CP-conserving unparticle phase effects on the unpolarized and polarized direct CP asymmetry in $b \to dl^+l^-$ transition

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We examine the unparticle CP-conserving phase effects on the direct CP asymmetry for both polarized and unpolarized leptons in the inclusive $b \to d\ell^+\ell^-$ transition, where the flavor-changing neutral currents are forbidden at tree level but are induced by one-loop penguin diagrams. The averaged polarized and unpolarized CP asymmetries depict strong dependency on the unparticle parameters. In particular, a sizable discrepancy corresponding to the standard model is achieved when the scale dimension value is $1 < d_{\mathcal{U}} < 2$. We see that the unparticle stuff significantly enhances, suppresses, or changes the sign of the CP asymmetry depending on the definite value of the scaling dimension $d_{\mathcal{U}}$. Especially, when $d_{\mathcal{U}} \sim 1.1$ the CP asymmetries vanish.

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I. INTRODUCTION

Georgi [1,2] has recently proposed unparticle stuff, which can couple to the standard model (SM) particles at the Tev scale. Unparticles are massless and invisible coming out of a scale-invariant sector with noninteger scaling dimension $d_{\mathcal{U}}$ when decoupled at a large scale. The propagator of these invisible unparticles includes a CP-conserving phase, which is dependent on the noninteger scaling dimension $d_{\mathcal{U}}$ [2]. The virtual unparticle propagation and its effects were first studied by Georgi himself [2]. Moreover, the CP-conserving phase of the unparticles and its effects in flavor-changing neutral-current (FCNC) processes, especially in hadronic and semileptonic B decays, have been studied in [3–6].

A phenomenological study needs a construction of the effective Hamiltonian to describe the interactions of unparticles with the SM fields in the low energy level [7] so that we can investigate the effects of the possible scale-invariant sector experimentally.

The direct search of the unparticles is based on the study of missing energies at various processes which can be measured at CERN LHC or a future International Linear Collider (ILC). The indirect search includes the dipole moments of fundamental particles, lepton flavor violation, and FCNC processes, where the virtual unparticles enter as a mediator. Note that the phenomenological studies considering the direct and indirect search on unparticles have been progressing [2–14]: their effects on the missing energy of many processes; the anomalous magnetic moments; the electric dipole moments; $D^0 - \bar{D}^0$ and $B^0 - \bar{B}^0$ mixing; lepton flavor-violating interactions; direct CP violation in particle physics; and the phenomenological implications in cosmology and in astrophysics.

It is well known that in a decay process the existence of direct CPA (A_{CP}) requires first at least two different terms in decay amplitude. Second, these terms must depend on two types of phases named weak (δ) and strong (ϕ) phases. The weak phase is CP-violating, and the strong phase is a CP-conserving phase. The A_{CP} depends on the interference of a different amplitude and is proportional to the phases, i.e.,

$$A_{CP} \propto \sin(\delta)\sin(\phi). \tag{1}$$

The sizable value of A_{CP} can be obtained if both phases are nonzero and large. The weak phase of the SM is a unique phase of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix. This weak phase is a free parameter of the SM and cannot be fixed by the SM itself, but it has been fixed by experimental methods [15]. Unlike the weak phase, the CP-conserving strong phase is process-dependent (not unique). The theoretical calculation of the strong phase is, in general, hard due to the hadronic uncertainty. The CP-conserving unparticle phase existing in the propagators beside the strong phase can affect the value of the A_{CP} in some decay processes [see Eq. (1)]. To explore this possibility, Chen *et al.* concentrated on some pure hadronic and pure leptonic B decays [3,8].

We aim to study the possible effects of the CP-conserving phase in semileptonic B decays. Rare semileptonic decays $b \to s(d)\ell^+\ell^-$ are more informative for this aim, since these decays are relatively clean compared to pure hadronic decays. It is well known that the matrix element for the $b \to s\ell^+\ell^-$ transition involves only one independent CKM matrix element, namely, $|V_{tb}V_{ts}^*|$, so the CP violation in this channel is strongly suppressed in the SM considering the above-mentioned requirements of the CPA, which requires the weak phase. However, the possibility of CP violation as a result of the new weak phase coming out of the physics beyond the standard model in $b \to s$ transition has been studied in supersymmetry [16,17], the fourth-generation standard model, [18–21] and the minimal extension of the SM [22]. The situation for $b \to d\ell^+\ell^-$ is totally different from the $b \to s\ell^+\ell^-$

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transition. In this case, all CKM matrix elements $|V_{td}V^*_{tb}|$, $|V_{cd}V^*_{cb}|$, and $|V_{ud}V^*_{ub}|$ are in the same order, and for this reason the matrix element of $b \to d\ell^+\ell^-$ transition contains two different amplitudes with two different CKM elements; therefore, sizable CPA is expected [23,24]. Here we study the effects of the CP-conserving unparticle phase on CP asymmetry in the $b \to d\ell^+\ell^-$ transition with unpolarized and polarized lepton cases.

This study encompasses four sections: In Sec. II, we present the effective Lagrangian and effective vertices which drive the FCNC decays with vector unparticle mediation. In Sec. III, we calculate the polarized and unpolarized CP asymmetries. Section IV is devoted to the discussion and our conclusions.

II. FLAVOR-CHANGING NEUTRAL CURRENTS MEDIATED BY VECTOR UNPARTICLE

The starting point of the idea is the interaction between two sectors, the SM and the ultraviolet sector with a nontrivial infrared fixed point, at a high energy level. The ultraviolet sector appears as new degrees of freedom, called unparticles, being massless and having nonintegral scaling dimension $d_{\mathcal{U}}$ around $\Lambda_{\mathcal{U}} \sim 1$ TeV. This mechanism results in the existence of the effective field theory with the effective Lagrangian in the low energy level. One may for simplicity assume that unparticles couple only to the flavor-conserving fermion currents [8], described by [1,2,10,11]

$$\frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}}\bar{f}\gamma_\mu(C_L^{\rm f}P_L + C_R^{\rm f}P_R)fO^\mu_{\mathcal{U}}, \tag{2}$$

where $O^\mu_{\mathcal{U}}$ is the unparticle operator. Similar to the SM, FCNCs such as $f \to f'\mathcal{U}$ can be induced by the charged weak currents at the quantum loop level, and, clearly, neutral current $f \to f\mathcal{U}$ is flavor diagonal.

The leading order of the effective Hamiltonian for Fig. 1 can be written as follows:

$$\mathcal{L}_{\mathcal{U}} = \frac{g^2}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}}V_{tb}V^*_{tq}C_L^{qb}\bar{q}\gamma_\mu P_L b O^\mu_{\mathcal{U}}, \tag{3}$$

where

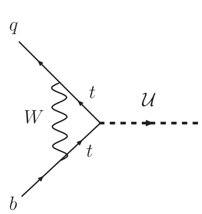

FIG. 1. Feynman diagram for $b \to q(s$ or $d)\mathcal{U}$, where t is the top quark.

$$C_L^{qb} = \frac{1}{(4\pi)^2}I(x_t),$$
$$I(x_t) = \frac{x_t(2C_R^t + C_L^t x_t)}{2(1-x_t)^2}(-1 + x_t - \ln x_t), \tag{4}$$

with $x_t = m_t^2/m_W^2$ [8].

To obtain the effective Hamiltonian for the $b \to qf\bar{f}$ transition, where unparticles enter as mediators, we must obtain the unparticle propagator, which is given by [1,2,10,11]

$$\int d^4x e^{ip\cdot x}\langle 0|T(O^\mu_{\mathcal{U}}(x)O^\nu_{\mathcal{U}}(0))|0\rangle = i\Delta_{\mathcal{U}}(p^2)e^{-i\phi_{\mathcal{U}}}, \tag{5}$$

where

$$\Delta_{\mathcal{U}}(p^2) = \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{-g^{\mu\nu} + ap^\mu p^\nu/p^2}{(p^2+i)^{2-d_{\mathcal{U}}}},$$
$$\phi_{\mathcal{U}} = (d_{\mathcal{U}} - 2)\pi, \tag{6}$$

where $a = 1$ for transverse $O^\mu_{\mathcal{U}}$ and $a = \frac{2(d-2)}{d-1}$ in the conformal field theories (CFTs) [12]. Note also that the contribution from the longitudinal piece $ap^\mu p^\nu/p^2$ in Eq. (6) can be dropped for massless or light external fermions. In this case, the Georgi [2] and Grinstein, Intriligator, and Rothstein [12] approaches provide the same result. Also,

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}}\frac{\Gamma(d_{\mathcal{U}}+1/2)}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}})}. \tag{7}$$

Note that in Eq. (5) the phase factor arises from $(-1)^{d_{\mathcal{U}}-2} = e^{-i\pi(d_{\mathcal{U}}-2)}$, and here the massless vector unparticle operator is conserved current, i.e., $\partial_\mu O^\mu_{\mathcal{U}} = 0$. The effective Hamiltonian for $b \to qf\bar{f}$ just with the contribution of the vector unparticle as a mediator can be given as

$$\mathcal{H}_{\mathcal{U}} = -\frac{G_F}{\sqrt{2}}V_{tb}V^*_{tq}\tilde{\Delta}_{\mathcal{U}}(p^2)e^{-i\phi_{\mathcal{U}}}\bar{q}\gamma_\mu P_L b\bar{f}$$
$$\times \gamma^\mu(C_L^{\rm f}P_L + C_R^{\rm f}P_R)f, \tag{8}$$

where

$$\tilde{\Delta}_{\mathcal{U}}(p^2) = 8C_L^{qb}\frac{A_{d_{\mathcal{U}}}}{2\sin d_{\mathcal{U}}\pi}\frac{m_W^2}{p^2}\left(\frac{p^2}{\Lambda^2_{\mathcal{U}}}\right)^{d_{\mathcal{U}}-1}. \tag{9}$$

Here f stands for fermions; i.e., f can be neutrinos or charged leptons or quarks.

III. $b \to d\ell^+\ell^-$ TRANSITION IN THE PRESENCE OF THE VECTOR UNPARTICLE AS A MEDIATOR

By the extension of the $b \to d\mathcal{U}$ to study the semileptonic decays of $b \to d\ell^+\ell^-$, the decay amplitude in the presence of the vector unparticle as a mediator can be obtained. Here again we assume that unparticles coupled to the leptons are flavor-conserving. The penguin diagram describing this decay is shown in Fig. 2. Because of the

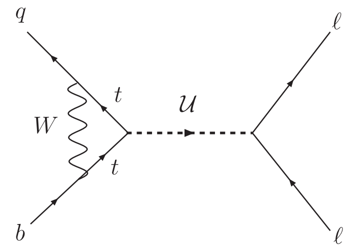

FIG. 2. $b \to q\ell^+\ell^-$ decays induced by the unparticle penguin diagram.

CKM suppression, the semileptonic decays with $b \to d$ are much less than those of $b \to s$. However, it is worth studying the $b \to d$ transition beyond the $b \to s$ one because the CKM matrix element V_{td} carries a CP-violating weak phase, which almost vanishes in the $b \to s$ transition. Thus, $b \to d\ell^+\ell^-$ decay could be even more interesting on CP violation in the framework of unparticle physics. We will focus on the CP-violating asymmetry in $b \to d\ell^+\ell^-$.

The QCD corrected effective Lagrangian for the decays $b \to d\ell^+\ell^-$ can be obtained by integrating out the heavy quarks, and the heavy electroweak bosons are as follows in the SM:

$$M = \frac{G_F \alpha_{\rm em} \lambda_t}{\sqrt{2}\pi}\Big[C_9^{\rm eff}(\bar{d}\gamma_\mu P_L b)\bar{\ell}\gamma_\mu \ell + C_{10}(\bar{d}\gamma_\mu P_L b)\bar{\ell}\gamma_\mu\gamma^5\ell - 2C_7 \bar{d} i\sigma_{\mu\nu}\frac{q^\nu}{q^2} \times (m_b P_R + m_s P_L) b \bar{\ell}\gamma_\mu \ell \Big], \tag{10}$$

In writing this, unitarity of the CKM matrix has been used, and the term proportional to $\lambda_t = V_{tb}^* V_{td}$ has been factored out, where q denotes the four momentum of the lepton pair and C_i's are Wilson coefficients. Neglecting the terms of $O(m_q^2/m_W^2)$, $q = u, d, c$, the analytic expressions for all Wilson coefficients, except $C_9^{\rm eff}$, can be found in [25]. The values of $C_7^{\rm eff}$ and C_{10} in leading logarithmic approximation are

$$C_7^{\rm eff} = -0.315, \qquad C_{10} = -4.642; \tag{11}$$

only $C_9^{\rm eff}$ has weak and strong phases, i.e.,

$$C_9^{\rm eff} = \xi_1 + \lambda_u \xi_2, \tag{12}$$

where the CP-violating parameter λ_u is as follows:

$$\lambda_u = \frac{V_{ub}^* V_{ud}}{V_{tb}^* V_{td}} = \frac{\rho(1-\rho) - \eta^2}{(1-\rho)^2 + \eta^2} - i\frac{\eta}{(1-\rho)^2+\eta^2} + \cdots. \tag{13}$$

The explicit expressions of functions ξ_1 and ξ_2 in $\mu = m_b$ can be found in [25–30]: Note that we neglect long-distance resonant contributions in $C_9^{\rm eff}$ for simplicity; a more complementary and supplementary analysis of the above decay has to take the long-distance contributions, which have their origin in real intermediate $c\bar{c}$ bound states, in addition to the short-distance contribution into account.

The Wilson coefficients of the SM are modified by the introducing the vector-type unparticles. It is easy to see that unparticles in this study are introduced in the way that new operators do not appear. In other words, the full operator set for the unparticle contributions is exactly the same as in the SM. The unparticle effects with the SM contributions can be derived by using $C_9^{\mathcal{U}}$ and $C_{10}^{\mathcal{U}}$, defined by

$$\begin{aligned} C_9^{\mathcal{U}}(q^2) &= C_9^{\rm eff} + \frac{\pi}{\alpha_{\rm em}}\frac{C_R^\ell + C_L^\ell}{2}\tilde{\Delta}_{\mathcal{U}}(q^2)e^{-i\phi_{\mathcal{U}}}, \\ C_{10}^{\mathcal{U}}(q^2) &= C_{10} + \frac{\pi}{\alpha_{\rm em}}\frac{C_R^\ell - C_L^\ell}{2}\tilde{\Delta}_{\mathcal{U}}(q^2)e^{-i\phi_{\mathcal{U}}}, \\ C_7^{\mathcal{U}}(q^2) &= C_7(q^2), \end{aligned} \tag{14}$$

instead of $C_9^{\rm eff}$ and C_{10}, respectively. Where C_7 remain the same as the SM, we can rewrite $C_i^{\mathcal{U}}$'s in the m_b scale [25]. Then $C_9^{\mathcal{U}}$ will be as

$$C_9^{\mathcal{U}} = \xi_1^{\mathcal{U}} + \lambda_u \xi_2, \tag{15}$$

where

$$\xi_1^{\mathcal{U}} = \xi_1 + \frac{\pi}{\alpha_{\rm em}}\frac{C_R^\ell + C_L^\ell}{2}\tilde{\Delta}_{\mathcal{U}}(q^2)e^{-i\phi_{\mathcal{U}}}. \tag{16}$$

Neglecting any low energy QCD corrections ($\sim 1/m_b^2$) [31,32] and setting the down quark mass to zero, the unpolarized differential decay width as a function of the invariant mass of the lepton pair is given by

$$\left(\frac{d\Gamma}{d\hat{s}}\right)_0 = \frac{G_F^2 m_b^5}{192\pi^3}\frac{\alpha_{\rm em}^2}{4\pi^2}|\lambda_t|^2(1-\hat{s})^2\sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}}\,\triangle\,\mathcal{U}, \tag{17}$$

with

$$\begin{aligned} \triangle^{\mathcal{U}}(\hat{s}) = {} & 4\frac{(2+\hat{s})}{\hat{s}}\left(1 + \frac{2\hat{m}_\ell^2}{\hat{s}}\right)|C_7^{\rm eff}|^2 \\ & + (1+2\hat{s})\left(1+\frac{2\hat{m}_\ell^2}{\hat{s}}\right)|C_9^{\mathcal{U}}|^2 \\ & + \left(1 - 8\hat{m}_\ell^2 + 2\hat{s} + \frac{2\hat{m}_\ell^2}{\hat{s}}\right)|C_{10}^{\mathcal{U}}|^2 \\ & + 12\left(1 + \frac{2\hat{m}_\ell^2}{\hat{s}}\right){\rm Re}(C_9^{\mathcal{U}*}C_7^{\rm eff}). \end{aligned} \tag{18}$$

The explicit expression for the unpolarized particle decay rate $(d\Gamma/d\hat{s})_0$ has been given in (17). Obviously, it can be written as a product of a real-valued function $r(\hat{s})$ times the function $\Delta(\hat{s})$, given in (18): $(d\Gamma/d\hat{s})_0 = r(\hat{s})\Delta(\hat{s})$. In the unpolarized case, the CP-violating asymmetry rate can be defined by

$$A_{CP}^{\mathcal{U}}(\hat{s}) = \frac{(\frac{d\Gamma}{d\hat{s}})_0 - (\frac{d\bar{\Gamma}}{d\hat{s}})_0}{(\frac{d\Gamma}{d\hat{s}})_0 + (\frac{d\bar{\Gamma}}{d\hat{s}})_0} = \frac{\Delta^{\mathcal{U}} - \bar{\Delta}^{\mathcal{U}}}{\Delta^{\mathcal{U}} + \bar{\Delta}^{\mathcal{U}}}, \tag{19}$$

where

$$\frac{d\Gamma}{d\hat{s}} \equiv \frac{d\Gamma(b \to d\ell^+\ell^-)}{d\hat{s}}, \qquad \frac{d\bar{\Gamma}}{d\hat{s}} \equiv \frac{d\bar{\Gamma}(\bar{b} \to \bar{d}\ell^+\ell^-)}{d\hat{s}}, \tag{20}$$

where $(d\bar{\Gamma}/d\hat{s})_0$ can be obtained from $(d\Gamma/d\hat{s})_0$ by making the replacement

$$C_9^{\mathcal{U}} = \xi_1^{\mathcal{U}} + \lambda_u \xi_2 \to \bar{C}_9^{\mathcal{U}} = \xi_1^{\mathcal{U}} + \lambda_u^* \xi_2. \tag{21}$$

Note that the term proportional to λ_u, the CP-violating parameter remains the same as the SM. Moreover, the CP-violating parameter just enters into the $C_9^{\mathcal{U}}$ expression the same as the SM ones. Consequently, the rate for antiparticle decay can be obtained by the following replacement in Eq. (18):

$$\bar{\Delta}^{\mathcal{U}} = \Delta^{\mathcal{U}}_{|\lambda_u \to \lambda_u^*}. \tag{22}$$

Using (19), the CP-violating asymmetry is evaluated to be

$$A_{CP}^{\mathcal{U}}(\hat{s}) = \frac{-2\mathrm{Im}(\lambda_u)\Sigma^{\mathcal{U}}(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s}) + 2\mathrm{Im}(\lambda_u)\Sigma^{\mathcal{U}}(\hat{s})} \approx -2\mathrm{Im}(\lambda_u)\frac{\Sigma^{\mathcal{U}}(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s})}. \tag{23}$$

In (23),

$$\Sigma^{\mathcal{U}}(\hat{s}) = \mathrm{Im}[\xi_1^{\mathcal{U}*}\xi_2] f_+(\hat{s}) + \mathrm{Im}(C_7^{\mathrm{eff}*}\xi_2) f_1(\hat{s}),$$
$$f_+(\hat{s}) = (1+2\hat{s})\left(1 + \frac{2\hat{m}_\ell^2}{\hat{s}}\right), \qquad f_1(\hat{s}) = 12\left(1 + \frac{2\hat{m}_\ell^2}{\hat{s}}\right). \tag{24}$$

Before turning to a derivation of CP-violating asymmetries for the case of polarized final state leptons, it is necessary to recall the calculation of the lepton polarization. The spin direction of a lepton can be described by setting a reference frame with three orthogonal unit vectors S_L, S_N, and S_T, such that

$$S_L = \frac{p^-}{|p^-|}, \qquad S_N = \frac{p_d \times p^-}{|p_d \times p^-|}, \qquad S_T = S_N \times S_L, \tag{25}$$

where p_d and p^- are the three momentum vectors of the d quark and the ℓ^- lepton, respectively, in the $\ell^+\ell^-$ center-of-mass system. For a given lepton ℓ^- spin direction $\vec{n}$, which is a unit vector in the ℓ^- rest frame, the differential decay spectrum is of the form [33]

$$\frac{d\Gamma(\hat{s}, \vec{n})}{d\hat{s}} = \frac{1}{2}\left(\frac{d\Gamma(\hat{s})}{d\hat{s}}\right)_0 [1 + (P_L e_L + P_T e_T + P_N e_N)\cdot \vec{n}], \tag{26}$$

where the polarization components P_i $(i = L, N, T)$ are obtained from the relation

$$P_i(\hat{s}) = \frac{d\Gamma(\vec{n} = e_i)/d\hat{s} - d\Gamma(\vec{n} = -e_i)/d\hat{s}}{d\Gamma(\vec{n} = e_i)/d\hat{s} + d\Gamma(\vec{n} = -e_i)/d\hat{s}} = \frac{\Delta_i^{\mathcal{U}}(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s})}. \tag{27}$$

The three different polarization asymmetries are

$$P_L(\hat{s}) = \frac{\Delta_L^{\mathcal{U}}(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s})} = \frac{v}{\Delta^{\mathcal{U}}(\hat{s})}[12\mathrm{Re}(C_7^{\mathrm{eff}} C_{10}^{\mathcal{U}*}) + 2\mathrm{Re}(C_9^{\mathcal{U}} C_{10}^{\mathcal{U}*})(1 + 2\hat{s})],$$
$$P_T(\hat{s}) = \frac{\Delta_T^{\mathcal{U}}(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s})} = \frac{3\pi\hat{m}_\ell}{2\Delta^{\mathcal{U}}(\hat{s})\sqrt{\hat{s}}}\Big[2\mathrm{Re}(C_7^{\mathrm{eff}} C_{10}^{\mathcal{U}*}) - 4\mathrm{Re}(C_7^{\mathrm{eff}} C_9^{\mathcal{U}*}) - \frac{4}{\hat{s}}|C_7^{\mathrm{eff}}|^2 + \mathrm{Re}(C_9^{\mathcal{U}} C_{10}^{\mathcal{U}*}) - |C_9^{\mathcal{U}}|^2\hat{s}\Big],$$
$$P_N(\hat{s}) = \frac{\Delta_N^{\mathcal{U}}(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s})} = \frac{3\pi\hat{m}_\ell v}{2\Delta^{\mathcal{U}}(\hat{s})}\sqrt{\hat{s}}\mathrm{Im}(C_9^{\mathcal{U}*} C_{10}^{\mathcal{U}}). \tag{28}$$

The study of the above-mentioned asymmetries is interesting in probing new physics. It is obvious that any alteration in the Wilson coefficients leads to changes in the polarization asymmetries.

Now we define the polarized CP asymmetry, which is

$$A_{CP}(\hat{s}) = \frac{\frac{d\Gamma(\hat{s},\vec{n})}{d\hat{s}} - \frac{d\bar{\Gamma}(\hat{s},\vec{\bar{n}})}{d\hat{s}}}{(\frac{d\Gamma(\hat{s})}{d\hat{s}})_0 + (\frac{d\bar{\Gamma}(\hat{s})}{d\hat{s}})_0}, \tag{29}$$

where

$$\frac{d\Gamma(\hat{s}, \vec{n})}{d\hat{s}} = \frac{d\Gamma(b \to d\ell^+\ell^-(\vec{n}))}{d\hat{s}},$$
$$\frac{d\bar{\Gamma}(\hat{s}, \vec{\bar{n}})}{d\hat{s}} = \frac{d\Gamma(\bar{b} \to \bar{d}\ell^+(\vec{\bar{n}})\ell^-)}{d\hat{s}}, \tag{30}$$

where $\vec{n}$ and $\vec{\bar{n}}$ are the spin directions for ℓ^- and ℓ^+ for b-decay and $\bar{b}$-decay, respectively, and $i = L, N, T$. Taking into account the fact that $\vec{\bar{e}}_{L,N} = -\vec{e}_{L,N}$ and $\vec{\bar{e}}_T = \vec{e}_T$, we obtain

$$A_{CP}(\vec{n} = \pm\vec{e}_i) = \frac{1}{2}\left\{\frac{(\frac{d\Gamma}{d\hat{s}})_0 - (\frac{d\bar{\Gamma}}{d\hat{s}})_0}{(\frac{d\Gamma}{d\hat{s}})_0 + (\frac{d\bar{\Gamma}}{d\hat{s}})_0} \pm \frac{(\frac{d\Gamma}{d\hat{s}})_0 P_i - ((\frac{d\Gamma}{d\hat{s}})_0 P_i)_{|\lambda_u \to \lambda_u^*}}{(\frac{d\Gamma}{d\hat{s}})_0 + (\frac{d\bar{\Gamma}}{d\hat{s}})_0}\right\}. \tag{31}$$

Using Eq. (28), we get from Eq. (31),

$$A_{CP}(\vec{n} = \pm\vec{e}_i) \approx \frac{1}{2}\left\{\frac{\Delta^{\mathcal{U}} - \bar{\Delta}^{\mathcal{U}}}{\Delta^{\mathcal{U}} + \bar{\Delta}^{\mathcal{U}}} \pm \frac{\Delta_i^{\mathcal{U}} - \bar{\Delta}_i^{\mathcal{U}}}{\Delta^{\mathcal{U}} + \bar{\Delta}^{\mathcal{U}}}\right\} = \frac{1}{2}\{A_{CP}(\hat{s}) \pm A_{CP}^i(\hat{s})\}, \tag{32}$$

where the upper sign in the definition of δA_{CP} corresponds to L and N polarizations, while the lower sign corresponds to T polarization.

The $A^i_{CP}(\hat{s})$ terms in Eq. (32) describe the modification to the unpolarized decay width, which can be written as

$$A^i_{CP}(\hat{s}) = \frac{-4\text{Im}(\lambda_u)\Sigma^i(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s}) + \bar{\Delta}^{\mathcal{U}}(\hat{s})} \approx -2\text{Im}(\lambda_u)\frac{\Sigma^i(\hat{s})}{\Delta^{\mathcal{U}}(\hat{s})}, \quad (33)$$

where the explicit expressions for $\Sigma^i(\hat{s})$ $(i = L, N, T)$ are as follows:

$$\begin{aligned}\Sigma^L(\hat{s}) &= v\text{Im}(C_{10}^{\mathcal{U}*}\xi_2)(1+2\hat{s}),\\ \Sigma^T(\hat{s}) &= \frac{3\pi\hat{m}_\ell}{2\sqrt{\hat{s}}}\bigg[2\text{Im}(C_7^{\text{eff}}\xi_2^*) + \frac{1}{2}\text{Im}(C_{10}^{\mathcal{U}*}\xi_2)\\ &\quad - \hat{s}\text{Im}(\xi_1^{\mathcal{U}*}\xi_2)\bigg],\\ \Sigma^N(\hat{s}) &= \frac{3\pi\hat{m}_\ell}{2\sqrt{\hat{s}}}v\bigg[\frac{\hat{s}}{2}\text{Re}(C_{10}^{\mathcal{U}*}\xi_2)\bigg].\end{aligned} \quad (34)$$

It is interesting to note that the polarized *CP* asymmetries have different combinations involving the imaginary and real parts of the $C_{10}^{\mathcal{U}}$ which do not appear in unpolarized *CP* asymmetry. The study of the polarized *CP* asymmetry beside the unpolarized *CP* asymmetry with unparticle contributions will give us more information about the unparticle parameters. In particular, when $C_L^\ell = -C_R^\ell$ in (14), the unparticle contribution vanishes in the $C_9^{\mathcal{U}}$. In such a situation, the unparticle effects in *CP* asymmetry just appear in the polarized *CP* asymmetries.

IV. NUMERICAL ANALYSIS AND DISCUSSION

We try to analyze the dependency of the unpolarized and polarized direct *CP* asymmetries on the unparticle parameters. We will use the next-to-leading order logarithmic approximation for the SM values of the Wilson coefficients C_9^{eff}, C_7^{eff}, and C_{10}^{eff} [34,35] at the scale $\mu = m_b$. It is worth mentioning that, beside the short-distance contribution, C_9^{eff} has also long-distance contributions resulting from the real $\bar{c}c$ resonant states of the J/ψ family. In the present study, we do not take the long-distance effects into account. Furthermore, one finds that significant contributions of unparticles occurs at a small region of $\hat{s}$ which is free of long-distance effects [obviously, the unparticle contributions for the μ channel is more significant than the τ channel since the small $\hat{s}$ region ($\hat{s} \sim 0.0$) for the τ channel is absent by kinematical consideration]. One can confirm the above statement by looking at Eqs. (2) and (5), where at the small $\hat{s} = q^2/m_b^2$ region the dependency of the propagator is as follows:

$$\bigg[\frac{1}{q^2}\bigg(\frac{q^2}{\Lambda_{\mathcal{U}}^2}\bigg)^{d_{\mathcal{U}}-1}\bigg]^2. \quad (35)$$

The SM parameters we used in this analysis can be seen in Table I.

TABLE I. The values of the input parameters used in the numerical calculations.

Parameter	Value
α_{em}	1/129 (GeV)
m_u	2.3 (MeV)
m_d	4.6 (MeV)
m_c	1.25 (GeV)
m_b	4.8 (GeV)
m_μ	0.106 (GeV)
m_τ	1.780 (GeV)

The allowed range for the Wolfenstein parameters is $0.19 \le \rho \le 0.268$ and $0.305 \le \eta \le 0.411$ [36] where, in the present analysis, they are set as $\rho = 0.25$ and $\eta = 0.34$.

The direct *CP* asymmetries depend on both $\hat{s}$ and the new parameters coming from unparticle stuff. We eliminate the variable $\hat{s}$ by performing an integration over $\hat{s}$ in the allowed kinematical region. The averaged direct *CP* asymmetries are defined as

$$\begin{aligned}\mathcal{B}_r &= \int_{4m_\ell^2/m_b^2}^{(1-\sqrt{\hat{r}_d})^2}\frac{d\mathcal{B}}{d\hat{s}}d\hat{s} \qquad \bigg(\hat{r}_d = \frac{m_d^2}{m_b^2}\bigg),\\ \langle A^i_{CP}\rangle &= \frac{\int_{4m_\ell^2/m_b^2}^{(1-\sqrt{\hat{r}_d})^2}A^i_{CP}\frac{d\mathcal{B}}{d\hat{s}}d\hat{s}}{\mathcal{B}_r}.\end{aligned} \quad (36)$$

At this stage, we discuss our restrictions for free parameters coming out of the unparticle.

(i) It is important to note that, while the discontinuity across the cut is not singular for integer $d_{\mathcal{U}} > 1$, the propagator [Eq. (6)] is singular because of the $\sin(d_{\mathcal{U}}\pi)$ in the denominator. Some researchers believe that this is a real effect [2]. These integer values describe multiparticle cuts, and the mathematics tells us that we should not try to describe them with a single unparticle field.
Moreover, the lower bounds for the scaling dimensions of the gauge-invariant vector operators of a CFT are $d_{\mathcal{U}} \ge 2$ and $d_{\mathcal{U}} \ge 3$ [12] for nonprimary and primary vector operators, respectively. We obtain that for $d_{\mathcal{U}} > 2$ the unparticle effects on physical observables (branching ratio, *CP* asymmetry, and so on) almost vanish because $\tilde{\Delta}(p^2)$ is negligible for $p < \Lambda_{\mathcal{U}}$ [see Eq. (9)].
We focus on $1 < d_{\mathcal{U}} < 2$, the bound that is allowed for transverse $O^\mu_{\mathcal{U}}$ or for non-gauge-invariant vector operators of the CFT. Also, it is consistent with the $b \to s\ell^+\ell^-$ rate [8] and B_s mixing [9]. We also assume that the virtual effects of unparticles are gentlest away from the integer boundaries. On the other hand, the momentum integrals converge for $d_{\mathcal{U}} < 2$ [13].

(ii) C_L^t is always associated with C_R^ℓ and C_R^ℓ [see Eq.]. For simplicity, we set $C_R^\ell = C_L^\ell$ or $C_R^\ell = -C_L^\ell$. We will set new parameters to be $C_L^t C_L^\ell = C_L^t C_R^\ell = \lambda_V^\ell$ and $C_L^t C_L^\ell = -C_L^t C_R^\ell = \lambda_A^\ell$ and choose the $\lambda_{V[A]}^\ell =$ 0.005, 0.01, and 0.05 which is consistent with the $b \to s\ell^+\ell^-$ rate [8].

(iii) We take the energy scale $\Lambda_{\mathcal{U}} = 1$(TeV) and study $d_{\mathcal{U}}$ dependence of the polarized and unpolarized CP asymmetry.

CP asymmetry is a good candidate (unlike the other physical observables, i.e., branching ratio, forward-backward asymmetry, etc.) to probe the unique unparticle phase. The other physical observables can be utilized to give strong constraints on the unparticle parameters except the phase, i.e., on the unparticle couplings to leptons such as $\lambda_{V(A)}^\ell \sim \{0.005–0.05\}$ [8]. Moreover, our numerical analysis confirm the result of [8], where the branching ratio (BR) of the $b \to s(d)\ell^+\ell^-$ decay depicts the strong enhancement at the low value of the scale dimension $d_{\mathcal{U}} \sim$ 1.1 with respect to the SM value. As a natural consequence of this feature, the averaged value of asymmetries will vanish unless they depict stronger enhancement than the BR.

The contributions of the unparticle to the CPA of $b \to d\ell^+\ell^-$ in terms of the values for the common parameters are presented in Figs. 3–10. The horizontal thin lines are the SM contributions; the dashed lines and dashed-dotted lines correspond to the different $\lambda_{A[V]}^\ell = 0.005$, 0.01, and 0.05, respectively. From these figures, we conclude that:

(i) $\langle A_{CP}\rangle$ for both μ and τ leptons depicts strong dependency on the unparticle effects (for the μ case, the dependency is stronger than the τ case as we discussed above). While it is suppressed to the zero value by the unparticle contributions at lower values of the scale dimension $d_{\mathcal{U}} \sim 1.1$, its value is close to the SM value at the higher values of the scale dimension $d_{\mathcal{U}} \sim 1.9$. Moreover, the sensitivity for different values of the λ_A is stronger and more interesting than the λ_V values. While for different λ_V values, $\langle A_{CP}\rangle$ is just decreasing in terms of the $d_{\mathcal{U}}$, for λ_A it is increasing, decreasing, and changing the sign (see Figs. 3 and 4).

(ii) $\langle A_{CP}^L\rangle$ for both μ and τ leptons shows strong dependency on the unparticle parameters. While it is suppressed to the zero value by the unparticle contributions at lower values of the scale dimension $d_{\mathcal{U}} \sim 1.1$ (see Figs. 5 and 6), its value is close to the SM value at the higher values of the scale dimension $d_{\mathcal{U}} \sim 1.9$. The situation for the μ leptons is much more interesting. While the SM value is about a few percent, it receives a sizable and measurable contribution up to 10% from unparticle effects (see Fig. 5). As $\langle A_{CP}^L\rangle$ and $\langle A_{CP}\rangle$ are sensitive to the $C_{10}^{\mathcal{U}}$ and $C_9^{\mathcal{U}}$, respectively, thus, the study of $\langle A_{CP}^L\rangle$ beside $\langle A_{CP}\rangle$ is supplementary and complementary to studying unparticle effects. More precisely, unlike $\langle A_{CP}\rangle$, $\langle A_{CP}^L\rangle$ shows stronger dependency on the different values of the λ_A than the λ_V values.

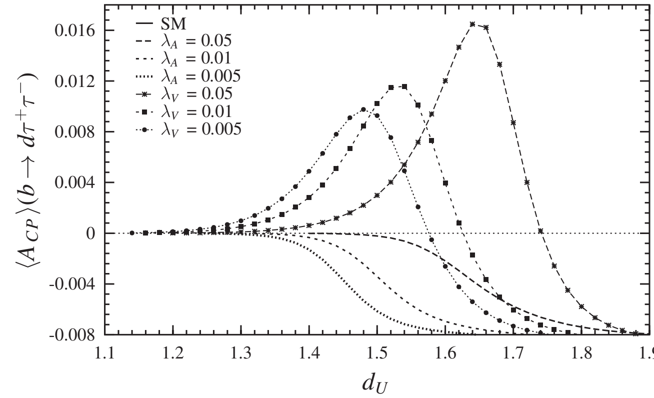

FIG. 4. The same as in Fig. 1 but for the τ lepton.

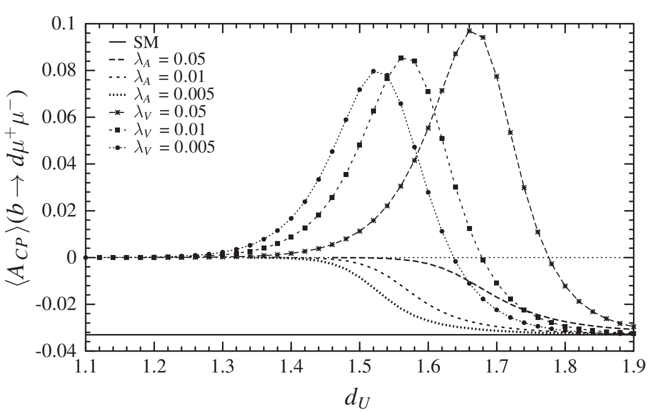

FIG. 3. The dependence of the $\langle \mathcal{A}_{CP}\rangle$ for the $b \to d\mu^+\mu^-$ decay on $d_{\mathcal{U}}$ for three different values of λ_V: 0.005, 0.01, and 0.05 and λ_A: 0.005, 0.01, and 0.05 in the fixed value of $\Lambda_{\mathcal{U}} =$ 1 TeV.

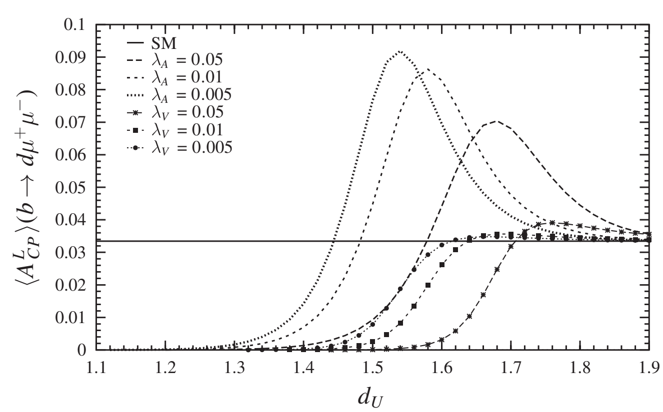

FIG. 5. The dependence of the $\langle \mathcal{A}_{CP}^L\rangle$ for the $b \to d\mu^+\mu^-$ decay on $d_{\mathcal{U}}$ for three different values of λ_V: 0.005, 0.01, and 0.05 and λ_A: 0.005, 0.01, and 0.05 in the fixed value of $\Lambda_{\mathcal{U}} =$ 1 TeV.

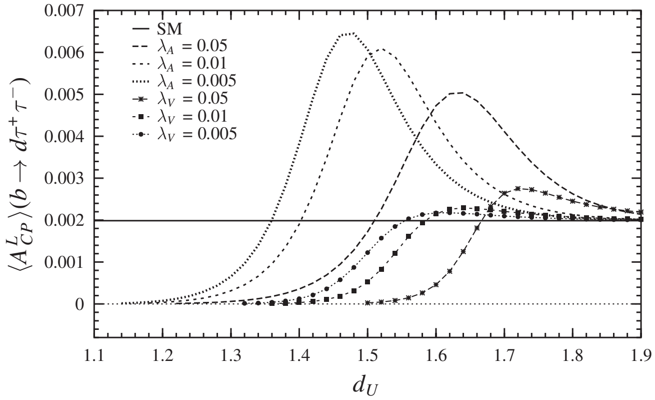

FIG. 6. The same as in Fig. 3 but for the τ lepton.

(iii) $\langle A^T_{CP}\rangle$ is generally sensitive to the unparticle contributions for both μ and τ channels. While the SM values of $\langle A^T_{CP}\rangle$ almost vanishes, the unparticle contributions lead to a sizable deviation from the SM values (see Figs. 7 and 8). This sizable discrepancy

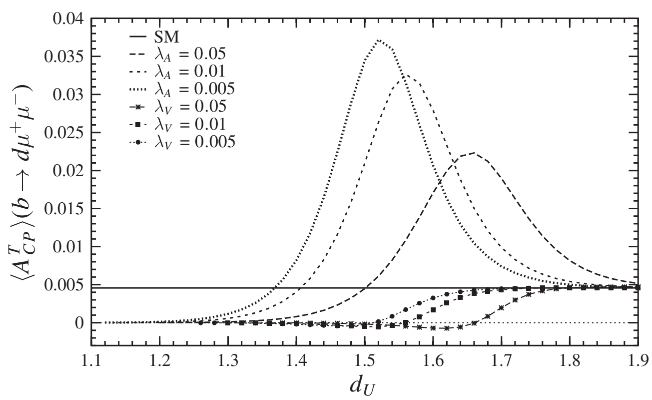

FIG. 7. The dependence of the $\langle \mathcal{A}^T_{CP}\rangle$ for the $b \to d\mu^+\mu^-$ decay on $d_{\mathcal{U}}$ for three different values of λ_V: 0.005, 0.01, and 0.05 and λ_A: 0.005, 0.01, and 0.05 in the fixed value of $\Lambda_{\mathcal{U}} =$ 1 TeV.

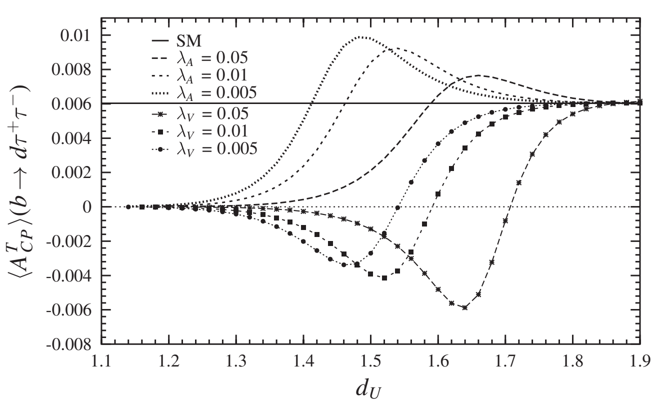

FIG. 8. The same as in Fig. 5 but for the τ lepton.

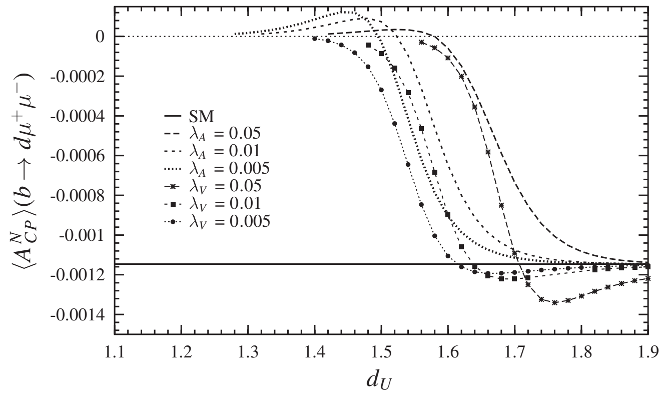

FIG. 9. The dependence of the $\langle \mathcal{A}^N_{CP}\rangle$ for the $b \to d\mu^+\mu^-$ decay on $d_{\mathcal{U}}$ for three different values of λ_V: 0.005, 0.01, and 0.05 and λ_A: 0.005, 0.01, and 0.05 in the fixed value of $\Lambda_{\mathcal{U}} =$ 1 TeV.

from the SM values can be measured in future experiments such as LHC and ILC.

(iv) Either the SM value or its value with unparticle contributions for $\langle A^N_{CP}\rangle$ is negligible (see Figs. 9 and 10).

At the end, the quantitative estimation about the accessibility to measure the various physical observables is in order. An observation of a 3σ signal for *CP* asymmetry of the order of 1% requires about $\sim 10^{10}$ $B\bar{B}$ pairs [33]. For the $b \to d\ell^+\ell^-$ measurement, a good *d*-quark tagging is necessary to distinguish it from the much more stronger $b \to s\ell^+\ell^-$ decay signal. Putting aside this challenging task, the number of $B\bar{B}$ pairs, expected to be produced at LHC, is about $\sim 10^{12}$. As a result of a comparison of these values, we conclude that a typical asymmetry of ($\mathcal{A} =$ 1%) is certainly detectable at LHC.

In conclusion, first, we obtain that the unparticle effects on physical observables, i.e., branching ratio and *CP* asymmetry for $b \to d(s)\ell^+\ell^-$, decay when $d_{\mathcal{U}} \geq 2$ vanish. Second, for $1 < d_{\mathcal{U}} < 2$, the *CP* asymmetry for polarized and unpolarized lepton cases is studied within the unparticle contributions in the CPA of the $b \to d\ell^+\ell^-$ decays. We obtain that the unpolarized and polarized *CP*

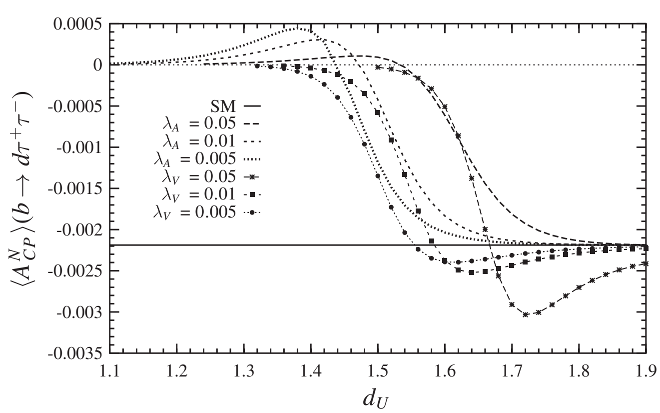

FIG. 10. The same as in Fig. 5 but for the τ lepton.

asymmetries are strongly sensitive to the unparticle effects. In particular, the CPA for small values of scale dimension $d_{\mathcal{U}} \sim 1.1$ suppresses to zero, and for its definite values the CPA enhances considerably and changes its sign with respect to the corresponding SM value. The other parameters of the scenario studied are the $\mathcal{U}$-fermion-fermion couplings, the energy scale, and the dependencies of the CPA to these free parameters and are also strong. We show that a measurement of the magnitude and sign of the unpolarized and polarized asymmetries can be instructive in order to test the possible signals coming from the unparticle physics.

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