant value [8,12,13]. However, one has to realize that the temperature $T_{\rm BH}^C$ of cosmological horizon is just an extension of $T_{\rm BH}^E$ of the event horizon and thus an important property of the degenerate horizon at $r = r_0$ may be lost for the thermodynamic purpose.

In this work, we investigate the thermal properties and phase transition of the SdS by introducing two temperatures. Specifically, we reexamine the thermal stability of the Nariai black hole which was considered in Ref. [10].

Our study is based on the on-shell observations of temperature, heat capacity, and free energy as well as the off-shell observations of generalized free energy, deficit angle, and β-function. In general, the on-shell thermodynamics implies equilibrium thermodynamics and thus the first-law of thermodynamics holds for this case. Hence it describes

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relationships among thermal equilibria but not the transitions between equilibria. On the other hand, the off-shell thermodynamics is designed for the description of off-equilibrium configurations [14,15]. This is suitable for the description of transitions between thermal equilibria. We note that the first law of thermodynamics does not hold for off-shell thermodynamics. We believe that the thermodynamic study on the SdS is very helpful to understand de Sitter spacetimes [16] because other analyses of perturbations under the SdS background [17] are more restrictive than thermodynamic analysis in de Sitter spacetimes.

II. THE STANDARD NORMALIZATION

We wish to study the thermal property of a black hole in de Sitter space. For this purpose, we consider the Schwarzschild-de Sitter black hole in five-dimensional spacetime [18]

$$ds^2_{\rm SdS} = -h(r)dt^2 + \frac{1}{h(r)}dr^2 + r^2 d\Omega_3^2, \tag{1}$$

where the metric function $h(r)$ is given by

$$h(r) = 1 - \frac{m}{r^2} - \frac{r^2}{\ell^2}. \tag{2}$$

Here m is a reduced mass of the black hole and ℓ is the curvature radius of de Sitter spacetime. In the case of $m = 0$ (no black hole), we have the pure de Sitter space with the largest cosmological horizon ($r_C = \ell$). $m \neq 0$ generates the SdS black hole. From the condition of $h(r_{C/E}) = 0$, one finds that the cosmological and event horizons are located at

$$r^2_{C/E} = \frac{\ell^2}{2}(1 \pm \sqrt{1 - m/m_0}) \tag{3}$$

with $m_0 = \ell^2/4 = r_0^2/2$. We classify three cases with $r_0 = \ell/\sqrt{2}$: $m = m_0(r = r_0)$, $m > m_0$, and $m < m_0$. The case of $m = m_0$ corresponds to the maximum black hole with the minimum cosmological horizon, the Nariai black hole. Here we have the degenerate horizon of $r_0 = r_E = r_C$. A large black hole of $m > m_0$ is not allowed in de Sitter space. The case of $m < m_0$ corresponds to a small black hole inside the cosmological horizon. In this case a cosmological horizon is located at $r_C \simeq \ell\sqrt{1 - m/4m_0}$, while an event horizon is at $r_E \simeq \sqrt{m}$. Hence, restrictions on r_E and r_C are given by

$$0 < r_E \leq r_0, \qquad r_0 \leq r_C < \ell. \tag{4}$$

One expects that as a reduced mass m approaches the maximum value of $m = m_0$, the small black hole increases to the Nariai black hole at $r_E = r_0$ by absorbing radiation ($\overrightarrow{EH}$), whereas the cosmological horizon decreases to the minimum value of $r_C = r_0$ by emitting radiation ($\overleftarrow{CH}$). This was the Hawking-Page transition (HP) for obtaining a large, stable black hole in de Sitter space [10]. Also we note that the size of black hole is closely related to the size of the cosmological horizon.

The energy and entropy for two horizons take the forms

$$\begin{aligned} E_{E/C} &= \pm\left(\frac{3V_3 m_{E/C}}{16\pi G_5} - E_0\right) \quad \text{with} \\ m_{E/C} &= r^2_{E/C}\left(1 - \frac{r^2_{E/C}}{\ell^2}\right), \\ S_{E/C} &= \frac{V_3 r^3_{E/C}}{4G_5}, \end{aligned} \tag{5}$$

where V_3 is the volume of a unit three-dimensional sphere Ω_3 and $E_0 = \frac{3V_3 m_0}{16\pi G_5}$ is the energy of the Nariai black hole. We note here that $E_E \leq 0$, while $E_C \geq 0$. In this case, there is no energy gap between two horizons ($E_E = E_C$) at $r_+ = r_0$. We use these definitions of energy since the fixed-ℓ ensemble of de Sitter space is similar to the fixed-charge Q ensemble in the Reissner-Norström-AdS black holes [19]. However, the energy was used without E_0 in Refs. [10,20–22]. The thermodynamic quantities of temperature, heat capacity, and free energy for the two horizons are given by

$$T_H^{E/C} = \mp\left(\frac{2r^2_{E/C} - \ell^2}{2\pi\ell^2 r_{E/C}}\right), \qquad C_{E/C} = 3\frac{2r^2_{E/C} - \ell^2}{2r^2_{E/C} + \ell^2} S_{E/C}, \tag{6}$$

$$F_{E/C} = \pm\frac{V_3 r^2_{E/C}}{16\pi G_5}\left(\frac{r^2_{E/C}}{\ell^2} + 1\right) \mp E_0. \tag{7}$$

Hereafter we use the normalization of $V_3/16\pi G_5 = 1$ for simplicity. It is easily checked that the first law of thermodynamics holds for two horizons,

$$dE_{E/C} = T_H^{E/C} dS_{E/C}. \tag{8}$$

Imposing the equilibrium condition $T = T_H$, we obtain a small, unstable black hole of size

$$r_u = \frac{\pi\ell^2 T}{2}\left[-1 + \sqrt{1 + \frac{8}{(2\pi\ell T)^2}}\right] \tag{9}$$

and a large, stable cosmological horizon of size

$$r_s = \frac{\pi\ell^2 T}{2}\left[1 + \sqrt{1 + \frac{8}{(2\pi\ell T)^2}}\right]. \tag{10}$$

As is shown in Fig. 1, the temperatures $T_H^{E/C}$ behave differently. We find two thermal equilibria for the range of $0 \leq T \leq T^C_{\max} = 1/2\pi\ell$. For $T > T^C_{\max}$, there exists one unstable equilibrium. Four temperatures $\{T\}$ are introduced to investigate the phase transition. For these temperatures, we have unstable equilibria of $\{r_u\} = \{5, 5.51, 6.33, 6.91\}$ and stable equilibria of $\{r_s\} = \{10, 9.07, 7.90, 7.23\}$. Even though the temperature graph shows the key property, we need to introduce other quantities for a complete analysis of thermodynamic stability and phase transition.

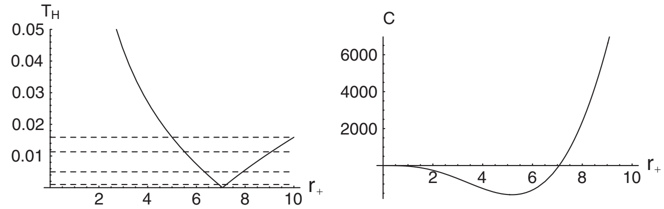

FIG. 1. Temperature and heat capacity for SdS with $\ell = 10$ and $r_0 = 7.07$. Here $r_+ = r_E(r_C)$ are confined to $0 < r_E \leq r_0(r_0 \leq r_C < \ell)$. At the left graph, the solid curve represents the temperature of the event horizon $T_H^{E/C}$, while the dashed lines denote four external temperatures of $T = T^C_{\rm max}(= 0.016)$, 0.011, 0.005, 0.001 from top to bottom.

From the graph of heat capacity, we find that the event horizon r_E is locally unstable because of negative heat capacity, whereas the cosmological horizon r_C is locally stable because of positive heat capacity. A global stability of black hole is achieved only when $C > 0$ and $F < 0$. The cosmological horizon of $r_C > r_0$ seems to be globally stable, as is shown in Fig. 2. However, such thermodynamic arguments describe relationships among thermal equilibria but not the transitions between equilibria. In order to describe transitions between thermal equilibria, we need to introduce the off-shell free energy, deficit angle, and off-shell β-function as [23]

$$F^{\rm off}_{E/C}(r_+, T) = E_{E/C} - TS_{E/C},$$
$$\delta_{E/C}(r_+, T) = 2\pi\left(1 - \frac{T_H^{E/C}}{T}\right), \tag{11}$$

$$\beta_{E/C}(r_+, T) = -6r_+^2 \delta_{E/C}(r_+, T). \tag{12}$$

We use the off-shell free energy to study the growth of a black hole [14]. In order to investigate the off-shell process explicitly, we consider the deficit angle $\delta_{E/C}$. The range of deficit angle is $0 \leq \delta_{E/C} \leq 2\pi$ for the proper transition between two black holes. $\delta_{E/C}$ has the maximum value of 2π at the extremal point and it is zero at the equilibrium point of $T = T_H$. This implies that the Nariai configuration at $r_+ = r_0$ has the narrowest cone of the shape ($\prec$) near the horizon, while the geometry at $T = T_H$ is a contractible manifold ($\subset$) without conical singularity. For any off-shell process of the growth of the black hole, we have $0 < \delta_{E/C} < 2\pi$ and a conical singularity of the shape ($<$) is allowed near the horizon [14,15,23]. Also, the off-shell β-function is introduced to measure the mass of a conical singularity at the event horizon [15]. Hereafter, we do not consider the β-function because it is proportional to the deficit angle $\delta_{E/C}$.

All equilibria of $\{r_u\}$ and $\{r_s\}$ could be reproduced by each condition of $F^{\rm off}_{E/C} = F_{E/C}$ and $\delta_{E/C} = 0$. We know that the black hole is quite different from the cosmological horizon because the former is unstable, while the latter is stable. The HP may occur for $T > T_c$ where $T_c = 0.011$ is determined from the equilibrium condition of $F^{\rm off}_C(r_c, T_c) = F_C(r_c) = F_E(0)$ at $r_c = 9.07$ with $F_E(0) = -75$. We note a sequence of free energy of $F_C(\ell) < F_E(0) < F_{E/C}(r_0)$, which means that the pure de Sitter space at $r_+ = \ell$ is globally stable and the Nariai black hole at $r_+ = r_0$ is unstable. As $T \to 0$, $F^{\rm off}(r_+, T)$ connects the point of $r_+ = 0$ to the Nariai case. On the other hand, as $T \to T^C_{\rm max}$, $F^{\rm off}(r_+, T)$ connects the point of $r_+ = 0$ to the pure de Sitter space through the unstable black hole at $r_+ = r_u$. For $T > T_c$, the pure de Sitter space is more favorable than the Nariai case, while for $T < T_c$, the pure de Sitter space is less favorable than the Nariai case. This implies that the HP of $\vec{EH}\,\overleftarrow{CH}$ is unlikely to occur by absorbing radiation, while the evaporating process of $\overleftarrow{EH}\,\vec{CH}$ is likely to occur by emitting radiation.

We note that for $T < T_C$, $F^{\rm off}_E(r_u, T)$ and $F^{\rm off}_C(r_s, T)$ are greater than $F_E(0)$. On the other hand, for $T > T_C$, $F_E(0)$ is between $F^{\rm off}_E(r_u, T)$ and $F^{\rm off}_C(r_s, T)$. There exists an evaporating process from $r_E = r_0$ to $r_E = 0$ even for $T \simeq 0$. This shows that the Nariai black hole is not a globally stable

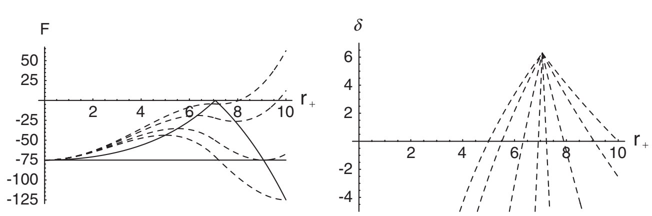

FIG. 2. The graphs of free energy and deficit angle $\delta_{E/C}$ for SdS. Here r_+ represents r_E for the event horizon and r_C for the cosmological horizon. At the left graph, the solid curve represents the free energy $F_{E/C}$, while the dashed lines denote off-shell free energy $F^{\rm off}_{E/C}(r_+, T)$ for temperatures of $T = 0.001$, 0.005, $T_c(= 0.011)$, 0.016 from top to bottom. The reverse order of T is for the deficit angle $\delta_{E/C}$.

object, whereas the pure de Sitter space is a globally stable object. The shapes of free energy and its off-shell free energy are similar to those for the Schwarzschild-AdS black hole. All of deficit angles $\delta_{E/C}$ are positive for proper transitions between r_u and r_s. The differences are the downward shift of free energy and the peak point at $r_+ = r_0$ as the extremal point. Hence, the HP of $\vec{EH}\,\overleftarrow{CH}$ may be excluded from the candidate for phase transition of the SdS. This is an opposite conclusion to the previous result based on the discontinuous free energy [10].

III. THE BOUSSO-HAWKING NORMALIZATION

The new temperatures based on the Bousso-Hawking normalization take the form [8,11–13]

$$T_{\rm BH}^{E/C} = \frac{T_H^{E/C}}{\sqrt{h(r_0)}} = \mp \frac{1}{\sqrt{1 - \frac{2r_+\sqrt{\ell^2 - r_+^2}}{\ell^2}}}\left(\frac{2r_+^2 - \ell^2}{2\pi\ell^2 r_+}\right), \quad (13)$$

where $r_+ = r_E(r_C)$ go with $0 \le r_E \le r_0 (r_0 \le r_C \le \ell)$. Here we check that in the Nariai limit, the Bousso-Hawking temperatures for event and cosmological horizons approach a constant value as

$$\lim_{r_+ \to r_0} T_{\rm BH}^{E/C} = \frac{1}{\pi\ell}. \quad (14)$$

Here one takes the limit from smaller value for computing $T_{\rm BH}^E(r_0)$, while for $T_{\rm BH}^C(r_0)$, one takes the limit from larger value.

Assuming that the first law of thermodynamics

$$d\tilde{E}_{E/C} = T_{\rm BH}^{E/C} dS_{E/C} \quad (15)$$

holds for this normalization, we obtain the corresponding energy from its integration over r_+ as

$$\tilde{E}_{E/C} = \mp\frac{2}{\ell}(\ell^2 + r_+\sqrt{\ell^2 - r_+^2})\sqrt{\ell^2 - 2r_+\sqrt{\ell^2 - r_+^2}}. \quad (16)$$

The heat capacity is defined to be

$$\tilde{C}_{E/C} = \frac{d\tilde{E}_{E/C}}{dT_{\rm BH}^{E/C}} = \frac{r_+(2r_+^2 - \ell^2)\sqrt{\ell^2 - r_+^2}(\ell^2 - 2r_+\sqrt{\ell^2 - r_+^2})}{\ell^2(2r_+^3 - 3\ell^2 r_+ + (2r_+^2 + \ell^2)\sqrt{\ell^2 - r_+^2})}. \quad (17)$$

The on-shell free energy is defined by

$$\tilde{F}_{E/C} = \tilde{E}_{E/C} - T_{\rm BH}^{E/C} S_{E/C}. \quad (18)$$

On the other hand, the off-shell free energy is

$$\tilde{F}_{E/C}^{\rm off} = \tilde{E}_{E/C} - TS_{E/C} \quad (19)$$

The equilibrium condition of $d\tilde{F}_{E/C}^{\rm off}/dr_+ = 0$ provides $T = T_{\rm BH}^{E/C}$, which shows in turn that $\tilde{F}_{E/C}^{\rm off} = \tilde{F}_{E/C}$. Similarly, we could define the deficit angle $\tilde{\delta}_{E/C}(r_+, T)$ using the temperatures $T_{\rm BH}^{E/C}$.

Now we are in a position to discuss the thermal properties of the SdS, which are based on $T_{\rm BH}^{E/C}$. From Fig. 3, it turns out that the cosmological horizon branch is just an extension of the event horizon branch. We have the temperature bound of $T_{\rm BH}^E \ge T_{\rm BH}^C$, where the equality holds for the Nariai black hole. Here we have the range of temperature for the cosmological horizon: $T_{\rm BH}^C(\ell = 10) \le T \le T_{\rm BH}^C(r_0 = 7.07)$. Both horizons are thermodynamically unstable because of $\tilde{C}_{E/C} < 0$. We have $\tilde{C}_C(\ell) = 0$ for the pure de Sitter case and $\tilde{C}_E(0) = 0$ for no black hole. From the free energy graph in Fig. 4, it follows that any Hawking-Page phase transition would not occur between two branches because thermal equilibria of the cosmological horizon are unstable points. We observe that the deficit angles $\tilde{\delta}_{E/C}$ are positive only for $r_u < r_+ < \ell$, where unstable equilibria $\{r_u\} = \{9.98, 9.68, 8.68, 7.07\}$ are determined by the condition of $T = T_{\rm BH}^C$. Actually, this region is beyond thermal equilibria. We point out that the Nariai back hole at $r_+ = r_0$ is nothing special in the thermodynamic aspect.

Finally, we mention that the Bousso-Hawking normalization does not provide attractive features for the thermodynamics of SdS because it does not make a significant distinction between the event and cosmological horizons.

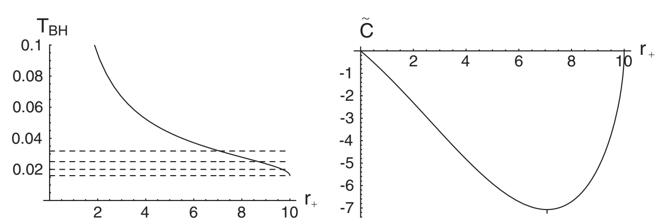

FIG. 3. Temperature and heat capacity $\tilde{C}_{E/C}$ for SdS with $\ell = 10$ and $r_0 = 7.07$. Here $r_+ = r_E(r_C)$ are confined to $0 < r_E \le r_0 (r_0 \le r_C < \ell)$. At the left graph, the solid curve represents the temperature of $T_{\rm BH}^{E/C}$, while the dashed lines denote the external temperatures of $T = 0.032, 0.025, 0.02, 0.016$, from top to bottom.

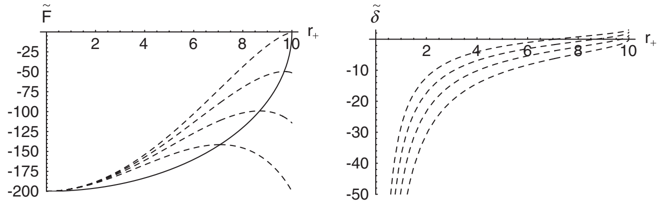

FIG. 4. The graph of free energy and deficit angle $\tilde{\delta}_{E/C}$ for SdS with $T_{\rm BH}^{E/C}$. Here r_+ represents r_E for the event horizon and r_C for the cosmological horizon. At the left graph, the solid curve represents the free energy $\tilde{F}_{E/C}$, while the dashed lines denote $\tilde{F}^{\rm off}_{E/C}(r_+, T)$ for external temperatures of $T = 0.016, 0.02, 0.025, 0.032$ from top to bottom. The reverse order of T is for the deficit angle $\tilde{\delta}_{E/C}$.

IV. DISCUSSION

We start to discuss two limiting cases: a very small black hole ($m \ll m_0$) and a nearly degenerate Schwarzschild-de Sitter case ($m \simeq m_0$).

For the first case, the effect of the radiation coming from the cosmological horizon is negligible, and one would expect the evaporating process to be similar to that of Schwarzschild black hole. Thus we expect to have the pure de Sitter space (no black hole) as the stable ending point.

The second case corresponds to the near-horizon thermodynamics of the degenerate horizon [24]. In case of the Nariai black hole, the two horizons have the same size and the same temperature. Hence they will be in thermal equilibrium. If one considers a perturbation of the geometry to cause the black hole to become hotter than the cosmological horizon, the thermal condition of the Nariai black hole becomes unstable. Actually, the thermal stability will be determined by the sign of heat capacity.

At this stage, we would like to mention the Nariai phase transition of the SdS at $T = 0$. A previous work has shown that the location $r_+ = r_0$ is not only the critical point of phase transition but also the position of the stable cosmological horizon. This arises because an inappropriate form of the discontinuous free energy was used to analyze the Nariai configuration [10]. In this work, we showed that the Nariai black hole is not a globally stable object when using the continuous free energy. Instead, the pure de Sitter space plays the role of a globally stable object. Consequently, the HP of $\overrightarrow{E}H\,\overleftarrow{C}H$ is unlikely to occur by absorbing radiation from the cosmological horizon, while the evaporating process of $\overleftarrow{E}H_{\rightarrow}$Accent is empty! is likely to occur by emitting radiation. This is consistent with intuitive thermodynamic arguments on the black hole in de Sitter space.

If one uses the Bousso-Hawking temperatures, the Nariai black hole is thermodynamically unstable because of their negative heat capacity. Furthermore, it seems inappropriate to describe either the Hawking-Page phase transition or the evaporation process by using these temperatures.

At this stage, we would like to comment on another temperature $\bar{T} \propto \sqrt{-h''(r_0)}$ of the SdS [25]. This temperature is valid for the near-horizon region only because the condition of $h''(r_0) \neq 0$ implies the near-horizon of the degenerate horizon. For the whole region, it would be better to use the temperature (4.9) in Ref. [8] for four dimensions or $T_{\rm BH}^{E/C}$ in Eq. (13) for five dimensions.

In conclusion, it turns out that the Nariai black hole of $r_E = r_0$ is a thermodynamically unstable object. The Hawking-Page phase transition from $r_E = 0$ to $r_E = r_0$ is unlikely to occur, while the evaporation process from $r_E = r_0 (= r_C)$ to $r_E = 0 (r_C = \ell)$ is likely to occur when using the Hawking and Gibbons-Hawking temperatures based on the standard normalization.

However, a small group of works [10,16,26] supports the stable Nariai black hole, while a large group of works [12,27–29] shows the instability of Nariai black hole. The former has used the standard normalization to support the stability of the Nariai black hole, the latter has used different schemes to show the instability. Here, we focus on the thermodynamic stability of the Nariai black hole. A black hole is thermodynamically unstable when its heat capacity is negative ($C < 0$). Furthermore, a global stability of black hole is achieved only when $C > 0$ and $F < 0$. As is shown Fig. 1, we have the zero heat capacity for the case of the Nariai black hole in the standard normalization, which means that the issue of thermal stability remains unclear, and it should be further resolved by choosing an appropriate free energy. We used the discontinuous free energy in Ref. [10], where the Nariai black hole was shown to be stable. In this work, we used an appropriate free energy to show the unstable Nariai black hole. On the other hand, we find from Fig. 3 that the heat capacity of the Nariai black hole is always negative. This means that the Nariai black hole is unstable when using the Bousso-Hawking normalization.

Finally, this work supports the instability of Nariai black hole.

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[1] D. J. Gross, M. J. Perry, and L. G. Yaffe, Phys. Rev. D **25**, 330 (1982).
[2] J. W. York, Phys. Rev. D **33**, 2092 (1986).
[3] S. W. Hawking and D. N. Page, Commun. Math. Phys. **87**, 577 (1983).
[4] J. D. Brown, J. Creighton, and R. B. Mann, Phys. Rev. D **50**, 6394 (1994).
[5] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
[6] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).
[7] P. H. Ginsparg and M. J. Perry, Nucl. Phys. **B222**, 245 (1983).
[8] A. Corichi and A. Gomberoff, Phys. Rev. D **69**, 064016 (2004).
[9] V. Balasubramanian, J. de Boer, and D. Minic, Phys. Rev. D **65**, 123508 (2002); R. G. Cai, Y. S. Myung, and Y. Z. Zhang, Phys. Rev. D **65**, 084019 (2002); Y. S. Myung, Mod. Phys. Lett. A **16**, 2353 (2001); A. M. Ghezelbach and R. B. Mann, J. High Energy Phys. 01 (2002) 005.
[10] Y. S. Myung, Phys. Lett. B **645**, 369 (2007).
[11] R. Bousso and S. W. Hawking, Phys. Rev. D **54**, 6312 (1996).
[12] R. Bousso and S. W. Hawking, Phys. Rev. D **57**, 2436 (1998).
[13] P. Kanti, J. Grain, and A. Barrau, Phys. Rev. D **71**, 104002 (2005).
[14] V. P. Frolov, D. V. Fursaev, and A. I. Zelnikov, Phys. Rev. D **54**, 2711 (1996).
[15] J. L. F. Barbon and E. Rabinovici, J. High Energy Phys. 03 (2002) 057; Found. Phys. **33**, 145 (2003); arXiv:hep-th/0407236.
[16] Q. G. Huang, K. Ke, and M. Li, J. High Energy Phys. 02 (2006) 045.
[17] W. T. Kim, J. J. Oh, and K. H. Yee, Phys. Rev. D **66**, 044017 (2002); M. Maceda, Phys. Rev. D **74**, 088501 (2006).
[18] R. G. Cai, Phys. Lett. B **525**, 331 (2002).
[19] A. Chamblin, R. Emparan, C. V. Johnson, and R. C. Myers, Phys. Rev. D **60**, 064018 (1999).
[20] R. G. Cai and Y. S. Myung, Phys. Rev. D **67**, 124021 (2003).
[21] S. Nojiri, S. D. Odintsov, and S. Ogushi, Int. J. Mod. Phys. A **18**, 3395 (2003).
[22] Y. S. Myung, Classical Quantum Gravity **21**, 1279 (2004).
[23] Y. S. Myung, Int. J. Mod. Phys. A **22**, 41 (2007).
[24] J. Cruz, A. Fabbri, D. J. Navarro, and J. Navarro-Salas, Phys. Rev. D **61**, 024011 (1999).
[25] J. H. Cho and S. Nam, J. High Energy Phys. 03 (2008) 027.
[26] S. Nojiri and S. D. Odintsov, Phys. Lett. B **523**, 165 (2001).
[27] O. J. C. Dias and J. P. S. Lemos, Phys. Rev. D **68**, 104010 (2003).
[28] R. Bousso, arXiv:hep-th/0205177.
[29] M. l. Liu, H. y. Liu, C. x. Wang, and Y. l. Ping, Int. J. Mod. Phys. A **22**, 4451 (2007).