# Effects of the regularization on the restoration of chiral and axial symmetries

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The effects of a type of regularization for finite temperatures on the restoration of chiral and axial symmetries are investigated within the SU(3) Nambu-Jona-Lasinio model. The regularization consists in using an infinite cutoff in the integrals that are convergent at finite temperature, a procedure that allows one to take into account the effects of high momentum quarks at high temperatures. It is found that the critical temperature for the phase transition is closer to lattice results than the one obtained with the conventional regularization, and the restoration of chiral and axial symmetries, signaled by the behavior of several observables, occurs simultaneously and at a higher temperature. The restoration of the axial symmetry appears as a natural consequence of the full recovering of the chiral symmetry that was dynamically broken. By using an additional ansatz that simulates instanton suppression effects, by means of a convenient temperature dependence of the anomaly coefficient, we found that the restoration of U(2) symmetry is shifted to lower values, but the dominant effect at high temperatures comes from the new regularization that enhances the decrease of quark condensates, especially in the strange sector.

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#### I. INTRODUCTION

Although the studies on QCD thermodynamics have contributed to the improvement of our understanding of the QCD phase diagram, many challenging questions remain open. In this concern, microscopic and phenomenological models have played a meaningful role and are expected to clarify various problems in the future. Phase transitions associated to deconfinement and restoration of chiral and axial  $U_A(1)$  symmetries are expected to occur at high density and/or temperature. A question that has attracted a lot of attention is whether these phase transitions take place simultaneously or not and which observables could signal their occurrence.

As it is well known, the Nambu-Jona-Lasinio (NJL) model has the drawback of being nonrenormalizable, its action containing ultraviolet divergences that should be regularized. Different types of regularizations may be found in the literature [1,2], and the sensitivity of different observables to the type of regularization or value of the cutoff has been discussed [3,4]. In the NJL model, the cutoff used to regularize the quark loop term is, in general, lower than 1 GeV, which limits the domain of applicability of the model. Several unpleasant features of the model are due to the fact that the number of levels of the Fermi sea occupied is restricted by the value of the cutoff, as discussed in [4], where the authors study the influence of the ultraviolet cutoff on the stability of cold nuclear matter.

Nevertheless, the usefulness of NJL-type models to explore a variety of problems is well recognized, and improvements have been achieved, in particular, concerning the regularization procedure. In fact, while the use of a

constant cutoff in all integrals was a standard procedure in the former applications of the model, in the past few years some authors have regularized the action in order to eliminate logarithmic or quadratic divergences only, which means that there is no need to cut the convergent integrals. This approach has been used in the evaluation of integrals associated to triangle or box diagrams, as in [5,6] to calculate the anomalous decays of  $\eta$  and  $\pi^0$  mesons and in [3] to calculate the  $\rho$  meson form factor and  $\pi\pi$  scattering lengths; in both cases, a better agreement with experimental results was obtained. Recently, a regularization method that consists in regularizing, even in the logarithmic and divergent integrals, only the divergent parts has been performed [7].

A similar approach is nowadays used at finite temperature, since some integrals, divergent in the vacuum, become convergent due to the presence of the Fermi functions; therefore, the ultraviolet cutoff  $\Lambda$  is used only in the divergent integrals and  $\Lambda \to \infty$  in the convergent ones. This procedure was shown to have advantages in the study of several thermodynamic properties [8–11]. However, the influence of this type of regularization (which from now on we denote as regularization I) on a crucial question such as restoration of symmetries has not yet been analyzed. This is the main goal of the present work.

We perform our calculations in the framework of the three-flavor NJL model whose Lagrangian includes the determinantal 't Hooft interaction that breaks the  $U_A(1)$  symmetry:

$$\mathcal{L} = \bar{q}(i\partial \cdot \gamma - \hat{m})q + \frac{g_S}{2} \sum_{a=0}^{8} [(\bar{q}\lambda^a q)^2 + (\bar{q}(i\gamma_5)\lambda^a q)^2]$$

+ 
$$g_D[\det[\bar{q}(1+\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]].$$
 (1)

Here q = (u, d, s) is the quark field with  $N_f = 3$  and

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 $N_c=3$ ,  $\hat{m}={\rm diag}(m_u,m_d,m_s)$  is the current quark mass matrix, and  $\lambda^a$  are the Gell-Mann matrices,  $a=0,1,\ldots,8$ ,  $\lambda^0=\sqrt{\frac{5}{3}}{\bf I}$ . The model is fixed by the coupling constants  $g_S$  and  $g_D$ , the cutoff parameter  $\Lambda$ , which regularizes the divergent integrals, and the current quark masses  $m_i$  (i=u,d,s). We use the parameter set [12]  $m_u=m_d=5.5$  MeV,  $m_s=140.7$  MeV,  $g_S\Lambda^2=3.67$ ,  $g_D\Lambda^5=-12.36$ , and  $\Lambda=602.3$  MeV, which is fixed by mesonic spectroscopy data:  $f_\pi=92.4$  MeV,  $M_\pi=135.0$  MeV,  $M_K=497.7$  MeV, and  $M_{\eta'}=960.8$  MeV.

The constituent quark masses are fixed in the vacuum by fitting the parameters of the model to physical observables, and, in hot and dense matter, these masses depend on temperature and density/chemical potential. A drawback of the NJL model with the former regularization (ultraviolet cutoff in all of the integrals—regularization II) is that, while, in the chiral limit, the constituent quark masses vanish at a critical temperature (density), away from this limit, the masses, although decreasing, never reach its current values. This means that chiral symmetry is always only approximately restored, since the quark condensates, the order parameters associated to the dynamical chiral symmetry breaking, never vanish. As a matter of fact, the constituent masses of nonstrange quarks go asymptotically to their current values, and the condensates become very small at high values of temperature (density), but the strange quark mass is always far from its current value, so it is hard to talk about of restoration of chiral symmetry, even partial, in the strange sector. Therefore, the use of a regularization that leads all of the constituent quark masses to their current values could have important consequences for the restoration of symmetries, especially the axial symmetry that we have shown to be particularly sensitive to the behavior of the strange quark mass [13–15].

The axial  $U_A(1)$  symmetry is explicitly broken at the quantum level by the axial anomaly that may be described at the semiclassical level by instantons. This effect is enough to generate a mass for the  $\eta'$  in the chiral limit, so this meson cannot be a remnant of a Goldstone boson in the real world. The  $U_A(1)$  anomaly is responsible for the flavor mixing effect that lifts the degeneracy between several mesons. Therefore, if there occurs restoration of  $U_A(1)$  symmetry, the behavior of meson masses and mixing angles should exhibit signals for this restoration. Another observable relevant in this concern is the topological susceptibility  $\chi$  and its slope [13,16–18]. The topological susceptibility is defined as

$$\chi = \int d^4x \langle T\{Q(x)Q(0)\}\rangle, \tag{2}$$

where Q(x) is the topological charge density. We remark that  $\chi$  is related to the  $\eta'$  mass through the Witten-Veneziano formula [19]. Since large instanton effects are supposed to be suppressed at high temperatures or densities, and interactions between instantons contribute to the

elimination of fluctuations of the topological charge, there are good reasons to expect that the  $U_A(1)$  symmetry might be restored [20].

Several lattice calculations (see [21] and references therein) indicate a sharp decrease of the topological susceptibility with temperature at zero density, and, more recently [22], it was shown that, at a fixed T and by varying the chemical potential  $\mu$ , a critical  $\mu$  is found, where the quark condensate and the topological susceptibility drop and the Polyakov loop raises; its derivatives vary sharply.

A long-standing question, for which there is yet no answer, is whether the axial symmetry is restored at a temperature higher than the critical temperature for the phase transition (scenario 1) or at about that temperature (scenario 2) [23,24], the two scenarios leading to different predictions concerning the behavior of chiral partners in the critical region. In other words, the question may be formulated as: is the restoration of chiral symmetry driven by quark condensates or instantons? If it is driven by instantons, scenario 2 would be likely, implying that around the critical point signals of the restoration of  $U_A(1)$  symmetry should already be present, as, for instance, large fluctuations in the  $\eta$  spectrum.

In order to simulate the fate of the anomaly, it is usually assumed that the anomaly coefficient  $g_D$  is a dropping function of temperature, whether the approach is phenomenological [25] or lattice-inspired [13,17,18]. In previous works [13,14], we studied the possible effective restoration of the axial symmetry. For the case of finite temperature and zero chemical potential, we explored the effects of an anomaly coupling temperature dependence  $g_D(T)$  using two different ansatz. When we modeled  $g_D(T)$  from lattice results for  $\chi$ , as a Fermi function, we found that: (i) at  $T \simeq$ 250 MeV the chiral partners  $(\pi^0, \sigma)$  as well as  $(a_0, \eta)$ become degenerate, which is a manifestation of the effective restoration of chiral symmetry<sup>1</sup>; (ii) at  $T \simeq 350$  MeV,  $\chi \to 0$ , the pair  $(\pi^0, \sigma)$  becomes degenerate with  $(a_0, \eta)$ and the mixing angles get close to the ideal values, indicating an effective restoration of the axial symmetry. Chiral symmetry is effectively restored before axial symmetry, but, as shown in [14], both symmetries can be restored at the same point if  $g_D(T)$  is chosen as an appropriate decreasing exponential, a choice based on the phenomenological argument that high temperature suppresses large instanton fluctuations. To model such a decreasing exponential, we endow the anomaly coefficient with a temperature dependence in the form [25]:

$$g_D(T) = g_D(0) \exp[-(T/T_0)^2],$$
 (3)

where  $T_0 \simeq 100-200$  MeV.

<sup>&</sup>lt;sup>1</sup>The inflection point for the quark condensates, in the  $T - \mu$  plane, is taken as the critical point for the phase transition associated with partial restoration of chiral symmetry; the point where the masses of chiral partners become degenerate signals the effective restoration of chiral or axial symmetries.

We verified that the whole U(3)  $\otimes$  U(3) symmetry is not effectively restored, in the range of temperatures considered for both lattice-inspired or decreasing exponential ansatz: (i) the strange quark condensate decreases slowly, and chiral symmetry in the strange sector remains broken; (ii) the mesons  $\eta'$  and  $f_0$  become purely strange, and their masses decrease moderately but do not show a tendency to converge or to get close to the other meson masses.

More recently, we performed a study of the relevant observables as functions of  $\mu$  for a fixed temperature, and we verified that the combined effect of finite T and  $\mu$  does not change the usual results [15]: chiral symmetry is effectively restored only in the nonstrange sector, and restoration of axial symmetry is not achieved, unless the anomaly coefficient is chosen as a dropping function of temperature or chemical potential.

In summary, neither the ansatz used for  $g_D(T)$  nor the combined effects of temperature and density lead to a restoration of chiral and axial symmetries in the strange sector, unless the unrealistic condition of equal current quark masses for all quarks  $m_q = m_s = 5.5$  MeV is imposed from the beginning [15].

A subject that deserves attention is, therefore, the role played by the strange quark regarding the restoration of chiral and axial symmetries and whether the restoration of the singlet chiral symmetry could be just a consequence of the restoration of chiral symmetry or not, i.e., whether the fate of instantons drives the mechanism of restoration of chiral symmetry or the opposite. Moreover, the possible changes that could be induced in the scenarios described above by allowing quark states of high momentum to be present at high temperature should be investigated. This is achieved by means of a regularization that consists, as already referred, in letting the ultraviolet three-momentum cutoff go to infinity in all of the convergent integrals (regularization I). Two situations, which are summarized in Table I, will be analyzed: the anomaly coefficient  $g_D =$  $g_D(0)$  is kept constant, meaning that the restoration of symmetry is driven by the decrease of the quark condensates (case A); in order to investigate the effect of a competitive mechanism, simulating the suppression of instantons at finite temperature, we endow the anomaly coefficient with a temperature dependence in the form of Eq. (3), with an appropriate choice of  $T_0$  (case B). Both results will be compared with those obtained by regulariz-

TABLE I. Different schemes of explicit axial symmetry breaking and regularizations. The anomaly coefficient  $g_D(T)$  is given by Eq. (3).

	Anomaly coefficient $g_D$	Regularization
Case A-I	$g_D(0)$	$\Lambda \to \infty$
Case A-II	$g_D(0)$	$\Lambda = Constant$
Case B-I	$g_D(T)$	$\Lambda \to \infty$
Case B-II	$g_D(T)$	$\Lambda = \text{Constant}$

ing all of the integrals with the cutoff  $\Lambda$  (regularization II). The effects of regularization on important observables, such as the pressure and energy, will also be analyzed.

A remark should be added concerning the choice made for  $g_D(T)$  given by Eq. (3). We do not know which pattern of axial symmetry restoration is chosen by nature, but the variation of  $g_D(T)$  should not be at all arbitrary. By using the decreasing exponential, we might choose  $T_0$  in order to have two scenarios: (i) restoration of axial symmetry close to chiral symmetry, provided  $T_0$  is small enough (around 100 MeV) [14]—this would give a critical temperature for the phase transition of about  $T_c \simeq 135$  MeV, much lower than the present accepted values— and (ii) to choose  $T_0$  in order to have a critical temperature within the interval of accepted values ( $T_0 = 170$  MeV leads to  $T_c \simeq 154$ –163 MeV). The last point of view will be followed here.

## II. NUMERICAL RESULTS

The calculations are done in a standard way [12]. With the help of the bosonization procedure, an effective action is obtained, allowing us to evaluate a gap equation for the constituent quark masses, the quark condensates, and the scalar and pseudoscalar mesons. To begin with, we analyze the results for the quark masses and quark condensates that are plotted in Fig. 1. The results with regularization II, whether  $g_D$  is constant (case A-II) or a decreasing function of the temperature (case B-II), exhibit the following effects: the nonstrange quark masses decrease, although never attaining the current values, and the condensates also decrease but never vanish; concerning the strange quarks, the mass and condensate decrease moderately and are always far from the current value for the mass and zero for the quark condensate.

The situation changes drastically when we use the new regularization (cases A-I and B-I): all of the quark masses, at  $T \simeq 333$  MeV, from now on denoted as  $T_{\rm eff}$ , go to their current values and the quark condensates vanish, which is an indication of the complete restoration of the dynamically broken chiral symmetry. This means that the contribution for the quark masses originated by dynamical symmetry breaking completely disappears; only the current masses, due to the explicit chiral symmetry breaking ab initio, remain.

We can also see that the difference between the cases  $g_D(0)$  and  $g_D(T)$  is relevant only at low temperatures. The new finding is that at high temperatures the dominant effect is no longer the instanton suppression but the decrease of quark condensates that is enhanced, especially in the strange sector, when high momentum quark states are allowed  $(\Lambda \to \infty)$ .

<sup>&</sup>lt;sup>2</sup>As matter of fact,  $M_u = m_u$  for T = 333 MeV and  $M_s = m_s$  for T = 335 MeV. Once they are close, we adopt  $T_{\rm eff} \simeq 333$  MeV.

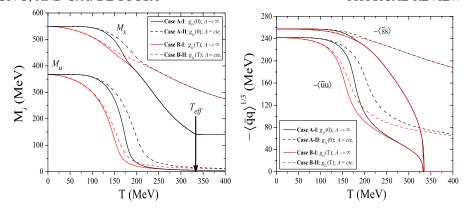


FIG. 1 (color online). Quark masses (left) and quark condensates (right) as functions of the temperature with two ansatz for  $g_D$  and two different regularizations (cases A-I–II and B-I–II.)

A remark is now in order concerning a nontrivial effect of regularization I. We notice that, above  $T_{\rm eff}$ , if we do not impose any restriction, the quark masses become lower than their current values and eventually become negative, an effect that has been found by other authors but without its implications discussed. One can argue that the quark masses are not observables and therefore the fact that they become negative is not meaningful. However, when the quark masses get lower than their current values, the quark condensates become negative, which is not physical since they are order parameters that should be zero in the phase of restored chiral symmetry. Therefore, if we want to keep calculating observables in this region, it seems sensible to impose the condition that the quark masses take their current values and the quark condensates remain zero. This is the approach used here.

In the left panel of Fig. 2, we plot the topological susceptibility, and we see that it decreases but does not vanish when regularization II is used; the effect of regularization I, whether we consider  $g_D(0)$  or  $g_D(T)$ , is the vanishing of the topological susceptibility at the same temperature as the quark condensates. Again we notice the same pattern found for the masses and quark condensates:

the behavior observed at high temperatures is dominated by the effect of the infinite cutoff. Since the vanishing of the topological susceptibility, only by itself, does not guarantee the restoration of the axial symmetry, we will analyze other observables.

The results of the right panel are also interesting. They show that, for both regularizations, the inflection points of the quark condensates and topological susceptibility occur approximately at the same temperature, a result that has already been found in [15,22]; the new finding is that the critical temperature with the new regularization is now  $T_c \simeq 177$  MeV, for case A, a value closer to the lattice result (see [26]) than the one obtained with regularization II ( $T_c \simeq 202$  MeV). For case B, the influence of the regularization on the value for the critical temperature is smaller than in case A (see Table II).

The behavior of the mixing angles (Fig. 3) and meson masses (Fig. 4) gives us complementary information on the effective restoration of the symmetries under study. Concerning the mixing angles (left panel) we see that they converge to the ideal values in case A-I exactly at  $T = T_{\rm eff} = 333$  MeV, the temperature at which all of the quark condensates vanish; for case B, independently of the regu-

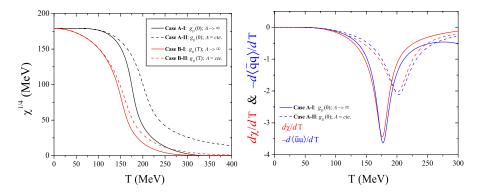


FIG. 2 (color online). Topological susceptibility (left) as function of the temperature for cases A-I–II and B-I–II. The derivatives of the quark condensates and of the topological susceptibility (right) are shown only for cases A-I–II; a similar pattern is found for cases B-I–II, with a shift of the inflection points for lower temperature.

TABLE II. Transition temperatures for the different cases.  $T_{\rm eff}$  is the transition temperature for the complete restoration of the dynamically broken chiral symmetry.

	Case A-I	Case A-II	Case B-I	Case B-II
Phase transition $(T_c)$	177 MeV	202 MeV	154 MeV	163 MeV
SU(2) chiral symmetry effective restoration	200 MeV	250 MeV	180 MeV	205 MeV
U(2) axial symmetry effective restoration	333 MeV	• • •	250 MeV	300 MeV
$T_{ m eff}$	333 MeV	•••	333 MeV	• • •

larization used, the mixing angles reach the ideal values at lower temperatures.

Prior to the analysis of splitting between the masses of the chiral partners, an observable that measures the degree of effective restoration of chiral symmetry, some preliminary remarks should be made. We notice that most of the theoretical insight into the problem of restoration of chiral and axial symmetries comes from lattice calculations for a pure gauge theory and from model calculations with massless quarks or in SU(2) models. Here we consider the physically relevant situation of explicit chiral symmetry breaking in SU(3) with the presence of the  $U_A(1)$  anomaly, and some care should be taken when making comparisons. The chiral partners  $(\pi^0, \sigma)$  and  $(\eta, a_0)$ , here studied, have their analogs in a  $SU(2) \otimes SU(2)$  world without strangeness, where the  $\sigma$  and  $\eta$  are nonstrange; the  $(\eta', f_0)$  exist only in  $SU(3) \otimes SU(3)$ . Therefore, the convergence of the two first chiral partners is driven by the restoration of  $SU(2) \otimes SU(2)$  symmetry, and the convergence of both pairs, which occurs when the mixing angles go to the ideal values and all four mesons are nonstrange, indicates the effective restoration of  $U(2) \otimes U(2)$  symmetry. The behavior of  $(\eta', f_0)$  is governed by the restoration of symmetry in the strange sector.

Concerning the meson masses (see Fig. 4), in case A-I, we observe the degeneracy of the chiral partners  $(\pi^0, \sigma)$  and  $(\eta, a_0)$  at  $T \simeq 200$  MeV, but both pairs get degenerate at  $T_{\rm eff} \simeq 333$  MeV, the temperature at which the quark condensates vanish. Comparing with case A-II, it can be

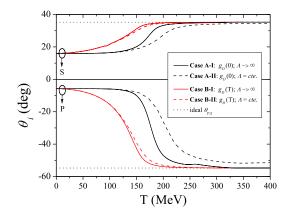


FIG. 3 (color online). Mixing angles and meson masses as functions of the temperature for cases A and B.

seen (Fig. 4) that the partners  $(\pi^0, \sigma)$  and  $(\eta, a_0)$  become degenerate at  $T \approx 250$  MeV (see also Table II). As expected, the axial symmetry is not restored.

In case B-I, the situation is qualitatively similar, but the temperatures for the phase transition and restoration of symmetries are shifted to lower values (see Table II), and the effective restoration of U(2) symmetry occurs at  $T \simeq 250$  MeV, which is consistent with the fact that the mixing angles go to the ideal values at this temperatures. The same effect is found with regularization II (case B-II) but at a larger temperature ( $T \simeq 300$  MeV).

Let us analyze the result with regularization I in more detail. We observe that the four mesons  $(\pi^0, \sigma, \eta, a_0)$  are nonstrange at the temperature where the mixing angles become ideal, even the mesons that had a component of strangeness in the vacuum, as  $\sigma$  and  $\eta$ . We verified that, when the condensates are zero, the following relations hold:  $M_{\sigma}^2 \simeq 4m_q^2 + M_{\pi}^2$  and  $M_{a_0}^2 \simeq 4m_q^2 + M_{\eta}^2$ . Since the nonstrange current quark masses are negligible as compared with the meson masses at this temperature, it is natural that the chiral partners become degenerated. Concerning the behavior of the chiral partners  $(f_0, \eta')$ , these mesons are completely strange at  $T_{\rm eff} \simeq 333$  MeV, and, since the strange current quark mass is high compared to that of the nonstrange quarks, it is natural that, although

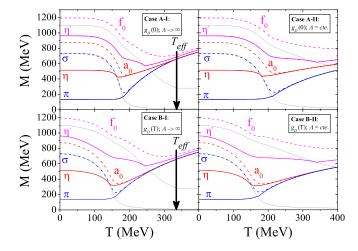


FIG. 4 (color online). Meson masses as functions of the temperature for cases A and B. The dotted lines indicate the temperature dependence of nonstrange (lower curves) and strange (upper curves)  $q\bar{q}$  thresholds.

their masses decrease meaningfully and get close to the other meson masses, they never converge with them. Even  $f_0$  and  $\eta'$  do not become degenerate between themselves, which is due to the high value of the current strange quark mass, since the analog of the previous relations between meson masses is now:  $M_{f_0}^2 \simeq 4m_s^2 + M_{\eta'}^2$ . In fact, the effects of dynamical symmetry breaking vanish, in both the strange and nonstrange sectors, but not those due to explicit symmetry breaking, which are negligible only in the nonstrange sector.

A question that could be raised now is if our results fit in one of the scenarios proposed by Shuryak [23] and in which of them [we recall that Shuryak's discussion was restricted to  $SU(2) \otimes SU(2)$ ]. Concerning the restoration of chiral  $SU(2) \otimes SU(2)$  and  $U(2) \otimes U(2)$  symmetries, the new regularization (case A-I) does not lead to a change of scenario with regards to the old regularization; however, considering the SU(3) sector, one can say that when high momentum quark states are allowed we have a scenario where the axial and chiral symmetry are effectively restored at the same temperatures  $T = T_{\rm eff} \simeq 333 \ {\rm MeV}$ . Nevertheless, we think that the relevant question concerning the two scenarios is not the classification but rather whether the restoration of chiral symmetry is driven by instantons or not; in the first case, signals of the restoration of axial symmetry should be observed at temperatures around the critical temperature for the phase transition  $(T_c)$ . This is not observed in any of the cases studied here. If we take into account a mechanism simulating instanton suppression (case B), the restoration of axial symmetry, although taking place after the restoration of chiral symmetry, is shifted to lower values ( $T \simeq 250 \text{ MeV}$ , instead of  $T \simeq 333$  MeV), but this mechanism is dominant only at low temperatures (where the infinite cutoff effect is not relevant) and in the nonstrange sector.

Finally, we check the usefulness of the present regularization by plotting the energy and the pressure as functions of the temperature (Fig. 5), which shows that the present results improve significantly with regards to those obtained with regularization II. So, although not reproducing the

lattice results, like the Nambu-Jona-Lasinio model coupled to the Polyakov loop [10,27], they are interesting from a qualitative point of view. In fact, our results follow the expected tendency and go to the free gas (Stefan-Boltzmann) values [28], a feature that was also found in other observables with this type of regularization [8].

#### III. SUMMARY AND CONCLUSIONS

We have studied the effects on the restoration of chiral and axial symmetries of a regularization that consists in using an infinite cutoff in the integrals that are convergent at finite temperature. When the decrease of the quark condensates is the dominant mechanism, we found that the critical temperature, signaling the phase transition associated with partial restoration of chiral symmetry, is lower than with the conventional regularization and closer to the lattice results. The main finding is that, with the implementation of the new cutoff procedure, restoration of chiral and axial symmetries can also be a phenomenon relevant in the strange sector. In fact, the dynamically broken chiral symmetry is completely recovered, in both the strange and nonstrange sectors, leading to the restoration of the axial symmetry at about the same temperature  $(T_{\mathrm{eff}})$ : the quark masses go to the current values, the quark condensates and topological susceptibility vanish, the mixing angles go to the ideal values, and the masses of the mesons, which are nonstrange at  $T_{\rm eff}$ , converge. When an ansatz that simulates independent suppression of instanton effects is taken into account, it is shown that this mechanism is relevant only for temperatures below  $T \approx$ 200 MeV and shifts the temperatures for the phase transition as well as effective restoration of symmetries in the nonstrange sector to lower values.

Regularization I also gives better results in the calculation of other observables, like the pressure and energy, that now have the expected tendency at high temperatures. The rich pattern of dynamical chiral symmetry breaking/restoration here presented and its relevance for other physical situations demands certainly further investigation. On the other hand, being aware of the simplicity of our approach,

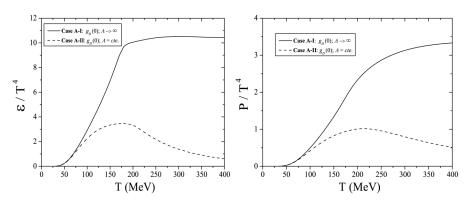


FIG. 5. Energy and pressure as functions of the temperature for cases A-I-II.

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we think that a deeper look into this problem is necessary. Work in this direction is in progress.

## ACKNOWLEDGMENTS

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