Newton's second law versus modified-inertia MOND: A test using the high-latitude effect

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The modified-inertia MOND is an approach that proposes a change in Newton's second law at small accelerations as an alternative to dark matter. Recently it was suggested that this approach can be tested in terrestrial laboratory experiments. One way of doing the test is based on the static high-latitude equinox modified-inertia effect: around each equinox date, 2 spots emerge on the Earth where static bodies experience spontaneous displacement due to the violation of Newton's second law required by the modified-inertia MOND. Here, a detailed theory of this effect is developed and estimates of the magnitude of the signal due to the effect are obtained. The expected displacement of a mirror in a gravitational-wave interferometer is found to be about 10^{-14} m. Some experimental aspects of the proposal are discussed.

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I. INTRODUCTION

In this paper, I examine the experimentally testable consequences of the hypothesis that Newton's second law should be changed for small accelerations. This hypothesis—called the modified Newtonian dynamics (MOND) with modified inertia—has been proposed by Milgrom [1] as an alternative to the more conventional dark matter explanation of the shape of the galactic rotation curves.

The main assumption of the modified-inertia MOND is that Newton's second law should be modified to read

$$\mathbf{F} = m\mathbf{a}\mu(a/a_0), \tag{1.1}$$

where a_0 is a fundamental acceleration of the order of $10^{-10}\ \mathrm{m\,s^{-2}}$ and μ is a function satisfying the two conditions: $\mu(a/a_0) \to 1$ at $a \gg a_0$ and[1] $\mu(a/a_0) \to a/a_0$ at $a \ll a_0$. (More details about μ and a_0 are in Secs. V and VI.)

Further, the universal gravitation law is assumed to keep its conventional form

$$F = \frac{Gm_1m_2}{r^2}. \tag{1.2}$$

At this stage, the modified-inertia MOND is only formulated in the context of Newtonian physics, i.e., a flat-space picture, which will be assumed throughout this paper.

At the moment the MOND approach is attracting constantly growing interest. Various astrophysical aspects of this approach are under active discussion (see [3] and the last listing of Ref. [1] for a comprehensive bibliography). However, until lately the analysis of terrestrial (as opposed to astrophysical) consequences of MOND has been missing in the literature. The unusual smallness of the acceleration a_0 makes it very difficult to think of a laboratory test. It also explains why we do not see any deviations from Newton's second law under ordinary circumstances. Surprising first results of such an analysis have recently been described in [2].

One of these results is the existence of a new effect that has been called static high-latitude equinox modified inertia or SHLEM: around each equinox date, 2 spots emerge on the Earth where static bodies experience spontaneous displacement due to the violation of Newton's second law required by the modified-inertia MOND. The laboratory observation of this effect may be accessible to current experimental capabilities.

Consequently, it could serve as a basis for a proposal to test the validity of Newton's second law for small accelerations in a laboratory-based experiment—the first of its kind. In fact, such a test could become a crucial experiment in view of the lack of conclusive astrophysical evidence either in favor or against the modified Newtonian dynamics.

Here, a detailed theory of this effect is developed and estimates of the magnitude of the signal due to the effect are obtained. The expected displacement of a mirror in a gravitational-wave interferometer is found to be about 10^{-14} m.

The nature of inertia has long been one of the most fundamental puzzles of physics. In recent times, this puzzle has widened to include also the problem of the origin of mass in the standard model. Despite concerted theoretical efforts over decades, the path to the solution is still to be found.

Thus the pursuit of MOND as an alternative explanation of the "dark matter" puzzle seems well motivated. If true, it will bring about the need to revise foundations of modern physics.

The paper is organized as follows. To make the exposition more understandable and self-contained, Sections II and III review and extend the background material from [2]. Section IV describes the SHLEM effect qualitatively. In Secs. V and VI the equation of motion is derived and its

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[1]The difference between this asymptotic condition and that of Ref. [2] is due to a misprint in the latter.

solution is obtained. In Sec. VII some experimental suggestions are discussed. Finally, the conclusions are presented in Sec. VIII.

II. ACCELERATION: "ABSOLUTE" OR "RELATIVE"?

First, I emphasize that to obtain laboratory-testable predictions, MOND needs to be formulated not only in inertial reference systems, but also in noninertial systems as well [2]. (In the MOND context all laboratory reference systems should be considered as noninertial.) Because the dynamical law is modified depending on the acceleration, the transition between inertial and noninertial systems in MOND becomes less straightforward than in the conventional mechanics.

Of particular interest are transformation properties of a_0. Logically, at least 2 options could be imagined. First, one can assume that the fundamental acceleration that determines the onset of the MOND regime equals a_0 only in the inertial reference systems. Second, it could be assumed that a_0 is invariant under transformations from inertial to noninertial systems. One would expect that these 2 types of theories would lead to drastically different experimental predictions.

For instance, the first type of theory requires that the MOND regime is reached as soon as the test body moves with a tiny acceleration $\lesssim a_0$ *with respect to the galactic reference frame*.[2]

On the other hand, the second type of theory implies that, in order to reach the MOND regime, we should try to ensure that the test body moves with a tiny acceleration $\lesssim a_0$ with respect to the laboratory reference frame.

However, it has been pointed out that the second version (invariant acceleration a_0) is not self-consistent [2]. The reason is that the invariance of a_0 is inconsistent with the kinematical rules of acceleration addition. Indeed, let us take two reference frames: an inertial S, and a noninertial S' that is in translational motion with acceleration $\mathbf{b}$ relative to S. Then, in the system S the equation of motion will be

$$\mathbf{F} = m\mathbf{a}\mu(a/a_0), \tag{2.1}$$

where $\mathbf{a}$ is the acceleration of a test body in the system S. If we assume that a_0 is invariant, then the equation of motion in the system S' is

$$\mathbf{F} = m\mathbf{a}'\mu(a'/a_0) + m\mathbf{b}, \tag{2.2}$$

[2]Although the choice of the galactic reference frame is a natural one, a question can be raised whether alternative choices are possible. For instance, what if we take a reference frame with the origin at the center of mass of the local group of galaxies? Fortunately, that would not significantly affect the results because the acceleration due to the neighboring galaxies is much smaller than a_0. For example, the acceleration due to the Andromeda galaxy is less than 10^{-12} m s^{-2}.

where $\mathbf{a}' = \mathbf{a} - \mathbf{b}$ is the acceleration of the test body in the system S'. However, Eqs. (2.1) and (2.2) cannot hold simultaneously for all $\mathbf{a}$ and $\mathbf{b}$: for instance, if we put $\mathbf{a} = 0$ then we obtain

$$m\mu(b/a_0) = m \tag{2.3}$$

for all $\mathbf{b}$ which means that $\mu(z) = 1$ for all z. Thus, the invariant-a_0 version of MOND is inconsistent.

In addition, this version has been ruled out experimentally [4]. In what follows, only the first version will be considered.

III. CONDITION OF ENTRY INTO SHLEM REGIME

We will now recall what conditions must be realized in order to obtain the SHLEM effect for test bodies at rest in the ground-based laboratory [2].

This question is easy to answer in the inertial system S_0. (It is the system with the origin in the center of mass of our Galaxy and the axes pointing to certain far-away quasars). In this system, we should ensure that the test body moves with a tiny acceleration $\mathbf{a}_{gal}$ with respect to S_0:

$$\mathbf{a}_{\rm gal} \approx 0. \tag{3.1}$$

In this section, the $\approx$ sign will mean that the difference between the left-hand side and the right-hand side of an equation is much less than the characteristic MOND acceleration a_0.

Next, we are going to the laboratory system with the help of

$$\mathbf{a}_{\rm gal} \approx \mathbf{a}_1(t) + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times (\mathbf{r} + \mathbf{r}_1)) + \mathbf{a}_2, \tag{3.2}$$

where $\mathbf{a}_1$ is the acceleration of the Earth's center with respect to the heliocentric reference frame, $\boldsymbol{\omega}$ is the angular velocity of the Earth's rotation, $\mathbf{a}_2$ is the Sun's acceleration with respect to S_0; $\mathbf{r}$ is the position of the test body with respect to the laboratory reference frame; $\mathbf{r}_1$ is the position vector of the origin of the lab frame with respect to the terrestrial frame with the origin at the Earth's center.[3] A number of terms have not been written out in Eq. (3.2) on account of their smallness. They include terms due to: the Coriolis acceleration of the Sun, the length-of-day (LOD) variation, precession and nutation of the Earth's rotation axis, polar motion and Chandler's wobble (see Table I for their approximate magnitudes).

From Eqs. (3.1) and (3.2) we obtain the necessary and sufficient condition for realization of the MOND regime in the laboratory:

$$\mathbf{a}_s(t) + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_1) \approx 0, \tag{3.3}$$

[3]As practical, high-precision realizations of these intermediate frames, one can take the International Celestial Reference System (ICRS) [5] and the International Terrestrial Reference System (ITRS) [6].

TABLE I. Accelerations that can be ignored.

Source	Approximate magnitude, m/s^2
"Galactic Coriolis"	2×10^{-11}
Short period LOD variation	3×10^{-12}
Annual LOD variation	1×10^{-13}
Precession	2×10^{-14}
Secular increase of LOD	3×10^{-15}
Nutation (main term, epoch 1900,0)	9×10^{-20}
Chandler's wobble	1×10^{-20}
Annual pole motion	4×10^{-21}
Secular polar motion	3×10^{-24}

where I have introduced $\mathbf{a}_s = \mathbf{a}_1 + \mathbf{a}_2$ for convenience and put $\mathbf{r} = 0$ (without loss of generality).

We note that this equation has no solutions unless $\mathbf{a}_s$ is orthogonal to $\boldsymbol{\omega}$, so we must first look for those instants t_p when

$$a_{s\|}(t_p) \approx 0 \quad \text{or} \quad a_{s\|}(t_p)| \ll a_0, \tag{3.4}$$

where $a_{s\|} = (\mathbf{a}_s\boldsymbol{\omega})/\omega$. A continuity argument shows that this equation has at least 2 solutions during each year. Indeed, at the instant of a (northern) summer solstice $a_{s\|} > 0$ whereas at the instant of a winter solstice $a_{s\|} < 0$. Therefore, there must be at least one instant during autumn and one instant during spring when $a_{s\|} = 0$ *exactly*. Neglecting the effects due to the Moon and planets, these instants would coincide exactly with the autumnal and vernal equinoxes. In reality, the instants will be shifted from the equinoxes. However, the above "existence theorem" guarantees that these instants t_p can be found with astronomical precision through a straightforward but time-consuming procedure using the lunar and planetary ephemerides. In addition, one can show that the off-equinox shift, in any case, should be less than a few days.

Once t_p is found and plugged into Eq. (3.3), the corresponding solution for the laboratory location is

$$\mathbf{r}_{1\perp} = \frac{\mathbf{a}_s(t_p)}{\omega^2}. \tag{3.5}$$

This key relation allows us to find both the latitude and the longitude of the right spot. If we again ignore the lunar and planetary effects, the relevant magnitude is $|\mathbf{a}_s(t_p)| \simeq$

TABLE II. Longitudes of the "SHLEM spots" calculated in the Earth-Sun approximation.

Date	Time (UT)	Longitude
2008 Mar 20	5:48	93° E
2008 Sept 22	15:44	56° W
2009 Mar 20	11:44	4° E
2009 Sept 22	21:18	139°30′ W
2010 Mar 20	17:32	83° W
2010 Sept 23	3:09	132°45′ E
2011 Mar 20	23:21	170°15′ W
2011 Sept 23	9:04	44° E
2012 Mar 20	5:14	101°30′ E
2012 Sept 22	14:49	42°15′ W
2013 Mar 20	11:02	14°30′ E
2013 Sept 22	20:04	121° W
2014 Mar 20	16:57	74°15′ W
2014 Sept 23	2:29	142°45′ E
2015 Mar 20	22:45	161°15′ W
2015 Sept 23	8:20	55° E
2016 Mar 20	4:30	112°30′ E
2016 Sept 22	14:21	35°15′ W
2017 Mar 20	10:28	23° E
2017 Sept 22	20:02	120°30′ W
2018 Mar 20	16:15	63°45′ W
2018 Sept 23	1:54	151°30′ E
2019 Mar 20	21:58	149°30′ W
2019 Sept 22	13:30	22°30′ W

$0.005\,93\ \mathrm{m\,s^{-2}}$ which gives the required latitude $\phi \simeq \pm 79°50'$. As for the longitude, it would generally vary from year to year. For instance, on the autumnal equinox of September 22, 2008, these spots would be at 56° W—one in Greenland, (79°50′ N), another in Antarctica (79°50′ S). Predictions for the years 2008–2019 are shown in Table II. The dates and times of the equinoxes are taken from Ref. [7].

The account of lunar perturbation can significantly change the longitude, but the latitude prediction is much more robust: it would not change by more than $\sim 6'$, or 10 km.

The question may arise why the SHLEM spots will be located on the Earth surface and not at some altitude or underground [8]. The answer is that the spots will be sitting not only on the surface, but also above and below. More exactly, these spots will occupy a straight line running parallel to the Earth rotation axis at a distance of $\mathbf{a}_s(t_p)/\omega^2 \simeq 1120$ km from the axis. (In the lab system this line is $\simeq 10°$ off vertical and points North in the northern hemisphere and South in the southern.)

This follows from Eq. (3.5) which is a straight line equation; it does not single out the Earth's radius. The two *surface* SHLEM spots emerge at those points where the line crosses the Earth's surface. Only if we want to find the coordinates of these *surface* spots, the Earth radius enters the game: in Eq. (3.5) we need to plug in $R_E \cos\phi$ instead of $\mathbf{r}_{1\perp}$ and then find ϕ.

To determine the latitudes of SHLEM spots at altitude $h > 0$ (or at depth $h < 0$ underground), all we need to do is to plug $(R_E + h)\cos\phi(h)$ instead of $R_E \cos\phi$ into Eq. (3.5). In this way we obtain the altitude/depth-dependent latitude $\phi(h)$ as

$$\cos\phi(h) = \frac{\mathbf{a}_s(t_p)}{\omega^2(R_E + h)}. \qquad (3.6)$$

As for the longitudes of SHLEM spots, they will all be equal to the longitude of the SHLEM line and therefore will not depend on h.

What happens if we now take into account perturbations due to the Moon and planets? These perturbations will slightly change the vector $\mathbf{a}_s(t_p)/\omega^2$ at the right-hand side of Eq. (3.5) but it will still be just a fixed vector. Consequently, the SHLEM line defined by Eq. (3.5) will shift around, but it will still remain a straight line running parallel to the Earth axis. As a result, it will still be possible to find the SHLEM spots on the Earth surface as well as underground or above ground.[4] The equality of longitudes for all these spots (for a given instant t_p) will still hold.

It is worth reemphasizing that the validity of this "straight line theorem" and its implications is guaranteed by purely topological arguments and is therefore rather general (e.g., it does not rely on the "Earth-Sun" approximation or any assumptions about the shape of the Earth).

IV. THE GENERAL PICTURE OF MOTION

The signature of the SHLEM effect would be a spontaneous displacement of the test body occurring around the instant t_p defined by Eq. (3.4), and we are now ready to start calculating its magnitude. The qualitative scenario runs as follows: around the instant t_p the test body and the reference body move according to two different laws of motion. Roughly speaking, MOND makes the test body "lose" its mass while the reference body keeps its normal mass (because it lies outside of the MOND regime). Therefore the test body will shift with respect to the reference body by a tiny but nonzero, time-dependent distance $x(t)$.

Throughout the rest of the paper, I will work in the Earth-Sun approximation in which the corrections due to the Moon and planets are ignored. How would account of these corrections change the results? As was shown previously [2] and discussed at the end of Sec. III, the time and location of the SHLEM event can change slightly, but the effect itself will survive because its existence is based on topological arguments.

Another worry is whether we can use Newtonian, not modified-inertia mechanics to calculate the instant t_p [9]. To address this, we note that this calculation rests on the fact that the motion of the Earth as a whole obeys the Newtonian mechanics. Of course, within the MOND approach this is not absolutely true. Moreover, the approach developed in [2] and here allows one to calculate the MOND corrections to the value of t_p as precisely as one wishes. However, these corrections appear to be so tiny that they can be completely neglected even without their precise calculation. Indeed, they would, in any case, include the suppressing factor of $M_{\mathrm{MOND}}/M_{\mathrm{Earth}}$ where M_{MOND} is the mass of the part of the Earth affected by MOND.

The MOND-affected mass can be (generously) bounded from above as follows:

$$M_{\mathrm{MOND}} \lesssim 2R_E \sin\phi(v_r \delta t)^2 \rho_E, \qquad (4.1)$$

where R_E, ρ_E, and v_r are the Earth radius, density, and linear rotation speed at the latitude $\phi \approx 80°$. The time scale δt was defined in [2] as

$$\delta t \sim (a_0/a_s)(4\epsilon/T)^{-1} \sim 1\ \mathrm{s}, \qquad (4.2)$$

where $\epsilon = 23°27' = 0.41$, $T = 1$ yr.

Altogether, the suppressing factor is

$$M_{\mathrm{MOND}}/M_{\mathrm{Earth}} \lesssim 10^{-10}, \qquad (4.3)$$

which means that, for the purposes of calculating the SHLEM instant and coordinates of the SHLEM spot, the Newtonian mechanics can be used quite safely.

[4] It is not inconceivable that the existence of these additional off-ground spots could somehow be helpful for experimenters.

V. EQUATION OF MOTION

We start by working in an inertial reference frame S_0 first. Denote by $\mathbf{f}$ the total physical force acting on the unit mass. Then the MOND equation of motion is

$$\mathbf{f} = \ddot{\mathbf{r}}\mu(|\ddot{\mathbf{r}}|/a_0). \quad (5.1)$$

Although the present treatment can be used with any interpolating function μ, to obtain a definite result one needs to fix its concrete form; the standard choice has been [1,10,11]

$$\mu(z) = \frac{z}{\sqrt{1+z^2}}. \quad (5.2)$$

It is a matter of current debate [12,13] whether this function satisfies the constraints derived from the precision solar system data [14]. In any case, a new function can only be introduced after a reanalysis of the galactic rotation curves and a corresponding reestimate of the acceleration scale a_0. (For instance, it would be inconsistent to use a new interpolating function with the old value of a_0.) For this reason, at the moment we have no choice for μ other than (5.2), and it will be used from now on.

Solving Eq. (5.1) for $\ddot{\mathbf{r}}$ we obtain

$$\ddot{\mathbf{r}} = \mathbf{f}\Phi(\mathbf{f}^2/a_0^2), \quad (5.3)$$

where

$$\Phi(\zeta) = \sqrt{\frac{1+\sqrt{1+\frac{4}{\zeta}}}{2}}. \quad (5.4)$$

Next, we go into the laboratory reference frame by adding the noninertial accelerations $\mathbf{f}_{\rm in}$:

$$\ddot{\mathbf{r}} = \mathbf{f}\Phi(\mathbf{f}^2/a_0^2) + \mathbf{f}_{\rm in}. \quad (5.5)$$

To obtain a more specific equation, let us make simplifying assumptions similar to those usually made when analyzing the response of the gravitational-wave detectors (these assumptions will be justified in the next section when the solution of the equation of motion is obtained):

(i) the body interacts with its environment (such as a support, suspension etc.) through an elastic force with negligible dissipation:

$$\mathbf{f}_{\rm el} = -\omega_0^2\mathbf{r}. \quad (5.6)$$

(ii) random forces (e.g., due to thermal and vibrational noises) are ignored.[5]

(iii) the test body is treated as a point mass, i.e., we ignore a small variation of f over its volume.

(iv) terms of the second and higher order in the small parameter $\omega(t-t_p)$ are neglected.

(v) displacement of the reference body is negligible. (The reference body is the body that plays the role of the origin of the laboratory reference frame. For example, in interferometers the reference body is one of the mirrors.)

Therefore, the total physical force will consist of the sum of gravitational forces and the elastic coupling:

$$\mathbf{f} = \sum \mathbf{f}_{\rm gr} + \mathbf{f}_{\rm el} = \mathbf{f}_{\Sigma{\rm gr}} - \omega_0^2\mathbf{r}, \quad (5.7)$$

and the resulting equation of motion will take the form

$$\ddot{\mathbf{r}} + \omega_0^2\mathbf{r}\Phi = \mathbf{f}_{\Sigma{\rm gr}}\Phi + \mathbf{f}_{\rm in}. \quad (5.8)$$

Let us now introduce the following coordinate system within the laboratory reference frame: x is the West-to-East axis, y—the South-to-North axis, and z is the vertical axis. Then in our approximation the projection of the above equation on these axes will give

$$\ddot{x} + \omega_0^2 x\Phi \simeq f(t)\Phi - f(t), \qquad \ddot{y} \simeq 0, \qquad \ddot{z} \simeq 0, \quad (5.9)$$

where

$$f(t) \simeq a_s\omega(t-t_p), \qquad \Phi \simeq \Phi[(f-\omega_0^2 x)^2/a_0^2]. \quad (5.10)$$

Thus we can ignore motion along the y and z axes and our problem becomes one dimensional.

VI. SOLUTION OF THE EQUATION OF MOTION

The obtained nonlinear equation of motion is best solved numerically. But first we need to fix the frequency $\omega_0 = 2\pi f_0$. It is determined by the suspension design: for example, $f_0 = 0.65$ Hz in the case of LIGO.[6] Therefore, $\omega_0 = 4.1\ {\rm s}^{-1}$ will be used.

At present there is some uncertainty in the magnitude of a_0, the fundamental parameter of the theory. For instance, in Refs. [10,11] it was found that $a_0 = (1.5 \pm 0.7) \times 10^{-10}\ {\rm m\,s^{-2}}$ and $a_0 = (1.35 \pm 0.51) \times 10^{-10}\ {\rm m\,s^{-2}}$, respectively.[7] Because of that, we will first assume that $a_0 = 2 \times 10^{-10}\ {\rm m\,s^{-2}}$ and then repeat the calculation with $a_0 = 1.2 \times 10^{-10}\ {\rm m\,s^{-2}}$ and $a_0 = 1 \times 10^{-10}\ {\rm m\,s^{-2}}$.

The resulting solution for $a_0 = 2 \times 10^{-10}\ {\rm m\,s^{-2}}$ is shown in Fig. 1.

The displacement amplitude has a maximum of $\simeq 3.8 \times 10^{-14}$ m. This maximum is reached $\simeq 0.76$ s after the SHLEM instant t_p.

Having obtained the solution, we can now fully justify the assumptions made earlier while deriving the equation of motion. First, the damping can indeed be completely

[5]The account of these would require a detailed knowledge of the experimental setup. If a gravitational-wave type of detector is considered, the account can be made using methods developed in that area.

[6]VIRGO has a very close frequency: $f_0 = 0.60$ Hz.

[7]Reference [11] also quotes a slightly different value: $a_0 = (1.21 \pm 0.27) \times 10^{-10}\ {\rm m\,s^{-2}}$. It is obtained if one outlier galaxy is removed from the sample on the basis of a possible error in the determination of its distance. The value $a_0 = 1.2 \times 10^{-10}\ {\rm m\,s^{-2}}$ is generally assumed in the literature.

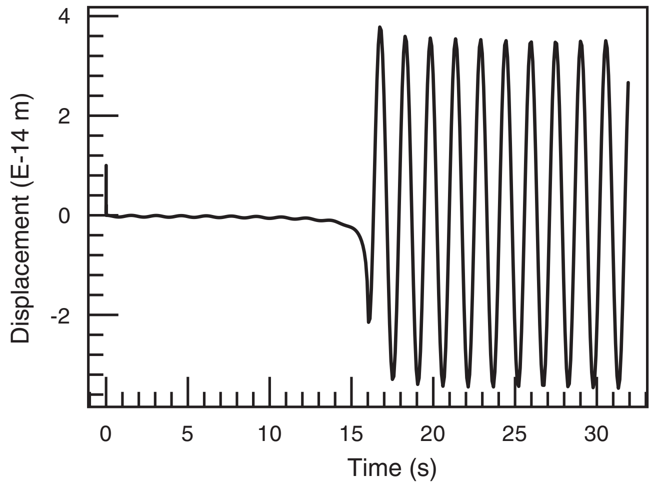

FIG. 1. Displacement of the mirror as function of time. The instant t_p corresponds to $t = 16$ s.

neglected given that the relevant quality factor is $Q =$ few $\times 10^3$ [15].

Second, the linear time dependence in $f(t)$ can only be used for "short" times t so that $\omega(t - t_p) \ll 1$, where ω is the Earth's angular velocity. In other words, the time intervals must be much less than 24 h$/2\pi \simeq 4$ hours. For our solution this condition obviously holds by a wide margin.

Third, the neglect of transverse motion is justified by the same condition $\omega(t - t_p) \ll 1$.

Fourth, let us confirm that the size of the mirror does not matter. Indeed, the difference in centrifugal accelerations that act on the mirror edges is

$$\Delta a = a_s \frac{x}{R_E \cos\phi}, \tag{6.1}$$

where x is the distance between edges, R_E is the Earth's radius, and ϕ is the lab latitude. In the case of LIGO, depending on the mirror orientation, x can take the values of 9.3 cm (the mirror thickness) or 25 cm (the mirror diameter). To ensure that this difference is insignificant, we must require that it be less than the "driving force" $f(t) = a_s \omega(t - t_p)$ in Eq. (5.9). It is easy to check that this requirement is always fulfilled except during a very short interval

$$\Delta t \sim \frac{x}{R_E \omega \cos\phi} \sim 3 \times 10^{-3} \left(\frac{x}{25 \text{ cm}}\right) \text{s} \tag{6.2}$$

around t_p. But the contribution of this interval to the total displacement of the mirror is quite negligible as can be seen from Fig. 1. This justifies our treatment of the mirror as a mass point.

Finally, we can check if the displacement of the second (reference) mirror can be neglected. At the SHLEM latitude $\phi \approx 80°$, the Earth rotates with a linear speed of $v_r = R_E \omega \cos\phi \approx 80$ m/s (eastward). This means that the SHLEM spot runs on the Earth's surface with the same speed (westward). Thus, if a reference mirror is farther than 80 m from the test mirror, then the reference mirror's motion can be neglected during an observation interval of about 1 s around the SHLEM instant t_p. Otherwise (i.e., if the distance is shorter than 80 m or the observation interval is longer than 1 s) the motion of the reference mirror must be taken into account. In that case, the signal will be equal to the difference between the two mirrors' displacements.

Now, it is of interest to see how sensitive is the obtained solution to the variation of a_0. If we use the value $a_0 = 1 \times 10^{-10}$ m s^{-2} instead of $a_0 = 2 \times 10^{-10}$ m s^{-2}, then the maximum of the displacement amplitude will drop to $\simeq 0.95 \times 10^{-14}$ m (i.e., by a factor of 4). Finally, if the currently favored value $a_0 = 1.2 \times 10^{-10}$ m s^{-2} is used, the maximum will be at $\simeq 1.4 \times 10^{-14}$ m. In all cases considered, it takes approximately the same time ($\simeq 0.76$ s counting from the SHLEM instant t_p) to reach the maximum.

It is also interesting to know how a change in the interpolating function would affect the result. We can generally expect that a smoother interpolating function, e.g.,

$$\mu(z) = \frac{z}{1+z}, \qquad z = \frac{a}{a_0} \tag{6.3}$$

could lead to a greater displacement, while a sharper function such as

$$\mu(z) = 1 - \exp(-z), \qquad z = \frac{a}{a_0} \tag{6.4}$$

could produce a weaker signal compared to the standard function with the same value of a_0.

However, we are not completely free in the choice of the interpolating function: as was pointed out in Ref. [1], it must not only describe the galactic rotation curves, but also satisfy the important additional constraints imposed, in particular, by the equivalence principle (EP) tests and the precision data on the planetary motions in the solar system.

For example, consider lunar laser ranging (LLR) which tests the EP at the level of 10^{-13} [16]. Take the interpolating function in the form (6.3).

The difference between 1 and μ is a crude upper bound on the violation of EP because μ can be interpreted as the ratio of the inertial mass to the gravitational mass. The orbital acceleration of the Earth is about $a \simeq 6 \times 10^{-3}$ m/s^2. That gives

$$1 - \mu \simeq \frac{1}{z} \simeq 0.3 \times 10^{-7}. \tag{6.5}$$

To finalize the estimate, we recall [16] that the LLR measures not μ itself but rather the difference between the inverse of that quantity for the Earth and for the Moon, i.e. $\mu_{\text{Earth}}^{-1} - \mu_{\text{Moon}}^{-1} \simeq \mu_{\text{Moon}} - \mu_{\text{Earth}}$. This produces an extra factor of $\sim 4L/S \simeq 10^{-2}$, where L is the Earth-Moon distance and S is the Earth-Sun distance. Altogether, we obtain $0.3 \times 10^{-7} \times 10^{-2} = 0.3 \times 10^{-9}$ which exceeds the experimental upper limit 10^{-13} by more than 3 orders of magnitude. Thus the function (6.3) does not pass the equivalence principle test.[8]

By contrast, the standard interpolation function $\mu(z)$ is not excluded by this kind of argument because it behaves as $\mu(z) \simeq 1 - \frac{1}{2z^2}$ at large z [1].

In addition, the function (6.3) is ruled out by the data on the planetary motion in the solar system [1,12,13].

If we now consider the function (6.4) which is sharper than the standard one and thereby avoids the above constraints, then we encounter the following difference: In order to find the analogue of the function Φ, we need to solve the equation

$$f = (1 - \exp(-z))z \qquad (6.6)$$

for z, which cannot be done analytically as in the standard case, Eq. (5.4). Thus a modified numerical scheme would be required.

Besides, the function (6.4), unlike the forms (5.2) and (6.3), has not been used extensively for the fitting of galactic rotation curves (see, e.g., [1,10,11]).

If we (arbitrarily) borrow the value of a_0 from the standard function, then we can reasonably expect that a weakening of the signal is possible. There are no substantial grounds to believe that some radically different predictions for the experiment will result. For all these reasons, we defer full treatment of this case until the time when it becomes more expedient.

VII. EXPERIMENTAL CONSIDERATIONS

A variety of approaches to the experimental searches for the modified-inertia effects have been proposed in Ref. [2]. Because we want to detect a tiny displacement/acceleration, it is natural to turn to the vast and vigorous area of gravitational experiments. In particular, it was pointed out that the existing gravitational-wave detectors could be a good starting point in designing the experiment.[9]

At present, there are 2 types of gravitational-wave facilities: the interferometers and resonators. The interferometers are generally more sensitive. So it is reasonable to start our discussion with them.[10]

The most sensitive of the currently operating interferometers are LIGO and Virgo [19–22]. The heart of such a detector is a suspended mirror whose position can be monitored with an ultrahigh accuracy $\delta l \sim 10^{-18}$ m. This is achieved by using a Michelson-type of interferometer with a long arm length: $L = 4$ km and $L = 2$ km for LIGO, or $L = 3$ km for Virgo. The detector's sensitivity is determined by the dimensionless ratio $\delta l/L \sim 10^{-21}$ which can be translated into the dimensionless amplitude of a detectable gravitational wave. In some respects, the expected SHLEM signal would be similar to the signal expected from a gravitational wave. However, there is a crucial difference between the two. The SHLEM signal is characterized by *dimensional* quantity—displacement of the test body δl—rather than a dimensionless ratio $\delta l/L$.

As a consequence, *having long arm length L is unnecessary for us* as it does not affect the detector sensitivity to the SHLEM signal. This opens up an interesting opportunity of considering *a "short-arm LIGO/Virgo-like" detector* that would be based on the same ideas and technical know-how as LIGO/Virgo themselves, but would have a much smaller size that would allow it to be more easily transported and installed in a required location.

In other words, we can imagine a setup that would be similar to LIGO/Virgo, but with a much shorter arm length (as is the case for TAMA300 [23], GEO [24], or ACIGA/AIGO [25], where the respective arm lengths are about 300 m, 600 m, and 80 m). LIGO's prototype interferometer on the Caltech campus has even a shorter arm length of 40 meters.[11]

It is quite possible that additional effort would be required to ensure that the shorter arm length does not compromise the overall sensitivity of the detector. For example, the issue of high laser power in a short arm would have to be carefully analyzed. At this stage, though, it seems premature to go deeper into these details.

To assess the future potential of an interferometer setup, we have to take into account the plans [15] to upgrade both LIGO and Virgo in the near future. The second generation detectors (enhanced LIGO and Virgo +) will have their sensitivities increased twofold or threefold by 2009. At the next stage (advanced LIGO, advanced Virgo, 2014) the increase of sensitivity will be tenfold. Further, the third generation underground detectors, such as the Einstein gravitational-wave telescope, are currently under active discussion.

Thus it does not seem unrealistic to think that a suitable laboratory based on the LIGO/Virgo technology can be conceived in which the position of the mirror can be monitored with an accuracy sufficient for detecting the SHLEM effect.

In that case, how should one analyze the data? Because of a similarity between a SHLEM signal and a gravitational

[8]This conclusion, based on the earlier LLR data, has been known since 1983 [1].

[9]Recently, an analog of the SHLEM effect for the satellites' motion was considered by McCulloch [17] starting from a different way of modifying inertia [18].

[10]The resonant detectors can be analyzed in a similar way, but technically there are some differences which will not be considered here.

[11]A lower bound on the arm length would be set by the desirability to keep the second mirror unaffected by SHLEM. As discussed at the end of Sec. VI, one way to do this is to make the distance between mirrors large enough. It is hard to work out the precise lower bound in advance because it would depend on the orientation of the arm and other unknowns.

wave, the approach to this problem could be similar to that used in Refs. [26–32]. In particular, the paper [32] by the joint LIGO-Virgo working group gives a detailed analysis of various statistical procedures aimed at detecting gravitational-wave bursts using LIGO and Virgo facilities. As an input, astrophysical calculations are used which give the waveforms and amplitudes of signals corresponding to different sources.

Since the distance, position, and nature of the source are not known in advance, one has to take into account many possibilities which lead to considerable uncertainty in the input data. By contrast, in our case, the input is unique: it is as if someone were able to predict everything about the burst—the source, its distance, right ascension and declination, and even the exact time of the burst. It makes the statistical analysis rather more certain and robust compared to the analysis required by a largely uncertain gravitational burst.

The complete information about the profile of the signal in our case is contained in Eq. (5.9) and Fig. 1 that give the time dependence of the test body's displacement. If one is interested in the signal spectrum, it can be obtained from the same equation.

Based on this information, the candidate filters can be constructed via methods similar to those employed in Refs. [26–32]. Their performance can then be tested using simulations with the signal shape known from Eq. (5.9) and Fig. 1 and thus the optimal filter can be selected.

As is well known, the coincidence of signals in different detectors plays an important role in the strategies of the gravitational-wave searches. In our case, various coincidence and anticoincidence schemes can be conceived due to the strict localization of the effect in time and space. For example, one can use the scheme in which the first detector is placed in the SHLEM spot in the northern hemisphere while the second detector is located symmetrically in the southern hemisphere.

We can therefore hope that the chances of detecting the SHLEM signal could be better than the chances of detecting a gravitational-wave burst of a similar magnitude.

Because the effect can only be observed at high latitudes, a question can be raised [9] if icebergs floating in the ocean nearby (say, $D \simeq 10$ km away) can trigger false alarms by creating an excess of gravity of the order of a_0.

To assess this possibility, we note that the short time scale of the effect needs to be taken into account. Namely, to create a false alarm, the external source must provide a *variable* gravity: its variation $\delta g_{\rm net}$ over the interval $\delta t \sim 1$ s must be of the order of a_0. Here $g_{\rm net}$ is the net excess gravity which the difference between the gravity due to the ice and the "negative" gravity due to the displaced water:

$$g_{\rm net} = g_{\rm ice} - g_{\rm water} \simeq g_{\rm ice}\left(\frac{d}{D}\right)^2 \simeq g_{\rm ice}\left(\frac{h}{D}\right)^2, \qquad (7.1)$$

where d is the distance between the centers of mass of the ice and the displaced water, h is the height of the above-water tip of the iceberg. The exact relation between d and h depends on the iceberg's shape. For instance, a cubic shape gives $d = h/2$. As we will see shortly, the "shape factor" would not greatly affect the final result so $d \sim h$ will be assumed.

Next, suppose that the iceberg's velocity is v. Then during the time δt it will shift by the distance $\delta D \sim v\delta t$ relative to the lab. That will lead to gravity variation of the order of

$$|\delta g_{\rm net}| \sim \frac{GMh^2}{D^5}\delta D \sim \frac{GMh^2}{D^5} v\delta t. \qquad (7.2)$$

Requiring $|\delta g_{\rm net}| \sim a_0$ leads to the following condition:

$$\frac{Mh^2 v}{D^5} \sim \frac{a_0}{G\delta t}. \qquad (7.3)$$

Therefore, the iceberg mass, tip height, and velocity must satisfy

$$Mh^2 v \simeq 3 \times 10^{20}\ \mathrm{kg\,m^3\,s^{-1}}. \qquad (7.4)$$

Such parameters appear to be unrealistically large. Indeed, a very large iceberg would have a mass of about $M \sim 10^{11}$ kg, with $h \sim 100$ m [33] so the product Mh^2v falls short by several orders of magnitude. Thus icebergs cannot trigger false alarms.

VIII. CONCLUSIONS

Astrophysically inspired laboratory-based experiments such as dark matter searches have become a common part of the physics landscape. The laboratory tests of MOND is a much needed, complementary activity. It is a new virgin territory waiting to be explored.

This paper shows that methods and installations used in the gravitational-wave research are likely to be useful also in the new area of searching for modified inertia and the SHLEM effect.

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[1] M. Milgrom, Astrophys. J. **270**, 365 (1983); **270**, 371 (1983); **270**, 384 (1983); Ann. Phys. (N.Y.) **229**, 384 (1994); Phys. Lett. A **253**, 273 (1999); arXiv: astro-ph/0510117; arXiv: astro-ph/0801.3131.
[2] A. Yu. Ignatiev, Phys. Rev. Lett. **98**, 101101 (2007).
[3] S. McGaugh's website: http://www.astro.umd.edu/~ssm/mond/.
[4] J. H. Gundlach *et al.*, Phys. Rev. Lett. **98**, 150801 (2007); A. Abramovici and Z. Vager, Phys. Rev. D **34**, 3240 (1986).
[5] http://aa.usno.navy.mil/faq/docs/ICRS_doc.html#REFS.
[6] http://itrf.ensg.ign.fr.
[7] http://aa.usno.navy.mil/data/docs/EarthSeasons.php.
[8] D. Castelvecchi (private communication).
[9] P. Ball, Nature (London) **446**, 357 (2007).
[10] M. Milgrom, Astrophys. J. **333**, 689 (1988); S. M. Kent, Astron. J. **93**, 816 (1987).
[11] K. G. Begeman, A. H. Broeils, and R. H. Sanders, Mon. Not. R. Astron. Soc. **249**, 523 (1991).
[12] M. Sereno and Ph. Jetzer, Mon. Not. R. Astron. Soc. **371**, 626 (2006).
[13] L. Iorio, arXiv:gr-qc/0711.2791.
[14] E. V. Pitjeva, Astron. Lett. **31**, 340 (2005); Solar System Research **39**, 176 (2005).
[15] G. Losurdo, HEP 2007 Conference, Manchester, 2007.
[16] J. G. Williams, S. G. Turyshev, and D. H. Boggs, Phys. Rev. Lett. **93**, 261101 (2004).
[17] M. E. McCulloch, arXiv:astro-ph/0712.3022.
[18] M. E. McCulloch, Mon. Not. R. Astron. Soc. **376**, 338 (2007).
[19] B. Abbott *et al.* (LIGO Scientific Collaboration), Classical Quantum Gravity **23**, S653 (2006).
[20] S. Waldman (LIGO Scientific Collaboration), HEP 2007 Conference, Manchester, 2007.
[21] F. Acernese *et al.* (Virgo Collaboration), Classical Quantum Gravity **24**, S381 (2007).
[22] E. Tournefier (Virgo Collaboration), HEP 2007 Conference, Manchester, 2007.
[23] D. Tatsumi *et al.*, Classical Quantum Gravity **24**, S399 (2007).
[24] B. Willke (LIGO Scientific Collaboration), Classical Quantum Gravity **24**, S389 (2007).
[25] C. Zhao *et al.*, J. Phys.: Conf. Ser. **32**, 368 (2006).
[26] B. Abbott *et al.* (LIGO Scientific Collaboration), Phys. Rev. D **69**, 102001 (2004).
[27] B. Abbott *et al.* (LIGO Scientific Collaboration), Phys. Rev. D **72**, 062001 (2005).
[28] B. Abbott *et al.* (LIGO Scientific Collaboration), Classical Quantum Gravity **23**, S29 (2006).
[29] B. Abbott *et al.* (LIGO Scientific Collaboration), Classical Quantum Gravity **24**, 5343 (2007).
[30] F. Acernese *et al.*. (Virgo Collaboration), Classical Quantum Gravity **23**, S197 (2006).
[31] F. Acernese *et al.* (Virgo Collaboration), Classical Quantum Gravity **24**, S415 (2007).
[32] F. Beauville *et al.* (LSC/Virgo Working Group), Classical Quantum Gravity **25**, 045002 (2008).
[33] International Ice Patrol website http://www.uscg.mil/lantarea/iip/faq/icebergs.