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Finding evidence for massive neutrinos using 3D weak lensing

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In this paper we investigate the potential of 3D cosmic shear to constrain massive neutrino parameters. We find that if the total mass is substantial (near the upper limits from large scale structure, but setting aside the Ly alpha limit for now), then 3D cosmic shear + Planck is very sensitive to neutrino mass and one may expect that a next generation photometric redshift survey could constrain the number of neutrinos N_ν and the sum of their masses $m_\nu = \sum_i m_i$ to an accuracy of $\Delta N_\nu \sim 0.08$ and $\Delta m_\nu \sim 0.03$ eV, respectively. If in fact the masses are close to zero, then the errors weaken to $\Delta N_\nu \sim 0.10$ and $\Delta m_\nu \sim 0.07$ eV. In either case there is a factor 4 improvement over *Planck* alone. We use a Bayesian evidence method to predict *joint* expected evidence for N_ν and m_ν. We find that 3D cosmic shear combined with a Planck prior could provide "substantial" evidence for massive neutrinos and be able to distinguish "decisively" between many competing massive neutrino models. This technique should "decisively" distinguish between models in which there are no massive neutrinos and models in which there are massive neutrinos with $|N_\nu - 3| \gtrsim 0.35$ and $m_\nu \gtrsim 0.25$ eV. We introduce the notion of marginalized and conditional evidence when considering evidence for individual parameter values within a multiparameter model.

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I. INTRODUCTION

In this paper we will investigate the potential of 3D weak lensing to constrain the properties of, and provide evidence for, massive neutrinos. The conclusion that neutrinos have mass, and the resolution of the actual masses, would have a profound impact on our understanding of particle physics and cosmology.

As a photon travels from a distant galaxy the path it takes is diverted, by the presence of large-scale structure along the line-of-sight resulting in the image of any galaxy being slightly distorted. This weak lensing effect depends on both the details of the matter power spectrum and growth of structure as well as the geometry of the observer-lens-source configuration. 3D weak lensing combines this weak lensing information with any redshift information available which then allows for evolving effects, for example, dark energy, to be investigated. Weak lensing (see [1] for a recent review) has been used to constrain cosmological parameters including dark energy parameters ([2,3]), measure the growth of structure [4,5], and map the dark matter distribution as a function of redshift [6]. It has been shown that 3D weak lensing has the potential to constrain dark energy parameters to an unprecedented degree of accuracy using upcoming and future surveys, e.g. Pan-STARRS [7] and DUNE [8]. In addition to dark energy parameters, galaxy redshift surveys [9–11] and weak lensing tomography (in which galaxies are binned in redshift) [12–14] have the potential to constrain the total neutrino mass. In this paper we consider a novel technique, 3D cosmic shear [2,15–17] in which galaxies are not binned in redshift and the full 3D shear field is used, thus maximizing the information extracted. We report the 3D cosmic shear performance in constraining neutrino properties and also present Bayesian evidence calculations that will show whether future weak lensing surveys will find convincing evidence for massive neutrinos.

In the standard model of particle physics, neutrinos must have zero mass by definition; if neutrinos have mass this may be a signal of nonstandard neutrino interactions or

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Higgs mechanisms, evidence that the standard model is not renormalisable, or extra (stringy) dimensions (we refer to neutrino mass and oscillation reviews [18–20] and references therein). In cosmology, neutrinos play a role in structure formation by damping structure on small scales. They can be categorized as a hot component of dark matter, though recent results (for example WMAP [21]) rule out massive neutrinos as the dominant dark matter component. Beyond the affect of neutrinos on large scale structure, they are of further interest in cosmology since by directly observing neutrinos they could provide a window on the early universe beyond the surface of last scattering to the epoch of electroweak unification. Theoretically, massive (majorana; $\nu_\alpha = \bar{\nu}_\alpha$) neutrinos may provide some explanation for the baryon asymmetry [22]. And indeed astronomical considerations first alluded to neutrino mass since the number of neutrinos detected from the Sun was much less than the expected number from the Sun's luminosity.

There is currently a substantial and growing amount of evidence that neutrinos have mass (for recent reviews see [18–20]). This conclusion has been reached using results from large particle physics experiments which have found that the oscillation of neutrinos from one flavor (e, μ, or τ) to another is needed to explain the observed data. Super-Kamiokande [23] found that only $\sim 1/3$ the flux of muon neutrinos ν_μ from cosmic ray collisions in the atmosphere were observed along the line-of-sight through the Earth implying an oscillation of ν_μ to some other flavor with a scale length comparable to the radius of the Earth. SNO [24] has observed both the total flux of neutrinos from the Sun as well as the flux of electron neutrinos ν_e and found that the "solar neutrino problem" is resolved by postulating that ν_e oscillate to other flavors in the high density environment of the Sun's core by gaining a small effective mass. Further evidence for neutrino oscillations comes from nuclear reactors (KamLAND [25]) and neutrino beam experiments (K2K [26]).

The observed oscillation of neutrinos is linked to the implication that neutrinos have mass via the lepton mixing matrix U^2 whose elements describe the probability for one neutrino flavor to oscillate to another

$$|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle; \qquad (1)$$

this relates the observed neutrino flavors $|\nu_\alpha\rangle$ to a hierarchy of neutrino mass states $|\nu_i\rangle$. In general the elements of U can be complex. The standard model has three neutrino flavors $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$. The lepton mixing matrix allows for more than three neutrino flavors; if there are more than three then the extra neutrinos are called "sterile" since they would not couple to any standard model electroweak interaction (i.e. would not be associated with electrons, muons or tau).

Assuming three neutrino flavors U can be parametrized as a product of three Euler rotations $U = R_{23}R_{13}R_{12}$ where each rotation describes how one neutrino mass state is coupled to another. The elements of the rotation matrices depend on the *mass difference* $\delta m_{ij} = m_i - m_j$ and an angle θ_{ij} on which the probability $P(\nu_i \rightarrow \nu_j)$ depends. Thus neutrino oscillation experiments can only measure the relative masses of neutrinos not the absolute mass scale.

Currently world neutrino data are consistent with a three-flavor mixing framework (see [27] and references therein), parametrized in terms of three neutrino masses (m_1, m_2, m_3) and of three mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$), plus a possible CP violating phase δ. Current constraints are $\delta m_{23} \sim 0.05$ eV and $\delta m_{12} \sim 0.007$ eV. There are currently no strong constraints on δm_{13} though upcoming experiments, for example, T2K [28] should measure $\Delta\theta_{13} \sim 0.05$. Thus current constraints allow for two possible orders of the massive neutrino hierarchy: $m_1 < m_2 < m_3$ or the inverted hierarchy $m_3 < m_1 < m_2$. There are planned particle physics experiments that will measure the absolute mass scale via the beta decay of tritium to constrain $m_{\nu e}$ [29–31], for example, KATRIN [32,33] should reach an accuracy of $\Delta m_{\nu e} \sim 0.35$ eV. There are arguments (see [19]) that the requirement on the accuracy of the absolute mass scale needed to break the hierarchy or inverted-hierarchy degeneracy is $\Delta m_\nu \lesssim 0.1$ eV. As discussed below this can be achieved by cosmological observations.

Cosmologically massive neutrinos play a role in structure formation since free-streaming neutrinos can suppress growth on small scales. Neutrinos streaming from an overdensity will reduce the amount of matter that can gravitationally accumulate by providing an extra effective pressure. In the nomenclature of cosmological parameter estimation massive neutrinos modify the matter power spectrum's growth rate by providing a suppression at small scales. It can thus leave key signatures in large scale structure data (see, eg., [34]) and, to a lesser extent, in CMB data (see, e.g. [35]). Very recently, it has also been shown that accurate Lyman-α (Lyα) forest data [36], taken at face value, can improve the current CMB + LSS constraints on m_ν by a factor of ~ 3, with important consequences on absolute neutrino mass scenarios [37]. Further cosmological neutrino mass probes include weak lensing of the CMB ([38,39]) (which would be very complementary to 3D weak lensing constraints) and the ISW-galaxy cross correlation [40]. Current cosmological constraints from WMAP CMB combined with SDSS BAO and including Lyman-α constraints provide a current upper limit on the total mass of $m_\nu \lesssim 0.42$–0.79 eV depending on the assumptions made, with a median value of $m_\nu \lesssim 0.67$ eV [21,41–44].

There has been substantial work in numerically estimating the growth rate including the presence of massive neutrinos (for example [34,40,45]). A degeneracy between neutrino mass and dark energy parameters arises because they both effectively suppress the matter power spectrum growth rate as highlighted by [45]. Optimistically, methods which can constrain dark energy parameters well should also be able to constrain the neutrino mass parameters, and by combining constraints from multiple methods (e.g. CMB, weak lensing) parameter degeneracies should be lifted.

It is currently believed that the hierarchical mass scale of neutrinos implies that the total mass of neutrinos will be approximately $m_\nu \sim 0.04$–0.1, therefore probes which are sensitive to this range of values are required to effectively constrain the neutrino mass. The remainder of this paper will highlight the possibility of using 3D cosmic shear to constrain massive neutrino properties. We will introduce the methodology and assumptions made in Sec. II and present results in Secs. III and IV, in Sec. V we will discuss our conclusions.

II. METHODOLOGY

The central quantity in cosmological neutrino mass constraints is the fraction of the total matter density that is attributed to massive neutrinos $f_\nu \equiv \Omega_\nu/\Omega_m$ which we take from [11] [Eq. (1)]

$$f_\nu = 0.05\left(\frac{m_\nu}{0.658\ \mathrm{eV}}\right)\left(\frac{0.14}{\Omega_m h^2}\right) \tag{2}$$

where $m_\nu = \sum_{i=0}^{N_\nu} m_i$ is the total neutrino mass, a sum over all neutrino species each with a mass m_i. The effect of massive neutrinos on the matter power spectrum is commonly expressed using $\Delta P(k)/P(k) = [P(k; f_\nu) - P(k; f_\nu = 0)]/P(k; f_\nu = 0)$ which decreases and is negative as the wave number k increases and power is suppressed due to the free-streaming of the neutrinos. We use the Eisenstein and Hu [34] fitting formula for the linear power spectrum, which depends on both the number of massive neutrino species N_ν and the total neutrino mass, and uses the modification of the linear growth factor $f \equiv d\ln\delta/d\ln a$ suggested by [45] which adds a further dependence on f_ν and in addition a dependence on the dark energy equation of state $w \equiv p/\rho$. We use the common parametrization of the dark energy equation of state $w(a) = w_0 + w_a(1 - a)$ [46]. One limitation of the Eisenstein and Hu [34] parametrization is that it assumes that each massive neutrino has the same mass, i.e. $m_\nu = N_\nu m_i$ where m_i is the same for all neutrino mass eigenstates. Further to this approximation we will also treat the number of massive neutrinos as a continuous parameter that is fitted by data, as opposed to an integer number. This can be justified since any light particle that does not couple to electrons, photons, or ions will contribute to the effective number. However these assumptions imply that the constraints and predicted evidence presented are meant to be indicative of the ability of 3D cosmic shear to constrain neutrino mass and not as entirely representative of the situation as it is currently understood—in which there are an integer number of mass eigenstates each with a different mass.

We use a k range of $k = 0.001$–$1\ \mathrm{Mpc}^{-1}$ for the weak lensing Fisher matrix calculations and use the Smith *et al.* [47] nonlinear correction to the linear power spectrum. Note that this is in the quasilinear régime and using wave numbers that are at the limit of the reliability of the linear power spectrum fitting formula for massive neutrinos (for a recent review of the effect of massive neutrinos on the nonlinear power spectrum see [48]).

A. 3D cosmic shear

The cosmological probes that we will consider in this paper are 3D cosmic shear and CMB. We use a CMB Planck Fisher matrix which is calculated using CMBFAST (version 4.5.1, [49]) using the method outlined in [6] and will effectively be used as a prior in the results presented for 3D cosmic shear. For a general discussion of using Planck to constrain neutrinos see [50].

The weak lensing method we use, 3D cosmic shear [2,15–17], uses the weak lensing shear and redshift information of every galaxy. The 3D shear field is expanded in spherical harmonics and the covariance of the transform coefficients can be used to constrain cosmological parameters. The transform coefficients for a given set of azimuthal ℓ and radial k [$h\mathrm{Mpc}^{-1}$] wave numbers are given by summing over all galaxies g;

$$\hat{\gamma}(k, \ell) = \sum_g \gamma^g k j_\ell(kr^g) \mathrm{e}^{-i\ell.\theta^g}, \tag{3}$$

following the conventions of [15], and we assume a flat sky approximation. θ^g is the angular position of a galaxy on the sky, r^g is the comoving distance to the galaxy, and j_ℓ are spherical Bessel functions. γ^g is the measured shear of the galaxy which parametrizes the amount of distortion that the galaxy image has obtained due to intervening large scale structure.

This is a novel approach over other 3D weak lensing analyses since galaxies are not binned in redshift which may cause problems at the bin boundaries and mean information loss in averaging over the bins. The binning approach, weak lensing tomography, creates a 2D map of the galaxy's distortions at each redshift and takes the cross correlation between each map to gain some extra 3D information. Conversely, 3D cosmic shear, presented here, uses the entire 3D shear field thus maximizing the potential for information to be extracted from the galaxy image distortions.

Since the mean of the coefficients in Eq. (3) is zero, the covariance is varied until it matches that of the data [2], i.e. the covariance is used as the "signal." The 3D cosmic shear covariance depends on the lensing geometry and the matter power spectrum, so the total parameter set that can be constrained is: Ω_m, Ω_{de}, Ω_b, h, σ_8, w_0, w_a, n_s, the running of the spectral index α_n, m_ν, and N_ν. We also

TABLE I. The parameters describing the surveys investigated.

Survey	DUNE (fiducial)
Area/sqdeg	20 000
$z_{\rm median}$	0.90
n_0/sqarcmin	35
$\sigma_z(z)/(1+z)$	0.025
σ_ϵ	0.25

include the tensor to scalar ratio $r = T/S$ and the optical depth to last scattering τ for the CMB Fisher matrix calculation. We do not assume spatial flatness and all results on individual parameters are fully marginalized over all other cosmological parameters. We use an $\ell_{\rm max} = 5000$ for the 3D cosmic shear analysis and a $k_{\rm max} = 1.0\ {\rm Mpc}^{-1}$, and use the same assumptions presented in [17]. For the Planck constraints we use a maximum ℓ of $\ell_{\rm max} = 2500$. We will present results for a fiducial weak lensing survey which is based on the DUNE weak lensing concept. The assumed survey parameters are outlined in Table I. The $z_{\rm median}$ is the median redshift of the number density distribution of galaxies with redshift, and n_0 is the observed surface number density of galaxies. $\sigma_z(z)$ describes how the average accuracy with which a galaxy's position in redshift is known, and σ_ϵ is the statistical variance of the intrinsic observed distortion of galaxies due to their random orientation. Note that we expect the photometric redshift error to have a small effect on the predicted statistical constraints as shown in [17], however a good photometric redshift error is required to reduce the effect of intrinsic alignment systematics as shown in [51,52].

B. Fisher matrix and Bayesian evidence

The results presented in this paper will use the Fisher matrix formalism to make predictions of the cosmological parameter errors. The Fisher matrix is defined as the second derivative of the likelihood surface about the maximum. In the case of Gaussian distributed data with zero mean, this is given by [53–55]

$$F_{\alpha\beta} = \tfrac{1}{2}{\rm Tr}[C^{-1}C_{,\alpha}C^{-1}C_{,\beta}] \tag{4}$$

where $C = S + N$ is the theoretical covariance of a particular method which consists of signal S and noise N terms. The commas in Eq. (4) denote derivatives with respect to cosmological parameters about a fiducial cosmology. The fiducial cosmology used in this paper is based on the WMAP results [21]; $\Omega_m = 0.27$, $\Omega_{de} = 0.73$, $\Omega_b = 0.04$, $h = 0.71$, $\sigma_8 = 0.80$, $w_0 = -1.0$, $w_a = 0.0$, $n_s = 1.0$, $\alpha_n = 0.0$. We consider two sets of fiducial value for the neutrino mass; one which is in agreement with current cosmological constraints, and one in which there are no massive neutrinos. This will allow for some discussion on the sensitivity of the predicted results to the fiducial values. The first $m_\nu = 0.66$ eV and $N_\nu = 3.0$ is high compared to the expected hierarchical mass scale. We justify this since we are using the current constraint on the neutrino mass from cosmology. The second set of fiducial values are $m_\nu = 0$ eV and N_ν(massless) $= 3.0$.

The predicted marginal errors are calculated by taking the inverse of the Fisher matrix, the error on the ith cosmological parameter is given by $\Delta\theta_i = \sqrt{(F^{-1})_{ii}}$. To combine constraints from multiple experiments, the Fisher matrices are summed e.g $F_{\rm total} = F_{\rm lensing} + F_{\rm CMB}$.

In addition to presenting the marginal error on the cosmological parameters, we will present the expected Bayesian *evidence* that the fiducial survey could achieve for massive neutrinos. Computing the evidence allows one to *distinguish* different models rather than constrain parameters *within* a model (e.g. [56]) as explained below. A procedure for calculating the expected evidence directly from the Fisher matrix was presented in [57]. In the case of massive neutrinos, there is a natural question that an evidence calculation can answer: does the data provide evidence for massive neutrinos? Note that this is distinct from assuming that massive neutrinos exist and using the data to constrain their expected properties.

The concept of evidence is derived from Bayes' theorem which relates the probability of model given the data $p(M|D)$, to the probability of the data given the model $p(D|M)$

$$p(M|D) = \frac{p(D|M)p(M)}{p(D)} \tag{5}$$

where $p(M)$ is the prior probability on any parameters within the model M. We assume two competing models M and M'. We also assume that M' is a simpler model than M, containing fewer parameters $n' < n$, and that the models are nested, i.e. the more complex model M is an extension of the simpler model M'. By marginalization $p(D|M)$, known as the *evidence*, is

$$p(D|M) = \int d\theta p(D|\theta, M)p(\theta|M). \tag{6}$$

The posterior relative probabilities of the two models, regardless of what their model parameters are, is

$$\frac{p(M'|D)}{p(M|D)} = \frac{p(M')}{p(M)} \frac{\int d\theta' p(D|\theta', M')p(\theta'|M')}{\int d\theta p(D|\theta, M)p(\theta|M)}. \tag{7}$$

By assuming uniform priors on the models, $p(M') = p(M)$, this ratio simplifies to the ratio of evidences which is called the *Bayes Factor*,

$$B \equiv \frac{\int d\theta' p(D|\theta', M')p(\theta'|M')}{\int d\theta p(D|\theta, M)p(\theta|M)}. \tag{8}$$

It is the evaluation of the Bayes factor that allows one to determine whether a data set can distinguish between competing models. We wish to forecast whether a future survey will be able to distinguish between models. This can be done using the Fisher matrix using the following expression, given in [57]:

$$B = (2\pi)^{-p/2} \frac{\sqrt{\det F}}{\sqrt{\det F'}} \exp\left(-\frac{1}{2}\delta\theta_\alpha F_{\alpha\beta}\delta\theta_\beta\right) \prod_{q=1}^{p} \Delta\theta_{n'+q}, \tag{9}$$

with $\delta\theta_\alpha$ given by

$$\delta\theta'_\alpha = -(F'^{-1})_{\alpha\beta} G_{\beta\zeta}\delta\psi_\zeta \qquad \alpha, \beta = 1 \ldots n, \zeta = 1 \ldots p \tag{10}$$

where $p \equiv n - n'$. Note that F and F^{-1} are $n \times n$ matrices, F' is $n' \times n'$, and G is an $n' \times p$ block of the full $n \times n$ Fisher matrix F. $\delta\psi$ are the differences in the parameter's values between models M and M'. $\Delta\theta$ are any prior ranges imposed on the parameters. We set $\Delta\theta = 1$ at all times. The expression in Eq. (9), given its implicit assumption of Gaussian likelihood surfaces, allows one to very quickly evaluate the expected evidence. This was done in [57] for the case of modified gravity, forecasting the expected evidence for a single extra parameter γ which parametrizes any deviation from general relativity. Here we will use Eq. (9) to calculate the *joint* expected evidence for two parameters m_ν and N_ν. The Bayesian evidence has been extensively studied in cosmology in general (for example [58–61]) and in the field of dark energy (for example [62–65]).

Throughout this paper we will use the Jeffreys [66] scale in which, $\ln B < 1$ is "inconclusive," $1 < \ln B < 2.5$ is described as "substantial" evidence in favor of a model, $2.5 < \ln B < 5$ is "strong," and $\ln B > 5$ is "decisive."

III. MARGINAL ERROR RESULTS

In this section, we will present that the predicted marginal errors on the massive neutrino parameters could be found using 3D cosmic shear in combination with a CMB Planck experiment.

Table II shows the marginalized cosmological parameter errors predicted for Planck alone and combined with the 3D cosmic shear constraints from the fiducial survey. We also include the dark energy pivot redshift error. The pivot redshift is the point at which the error on w_p minimizes, this is defined by rewriting the dark energy parametrization used $w(a) = w_0 + w_a(1 - a)$ to $w(a) = w_p + w_a(a_p - a)$ where $a_p = a_p(z_p)$ can correspond to any redshift. The Dark Energy Task Force (DETF; [67]) Figure of Merit (FoM) is defined as being proportional to the reciprocal of the area constrained in the (w_p, w_a) plane at the pivot redshift

$$\text{FoM} = 1/\Delta w_p \Delta w_a. \tag{11}$$

Note we use the reciprocal of the 1-σ two-parameter ellipse, the DETF use the 2-σ two-parameter ellipse which differs from Eq. (11) by a constant factor.

A. Error Forecasts

By combining 3D cosmic shear constraints with Planck, the massive neutrino parameters could be constrained with marginal errors of $\Delta m_\nu \sim 0.03$ eV and $\Delta N_\nu \sim 0.08$ if the neutrinos are massive. This is a factor of 4 improvement over Planck alone. The dramatic improvement comes from the lifting of parameter degeneracies when the extra constraints are added. As shown in [52], without massive

TABLE II. Predicted marginalized cosmological parameter errors for Planck alone and combined with the 3D cosmic shear constraints from the fiducial survey. We also show the dark energy pivot redshift error and the DETF FoM. We show results assuming two different sets of fiducial values for the massive neutrino parameters, one in which neutrinos have mass and one in which neutrinos are massless.

Fiducial values:	$m_\nu = 0.66$ eV and $N_\nu = 3.00$		$m_\nu = 0$ eV and $N_\nu = 3.00$	
Parameter	Planck alone	3D Cosmic Shear + Planck	Planck alone	3D Cosmic Shear + Planck
$\Delta\Omega_m$	0.0014	0.0008	0.0104	0.0041
$\Delta\Omega_{de}$	0.0015	0.0012	0.0041	0.0021
Δh	0.0167	0.0055	0.0148	0.0060
$\Delta\sigma_8$	0.0965	0.0040	0.0999	0.0202
$\Delta\Omega_b$	0.0019	0.0006	0.0014	0.0007
Δw_0	0.5622	0.0442	0.6031	0.0309
Δw_a	1.8679	0.2277	1.9158	0.1853
Δn_s	0.0103	0.0018	0.014 35	0.0039
$\Delta\alpha_n$	0.0083	0.0044	0.0074	0.0046
Δr	0.0199	0.0193	0.0207	0.0202
$\Delta\tau$	0.0084	0.0078	0.0080	0.0078
Δm_ν/eV	0.2815	0.0324	0.3815	0.0728
ΔN_ν	0.1144	0.0836	0.2807	0.1042
Δw_p	0.1177	0.0110	0.1879	0.0112
DETF FoM	5	400	3	490

neutrinos the fiducial survey design could provide a dark energy FoM = 475. The inclusion of the massive neutrino parameters does not degrade this FoM substantially, since the extra parameters are well constrained. If there are no massive neutrinos, then the marginal errors on these parameters degrade.In this case the mass and number could be constrained to $\Delta m_\nu \sim 0.07$ eV and $\Delta N_\nu \sim 0.10$, however this is still a factor of 4 improvement over Planck alone.

This degradation in the marginal error occurs because the effect of massive neutrinos on the matter power spectrum and hence on 3D weak lensing is nonlinear. If the mass of neutrinos is larger then the amount of suppression at a given scale increases, furthermore the effect on the linear growth factor as a function of f_ν given by [45] is nonlinear. In addition, the scale at which power is suppressed due to free-streaming varies as a function of neutrino mass; neutrinos with lighter masses suppress growth at larger scales (higher k) at a given redshift. Reference [11] investigated the effect of the neutrino mass fiducial model when making predictions on future marginal errors for a galaxy redshift survey and found that if the fiducial value of $f_\nu \lesssim 0.01$ then the marginal error on N_ν depends strongly on the assumed value of f_ν (e.g. [11], Fig. 3). One should only expect parameter's fiducial values to have a small effect on the predicted marginal errors if they have a linear effect on the covariance of a method, i.e. so that the derivatives in the Fisher matrix [Eq. (4)] do not change as the point around which the derivative is taken changes. The assumed fiducial value of the neutrino parameters has a small effect on the errors of most other cosmological parameters, given that the fiducial models are so different, except for Ω_m. This is because the contribution of massive neutrinos to the total mass energy behaves with redshift like a extra matter density.

Hannestad *et al.* [12] find that, by combining the weak lensing tomography constraints from their wide-5 survey (which is similar to our fiducial survey) with Planck, $\Delta m_\nu = 0.043$ eV and $\Delta N_\nu = 0.067$. They assume a massive neutrino fiducial model with $m_\nu = 0.07$ eV and $N_\nu = 3.0$. We find good agreement between this result and our massive neutrino fiducial model despite using different weak lensing methods, slightly different survey designs, and different fiducial models. They find Planck-only predicted errors of $\Delta m_\nu = 0.48$ eV and $\Delta N_\nu = 0.19$, although they do not include B modes which we do in our CMB Fisher matrix [6].We also checked the CMB Fisher matrices with an MCMC analysis and found that the error on N_ν for Planck, assuming that the neutrino mass is fixed, was $\Delta N_\nu = 0.084$ and for the Fisher matrix analysis we find $\Delta N_\nu = 0.10$, assuming that the mass is fixed.

B. Forecast uncertainties

A common concern which arises in predictive weak lensing studies is that of uncertainties in the nonlinear matter power spectrum—the use of numerical nonlinear prescriptions (e.g. [47]) which are known to be accurate only to ~8%, and the concern that the numerical prescriptions do not contain sufficiently realistic prescriptions for clustering on nonlinear scales. Therefore the use of nonlinear modes may yield predicted cosmological parameter constraints which are too optimistic, or simply unreliable. Reference [14] shows that weak lensing forecasts are dependent on the nonlinear power spectrum. Reference [68] shows that there is at best a 5% difference in the power spectrum derived from various matter-only simulations on nonlinear scales of $k \gtrsim 1$ Mpc^{-1}. References [69,70] show that there are additional uncertanties in the weak lensing power spectrum due to cool baryon infall at scales of $\ell \gtrsim 1000$. References [71,72] have also highlighted that baryon contraction needs to be accurately modeled for the weak lensing power spectrum to be reliable at $\ell \gtrsim 1000$. When utilizing the effect of neutrinos on the power spectrum, the clustering of these neutrinos in massive halos should also be taken into consideration [73].

Throughout this paper we use a $\ell_{\max} = 5000$, and use the prescription in [47] to modify the matter power spectrum for nonlinear scales, so that we have used quasilinear and nonlinear modes in our predictions. In Table III we present forecasts for which we take $\ell_{\max} = 1000$ and compare these with the baseline predictions for which $\ell_{\max} = 5000$. The $\ell_{\max} = 1000$ results are pessimistic since they assume that modelling uncertainties will be so severe that nonlinear modes could not be used. It can be seen that the

TABLE III. Predicted marginalized massive neutrino and dark energy cosmological parameter errors for 3D Cosmic Shear + Planck from the fiducial survey using $\ell_{\max} = 1000$ and $\ell_{\max} = 5000$. Note that these errors are marginalized over all other cosmological parameters. The $\ell_{\max} = 1000$ results are the worst-case scenario for the predicted cosmological parameter errors, where no information on nonlinear scales is used.

Fiducial values:	$m_\nu = 0.66$ eV & $N_\nu = 3.00$		$m_\nu = 0.00$ eV and $N_\nu = 3.00$	
Parameter	$\ell_{\max} = 1000$	$\ell_{\max} = 5000$	$\ell_{\max} = 1000$	$\ell_{\max} = 5000$
Δm_ν/eV	0.0377	0.0324	0.1422	0.0728
ΔN_ν	0.1100	0.0836	0.1102	0.1042
Δw_0	0.0536	0.0442	0.0481	0.0309
Δw_a	0.2725	0.2277	0.2556	0.1853

average degradation in the predicted constraint is a factor of ~1.4 when $\ell_{max} = 1000$ is used in comparison with $\ell_{max} = 5000$. Even when nonlinear modes are neglected entirely, the predicted marginal errors remain reassuringly robust, and this is consistent over the two sets of fiducial parameter values that we have investigated.

Photometric weak lensing surveys will also have inherent systematic effects that could degrade the predicted statistical parameter errors. However as shown in [52] it can be expected that realistically the dark energy FoM constraints from 3D cosmic shear combined with a Planck prior will be reduced by approximately a factor of 2 due to photometric, intrinsic alignment, and shape measurement systematic effects, this correspond to a factor of $\sqrt{2}$ for each dark energy parameter. Since, as highlighted by [45], the neutrino mass parameters affect the power spectrum in a similar way to dark energy, one should realistically expect *at most* a factor of $\sqrt{2}$ reduction in the combined constraints due to systematics. Using this heuristic approximation this yields constraints of $\Delta m_\nu \sim 0.04$ eV and $\Delta N_\nu \sim 0.11$ for a massive neutrino fiducial model.

IV. BAYESIAN EVIDENCE RESULTS

In the interpretation of the results in the following section one should keep in mind that the magnitude of the Bayes factor shown is a prediction of an experiments *ability to distinguish* one model over another, i.e. to what level could the experiment in consideration will be able to provide evidence for the fiducial model over another competing model (or vice versa) where the models are distinguished by changes in the parameter values $\delta\psi_i$.

Note that in the evidence calculation we use the $m_\nu = 0$ eV and N_ν(massless) $= 3.0$ fiducial values for the Fisher matrices since this represents the "simple" model as described in Sec. II B and will allow for statements to be made about whether the data could provide evidence for a more complicated model containing massive neutrinos over a simpler model with no massive neutrinos.

A. Multiparameter expected evidence

Figure 1 shows the expected evidence contours for m_ν and N_ν jointly for the fiducial survey design. Note that a $\delta N_\nu = 0$ means that $N_\nu = 3$ and $\delta m_\nu = 0$ means that $m_\nu = 0$ eV, i.e. at the fiducial values. The figure shows that there is a substantial improvement in combing Planck with 3D weak lensing data. On its own, Planck could only provide at most substantial evidence massive neutrinos for models with a large range of massive neutrinos parameter values. The fiducial survey using 3D cosmic shear combined with a Planck prior will:

(i) Provide substantial evidence for massive neutrinos over models in which there are no massive neutrinos, and if the neutrino mass is small $\delta m_\nu \lesssim 0.1$ eV then there will be substantial evidence for these models.

(ii) Be able to decisively distinguish between models in which there are no massive neutrinos and models in which $N_\nu \lesssim 3.00 - 0.40$ or $N_\nu \gtrsim 3.00 + 0.40$ and $m_\nu \gtrsim 0.25$ eV.

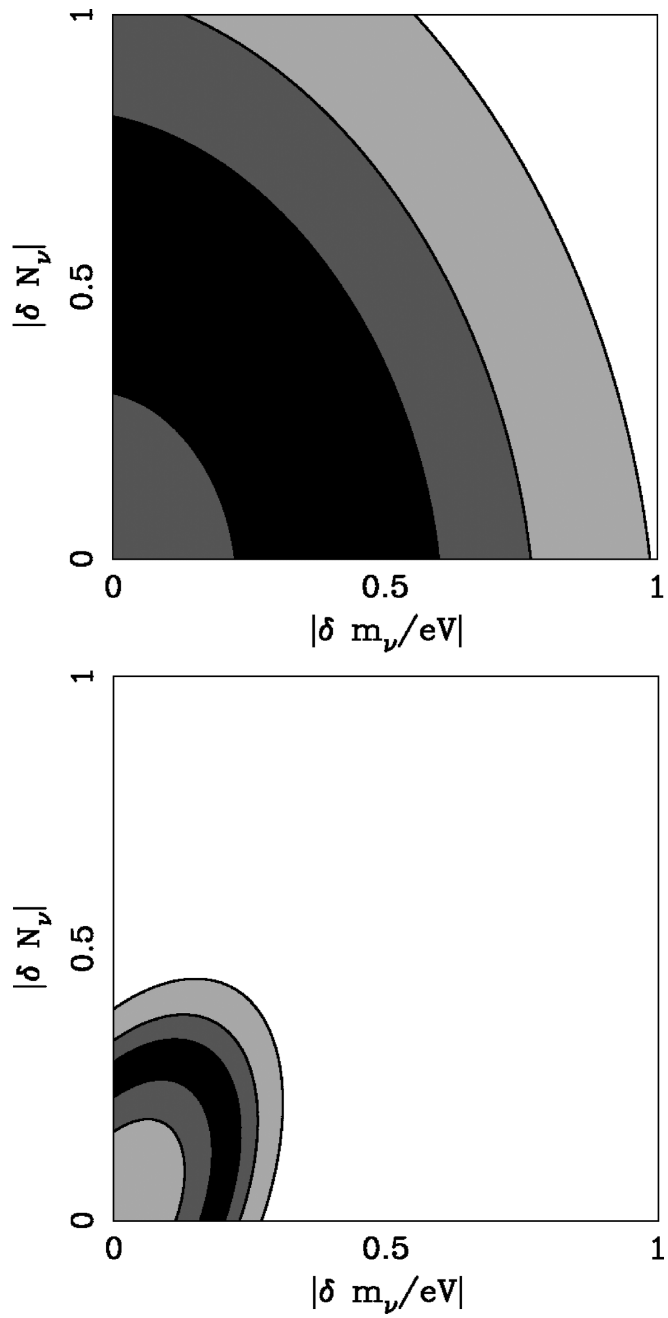

FIG. 1. The expected joint evidence for the number N_ν and mass m_ν of neutrinos using 3D cosmic shear and the fiducial survey design. White = decisive, lightest gray = substantial, darkest gray = strong, and black = inconclusive. The upper panel shows the constraints from Planck alone, the lower panel shows the constraints when 3D cosmic shear from the fiducial survey is combined with Planck.

(iii) Specifically the experiment could decisively distinguish between models in which there are three massless neutrinos and (i) models in which there are few $N_\nu < 2.6$ (possibly zero) massive $m_\nu > 0.25$ eV neutrinos, (ii) models in which there are many $N_\nu > 3.4$ massive $m_\nu > 0.25$ eV neutrinos, (iii) models in which there are few $N_\nu < 2.6$ massless $m_\nu = 0$ eV neutrinos, and (iv) models in which there are many $N_\nu > 3.4$ massless $m_\nu = 0$ eV neutrinos.

There is a band in which the expected evidence is inconclusive (the black band in Fig. 1), this represents the boundary between where the data would favor the simpler fiducial model and the situation in which the data would favor a different model (i.e. where the probability of either the fiducial or a different model being correct is equal).

B. Single-parameter expected evidence

As well as the joint expected evidence on the two massive neutrino parameters, we can also investigate the expected evidence for either parameter individually. When this is done there are two ways in which the other (hidden) parameter(s) can be dealt with:

(i) The hidden parameters can either be assumed to be fixed at their fiducial values. We will refer to this as the *conditional* evidence. In this case the expected evidence presented has the implicit assumption that the hidden parameters have the value chosen. This basically creates an intermediate model with one extra parameter.

(ii) The evidence can be integrated over the hidden parameters to obtain what we will refer to as the *marginal* evidence. For a multiparameter model which depends on $\theta_{i=1,\ldots,n}$ parameters, the total expected evidence is a function of all these parameters $B(\theta_{i=1,\ldots,n})$. The marginal evidence on one of the parameters θ_j is given by

$$B(\theta_j) = \int d\theta_1 \ldots d\theta_{j-1} d\theta_{j+1} \ldots d\theta_n B(\theta_{i=1,\ldots,n}). \tag{12}$$

In the case presented here, that of two parameters, the total evidence $B(N_\nu, m_\nu)$ can be integrated to obtain the marginal evidence $B(N_\nu)$ or $B(m_\nu)$.

The meaning of this is that the expected evidence is the probability of the simple model being favored, given that the more complex model is the true one, with given extra parameters δN_ν, δm_ν. Hence if we marginalize over one parameter, we compute the probability of the simpler model being favored, given that one extra parameters is fixed, and the other nonzero, but unknown with a flat prior.

Figure 2 shows the one-dimensional expected evidence for 3D cosmic shear combined with a Planck prior. It can be seen from both panels in this figure that by using the conditional evidence one can over estimate the evidence when the deviation between models is small by up to a factor of ~5, however when the models being compared are very different (large values of $\delta\psi$) the marginal and conditional evidences converge. The results drawn from these plots are similar to those from the full joint evidence. 3D cosmic shear should find substantial evidence that neutrinos are massless if this is the case. Furthermore

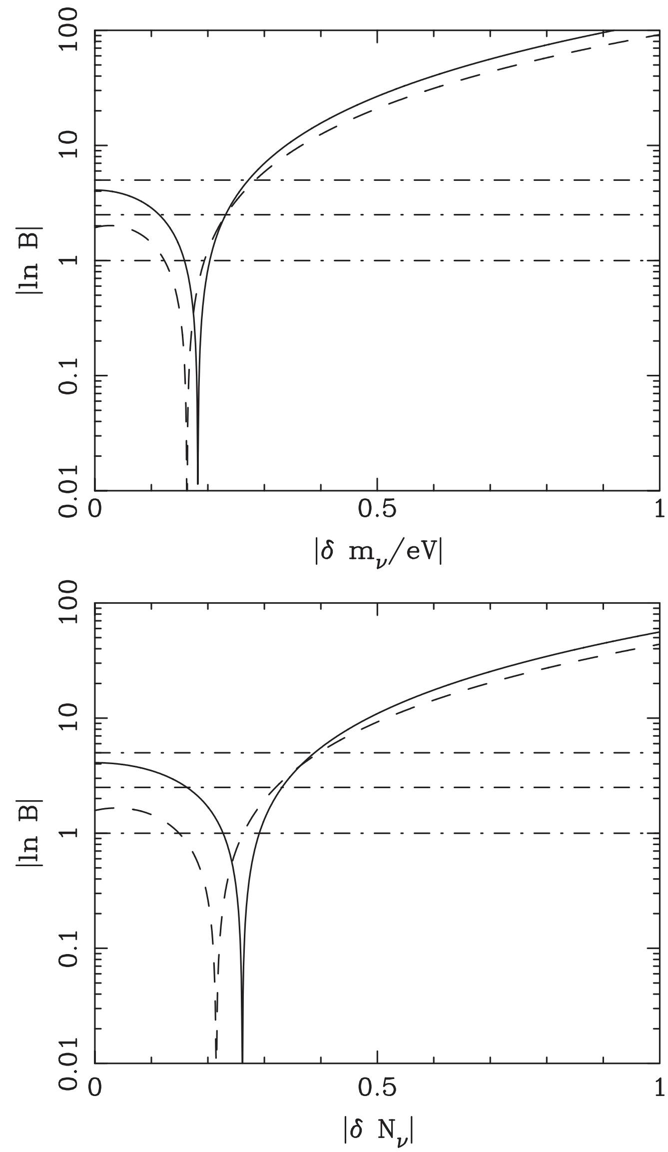

FIG. 2. The predicted evidence for the number N_ν and total mass m_ν of neutrinos individually for 3D cosmic shear using the fiducial survey combined with a Planck prior. In each plot the solid line shows the conditional evidence assuming that the other parameter is fixed at its fiducial value, the dashed line shows the marginal expected evidence when the possible values of the hidden parameter are taken into account, see Eq. (12). The dot-dashed lines show the defining evidence limits on the Jeffery's scale where $\ln B < 1$ is "inconclusive", $1 < \ln B < 2.5$ is "substantial" evidence in favor of a model, $2.5 < \ln B < 5$ is "strong", and $\ln B > 5$ is "decisive".

this experiment could decisively distinguish between models in which there are no massive neutrinos and models in which there are massive neutrinos with $m_\nu \gtrsim 0.25$ eV, or if the number of neutrinos differs by $|N_\nu - 3| \gtrsim 0.35$.

If we use $\ell_{\max} = 1000$, thereby removing the forecast uncertainty associated with nonlinear modes, then the fiducial experiment could decisively distinguish models for which $m_\nu \gtrsim 0.5$ eV or $|N_\nu - 3| \gtrsim 0.40$. However this is the worst-case scenario in which the nonlinear scales could not be used and are completely neglected. The increase in these values is consistent with the results presented in Table III where, for the $m_\nu = 0.0$ fiducial model, the marginal error on m_ν is most affected by the change in the maximum ℓ used.

V. CONCLUSION

In this paper we have shown that 3D cosmic shear has the ability to measure the effect that massive neutrinos can have on the matter power spectrum, and use this effect to place constraints on the total mass of neutrinos $m_\nu = \sum_i m_i$ and number of these massive neutrinos N_ν. By combining the results using 3D cosmic shear from a next generation photometric redshift survey with the constraints from the Planck CMB experiment, one could expect marginalized errors for the massive neutrino parameters of $\Delta m_\nu \sim 0.03$ eV and $\Delta N_\nu \sim 0.08$ which is a factor of 4 improvement over the constraints using the CMB alone. We found that if one assumes in this calculation that neutrinos are massless then the predicted marginal error on these parameters is substantially degraded to $\Delta m_\nu \sim 0.07$ eV and $\Delta N_\nu \sim 0.10$, however this is still an improvement of a factor of 4 over the marginal errors from Planck alone using the same fiducial model. This increase in the marginal errors occurs because the power spectrum is affected by neutrino mass in a nonlinear way.

Even by including heuristically systematic effects, using a rule-of-thumb from Kitching *et al.* [52], the improvement over Planck alone is still a factor of 3. Comparing with other probes we find that 3D weak lensing is competitive; in Ref. [11] we find that using a galaxy redshift survey combined with a Planck prior $\Delta m_\nu \gtrsim 0.025$ eV, in Ref. [74] we find that in combination with the Planck constraint, Lyman-α experiments could constrain $\Delta m_\nu \lesssim 0.06$ eV. However we note that these constraints should be dependent on the fiducial value of the neutrino mass chosen.

We explicitly presented results for a fiducial survey which has the characteristics of DUNE, however other forthcoming surveys are also well suited to do 3D weak lensing and should have a similar sensitivity to neutrino mass, for example, Pan-STARRS-1 should yield constraint roughly twice that of DUNE [52] $\Delta m_\nu \sim 0.06$ eV and $\Delta N_\nu \sim 0.16$. The LSST should yield constraints of roughly the same order of magnitude as DUNE.

Using the expected evidence calculation from [57], we have shown that one can expect substantial evidence for massive neutrinos if they exist, and furthermore that one could decisively distinguish between models in which there are no massive neutrinos and models in which there are massive neutrinos with $|N_\nu - 3| \gtrsim 0.35$ and $m_\nu \gtrsim 0.25$ eV.

We have introduced the concept of marginal and conditional evidence and shown that by assuming the value of one parameter in a model to be fixed, the one-parameter evidence can be under or over estimated by up to a factor 5. These evidence calculations can be generalized to models with an arbitrary number of parameters, and the simple application of this algorithm using only the Fisher matrix can allow predictions to be made which would be prohibitively time consuming using traditional evidence calculations (with the caveat that Gaussianity is assumed).

If the constraints predicted in this paper were realized, then our understanding of massive neutrinos could be revolutionized allowing the physics beyond the standard model which this implies to be understood more entirely.

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