Effects of hadronic loops on the direct *CP* violation of B_c

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It is well-known that the final state interaction plays an important role in the decays of B meson. The contribution of the final state interaction, which is supposed to be long-distance effects, to the concerned processes can interfere with that of the short-distance effects produced via the tree and/or loop diagrams at quark-gluon level. The interference may provide a source for the direct *CP* violation $\mathcal{A}_{CP}$ in the process $B_c^+ \to D^0\pi^+$. We find that a typical value of $\mathcal{A}_{CP}$ when the final state interaction effect is taken into account can be about -22% which is different from that without the final state interaction effect. Therefore, when we extract information on *CP* violation from the data which will be available at the Large Hadron Collider beauty experiment (LHCb) and the new experiments in B factories, the contribution from the final state interaction must be included. This study may be crucial for searching new physics in the future.

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I. INTRODUCTION

One of the most intriguing goals in the high energy physics is to look for new physics beyond the standard model (SM) via heavy hadron production and decay processes. The reason is that new physics which generally has a higher energy scale may be observed at the processes involving heavy flavors. Among all the possible quantities which are experimentally measurable, *CP* violation provides a more sensitive window to the new physics effects. Direct *CP* violation at B physics has been observed by the *BABAR* and Belle Collaborations [1,2], which is indeed a great success after confirmation of nonzero ϵ'/ϵ at K systems. Another promising place to study *CP* violation is the meson B_c, which is composed of different heavy flavors.

Since the CDF Collaboration observed B_c meson in the semileptonic decay $B_c \to J/\psi + l + \nu$ [3], studies on B_c have drawn great interests from both theorists and experimentalists of high energy physics. Decays of B_c can be realized via b decay, $\bar{c}$ decay, and annihilation of b and $\bar{c}$ [4]. Many theoretical works have been dedicated to study the decays of B_c [5–8]. A relatively complete discussion about its spectrum, production, and decays was presented in a review [9]. Because of the specific characteristics of its decay modes, the direct *CP* violation is an important observable which may provide valuable information towards the mechanism governing the transition and probably unveils a trace to the new physics beyond the SM.

In this work, we are just looking for a new source for the direct *CP* violation in B_c decays. The direct *CP* violation is caused in general by an interference among at least two channels which have the same final state, but different weak and strong phases. The *CP* quantity A_{CP} is proportional to

$$A_{CP} = \frac{2|A_1||A_2|\sin(\theta_1-\theta_2)\sin(\alpha_1-\alpha_2)}{|A_1|^2+|A_2|^2+2|A_1|A|_2|\cos(\theta_1-\theta_2)\cos(\alpha_1-\alpha_2)},$$

where A_1, A_2 are the amplitudes of the two distinct channels and θ_1, θ_2, α_1, α_2 are their strong phases and weak phases, respectively.

These phase differences come from either quark level or hadron level. At the quark level the strong phase difference usually occurs via the absorptive part of the loops involved in the calculation. The strong phase may also occur at the hadron level. As a matter of fact, it is well-known in the kaon system. When one studies the direct *CP* violation, i.e. ϵ'/ϵ, the phase shifts in the $\pi\pi$ scattering provide the strong phase which is necessary to result in *CP* violation. But recently most of the works to study direct *CP* violation concentrate on the strong phase induced by the absorptive part of the loops. Especially, the strong phase is coming from the absorptive part of the penguin diagram(s) which contribute along with the tree diagram to the amplitude. In that case the *CP* violation is induced by the interference between the contribution of the tree diagram and that of penguin.

The total width is related to $|A_1 + A_2|^2$. If one of the amplitudes is much smaller than the other one, the width should only depend on the larger one, say A_1, thus one can ignore the smaller one when calculating the decay width. However, even though $|A_2| \ll |A_1|$, the numerator of A_{CP} is proportional to their product, so one cannot ignore the smaller contribution, otherwise he would get null *CP* asymmetry.

In that case, obviously the contribution from the penguin diagram is much smaller than that from the tree diagram, so that if only the decay width is needed, one can completely ignore the contribution of the penguin. However, for evaluating the CP violation, he by no means can dismiss the penguin contribution.

In the SM, the weak phase originates from the Cabibbo-Kabayashi-Maskawa (CKM) matrix, and the strong phase is induced by the absorptive part of loops. At the quark-gluon level which is responsible for the short-distance effects, the strong phases may originate from the absorptive part of loops, for example, the penguin diagrams. On the other aspect, the final state interaction (FSI) plays an important role in B physics, as fully discussed in the literature [10]. At the short distance, the direct CP violation usually is caused by an interference between the tree-level contribution and the loop-induced one because they have different weak and strong phases (in fact the tree diagrams do not contribute a strong phase). Therefore an interference of the long-distance contribution with the short-distance ones may change the theoretical prediction on the CP violation. In fact, the FSI effect is extensively applied to the discussion of the CP violation of B and D decays [11–13].

Indeed, by the quantum field theory, the Lagrangian can be a combination of various pieces and each of them corresponds to different processes. For our transition matrix element $M = {}_{\rm out}\langle f|i\rangle_{\rm in}$, one has

$$M = \langle f|L^{(1)}_{\rm PQCD} + T[L_{\rm had}L^{(2)}_{\rm PQCD}]|i\rangle,$$

where $L^{(1),(2)}_{\rm PQCD}$ corresponds to the Lagrangian which includes QCD and weak or electromagnetic interactions, the superscripts (1) and (2) denote the Lagrangians which can lead to different final states, whereas $L_{\rm had}$ is the Lagrangian at hadron level. Then we further write the matrix element as

$$M = \langle f|L^{(1)}_{\rm PQCD}|i\rangle + \sum_n \langle f|L_{\rm had}|n\rangle\langle n|L^{(2)}_{\rm PQCD}|i\rangle, \qquad (1)$$

where the intermediate states $|n\rangle$ are a complete set of hadrons with proper quantum numbers and the matrix element $\langle f|L_{\rm had}|n\rangle$ is just the hadronic scattering process and corresponds to the hadronic loops in our work.

Generally, the long-distance effects due to the FSI refer to the rescattering of the intermediate hadrons which emerge at the direct decays, into the concerned final state and it is depicted by the term $\langle n|L^{(2)}_{\rm PQCD}|i\rangle$. In these channels with the intermediate hadronic intermediate states may have different weak phases from that of the short-distance production channel occurring at quark-gluon level. In the rescattering processes, phase shifts exist due to strong interaction and thus can offer strong phases. And an extra strong phase, which is definitely different from that induced by the quark-level loops, occurs from the hadron rescattering processes $\langle f|L_{\rm had}|n\rangle$.

Since the loop contribution is suppressed by the loop integration, generally the second term of the above equation is smaller than the first one which we may refer to as the "tree" level contribution (but maybe not the tree diagram in the common sense).

The traditional perturbative quantum chromodynamics (PQCD) calculation only takes care of the first term $\langle f|L^{(1)}_{\rm PQCD}|i\rangle$ and $\langle n|L^{(2)}_{\rm PQCD}|i\rangle$ in the second one, but leaves the part $\langle f|L_{\rm had}|n\rangle$ to be dealt with in other theories (for example, the chiral Lagrangian and etc.) at the hadron level. This picture is clearly depicted in Cheng's paper [10]. It indicates that unless the tree contribution (i.e. the first term) is suppressed by some mechanism, the first term corresponds to the direct process, so that is always dominating for the total amplitude. If we only need to consider the total decay width, the second term may contribute a smaller portion (sometimes it might be enhanced by some mechanism, but generally is much smaller). However, as we deal with the CP violation and need at least two different channels, their interference forces us not to abandon this term even though it might be much smaller than the first one. Indeed, one may argue that the loop diagram, such as penguin, can also contribute a strong phase and interfere with the tree contribution to result in a direct CP violation. The loop contribution may have a similar order as we considered here and possibly even smaller. At least as we state above, we are looking for a possible source of CP violation in B_c decays, i.e. the hadronic loops may contribute strong phases and cause sizable effects on CP violation as our numerical results given in the paper indicate.

Therefore we would say that the PQCD framework works well, but we instead are looking for a supposed-to-be smaller effect which can result in observable CP violation. If there is a small double-counting possible, that is because the wave function adopted in the calculation is not well-defined. In fact because $|n\rangle$ generally are not the same as $|f\rangle$, the double-counting does not appear.

At present the direct CP violation in B_c decays due to short-distance contribution has been studied by many authors [14–18]. But so far, the studies of the FSI effects on the direct CP violation of B_c are absent. In this work, by taking into account the long-distance contribution caused by the FSI, we would reevaluate the direct CP violation in the decays of B_c. Namely, we add a new contribution to the amplitude which has different strong and weak phases from that of short-distance contributions which were calculated by many authors. The interference of these contributions will significantly change the value of CP asymmetry in decays of B_c.

Indeed, before one can claim a discovery of new physics, he must exhaust all possibilities which the SM can provide. Therefore this work is also serving for the purpose of determining if the FSI can result in a sizable contribution to the direct CP violation of B_c.

We choose the channel $B_c^+ \to D^0\pi^+$ which should be one of the dominant decay modes of B_c. In this channel, there exist hadronic intermediate states which are mainly composed of $D^{(*)+}$ and J/ψ.

This paper is organized as follows. We present the formulation about $B_c^+ \to D^{(*)+} J/\psi \to D^0\pi^+$ in Sec. II. Then we present our numerical results. The last section is a short conclusion and discussion.

II. FORMULATION

The effective Hamiltonian related to B_c decays is [19]

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cd}^\dagger \{\mathcal{C}_1^b(\mu)(\bar{c}b)_{V-A})(\bar{d}c)_{V-A} + \mathcal{C}_2^b(\mu) \times (\bar{d}b)_{V-A})(\bar{c}c)_{V-A}\} + \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^\dagger \{\mathcal{C}_1^b(\mu) \times (\bar{u}b)_{V-A})(\bar{d}u)_{V-A} + \mathcal{C}_2^b(\mu)(\bar{d}b)_{V-A})(\bar{u}u)_{V-A}\}, \tag{2}$$

where the subscript $V - A$ denotes the left-chiral current $\gamma^\mu(1-\gamma^5)$. $\mathcal{C}_{1,2}^b(\mu)$ denote the Wilson coefficients.

First we calculate the transition amplitude of $B_c \to D^{(*)+} J/\psi$ at the quark level and the hadronization would be described by a few phenomenological parameters. The definitions of the relevant hadronic matrix elements are

$$\langle 0|\mathcal{J}_\mu|\mathcal{P}(k)\rangle = -if_{\mathcal{P}} k_\mu, \tag{3}$$

$$\langle 0|\mathcal{J}_\mu|\mathcal{V}(k,\epsilon)\rangle = f_{\mathcal{V}} \epsilon_\mu m_{\mathcal{V}}, \tag{4}$$

where $f_{\mathcal{P}}$ and $f_{\mathcal{V}}$ respectively stand for leptonic decay constants of pseudoscalar and vector mesons. k_μ is the four-momentum of the concerned hadron and ϵ_μ denotes the polarization of the vector meson. One has $\mathcal{J}_\mu = \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2$.

In addition, the hadronic matrix elements of B_c transiting into two mesons can be expressed in terms of a few form factors as [19]

$$\langle \mathcal{P}(k_2)|\mathcal{J}_\mu|B_c(k_1)\rangle = P_\mu f_+(Q^2) + Q_\mu f_-(Q^2), \tag{5}$$

$$\frac{1}{i}\langle \mathcal{V}(k_2,\epsilon)|\mathcal{J}_\mu|B_c(k_1)\rangle = \frac{\epsilon_\nu^*}{m_1+m_2} \times \{i\varepsilon^{\mu\nu\alpha\beta} P_\alpha Q_\beta F_V(Q^2) - g^{\mu\nu}(P\cdot Q)F_0^A(Q^2) + P^\mu P^\nu F_+^A(Q^2) + Q^\mu P^\nu F_-^A(Q^2)\} \tag{6}$$

with $P_\mu = (k_1+k_2)_\mu$ and $Q_\mu = (k_1-k_2)_\mu$. With the above formulas, we obtain

$$\mathcal{M}[B_c^+(p) \to D^+(p_1) J/\psi(p_2)] = \frac{iG_F}{\sqrt{2}} V_{cb} V_{cd}^\dagger \Big\{ a_1 f_D \frac{p_{1\sigma}}{m_{B_c}+m_\psi} [-g^{\sigma\lambda}(p+p_2)\cdot(p-p_2)F_0^A(q_1^2) + (p+p_2)^\sigma (p+p_2)^\lambda F_+^A(q_1^2) + (p-p_2)^\sigma(p+p_2)^\lambda F_-^A(q_1^2)] + a_2 f_\psi m_\psi [(p+p_1)^\lambda f_+(q_2^2) + (p-p_1)^\lambda f_-(q_2^2)]\Big\}, \tag{7}$$

and

$$\mathcal{M}[B_c^+(p) \to D^{*+}(p_1) J/\psi(p_2)] = \frac{iG_F}{\sqrt{2}} V_{cb} V_{cd}^\dagger \Big\{ a_1 f_{D^*} m_{D^*} \frac{i}{m_{B_c}+m_\psi} [i\varepsilon^{\sigma\omega\tau\delta}(p+p_2)_\tau (p-p_2)_\delta F_V(q_1^2) - g^{\sigma\omega}(p+p_2)\cdot(p-p_2)F_0^A(q_1^2) + (p+p_2)^\sigma(p+p_2)^\omega F_+^A(q_1^2) + (p-p_2)^\sigma(p+p_2)^\omega F_-^A(q_1^2)] + a_2 f_\psi m_\psi \frac{i}{m_{B_c}+m_{D^*}} \times [i\varepsilon^{\omega\sigma\tau\delta}(p+p_1)_\tau(p-p_1)_\delta F_V(q_2^2) - g^{\omega\sigma}(p+p_1)\cdot(p-p_1)F_0^A(q_2^2) + (p+p_1)^\omega(p+p_1)^\sigma F_+^A(q_2^2) + (p-p_1)^\omega(p+p_1)^\sigma F_-^A(q_2^2)]\Big\} \tag{8}$$

with $q_1 = p - p_2$ and $q_2 = p - p_1$. The values of $a_{1,2}$ will be given in next subsection.

A. Absorptive part of hadronic loop for $B_c^+ \to D^{(*)+} J/\psi \to D^0\pi^+$

Now let us turn to evaluate the contribution from the long-distance effects which occur at the hadron level. The diagrams shown in Fig. 1 depict sequent processes $B_c^+ \to D^{(*)+} J/\psi \to D^0\pi^+$.

The effective Lagrangian at the hadronic level is suggested to be in the following forms as [20]

$$\mathcal{L}_{\mathcal{D}^*\mathcal{D}\pi} = ig_{\mathcal{D}^*\mathcal{D}\pi}(\mathcal{D}_\mu^* \partial^\mu \pi \bar{\mathcal{D}} - \mathcal{D}\partial^\mu \pi \bar{\mathcal{D}}_\mu^*), \tag{9}$$

$$\mathcal{L}_{\mathcal{D}^*\mathcal{D}^*\pi} = -g_{\mathcal{D}^*\mathcal{D}^*\pi} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu \mathcal{D}_\nu^* \pi \partial_\alpha \bar{\mathcal{D}}_\beta^*, \tag{10}$$

$$\mathcal{L}_{\psi\mathcal{D}\mathcal{D}} = ig_{\psi\mathcal{D}\mathcal{D}} \psi_\mu (\partial^\mu \mathcal{D}\bar{\mathcal{D}} - \mathcal{D}\partial^\mu \bar{\mathcal{D}}), \tag{11}$$

$$\mathcal{L}_{\psi\mathcal{D}^*\mathcal{D}} = -g_{\psi\mathcal{D}^*\mathcal{D}}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\psi_\nu(\partial_\alpha\mathcal{D}^*_\beta\bar{\mathcal{D}} + \mathcal{D}\partial_\alpha\bar{\mathcal{D}}^*_\beta) \tag{12}$$

with $\pi = \boldsymbol{\tau}\cdot\boldsymbol{\pi}$, where fields $\mathcal{D}^{(*)}$ and $\bar{\mathcal{D}}^{(*)}$ are defined as $\mathcal{D}^{(*)} = (D^{(*)0}, D^{(*)+})$ and $\bar{\mathcal{D}}^{(*)T} = (\bar{D}^{(*)0}, \bar{D}^{(*)-})$.

The process shown in Fig. 1(a) is $B_c^+ \to D^+(p_1)J/\psi(p_2) \to \pi^+(p_3)D^0(p_4)$ where D^{*0} is exchanged at t channel, and its amplitude reads

$$\begin{aligned}\mathrm{Abs}^{(a)} &= \frac{1}{2}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(m_{B_c}-p_1-p_2)\mathcal{M}[B_c^+(p)\to D^+(p_1)J/\psi(p_2)]\\ &\quad\times[-g_{D^*D\pi}ip_3^\xi][-ig_{J/\psi D^*D}\varepsilon^{\mu\nu\alpha\beta}(-ip_{2\mu})iq_\alpha]\left(-g_{\lambda\nu}+\frac{p_{2\lambda}p_{2\nu}}{m_\psi^2}\right)\left(-g_{\beta\xi}+\frac{q_\beta q_\xi}{m_{D^*}^2}\right)\frac{i}{q^2-m_{D^*}^2}\mathcal{F}^2[q^2,m_{D^*}^2].\end{aligned} \tag{13}$$

Obviously, the conservation of angular momentum demands the contribution from Fig. 1(a) to be zero.

The amplitude corresponding to the process of $B_c^+ \to D^{*+}(p_1)J/\psi(p_2) \to \pi^+(p_3)D^0(p_4)$ where D^0 is exchanged at t channel reads as

$$\begin{aligned}\mathrm{Abs}^{(b)} &= \frac{1}{2}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(m_{B_c}-p_1-p_2)\mathcal{M}[B_c^+(p)\to D^{*+}(p_1)J/\psi(p_2)][-g_{D^*D\pi}(-ip_3^\xi)]\\ &\quad\times[-g_{\psi DD}(ip_4-iq)^\mu]\left(-g_{\sigma\xi}+\frac{p_{1\sigma}p_{1\xi}}{m_{D^*}^2}\right)\left(-g_{\omega\mu}+\frac{p_{2\omega}p_{2\mu}}{m_\psi^2}\right)\frac{i}{q^2-m_D^2}\mathcal{F}^2[q^2,m_D^2].\end{aligned} \tag{14}$$

For Fig. 1(c), $B_c^+ \to D^{*+}(p_1)J/\psi(p_2) \to \pi^+(p_3)D^0(p_4)$ where D^{*0} is exchanged at t channel, the amplitude is

$$\begin{aligned}\mathrm{Abs}^{(c)} &= \frac{1}{2}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(m_{B_c}-p_1-p_2)\mathcal{M}[B_c^+(p)\to D^{*+}(p_1)J/\psi(p_2)]\\ &\quad\times[-ig_{D^*D^*\pi}\varepsilon^{\mu\nu\alpha\beta}(-ip_{1\mu})(-iq_\alpha)][-ig_{J/\psi D^*D}\varepsilon^{\xi\lambda\kappa\rho}(-ip_{2\xi})iq_\kappa]\left(-g_{\beta\rho}+\frac{q_\beta q_\rho}{m_{D^*}^2}\right)\left(-g_{\sigma\nu}+\frac{p_{1\sigma}p_{1\nu}}{m_{D^*}^2}\right)\\ &\quad\times\left(-g_{\omega\lambda}+\frac{p_{2\omega}p_{2\lambda}}{m_\psi^2}\right)\frac{i}{q^2-m_{D^*}^2}\mathcal{F}^2[q^2,m_{D^*}^2].\end{aligned} \tag{15}$$

In the above amplitudes, $q = p_3 - p_1$ and $\mathcal{F}(q^2, m_i)$ etc. denote the form factors which compensate the off-shell effects of mesons at the effective vertices and may be described by the possible pole structures [10]

$$\mathcal{F}(q^2, m_i) = \left(\frac{\Lambda^2 - m_i^2}{\Lambda^2 - q^2}\right)^n, \tag{16}$$

where Λ is a phenomenological parameter to be determined. As $q^2 \to 0$ the form factor becomes a number. If $\Lambda \gg m_i$, it becomes a unity. As $q^2 \to \infty$, the form factor approaches zero. It reflects the fact that as the distance between the mesons becomes very small, their inner structures would overlap and the whole picture of hadron interaction breaks down. Hence the form factor vanishes at large q^2 and effectively plays a role to cut off the ultraviolet divergence. The expression of Λ is suggested to be [10]

$$\Lambda(m_i) = m_i + \alpha\Lambda_{\mathrm{QCD}}, \tag{17}$$

where m_i denotes the mass of the exchanged meson and α is a phenomenological parameter. In this work, we adopt the dipole form factor $\mathcal{F}(q^2, m_i) = (\Lambda^2 - m_i^2)^2/(\Lambda^2 - q^2)^2$.

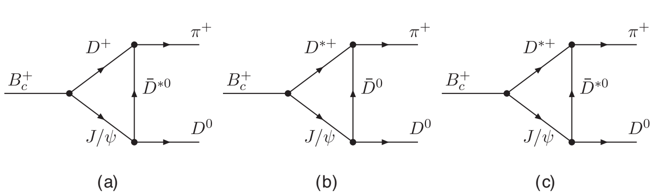

FIG. 1. The final state interaction contributions to $B_c^+ \to D^0\pi^+$.

B. Dispersive part of $B_c^+ \to D^{(*)+}J/\psi \to D^0\pi^+$

In the above subsection, the absorptive part of the triangle diagram to the amplitude of the sequent process $B_c^+ \to D^{(*)+}J/\psi \to D^0\pi^+$ can be easily obtained from the integrals (13)–(15). The dispersive part of $B_c^+ \to D^{(*)+}J/\psi \to D^0\pi^+$ can be related to the absorptive part via the dispersive relation [10,21]

$$\mathrm{Dis}[B_c^+ \to D^0\pi^+] = \frac{1}{\pi}\int_{s_1}^\infty \frac{\mathrm{Abs}[B_c^+ \to D^0\pi^+]}{s - m_{B_c}^2}ds. \tag{18}$$

However, the cutoff which is phenomenologically introduced and the complicated integral in Eq. (18) would cause unavoidable uncertainties to the dispersive part. In some of the former works, for estimating the decay width, the contribution of the dispersive part was assumed to be small (compared with that of the absorptive part) and ignored. However, for the direct CP violation, we must estimate the dispersive part and determine the strong phase induced by the triangle diagram, otherwise the strong phase would be exactly $\pi/2$.

In this work, adopting the method in our previous work [22], we obtain the dispersive part of $B_c^+ \to D^{(*)+}J/\psi \to D^0\pi^+$ by directly calculating the triangle where the intermediate hadrons are not on their mass shells. The amplitudes corresponding to the process of $B_c^+ \to D^{*+}(p_1)J/\psi(p_2) \to \pi^+(p_3)D^0(p_4)$ where D^0 or D^{*0} are exchanged are

$$\mathrm{Dis}^{(b)} = \int \frac{d^4q}{(2\pi)^4}\mathcal{M}[B_c^+(p) \to D^{*+}(p_1)J/\psi(p_2)][-g_{D^*D\pi}(-ip_3^{\xi})][-g_{\psi DD}(ip_4 - iq)^{\mu}](-g_{\sigma\xi})(-g_{\omega\mu}) \times \frac{i}{p_1^2 - m_D}\frac{i}{p_2 - m_{J/\psi}}\frac{i}{q^2 - m_D^2}\mathcal{F}^2[q^2, m_D^2], \tag{19}$$

and

$$\mathrm{Dis}^{(c)} = \int \frac{d^4q}{(2\pi)^4}\mathcal{M}[B_c^+(p) \to D^{*+}(p_1)J/\psi(p_2)][-ig_{D^*D^*\pi}\varepsilon^{\mu\nu\alpha\beta}(-ip_{1\mu})(-iq_{\alpha})] \times [-ig_{J/\psi D^*D}\varepsilon^{\xi\lambda\kappa\rho}(-ip_{2\xi})iq_{\kappa}](-g_{\beta\rho})(-g_{\sigma\nu})(-g_{\omega\lambda})\frac{i}{p_1^2 - m_D}\frac{i}{p_2 - m_{J/\psi}}\frac{i}{q^2 - m_{D^*}^2}\mathcal{F}^2[q^2, m_{D^*}^2]. \tag{20}$$

Because of the existence of the dipole form factors $\mathcal{F}^2[q^2, m_D^2]$ and $\mathcal{F}^2[q^2, m_{D^*}^2]$ the ultraviolet behavior of the triangle loop integration is benign. These form factors play an equivalent role to the Λ-related terms introduced in the Pauli-Villas renormalization scheme [23,24]. Because the final expressions of Eqs. (19) and (20) are complicated, we will collect some useful formulas in the Appendix.

C. Direct *CP* violation

The observable direct CP violation is defined as

$$\mathcal{A}_{CP} = \frac{|\mathcal{M}|^2 - |\bar{\mathcal{M}}|^2}{|\mathcal{M}|^2 + |\bar{\mathcal{M}}|^2} \tag{21}$$

with

$$\mathcal{M} = \mathcal{M}^{\mathrm{dir}}(B_c^+ \to D^0\pi^+) + \mathcal{M}^{\mathrm{FSI}}(B_c^+ \to D^0\pi^+),$$
$$\bar{\mathcal{M}} = \mathcal{M}^{\mathrm{dir}}(B_c^- \to \bar{D}^0\pi^-) + \mathcal{M}^{\mathrm{FSI}}(B_c^- \to \bar{D}^0\pi^-),$$

where $\mathcal{M}^{\mathrm{dir}}(B_c^+ \to D^0\pi^+)$ and $\mathcal{M}^{\mathrm{dir}}(B_c^- \to \bar{D}^0\pi^-)$ were calculated in the approach of PQCD by the many authors [18] and written as

$$\mathcal{M}^{\mathrm{dir}}(B_c^+ \to D^0\pi^+) = V_u(T_u + P)[1 - ze^{i(-\gamma+\delta)}], \tag{22}$$

$$\mathcal{M}^{\mathrm{dir}}(B_c^- \to \bar{D}^0\pi^-) = V_u^*(T_u + P)[1 - ze^{i(\gamma+\delta)}] \tag{23}$$

with

$$z = \left|\frac{V_c}{V_u}\right|\left|\frac{T_c + P}{T_u + P}\right| \quad \text{and} \quad \delta = \arg\left[\frac{T_c + P}{T_u + P}\right],$$

where $V_u = V_{ud}V_{ub}^*$ are the Cabibbo-Kabayashi-Maskawa entries, $|V_c/V_u| = \frac{\lambda}{1-\lambda^2/2}|V_{cb}/V_{ub}|$. The values of $T_{u,c}$, P, γ, λ, z, and δ are given in Ref. [18] and listed in Table I.

The amplitude of $B_c^+ \to D^0\pi^+$ induced by the FSI effect which is denoted by the superscript FSI is:

$$\mathcal{M}^{\mathrm{FSI}}(B_c^+ \to D^0\pi^+) = \mathrm{Dis} + i\sum_{j=a,b,c}\mathrm{Abs}^{(j)}, \tag{24}$$

and

$$\mathcal{M}^{\mathrm{FSI}}(B_c^+ \to D^0\pi^+) = \mathcal{M}^{\mathrm{FSI}}(B_c^- \to \bar{D}^0\pi^-). \tag{25}$$

TABLE I. These values are taken from Ref. [18]. Here $T_{u,c}$ and P are in units of 10^{-3} GeV. In this work, we need to multiply a factor $\sqrt{m_{B_c}^5}G_F/\sqrt{2|\mathbf{k}|} \sim 4.83 \times 10^{-4}$ to $T_{u,c}$ and P, because the formula for the decay widths adopted in this work takes a different normalization from that in Ref. [18].

T_u	$22.621 + 0.863i$	δ	123°
T_c	$-0.83 + 3.57i$	γ	55°
P	$-0.474 - 1.722i$	$\lvert\frac{V_{ub}}{V_{cb}}\rvert$	0.085
z	0.28		

III. NUMERICAL RESULTS

The input parameter set which we are going to use in this work includes: $m_{B_c} = 6.286$ GeV, $m_{J/\psi} = 3.097$ GeV, $m_{D^+} = 1.869$ GeV, $m_{D^{*+}} = 2.01$ GeV, $m_{D^0} = 1.865$ GeV [25]; $f_\psi = 405 \pm 17$ MeV [25];, $f_D = 222.6 \pm 16.7^{+2.8}_{-3.4}$ MeV, $f_{D^*} = 245 \pm 20^{+3}_{-2}$ MeV [26]; $V_{ud} = 0.974$, $V_{cd} = 0.230$, $V_{cb} = 0.0416$ [25]; $g_{D^*D\pi} = 17.3$, $g_{D^*D^*\pi} = 8.9$ GeV^{-1}, $g_{DD\psi} = 7.9$, $g_{D^*D\psi} =$

TABLE II. The values of $F(0)$, a, and b in the form factors of $B_c \to D^{(*)}$ and $B_c \to J/\psi$ [6,7].

		f_+	f_-	F_+^A	F_-^A	F_0^A	F_V
D	$F(0)$	0.189	−0.194	⋯	⋯	⋯	⋯
	a	2.47	2.43	⋯	⋯	⋯	⋯
	b	1.62	1.54	⋯	⋯	⋯	⋯
D^*	$F(0)$	⋯	⋯	0.158	−0.328	0.284	0.296
	a	⋯	⋯	2.15	2.40	1.30	2.40
	b	⋯	⋯	1.15	1.51	0.15	1.49
J/ψ	$F(0)$	⋯	⋯	0.66	−1.13	0.68	0.96
	a	⋯	⋯	1.13	1.23	0.59	1.24
	b	⋯	⋯	−0.067	0.006	−0.483	−0.002

4.2 GeV^{-1} [27]; $a_1 = 1.14$, $a_2 = -0.20$ [5]. $V_{ub} = A\lambda^3(\rho - i\eta) = 0.002\,18 - 0.003\,35i$. The Wolfenstein parameters of CKM matrix elements: $\lambda = 0.2272$, $A = 0.818$, $\bar{\rho} = 0.221$, and $\bar{\eta} = 0.340$ with $\bar{\rho} = \rho(1 - \frac{\rho}{2})$ and $\bar{\eta} = \eta(1 - \frac{\rho}{2})$. $G_F = 1.166\,37 \times 10^{-5}$ GeV^{-2} [25].

The form factors in processes $B_c \to D^{(*)}$ and $B_c \to J/\psi$ possess pole structures [6,7]

$$F(q^2) = \frac{F(0)}{1 - a\zeta + b\zeta^2} \tag{26}$$

with $\zeta = q^2/m_{B_c}^2$. The values of $F(0)$, a, and b are evaluated by some authors and for readers' convenience we list their results in Table II.

In Fig. 2, we plot the dispersive part and absorptive part of the amplitudes of $B_c^+ \to D^{(*)+} J/\psi \to D^0\pi^+$ versus α which is allowed to vary within $\alpha = 0.5 \sim 3$. In Fig. 3, we also list $\mathcal{A}_{CP}$ with several typical values of α. For a clear comparison, in this figure, we also give the value of $\mathcal{A}_{CP}$ calculated in Ref. [18] by the PQCD approach which is purely induced by the short-distance contribution (without considering the FSI). For clarity we also list some typical values of $\mathcal{A}_{CP}$ with various α in Table III.

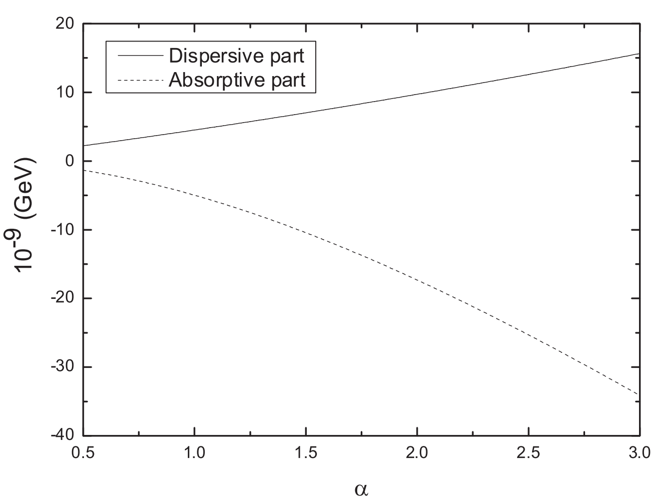

FIG. 2. The dispersive part and absorptive part of the amplitudes of $B_c^+ \to D^{(*)+} J/\psi \to D^0\pi^+$.

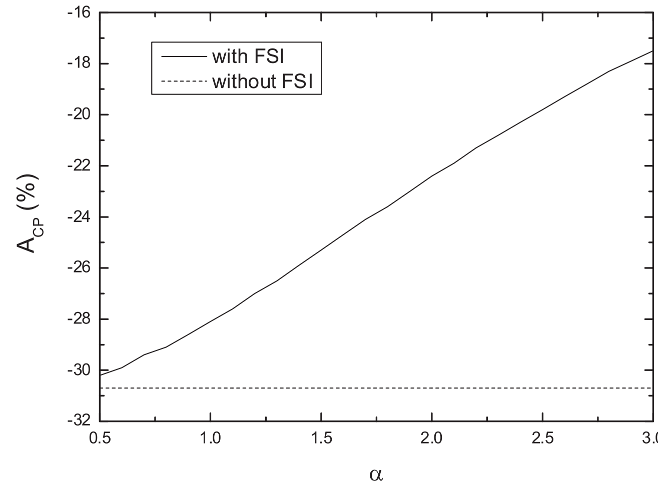

FIG. 3. The direct CP violation. Solid line and dashed line correspond to the direct CP with FSI effect and without FSI effect, respectively.

TABLE III. The typical values of $\mathcal{A}_{CP}$.

α	0.5	1.0	1.5	2.0	2.5	3.0
$\mathcal{A}_{CP}$	−30.2%	−28.1%	−25.3%	−22.4%	−19.8%	−17.5%

IV. DISCUSSION AND CONCLUSION

Recently the direct CP violation in B decays has been observed and it is expected to open a window for exploring new physics beyond the SM by which many theorists and experimentalists feel very inspired. Obviously, investigation of direct CP violation at B_c decays would be of special interest because it is composed of two heavy flavors and may be more sensitive to new physics. On another aspect, before one can claim to find a trace of new physics, he must exhaust all possibilities in the framework of the SM. As indicated in the literature, the FSI plays an important role in B decays, therefore one has a full reason to expect that it is also significant at B_c decays. In this work, we carefully study the contribution of the FSI to the direct CP violation via its interference with the contribution from the short-distance effects which are induced by the tree and loop diagrams. Concretely, in this work, we calculate the amplitudes for $B_c^+ \to D^0\pi^+$ via sequent processes $B_c^+ \to D^{(*)+} J/\psi \to D^0\pi^+$ and determine its strong and weak phases.

Here we need to add some interpretation about application of PQCD. Even though we indicate the significance of the FSI for evaluating direct CP violation in B_c decays, their absolute contribution is much smaller than that from the direct process which is calculated in the framework of

PQCD. Therefore if only the decay width of B_c is needed, one can ignore the contribution from the hadronic rescattering, but as the *CP* violation is concerned, as we see above, its contribution might be significant.

Our numerical results indicate that the typical value of $\mathcal{A}_{CP}$ with FSI effect is about -22%, which is different from the value -30.7% estimated in the PQCD approach without FSI [18]. On another aspect, one can also observe from Fig. 3 that the effect of the hadronic rescattering on A_{CP} may change quite diversely depending on the input parameter. Especially, as one adopts $\alpha = 0.5$, A_{CP} is about -30.2% which only slightly deviates from the value obtained in the framework of PQCD. However, as $\alpha = 3$ (even though $\alpha = 3$ seems too large to be very reasonable, this effective coupling indeed can exceed 1 for hadron interaction, in fact, in some applications its value is set to be very large for fitting data), A_{CP} would change to -17.5% and obviously deviates from the value of PQCD. It indicates that the contribution of FSI to A_{CP} is of opposite sign with that from the quark loops and the cancellation may cause remarkable effects when one analyzes the data achieved in a rather precise measurement. Thus our conclusion is that the contribution from the FSI is not negligible.

In future experiments, especially the Large Hadron Collider beauty experiment (LHCb), a great amount of data on B_c will be accumulated and there may be a possibility to measure the direct *CP* violation of B_c. If nonzero $\mathcal{A}_{CP}$ is well-measured as is expected, one can look for a trace of new physics by comparing the measured value with the theoretical result. When one compares the data which will be available at LHCb and/or other experiments with theoretical predictions, the contribution from the FSI must be included. This observation may be crucial for searching new physics in the future.

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APPENDIX

Some useful formulas in the calculation of Eqs. (19) and (20):

$$\int \frac{d^4q}{(2\pi)^4} \frac{1}{(p_1^2 - m_1^2)(p_2 - m_2)(q^2 - m^2)} \left(\frac{\Lambda^2 - m^2}{q^2 - \Lambda^2}\right)^4 = \frac{i}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \left\{ \frac{(\Lambda^2 - m^2)y}{\Delta^2(m_1, m_2, \Lambda)} + \frac{1}{\Delta(m_1, m_2, \Lambda)} - \frac{1}{\Delta(m_1, m_2, m)} - \frac{(-\Lambda^4 - m^4 + 2m^2\Lambda^2)y^2}{\Delta^3(m_1, m_2, \Lambda)} + \frac{(\Lambda^6 - 3m^2\Lambda^4 + 3m^4\Lambda^2 - m^6)y^3}{\Delta^4(m_1, m_2, \Lambda)} \right\}.$$

$$\int \frac{d^4q}{(2\pi)^4} \frac{l^2}{(p_1^2 - m_1^2)(p_2 - m_2)(q^2 - m^2)} \left(\frac{\Lambda^2 - m^2}{q^2 - \Lambda^2}\right)^4 = \frac{i}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \left\{ \frac{-2y(\Lambda^2 - m^2)}{\Delta(m_1, m_2, \Lambda)} + 2\ln\left[\frac{\Delta(m_1, m_2, \Lambda)}{\Delta(m_1, m_2, m)}\right] + \frac{(-\Lambda^4 - m^4 + 2m^2\Lambda^2)y^2}{\Delta^2(m_1, m_2, \Lambda)} - \frac{2y^3(\Lambda^6 - 3m^2\Lambda^4 + 3m^4\Lambda^2 - m^6)}{3\Delta^3(m_1, m_2, \Lambda)} \right\}.$$

$$\int \frac{d^4q}{(2\pi)^4} \frac{l^4}{(p_1^2 - m_1^2)(p_2 - m_2)(q^2 - m^2)} \left(\frac{\Lambda^2 - m^2}{q^2 - \Lambda^2}\right)^4 = \frac{i}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \left\{ 6y(\Lambda^2 - m^2) \ln\left[\frac{1}{\Delta(m_1, m_2, \Lambda)}\right] - 4y(\Lambda^2 - m^2) + 6\left[\Delta(m_1, m_2, \Lambda) \ln\left(\frac{1}{\Delta(m_1, m_2, \Lambda)}\right)\right] - \Delta(m_1, m_2, m) \ln\left[\frac{1}{\Delta(m_1, m_2, m)}\right] - \frac{(3y^2\Lambda^4 - m^4 + 2m^2\Lambda^2)}{\Delta(m_1, m_2, \Lambda)} + \frac{y^3(\Lambda^6 - 3m^2\Lambda^4 + 3m^4\Lambda^2 - m^6)}{\Delta^2(m_1, m_2, \Lambda)} \right\}.$$

Here

$$q = l - p_4 x + p_3(1 - x - y),$$

and

$$\Delta(a, b, c) = a^2(1 - x - y) + b^2 x + c^2 y + m_3^2(x^2 + y^2 - x - y + 2xy) + m_4^2(x^2 - x) + p_3 \cdot p_4(2x^2 + 2xy - 2x).$$

[1] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **87**, 091801 (2001).

[2] K. Abe *et al.* (Belle Collaboration), Phys. Rev. Lett. **87**, 091802 (2001).

[3] F. Abe *et al.* (CDF Collaboration), Phys. Rev. D **58**, 112004 (1998); Phys. Rev. Lett. **81**, 2432 (1998).

[4] M. Lusignoli and M. Masetti, Z. Phys. C **51**, 549 (1991).

[5] M. A. Ivanov, J. G. Körner, and P. Santorelli, Phys. Rev. D **73**, 054024 (2006).

[6] M. A. Ivanov, J. G. Körner, and O. N. Pakhomova, Phys. Lett. B **555**, 189 (2003).

[7] M. A. Ivanov, J. G. Körner, and P. Santorelli, Phys. Rev. D **63**, 074010 (2001).

[8] P. Colangelo, G. Nardulli, and N. Paver, Z. Phys. C **57**, 43 (1993); E. Jenkins, M. E. Luke, A. V. Manohar, and M. J. Savage, Nucl. Phys. **B390**, 463 (1993); V. V. Kiselev, A. K. Likhoded, and A. V. Tkabladze, Phys. At. Nucl. **56**, 643 (1993); V. V. Kiselev and A. V. Tkabladze, Phys. Rev. D **48**, 5208 (1993); C. H. Chang and Y. Q. Chen, Phys. Rev. D **49**, 3399 (1994); G. R. Lu, Y. D. Yang, and H. B. Li, Phys. Lett. B **341**, 391 (1995); Ikaros I. Y. Bigi, Phys. Lett. B **371**, 105 (1996); G. R. Lu, Y. D. Yang, and H. B. Li, Phys. Rev. D **51**, 2201 (1995); M. T. Choi and J. K. Kim, Phys. Rev. D **53**, 6670 (1996); C. H. Chang, J. P. Cheng, and C. D. Lu, Phys. Lett. B **425**, 166 (1998); V. V. Kiselev, A. K. Likhoded, and A. I. Onishchenko, Nucl. Phys. **B569**, 473 (2000); A. AbdEl-Hady, J. H. Munoz, and J. P. Vary, Phys. Rev. D **62**, 014019 (2000); V. V. Kiselev, A. E. Kovalsky, and A. K. Likhoded, Nucl. Phys. **B585**, 353 (2000).

[9] V. V. Kiselev, in CERN Yellow Report N. Brambilla *et al.*, CERN, Geneva CERN-2005-005, 2005; C. H. Chang, Int. J. Mod. Phys. A **21**, 777 (2006).

[10] H. Y. Cheng, C. K. Chua, and A. Soni, Phys. Rev. D **71**, 014030 (2005).

[11] Z. Z. Xing and D. S. Du, Phys. Lett. B **270**, 51 (1991); **276**, 511 (1992); D. S. Du, X. Q. Li, Z. T. Wei, and B. S. Zou, Eur. Phys. J. A **4**, 91 (1999); Y. S. Dai, D. S. Du, X. Q. Li, Z. T. Wei, and B. S. Zou, Phys. Rev. D **60**, 014014 (1999).

[12] N. G. Deshpande, X. G. He, W. S. Hou, and S. Pakvasa, Phys. Rev. Lett. **82**, 2240 (1999).

[13] N. Isgur, K. Maltman, J. Weinstein, and T. Barnes, Phys. Rev. Lett. **64**, 161 (1990); M. P. Locher, V. E. Markusin, and H. Q. Zheng, Report No. PSI-PR-96-13 (unpublished); H. Lipkin, Nucl. Phys. **B244**, 147 (1984); Phys. Lett. B **179**, 278 (1986); Nucl. Phys. **B291**, 720 (1987); H. J. Lipkin and B. S. Zou, Phys. Rev. D **53**, 6693 (1996); P. Geiger and N. Isgur, Phys. Rev. Lett. **67**, 1066 (1991); V. V. Anisovich, D. V. Bugg, A. V. Sarantsev, and B. S. Zou, Phys. Rev. D **51**, R4619 (1995); X. Q. Li, D. V. Bugg, and B. S. Zou, Phys. Rev. D **55**, 1421 (1997); X. Liu, B. Zhang, and S. L. Zhu, Phys. Lett. B **645**, 185 (2007).

[14] M. Masetti, Phys. Lett. B **286**, 160 (1992).

[15] J. F. Liu and K. T. Chao, Phys. Rev. D **56**, 4133 (1997).

[16] Y. S. Dai and D. S. Du, Eur. Phys. J. C **9**, 557 (1999).

[17] V. V. Kiselev, J. Phys. G **30**, 1445 (2004).

[18] J. F. Cheng, D. S. Du, and C. D. Lü, Eur. Phys. J. C **45**, 711 (2006).

[19] M. A. Ivanov, J. G. Körner, and P. Santorelli, Phys. Rev. D **73**, 054024 (2006).

[20] Y. Oh, T. Song, and S. H. Lee, Phys. Rev. C **63**, 034901 (2001).

[21] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).

[22] X. Liu, X. Q. Zeng, and X. Q. Li, Phys. Rev. D **74**, 074003 (2006).

[23] C. Itzykson and J. B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).

[24] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, MA, 1995).

[25] W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).

[26] M. Artuso *et al.* (CLEO Collaboration), Phys. Rev. Lett. **95**, 251801 (2005).

[27] C. Isola, M. Ladisa, G. Nardulli, and P. Santorelli, Phys. Rev. D **68**, 114001 (2003).