$\Upsilon(1s) \to \gamma(\eta\prime, \eta)$ decays

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Radiative decays $\Upsilon(1s) \to \gamma(\eta', \eta)$ are studied by an approach which has successfully predicted the ratio $\frac{\Gamma(J/\psi \to \gamma\eta')}{\Gamma(J/\psi \to \gamma\eta)}$. Strong dependence on quark mass has been found in the decays $(J/\psi, \Upsilon(1s)) \to \gamma(\eta', \eta)$. Very small decay rates of $\Upsilon(1s) \to \gamma(\eta', \eta)$ are predicted.

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New small upper limits of the branching ratios of radiative $\Upsilon(1s)$ decays into η and η' have been reported by CLEO [1]

$$B(\Upsilon(1s) \to \gamma\eta) < 1.0 \times 10^{-6}, \quad B(\Upsilon(1s) \to \gamma\eta') < 1.9 \times 10^{-6} \tag{1}$$

at the 90% C.L. In an early search for $\Upsilon(1s) \to \gamma\eta'$ by CLEO [2] the upper limit of the branching ratio is determined to be 1.6×10^{-5} at the 90% C.L. $B(\Upsilon(1s) \to \gamma\eta) < 2.1 \times 10^{-5}$ is obtained in a previous CLEO experiment at the 90% C.L. [3]. Comparing with $B(J/\psi \to \gamma\eta'(\eta))$, the branching ratios of $\Upsilon(1s) \to \gamma(\eta', \eta)$ are very small.

J/ψ and $\Upsilon(1s)$ are heavy vector mesons. It is known that the decay of heavy vector meson is a test ground of perturbative QCD (pQCD). The decay of a heavy vector meson to light hadrons is described as $V \to ggg$ and at the leading order the decay width is expressed as

$$\Gamma = b m_V \alpha_s^3(m_V),$$

where b is a constant which is independent of m_V, $\alpha_s(m_{J/\psi}) \sim 0.3$, and $\alpha_s(m_{\Upsilon(1s)}) \sim 0.18$ [4], respectively. In the process $V \to ggg \to$ hadrons $q\bar{q}$ pairs are produced by the three gluons and light hadrons are formed by recombinations of the quark pairs. In pQCD $V \to \gamma gg$ is the process for radiative decay of a heavy vector meson. The two gluons are color singlets and they can form 0^{++}, 0^{-+}, and 2^{++} states. J/ψ radiative decay has long been regarded as a fertile hunting ground for glueballs [5]. So far, the existence of glueballs has not been established. The efforts in the search for glueballs are complicated by mixture between the components of gluons and quarks. It is argued in Ref. [5(a)] that for V(heavy) $\to \gamma R$ one expects

$$B(R[q\bar{q}] \to gg) \sim O(\alpha_s^2), \qquad B(R[G] \to gg) \sim 1,$$

where $R[q\bar{q}]$ and R[G] are $q\bar{q}$ state and glue state, respectively. The production of the quark state in radiative decay of heavy vector meson is suppressed by $O(\alpha_s^2)$. It is shown in Ref. [5(b)] that the gluon content of a meson can be determined by using productions of this meson in both radiative decays of heavy vector quarkonium and $\gamma\gamma$ collisions. The relation between $B(V_{q\bar{q}} \to \gamma + R)$ and $B(R \to gg)$ has been proposed in Ref. [5(a)].

The upper limit of $B(\Upsilon \to \gamma\eta')$ (1) is smaller than $\mathrm{BR}(J/\psi \to \gamma\eta') = (4.71 \pm 0.27) \times 10^{-3}$ by almost 3 orders of magnitude. This is a challenge to the study of radiative decay of heavy vector mesons. The difference between the radiative decays of J/ψ and $\Upsilon(1s)$ is caused by the mass difference of c and b quark. As pointed out in Ref. [1], the $B(\Upsilon(1s) \to \gamma\eta')$ predicted by naive scaling is too large. There are different theoretical approaches in the study of the branching ratios of $\Upsilon(1s) \to \gamma(\eta', \eta)$. Very small branching ratios for $\Upsilon \to \gamma\eta, \gamma\eta'$ (10^{-6}–10^{-7}) are obtained by employing the vector meson dominance model in Ref. [6]. $B(V \to \gamma\eta(\eta') \sim (5\text{–}10) \times 10^{-5})$ obtained in a nonrelativistic quark model [7] are too large. By using the QCD sum rule and the U(1) anomaly, $B(\Upsilon \to \gamma\eta') = 3.3 \times 10^{-5}$ and $B(\Upsilon \to \gamma\eta) = 4.4 \times 10^{-6}$ are determined in Ref. [8]. They are larger than the upper limits (1). It is pointed out in Ref. [9] that the quark mass dependence of the decay width of V(heavy) $\to \gamma\eta'$ obtained in Ref. [8] is proportional to $\frac{1}{m_q^4}$ and this factor does not lead to small $B(\Upsilon \to \gamma\eta')$ [1]. Using $m_\Upsilon \sim 2m_b$ and $m_{J/\psi} \sim 2m_c$,

$$B(\Upsilon(1s) \to \gamma + \eta')/B(J/\psi \to \gamma + \eta') \sim 1.31(Q_b^2 m_c^6)/(Q_c^2 m_b^6)(\alpha(m_c)/\alpha_s(m_b)),$$
$$B(\Upsilon(1s) \to \gamma\eta) \sim 3.3 \times 10^{-7},$$
$$B(\Upsilon(1s) \to \gamma\eta') \sim 1.7 \times 10^{-6}$$

are obtained in Ref. [9]. They are compatible with Eq. (1). In 1984 we studied $J/\psi \to \gamma\eta, \gamma\eta'$ [10]. In this study η' and η are taken as mixing states of glueballs and $q\bar{q}$ mesons and the gluon contents make dominant contributions to $J/\psi \to \gamma\eta, \gamma\eta'$. The ratio $\frac{\Gamma(J/\psi \to \gamma\eta')}{\Gamma(J/\psi \to \gamma\eta)}$ has been successfully predicted [10]. In this short paper the same approach is applied to study $\Upsilon \to \gamma\eta'(\eta)$.

It has been known for a long time that η' contains substantial gluon content. The U(1) anomaly of the η' meson [11] shows that there is strong coupling between η' and two gluons. The contribution of the quark components to the mass of the η' meson is

$$\tfrac{1}{3}(m_{K^+}^2 + m_{K^0}^2 + m_\pi^2) = 0.17\ \mathrm{GeV}^2$$

which is much less than $m^2_{\eta'} = 0.918\ \mathrm{GeV}^2$. Therefore, gluon components in η' are required and they should make an important contribution to $m^2_{\eta'}$. Experimental data [4] shows that $\mathrm{BR}(J/\psi \to \omega(\phi)\eta) > \mathrm{BR}(J/\psi \to \omega(\phi)\eta')$. In pQCD $J/\psi \to$ light hadrons is described as $J/\psi \to ggg$, $ggg \to q\bar{q}$ pairs. This experimental result can be understood since the quark content in η is more than the one in η'. On the other hand, $B(J/\psi \to \gamma\eta') > B(J/\psi \to \gamma\eta)$ has been reported [4]. It is known that the gluon contents of η' and η make dominant contributions to $J/\psi \to \gamma\eta'(\eta)$, and η' contains more gluons than η does. The larger branching ratio of $J/\psi \to \gamma\eta'$ can be explained by these arguments. The experimental data and theoretical arguments mentioned above support that in radiative decays of J/ψ to (η', η) quark contents of η' and η are suppressed and the gluon contents of η' and η play dominant roles. J/ψ, $\Upsilon \to \gamma\pi\pi$ are another example of suppression of quark components. The resonance structure of the $\pi\pi$ channel is complicated. We concentrate in the region of $\sigma(600)$. σ is a 0^{++} mesonic state made of quarks, and $\sigma \to \pi\pi$ is the main decay channel. σ has been observed in $J/\psi \to \omega\sigma$, $\sigma \to \pi\pi$ with a large branching ratio by the DM2 Collaboration [12] and by the BES Collaboration [13], respectively. However, $J/\psi \to \gamma\sigma$, $\sigma \to \pi\pi$ has not been reported by BES [14]. In Ref. [15] the decay $\Upsilon \to \gamma\pi\pi$ has been studied by using the soft-collinear effective theory and nonrelativistic QCD. The ratio $\frac{B(\Upsilon \to \gamma\pi\pi)}{B(J/\psi \to \gamma\pi\pi)} \sim 0.01$–$0.02$ has been predicted in the range of low $m^2_{\pi\pi}$ [15]. Because $J/\psi \to \gamma\sigma$, $\sigma \to \pi\pi$ has not been observed, the theory [15] predicts very small $B(\Upsilon \to \gamma\pi\pi)$ in the same region. No $\Upsilon \to \gamma\pi\pi$ events have yet been found in the σ region [16]. These experimental results and theoretical study are consistent with that because the suppression of $q\bar{q}$ $B(V(\mathrm{heavy}) \to \gamma R[q\bar{q}])$ is small.

A brief review of the study done in Ref. [10] is presented below. There are two parts: the amplitude of $J/\psi \to \gamma gg$ is calculated by pQCD and the couplings $gg \to \eta'(\eta)$ are dominated by the gluon contents of η' and η. The flavor singlet part of η and η' is a mixing state of glueballs and quark singlets

$$|\eta_0\rangle = \sin\phi|G\rangle + \cos\phi|q\bar{q}\rangle,$$

where ϕ is the mixing angle. It is argued in Ref. [10] that the quark contents of η and η' are suppressed in $J/\psi \to \gamma\eta(\eta')$ by $O(\alpha_s^2)$. Therefore, only the glueball content is taken into account in the calculations of $\Gamma(J/\psi \to \gamma\eta(\eta'))$. The coupling between state G and two gluons is expressed as

$$\langle G|T\{A^a_\alpha(x_1)A^b_\beta(x_2)\}|0\rangle = \frac{\delta_{ab}}{\sqrt{2E_G}}\epsilon_{\alpha\beta\mu\nu}(x_1 - x_2)^\mu p^\nu f_G(x_1 - x_2)e^{(i/2)p_G(x_1+x_2)}. \tag{2}$$

The function $f_G(x_1 - x_2)$ is unknown. In order to see the effect of this function $f_G(x_1 - x_2)$ is taken as a harmonic function at first to calculate the decay width, then instead $f_G(x_1 - x_2)$ $f_G(0)$ is taken. $\Gamma(J/\psi \to \gamma\eta(\eta'))$ obtained from these two expressions of $f_G(x_1 - x_2)$ are different by about 10% [10]. The possible dependence of $f_G(x_1 - x_2)$ on $x_1 - x_2$ has been ignored and f_G is taken as a parameter. The decay widths are derived as

$$\Gamma(J/\psi \to \gamma\eta') = \cos^2\theta\sin^2\phi\frac{2^{11}}{81}\alpha\alpha_s^2(m_c)\psi_J^2(0)f_G^2\frac{1}{m_c^8} \times \frac{(1 - \frac{m^2_{\eta'}}{m_J^2})^3}{\{1 - 2\frac{m^2_{\eta'}}{m_J^2} + \frac{4m_c^2}{m_J^2}\}^2} \times \left\{2m_J^2 - 3m^2_{\eta'}\left(1 + \frac{2m_c}{m_J}\right) - 16\frac{m_c^3}{m_J}\right\}^2, \tag{3}$$

$$\Gamma(J/\psi \to \gamma\eta) = \sin^2\theta\sin^2\phi\frac{2^{11}}{81}\alpha\alpha_s^2(m_c)\psi_J^2(0)f_G^2\frac{1}{m_c^8} \times \frac{(1 - \frac{m^2_{\eta}}{m_J^2})^3}{\{1 - 2\frac{m^2_{\eta}}{m_J^2} + \frac{4m_c^2}{m_J^2}\}^2} \times \left\{2m_J^2 - 3m^2_{\eta}\left(1 + \frac{2m_c}{m_J}\right) - 16\frac{m_c^3}{m_J}\right\}^2, \tag{4}$$

where θ is the mixing angle between η and η', and $\psi_J(0)$ is the wave function of J/ψ at the origin, which is determined by the decay rate of $J/\psi \to ee^+$

$$\psi_J^2(0) = \frac{27}{64\pi\alpha^2}m_J^2\Gamma_{J/\psi\to ee^+}. \tag{5}$$

$m_c = 1.25 \pm 0.09$ GeV are presented in Ref. [4]. In the study of $J/\psi \to \gamma + f(1273)$ [17], it is found that $m_c = 1.3$ GeV fits the data very well and this value is in the range of m_c [4]. In the ratio $\frac{\Gamma(J/\psi\to\gamma\eta')}{\Gamma(J/\psi\to\gamma\eta)}$, the unknown parameter f_G is canceled. The mixing angle of $\eta - \eta'$ is determined by the following equations:

$$\tan^2\theta = \frac{m_{88}^2 - m_\eta^2}{m_{\eta'}^2 - m_{88}^2}, \qquad m_{88}^2 = \frac{1}{3}(4m_K^2 - m_\pi^2),$$
$$\theta = -10.75^0 \pm 0.05^0.$$

Taking $m_c = 1.3$ GeV and the value of the mixing angle, it is predicted

$$\frac{\Gamma(J/\psi \to \gamma\eta')}{\Gamma(J/\psi \to \gamma\eta)} = 5.30 \pm 0.05, \tag{6}$$

where the errors are from the mixing angle only. The prediction agrees with current experimental value $4.81(1 \pm 0.15)$ [4] (in Ref. [10] $\theta = -11^0$ is taken and the ratio is predicted to be 5.1).

The factor $\frac{1}{m_c^8}$ in Eqs. (3) and (4) shows strong dependence of the decay rate on m_c (3) and (4). Because of the cancellation between m_J and m_c in the factors

$$\left\{2m_J^2 - 3m_{\eta'}^2\left(1+\frac{2m_c}{m_J}\right) - 16\frac{m_c^3}{m_J}\right\}^2 (3),$$
$$\left\{2m_J^2 - 3m_{\eta}^2\left(1+\frac{2m_c}{m_J}\right) - 16\frac{m_c^3}{m_J}\right\}^2 (4),$$

the decay rates are very sensitive to the value of m_c. This sensitivity has been found in the study of the decay $J/\psi \to \gamma + f(1273)$ [17] too.

By changing corresponding quantities, the decay rates of $\Upsilon(1s) \to \gamma\eta', \gamma\eta$ are obtained from Eqs. (3) and (4). The ratio is determined to be

$$R_{\eta'} = \frac{B(\Upsilon \to \gamma\eta')}{B(J/\psi \to \gamma\eta')} = \frac{1}{4}\frac{\alpha_s^2(m_b)}{\alpha_s^2(m_c)}\frac{\psi_\Upsilon^2(0)}{\psi_J^2(0)}\frac{m_c^8}{m_b^8}\frac{(1-\frac{m_{\eta'}^2}{m_\Upsilon^2})^3}{(1-\frac{m_{\eta'}^2}{m_J^2})^2}\frac{(1-2\frac{m_{\eta'}^2}{m_J^2}+4\frac{m_c^2}{m_J^2})^2}{(1-2\frac{m_{\eta'}^2}{m_\Upsilon^2}+4\frac{m_b^2}{m_\Upsilon^2})^2} \times \frac{\{2m_\Upsilon^2 - 3m_{\eta'}^2(1+\frac{2m_b}{m_\Upsilon}) - 16\frac{m_b^3}{m_\Upsilon}\}^2}{\{2m_J^2 - 3m_{\eta'}^2(1+\frac{2m_c}{m_J}) - 16\frac{m_c^3}{m_J}\}^3}\frac{\Gamma_{J/\psi}}{\Gamma_\Upsilon}, \quad (7)$$

where

$$\frac{\psi_\Upsilon^2(0)}{\psi_J^2(0)} = 4\frac{\Gamma_{\Upsilon\to ee^+}}{\Gamma_{J/\psi\to ee^+}}\frac{m_\Upsilon^2}{m_J^2}. \quad (8)$$

In order to compare with the result obtained in Ref. [9] $m_J = 2m_c$ and $m_\Upsilon = 2m_b$ are taken in Eq. (7)

$$R_{\eta'} = \frac{B(\Upsilon \to \gamma\eta')}{B(J/\psi \to \gamma\eta')} = \frac{\Gamma(\Upsilon \to \gamma\eta')/\Gamma(\Upsilon \to \text{light hadrons})}{\Gamma(J/\psi \to \gamma\eta')/\Gamma(J/\psi \to \text{light hadrons})} \times \frac{B(\Upsilon \to \text{light hadrons})}{B(J/\psi \to \text{light hadrons})}$$
$$= \frac{\alpha_s(m_c)}{\alpha_s(m_b)}\left(\frac{m_c}{m_b}\right)^7 \frac{1-\frac{m_{\eta'}^2}{4m_b^2}}{1-\frac{m_{\eta'}^2}{4m_c^2}}\frac{B(\Upsilon \to \text{light hadrons})}{B(J/\psi \to \text{light hadrons})} \times \frac{\Gamma_{\Upsilon\to ee}}{\Gamma_{J/\psi\to ee}} = 0.29\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\left(\frac{m_c}{m_b}\right)^7. \quad (9)$$

Stronger dependence on quark masses and small coefficients are obtained by this approach.

Both $\Gamma(J/\psi \to \gamma\eta', \gamma\eta)$ and $\Gamma(\Upsilon \to \gamma\eta', \gamma\eta)$ are sensitive to the values of m_c and m_b, respectively. $m_J = 2m_c$ is not a good approximation. In Ref. [4], $m_b = (4.7 \pm 0.07)$ GeV is listed. $m_\Upsilon = 2m_b$ works well. In this paper, $m_b = 4.7$ GeV and $m_c = 1.3$ GeV are taken. Inputting $B(J/\psi \to \gamma\eta')$ into Eq. (7),

$$B(\Upsilon \to \gamma\eta') = R_{\eta'} \qquad B(J/\psi \to \gamma\eta') = 1.04\times^{-7} \quad (10)$$

is obtained. Replacing $m_{\eta'}$ by m_η in Eq. (7), the ratio R_η is obtained. Inputting $B(J/\psi \to \gamma\eta)$ into R_η,

$$B(\Upsilon \to \gamma\eta) = 0.022 \qquad B(\Upsilon \to \gamma\eta') = 0.23 \times 10^{-8} \quad (11)$$

is determined. Both branching ratios are less than the experimental upper limits [1].

The approach used in Ref. [10] is applied to study the decays of $\Upsilon(1s) \to \gamma\eta'(\eta)$. Very strong dependence of the decay rate on quark mass is revealed. The ratio $\frac{\Gamma(J/\psi\to\gamma\eta')}{\Gamma(J/\psi\to\gamma\eta)}$ and very small branching ratios of $\Upsilon(1s) \to \gamma\eta'(\eta)$ are predicted. The predictions agree with the data very well.

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