Note on the strong CP problem from a 5-dimensional perspective

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We consider 5-dimensional gauge theories where the 5th direction is compactified on an interval. The Chern-Simons (CS) terms (favored by the naive dimensional analysis) are discussed. A simple scenario with an extra $U(1)_X$ gauge field that couples to $SU(3)_{\rm color}$ through a CS term in the bulk is constructed. The extra component of the Abelian gauge field plays a role of the axion (gauge-axion unification), which in the standard manner solves the strong CP problem easily avoiding most of the experimental constraints. The possibility of discovering the gauge-unification at the LHC is discussed.

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I. INTRODUCTION

In the standard model (SM), the Higgs mechanism is responsible for generating fermion and vector-boson masses. Although the model is renormalizable and unitary, it has severe naturality problems associated with the so-called "hierarchy problem." At loop level this problem reduces to the fact that the quadratic corrections tend to increase the Higgs boson mass up to the UV cutoff of the theory. Extra-dimensional extensions of the SM offer a novel approach to gauge symmetry breaking in which the hierarchy problem could be either solved or at least reformulated in terms of the geometry of the higher-dimensional space.

Other inherent problems of the SM could also be addressed in extra-dimensional scenarios. For instance, within the SM the amount of CP violation is not sufficient to explain the observed baryon asymmetry [1], the gauge-Higgs unification scenario offers a possible solution since in such models the geometry can be a new source of explicit and spontaneous CP violation [2]. In this note we shall prove that the strong CP problem could be solved by introducing appropriate Chern-Simons (CS) terms in five dimensional (5D).[1] The scenario leads to an attractive possibility of gauge-axion unification.

II. HIERARCHY OF EFFECTIVE OPERATORS

We will first consider models in $D = 5$ dimensions with fermions, gauge bosons, and scalars propagating throughout the D-dimensional bulk, and some unspecified matter localized on lower dimensional manifolds (branes). Though these models are nonrenormalizable it is possible to define a hierarchy of possible terms in the Lagrangian that allows for a proper perturbative expansion; the procedure is a simple application of the arguments used in the naive dimensional analysis (NDA) [4] (see the Appendix). This hierarchy is specified by assigning to each gauge invariant operator an index $s = d_c + b' + (3f/2) - 4$ (d_c is the number of covariant derivatives, and f and b' are the number of fermion and scalar fields). As it is shown in the Appendix the least suppressed operators are those that have the index $s = 0$:

$$F^2; \qquad \bar{\psi} D \psi; \qquad |D\phi|^2; \qquad \bar{\psi}\phi\psi; \qquad \phi^4, \tag{1}$$

where F denotes the generic gauge tensor, ϕ a generic scalar, and ψ generic fermions.

The $s = 1$ operators not containing scalar fields are (A denotes a generic gauge field)

$$AF^2; \qquad \bar{\psi} F \psi, \tag{2}$$

whose coefficients are naturally suppressed by $1/(24\pi^3)$, together with all brane terms, presumably including the SM Lagrangian multiplied by $l_4^{-1}\delta(y - y_o)$. The first operator in (2) corresponds to the 5-dimensional CS term, while the second includes all magnetic-type couplings. Operators of index $s = 1$ containing ϕ are of the form $D^4\phi$, $D^2\phi^3$, or $D\bar{\psi}\psi\phi$.

The NDA argument favors the presence of a CS term (if only 5D vector bosons are present the CS term is the only bulk operator with index $s = 1$) with a coefficient as large as $1/(24\pi^3)$. Of course, it is still possible that there exist additional symmetries that forbid this term; however, if present, the CS term can generate interesting effects.

Hereafter we shall consider a 5D model containing $U(1)_X$ and $SU(3)_{\rm color}$ bulk gauge fields, denoted by X and G, respectively. Application of the NDA for this case (where there are no bulk fermions) yields the following action up to index $s = 1$

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[1] For other attempts to solve the strong CP problem by 5 dimensions see [3].

$$S = \int_{X^5} d^5x\Big\{-\frac{1}{4}X_{MN}X^{MN} - \frac{1}{2}\,\mathrm{Tr}[G_{MN}G^{MN}] + -\frac{1}{24\pi^3}\epsilon^{LMNPQ}\Big[c_1 g_5' g_5^2 X_L \mathrm{Tr}(G_{MN}G_{PQ}) + c_2 g_5'^3 X_L X_{MN} X_{PQ} + c_3 g_5^3 \mathrm{Tr}\Big(G_L G_{MN} G_{PQ} + \frac{i}{2} G_L G_M G_N G_{PQ} - \frac{1}{10} G_L G_M G_N G_P G_Q\Big)\Big]\Big\} + \frac{1}{16\pi^2} S_{\mathrm{brane}}, \tag{3}$$

where X_{MN} and G_{MN} are, respectively, the field strength tensors for the Abelian and non-Abelian groups[2] with the 5D gauge couplings, respectively, denoted by g_5' and g_5; $c_{1,2,3}$ are undetermined numerical constants, presumably of $O(1)$. In our specific applications we will consider models constructed on the space-time $X^5 = M^4 \times [0, R]$, and we will concentrate on the "mixed" Chern-Simons term proportional to $g_5' g_5^2$. We will assume that all SM fields are neutral under $U(1)_X$. Hereafter, whenever possible, in order to make the analysis as model independent as possible, we will avoid referring to any details of the embedding of the SM into 5D. The only assumption we make is that the SM is localized on one or perhaps both ends of the interval $[0, R]$.

III. SOLVING THE STRONG *CP* PROBLEM FROM A 5D PERSPECTIVE

As shown above, the NDA favors the CS term as an operator of index $s = 1$. We will argue that the presence of this term allows for a simple solution to the strong *CP* problem.

As it is well known, in a basis where the Yukawa matrices are diagonal, the phases of the Kobayashi-Maskawa matrix are responsible for all electroweak *CP* violation effects. There is, however, an additional ("strong") *CP*-violating term allowed by the symmetries of the 4D SM Lagrangian:

$$\mathcal{L}_{\mathrm{QCD}\,CP} = \theta \frac{\alpha_s}{16\pi} \mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}), \tag{4}$$

where $G_{\mu\nu}$ is the QCD field strength tensor, $\tilde{G}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}/2$, and $\alpha_s \equiv g^2/(4\pi)$ for g the SM 4D QCD gauge coupling constant. In the process of diagonalizing the Yukawa matrices, quark fields undergo a chiral rotation, which generates the same structure as in (4) (within the path-integral formulation this results from a nontrivial Jacobian for the fermionic measure [5]); therefore, the total effect of the strong *CP* violation is parameterized by the effective coefficient $\theta_{\mathrm{eff}} \equiv \theta + \theta_{\mathrm{weak}}$. The experimental data (electric dipole moment of the neutron) indicates that $|\theta_{\mathrm{eff}}| \lesssim 10^{-9}$ [6]; this is referred to as the strong *CP* "problem" since none of the symmetries of the SM requires such a strong suppression.

Models in extra dimensions offer new possibilities to solve this problem due to the possibility of constructing the Chern-Simons terms. Specifically, we will assume that the color gauge fields G_N^a propagate in the bulk, but that the rest of the SM fields are confined to one or two branes located at $y = 0$ and $y = R$. In addition, we assume the presence of an Abelian gauge field X_N also propagating in the bulk. For the 5D models being considered here, the QCD strong *CP* term (4) can be written as follows:

$$S_{\mathrm{brane}} = \frac{\alpha_s}{16\pi^2}\int d^5x[\theta_L\delta(y) + \theta_R\delta(y-R)]\mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}), \tag{5}$$

where $\theta_{R,L}$ are constant parameters.

Among the various terms in (3) we will concentrate on the effects of the mixed CS term:

$$S_{\mathrm{CS}} = -\frac{g_5' g_5^2 c_1}{24\pi^3}\int_{X^5} d^4x dy \epsilon^{LMNPQ} X_L \mathrm{Tr}(G_{MN}G_{PQ}). \tag{6}$$

The action (6) is not automatically gauge invariant under $U(1)_X$. However, using the Bianchi identity $\epsilon^{NMQPR}D_Q G_{PR} = 0$, one can show that under the Abelian transformation

$$X_L \to X_L' = X_L + \partial_L \lambda_X, \tag{7}$$

the change in S_{CS} is localized on the boundary of the space.[3]

$$\delta S_{\mathrm{CS}} = \frac{g_5' g_5^2 c_1}{24\pi^3}\int_{M^4} d^4x \lambda_X \epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}(G_{\mu\nu}G_{\alpha\beta})\big|_{y=0}^{y=R}; \tag{8}$$

there are various ways of insuring that this vanishes. One can, for example, add an appropriate set of chiral fermions on the two branes; in this case the anomaly generated by these fermions can be adjusted so that it cancels (8); see e.g. [7]. Brane scalars can be also arranged to have the same effect [3,7] provided they couple to $\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}(G_{\mu\nu}G_{\alpha\beta})$. A simpler alternative, which we will adopt here, is to impose appropriate boundary conditions such as $\lambda_X \mathrm{Tr}(G^2)|_{y=0} = \lambda_X \mathrm{Tr}(G^2)|_{y=L}$.

Variation of the total action (3) with $c_2 = c_3 = 0$ and $c_1 = 1$ leads to the following equations of motion for the gauge fields:

$$D_B G^{BA} = J^A + \text{brane terms} \quad \text{and} \quad \partial_B X^{BA} = j^A + \text{brane terms}, \tag{9}$$

[2]The convention for the antisymmetric tensors which we follow is such that $\epsilon_{01234} = \epsilon_{0123} = 1$ for the metric tensor $\eta_{MN} = \mathrm{diag}(1, -1, -1, -1, -1)$ and $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. We assume that the non-Abelian group generators, T^a are Hermitian and normalized according to $\mathrm{Tr}T^aT^b = 2^{-1}\delta_{ab}$.

[3]This assumes that λ_X is not a constant.

with the following Chern-Simons currents:

$$J^A = \frac{g_5' g_5^2}{24\pi^3}\epsilon^{ABCDE}X_{BC}G_{DE};$$
$$j^A = \frac{g_5' g_5^2}{24\pi^3}\epsilon^{ABCDE}\mathrm{Tr}(G_{BC}G_{DE}). \tag{10}$$

The brane terms in (9) originate from possible couplings of the bulk gauge fields to the fields localized on the branes.

For the extremum of the action the following boundary conditions (BC) must be fulfilled:

$$\mathrm{tr}\left[\left(G_{4\mu} - \frac{g_5' g_5^2}{6\pi^3}X^\nu \tilde{G}_{\mu\nu}\right)\delta G^\mu\right]\Big|_{y=0}^{y=R} = 0$$
$$\text{and}\quad X^{4\mu}\delta X_\mu|_{y=0}^{y=R} = 0. \tag{11}$$

Here we will restrict ourselves to theories containing massless zero-modes (gluons) of the non-Abelian gauge field. This implies a unique choice of BC for $SU(3)_{\mathrm{color}}$:

$$\partial_y G_\mu^a|_{y=0,R} = 0, \qquad G_4^a|_{y=0,R} = 0; \tag{12}$$

these conditions imply $G_{4\mu}^a|_{y=0,R} = 0$. For the Abelian field we require

$$X_\mu|_{y=0,R} = 0, \qquad \partial_y X_4|_{y=0,R} = 0, \tag{13}$$

so that $X_{\mu\nu}|_{y=0,R} = 0$. It follows that the BC (11) are satisfied.

The resulting Kaluza-Klein (KK) expansions read

$$G_\mu^a(x,y) = R^{-1/2}\sum_{n=0} d_n G_\mu^{a(n)}(x)\cos m_n y,$$
$$G_4^a(x,y) = R^{-1/2}\sqrt{2}\sum_{n=1} G_4^{a(n)}(x)\sin m_n y,$$
$$X_\mu(x,y) = R^{-1/2}\sqrt{2}\sum_{n=1} X_\mu^{(n)}(x)\sin m_n y,$$
$$X_4(x,y) = R^{-1/2}\sum_{n=0} d_n X_4^{(n)}(x)\cos m_n y, \tag{14}$$

where $m_n = \pi n/R$ and $d_n = 2^{(1-\delta_{n,0})/2}$. The zero-mode $G_\mu^{a(0)}(x)$ is the standard 4D gluon; it is also clear that the model also contains a massless 4D scalar $X_4^{(0)}(x)$.

Let us focus now on the Abelian gauge transformations. In order to preserve the BC, the gauge function $\lambda_X(x,y)$ must satisfy the following constraints:

$$\partial_\mu \lambda_X|_{y=0,R} = 0, \qquad \partial_y^2 \lambda_X|_{y=0,R} = 0. \tag{15}$$

That implies a corresponding KK expansion for the Abelian gauge function

$$\lambda_X(x,y) = \sum_{n=1}\lambda_X^{(n)}(x)\sin m_n y + \beta y, \tag{16}$$

where β is a constant. The 4D vector and scalar fields transform as

$$X_\mu^{(n)} \to X_\mu^{(n)} + \frac{1}{\sqrt{2}}\partial_\mu \lambda_X^{(n)}$$
$$X_4^{(n)} \to \begin{cases} X_4^{(0)} + \beta & \text{for } n = 0 \\ X_4^{(n)} + \frac{m_n}{\sqrt{2}}\lambda_X^{(n)} & \text{for } n > 0. \end{cases} \tag{17}$$

In the following we will take $\beta = 0$, which is the simplest condition ensuring the gauge symmetry of the CS action.[4]

In order to discuss phenomenological predictions of the model let us expand the CS action into KK modes:

$$S_{\mathrm{CS}} = \frac{R}{12\pi^3}\frac{g_5'}{R^{1/2}}\frac{g_5^2}{R}c_1\int d^4x\Big[X_4^{(0)}\mathrm{Tr}G_{\mu\nu}^{(0)}\tilde{G}^{\mu\nu(0)}$$
$$+ 2\partial_\mu X_4^{(0)}\sum_{n=1}^{\infty}\mathrm{Tr}G_\nu^{(n)}D_\rho G_\sigma^{(n)}\epsilon^{\mu\nu\rho\sigma}$$
$$- 4\,\mathrm{Tr}\tilde{G}^{\mu\nu(0)}\sum_{n=1}^{\infty}\Theta_{\mu\nu}^{(n)} + \cdots\Big], \tag{18}$$

where

$$D_\mu \equiv \partial_\mu + ig[G_\mu^{(0)},\cdots],$$
$$G_{\mu\nu}^{(0)} \equiv \partial_\mu G_\nu^{(0)} - \partial_\nu G_\mu^{(0)} + ig[G_\mu^{(0)}, G_\nu^{(0)}] \tag{19}$$

for $g = g_5/\sqrt{R}$ and

$$\Theta_{\mu\nu}^{(n)} \equiv \frac{1}{2}[(\partial_\mu X_4^{(n)}G_\nu^{(n)} - \partial_\nu X_4^{(n)}G_\mu^{(n)}) - (\partial_\mu X_\nu^{(n)}G_4^{(n)}$$
$$- \partial_\nu X_\mu^{(n)}G_4^{(n)}) - m_n(X_\mu^{(n)}G_\nu^{(n)} - X_\nu^{(n)}G_\mu^{(n)})]. \tag{20}$$

The ellipsis in (18) stands for terms (irrelevant for any practical applications) that involve four nonzero KK modes. Expanding the kinetic terms of (3), one can verify that indeed $G_{\mu\nu}^{(0)}$ corresponds to the SM QCD gluon [which is present due to our having adopted (12)], while $X_4^{(0)}(x) = a(x)$ can play the role of the axion. The lowest-order terms conform to the usual QCD action, the axion kinetic term, and the axion-gluon interactions[5]:

$$S_{\mathrm{low}}^{(0)} = \int_{M^4}\Big\{-\frac{1}{2}\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}) + \frac{1}{2}\partial_\mu a\partial^\mu a$$
$$+ \frac{\alpha_s}{16\pi}\left(\frac{a}{f_a} + \theta_{\mathrm{eff}}\right)\mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu})\Big\}, \tag{21}$$

where $\theta_{\mathrm{eff}} \equiv \theta_L + \theta_R$ and we dropped the (0) superscript in G. Adopting the NDA estimation of the CS coefficient, one obtains for the axion decay constant

$$f_a^{-1} = \frac{16g'}{3\pi}R, \tag{22}$$

[4]This is also a natural choice for S^1/Z_2 orbifold models since it insures that $X_\mu(x,-y) = -X_\mu(x,y)$, $X_4(x,-y) = X_4(x,y)$ and $X_N(x,y+2R) = X_N(x,y)$ are preserved under gauge transformations.

[5]It turns out that each term in the KK expansion of (5) is a total derivative (as they emerge from the full derivative $\mathrm{Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}]$). Only the zero-mode contribution will be relevant, as it contributes to the effective nonperturbative axion potential; other terms could be dropped.

where g' is the 4D Abelian gauge coupling, $g' = g'_5/\sqrt{R}$, and $\alpha_s = g^2/(4\pi)$. Note that for this mechanism of axion generation to work, the extra Abelian gauge symmetry must be broken by the boundary conditions (Scherk-Schwarz breaking) so that no additional massless vector-boson associated with X_μ is present. The only low-energy remnant of X_M is the axion $a(x)$. The crucial advantage of the model presented here is the unification of the axion and the $U(1)$ 5D gauge field. There are serious attempts to construct in 5D a realistic gauge-unification theory [8]. Those models combined with the scenario discussed here could provide an interesting alternative for a theory of electroweak interactions that offers the scalar sector of 4D theory fully unified with the gauge fields (solving the hierarchy problem [8] and the strong CP problem at the same time). As it will be discussed below, the gauge-axion unification is consistent with the existing experimental constraints and there is a chance to test the scenario at the LHC.

As in the standard Peccei-Quinn scenario, the effective axion coupling $(a/f_a + \theta_{\rm eff})$ relaxes to zero through instanton effects, solving the strong CP problem dynamically. The axion mass is generated in a standard manner [9]

$$m_a = \frac{f_\pi m_\pi}{f_a}\frac{\sqrt{m_u m_d}}{m_u + m_d} = 0.6\ \text{eV}\frac{10^7\ \text{GeV}}{f_a}, \qquad (23)$$

and no strictly massless scalars remain in the spectrum.

Let us discuss the consequences of the remaining interactions in the 5D CS term (6) that consists of quadratic and quartic terms in the nonzero KK modes. We will focus (for obvious phenomenological reasons) on the quadratic terms shown explicitly in (18). Of course, there are other terms involving the heavy fields generated by the kinetic part of the action (3), those have been considered previously in the literature, see e.g. [10].

Because of its relatively large coupling, the very last term ($\propto m_n$) in (18), will produce the most noticeable effects at the LHC. Therefore let us consider the production of heavy gluons $G_\mu^{(n)}$ and vector bosons $X_\mu^{(n)}$ (with $n \geq 1$) at the LHC. At the partonic level the leading contributions are the following: $GG \to G^\star \to G^{(n)}X^{(n)}$ and $GG \to G^{(n)}X^{(n)}$. Since the SM fields do not carry $U(1)_X$ quantum numbers, the $X_\mu^{(n)}$ bosons are stable at the tree level; on the other hand, heavy gluons $G_\mu^{(n)}$ couple to SM quarks located on a brane. Therefore, the experimental signature for the above reactions would be missing energy, momentum (carried away by the stable $X_\mu^{(n)}$), and two jets from the $G_\mu^{(n)}$ decays. Let us compare the amplitude strength for this process with the standard QCD two-jet production amplitude. Adopting the estimate of the CS coupling from the NDA in (18), we find that the ratio of the $X_\mu^{(n)}G_\nu^{(n)}G_\alpha$ coupling to the SM triple gluon vertex is of the order of

$$\frac{g'}{g}\frac{\alpha_s}{3\pi}n \sim \frac{g'}{g}10^{-2}n. \qquad (24)$$

Since $n \sim 1$ (otherwise KK modes are too heavy to be produced), it seems that it may be difficult to detect $G^{(n)}X^{(n)}$ over the two-jet QCD background. Nevertheless, it should be noticed that the huge amount ($\sim$ TeV) of missing energy (carried away by the stable and heavy $X_\mu^{(n)}$) may enhance the signal relative to the QCD background very efficiently, and that the large gluon luminosity of the LHC could be sufficient to provide enough events to test the scenario. Though these expectations are supported by the results for similar processes at the Tevatron [11], a dedicated Monte Carlo study would be needed to resolve this issue definitively; this, however, lies beyond the scope of this paper.

Another possible signature of the axion being the 4th component of 5D gauge field could be the heavy gluon production process through a virtual axion exchange: $GG \to a^\star \to G^{(n)}G^{(n)}$ for $n \geq 1$. The amplitude for this process is generated by the first two terms in (18). It is straightforward to find that the order of magnitude for the amplitude normalized to two gluon (GG) production is the following:

$$\frac{\alpha'}{9\pi^2}\alpha_s n^2 \sim 10^{-3}\alpha' n^2, \qquad (25)$$

where $\alpha' \equiv g'^2/(4\pi)$. If $\alpha' \sim \alpha_s$, then for small n the amplitude is suppressed by the factor 10^{-4}. Since both $G^{(n)}G^{(n)}$ and GG states decay roughly the same way (the signature is $n \geq 4$ jets in the final state), it would be a real challenge to see the axion exchange over the standard QCD background.[6]

Let us assume that the axion mass m_a (or equivalently the decay constant f_a) is known. Then the definite test of the model discussed here would be a verification of the gauge-axion unification that is caused by the fact that the axion is a component of the 5D gauge field X_M. The important consequence of the unification is that the total cross section for $G^{(n)}X^{(n)}$ production is predicted including the normalization. Therefore, the measurement of $\sigma_{\rm tot}(G^{(n)}X^{(n)})$ shall provide the definite experimental test of the model.

Concluding the review of various possible experimental tests of gauge-axion unification discussed here, one can say that, because of a huge missing energy ($\sim$ TeV), the process $GG \to G^{(n)}X^{(n)}$ provides the cleanest signature, which makes the observation of the signal plausible.

For the model being considered here, the axion decay constant f_a is determined by the geometrical scale R^{-1} (if the NDA arguments are applied; therefore, experimental limits on f_a constrain the size of the compact dimension. However, it should be emphasized that most of these constraints rely on effects produced by the coupling of the axion to two photons, and this coupling is *absent* in our

[6]Note also that the amplitude receives contributions from the other terms in the action.

model (to leading order). (For a review of experimental constraints, see [6].) Nevertheless, there exists a bound that should be also obeyed by our photophobic axion; this is the so-called "misalignment" lower axion mass limit that originates from the requirement that the contribution to the cosmic critical density from the relaxation of the axion field ($\theta_{\rm eff} \to 0$) does not overclose the universe. The resulting constraint [6], $m_a > 10^{-6}$ eV, leads to $R^{-1} \lesssim 10^{13}$ GeV, having used (22) and (23) and taken $g' = \mathcal{O}(1)$. Note that the NDA estimate of the CS coupling was crucial to derive the limit on R.

IV. CONCLUSIONS

We have shown that an extension of naive dimensional analysis to 5D gauge theories naturally allows relatively large coefficients in front of CS terms. The strong *CP* problem was discussed within a simple scenario containing a new $U(1)_X$ gauge field and the $SU(3)_{\rm color}$ gauge fields propagating in the bulk, and interacting through a mixed CS term. Adopting appropriate boundary conditions, the CS term was shown to be gauge invariant (without any need for brane matter). The zero-mode of the extra component of the new Abelian gauge field was seen to play a role of the axion (gauge-axion unification), which in the standard manner receives the instanton-induced potential, so that the strong *CP* problem (localized on the branes) disappears while the axion receives a mass. In the effective low-energy regime, the axion couples only to gluons; therefore, most of the limits on the axion decay constant do not apply in the context of this model. It was shown that the most promising test of the gauge-axion unification is the process of $G^{(n)}X^{(n)}$ production: $GG \to G^{(n)}X^{(n)}$. The huge missing energy ($\sim$ TeV) carried away by the stable and heavy $X_\mu^{(n)}$ is believed to provide a sufficiently clean signature of the final state.

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APPENDIX

In this appendix we provide, for completeness, a summary of the application of naive dimensional analysis (NDA) to higher-dimensional models. The NDA allows us to determine the scale Λ at which the theory becomes strongly interacting. For that purpose let us compare two graphs with the same number of external legs, one of which has an additional gauge-boson propagator. This second graph will be suppressed with respect to the first by the factor

$$\Lambda^\delta g^2 l_{4+\delta}^{-1}; \qquad l_D = (4\pi)^{D/2}\Gamma(D/2), \tag{A1}$$

where g denotes the gauge coupling constant, and l_D is the geometric loop factor obtained form integrating over momentum directions (note that in $D = 4 + \delta$ dimensions g has a mass dimension of $-\delta/2$). For a strongly interacting theory we impose the NDA requirement that the loop corrections be of the same order as the lowest-order value; this requires

$$\Lambda \sim (l_{4+\delta} g^{-2})^{1/\delta}. \tag{A2}$$

The same NDA requirement allows an estimate of the coefficients in front of effective operators. For this we consider a generic vertex of the form

$$\mathcal{V} = \lambda\Lambda^D (2\pi)^D \delta^D\Big(\sum p_i\Big)\left(\frac{g\psi}{\Lambda_\psi^{3/2}}\right)^f \left(\frac{p}{\Lambda}\right)^d \left(\frac{gA_M}{\Lambda}\right)^b \left(\frac{g\phi}{\Lambda_\phi}\right)^{b'}, \tag{A3}$$

where scale appropriate for the vector fields and derivatives (they enter together through the covariant derivative) was chosen to be Λ, while the coefficient λ, the fermionic scale (Λ_ψ), and the scalar scale (Λ_ϕ) are to be determined. The requirement to reproduce the starting operator by radiative corrections determines the maximal value of λ and minimal scales Λ_ψ, Λ_ϕ that are allowed by perturbativity

$$\lambda = l_{4+\delta}^{-1} \quad \text{and} \quad \Lambda_\psi = \Lambda_\phi = \Lambda. \tag{A4}$$

Let us now restrict ourselves to 5D theories, $\delta = 1$, and define the "index" of a vertex by

$$s = d_c + b' + \frac{3}{2}f - 4; \qquad d_c = d + b, \tag{A5}$$

where d_c is the number of covariant derivatives present in the vertex $\mathcal{V}$. If an L-loop graph contains V_n vertices with indices s_n, then the vertex corresponding to this graph has an index

$$s = L + \sum_n V_n s_n. \tag{A6}$$

In terms of s the coefficient of a given operator is (see also [12])

$$\left(\frac{1}{24\pi^3}\right)^s \times (\text{the powers of } g \text{ needed to get a dimension 5 object}); \tag{A7}$$

and $\Lambda = 24\pi^3/g^2$.

If the indices of all vertices are non-negative, then it follows from (A6) that $s \geq s_n$ for all n. This implies that if

$\mathcal{V}$ has index s, then only operators with indices $\leq s$ can renormalize the coefficient of $\mathcal{V}$ and we can then define a hierarchy according to the value of s, in the sense that we can consistently assume that operators with higher indices are generated only by higher orders in the loop expansion. This would be spoiled if the theory has vertices with negative indices (an addition of an internal line attached by vertices with $s_n < 0$ decreases s, so an extra loop leads to a less suppressed operator), which corresponds to the case $d_c = f = 0$, $b' = 3$, according to the definition (A5). In order to define a hierarchy one should accordingly require that all cubic terms in the scalar fields be absent[7] due to an additional symmetry such as a discrete $\mathbb{Z}_2$ under which the ϕ are odd, by gauge invariance (as in the SM) or just by an absence of scalar fields (as in this paper where we are considering only vector bosons in 5D, therefore, the cubic scalar interactions cannot be constructed and the hierarchy of operators is given just by (A7) without any other constraints). Fermion fields are assumed to transform appropriately under this symmetry, so as to allow all desirable scalar-fermion couplings.

In order to consistently include possible brane terms in the hierarchy, we note that these types of interactions are naturally generated by the bulk terms in a compactified space at the one loop level [13]. It is then natural to add 1 to s whenever a localizing factor of the form $\delta(y - y_o)$ is present. In addition, the geometric suppression factor for these terms equals $l_4 = 16\pi^2$ that replaces $l_5 = 24\pi^3$ present in (A7); see also [14].

[7]This statement holds within NDA; of course, if the coefficients of super-renormalizable operators are tuned to be small, then their effects are suppressed so that the problem of consistency disappears.

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