Minimal flavor violations and tree level flavor changing neutral currents

Anjan S. Joshipura* and Bhavik P. Kodrani*

Physical Research Laboratory, Navarangpura, Ahmedabad 380 009, India

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Consequences of a specific class of two Higgs doublet models in which the Higgs induced tree level flavor changing neutral currents (FCNC) display minimal flavor violation are considered. These FCNC are fixed in terms of the Cabibbo Kobayashi Maskawa (CKM) matrix elements and the down quark masses. The minimal model in this category with only two Higgs doublets has no extra *CP* violating phases, but such a phase can be induced by adding a complex singlet Higgs. The FCNC contribute significantly to *B* meson mixing and *CP* violation, but similar contributions in case of the *K* mesons are suppressed. Detailed numerical analysis to determine the allowed Higgs contributions to neutral meson mixings and the CKM parameters $\bar{\rho}$, $\bar{\eta}$ in their presence is presented. The Higgs induced phase in the $B_{d,s}^0 - \bar{B}_{d,s}^0$ transition amplitude $M_{12}^{d,s}$ is predicted to be equal for the B_d and the B_s systems. There is a strong correlation between $|V_{ub}|$ and phases $\phi_{d,s}$ in $M_{12}^{d,s}$. A measurable *CP* violating phase $\phi_s = -0.18 \pm 0.08$ is predicted on the basis of the observed phase ϕ_d in the B_d system if $|V_{ub}|$ is large and close to its value determined from the inclusive *b* decays.

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I. INTRODUCTION

The Cabibbo Kobayashi Maskawa (CKM) matrix V provides a unique source of flavor and CP violations in the standard model (SM). It leads to flavor changing neutral currents (FCNC) at the one loop level. K and B meson decays and mixing have provided stringent tests of these FCNC induced processes and the SM predictions have been verified with some hints for possible new physics contributions [1–3]. Any new source of flavor violations resulting from the well-motivated extensions of the SM (e.g. supersymmetry) is now constrained to be small [4–6].

Several extensions of the SM share an important property termed as minimal flavor violation (MFV) [7,8]. According to this, all flavor and CP violations are determined by the CKM matrix even when the SM is extended to include other flavor violating interactions. In the extreme case (termed as the constrained MFV [9]) the operators responsible for the flavor violations are also the same as in the SM. In more general situations, MFV models contain more operators with coefficients determined in terms of the elements of V. Some scenarios [10] termed as the next to minimal flavor violation contain new phases not present in V.

A simple example of MFV is provided by a two Higgs doublet model with natural flavor conservation (NFC) [11]. A discrete symmetry is imposed in this model to ensure that all the quarks of a given charge obtain their masses from a single Higgs field. As a result of this, the neutral Higgs couplings become flavor diagonal in the quark mass basis and there are no tree level FCNC. The same discrete symmetry also prevents any *CP* violation coming from the

Higgs potential and the CKM matrix provides a unique source of *CP* and flavor violations in these models. The MFV in these models can be explicitly seen by considering the $B_q^0 - \bar{B}_q^0$ (q = d, s) transition amplitude M_{12}^q as an example. The charged Higgs boson in the model gives additional contributions to the SM amplitude and the dominant top quark dependent part can be written [12] as

$$M_{12}^{q} = \frac{G_{F}^{2} M_{W}^{2} m_{B_{q}} B_{q} f_{B_{q}}^{2} \eta_{B}(x_{t}) (V_{33} V_{3q}^{*})^{2} S_{0}(x_{t})}{12\pi^{2}} (1 + \kappa_{H}^{+}).$$
(1)

 B_q refers to correction to the standard vacuum saturation approximation used in evaluating M_{12}^q . m_{B_q} and f_{B_q} refer to the mass and the decay constant of the B_q^0 mesons. $S_0(x_t)$ is the standard function [13] entering the box diagram calculation and $x_t = \frac{m_t^2}{M_W^2}$. κ_H^+ denotes the ratio of the charged Higgs and the SM contribution to M_{12}^q and is given by

$$\kappa_{H}^{+} \equiv \frac{1}{4S_{0}(x_{t})} \frac{\eta_{B}(x_{t}, y_{t})}{\eta_{B}(x_{t})} (\cot^{4}\theta S_{HH}(y_{t}) + \cot^{2}\theta S_{HW}(x_{t}, y_{t})),$$
$$\approx \frac{\eta_{B}(x_{t}, y_{t})}{\eta_{B}(x_{t})} (0.12 \cot^{4}\theta + 0.53 \cot^{2}\theta), \tag{2}$$

where q = 1, 2 corresponds to the down and strange quark, η_B are the QCD corrections [14,15], $\tan\theta$ is the ratio of the Higgs vacuum expectation values, and $y_t = \frac{m_t^2}{M_{H^+}^2}$. The functions appearing above can be found, for example, in [15,16] and the last line corresponds to the obtained numerical values in case of the charged Higgs mass $M_{H^+} = 200$ GeV. Flavor and *CP* violations are still governed by the same combinations of the CKM matrix elements that appear in the SM box diagram. The only effect of the

^{*}anjan@prl.res.in

⁺bhavik@prl.res.in

charged Higgs boson is an additional contribution to the function $S_0(x_t)$. The same happens in case of other observables and one can parametrize all the FCNC induced processes in terms of seven independent functions in MFV models [8].

Two Higgs doublet models (2HDM) with NFC lead to MFV but they do not represent the most generic possibilities. More general 2HDM will contain additional sources of *CP* and flavor violations through the presence of FCNC. The principle of NFC now appears to conflict [17] with the idea of the spontaneous CP violation at low energy and both cannot coexist together. Indeed, if NFC and the spontaneous CP violation are simultaneously present in multi-Higgs doublet models then the CKM matrix is implied to be real [18]. In contrast, the detailed model independent fits to experimental data requires the Wolfenstein parameter $\bar{\eta} = 0.386 \pm 0.035$ according to the latest fits by the UTfit collaboration [5]. Thus, the CKM matrix is proven to be complex under very general assumptions [19]. Attractive idea of low energy spontaneous CP violation can only be realized by admitting the tree level FCNC [20]. Independent of this, the 2HDM without NFC become phenomenologically interesting if there is a natural mechanism to suppress FCNC. The phenomenology of such models has been studied in variety of contexts [21].

This paper is devoted to discussion of models in which FCNC are naturally suppressed and show strong hierarchy [22–24]. Specifically, the FCNC couplings F_{ij}^d between the *i* and the *j* generations obey

$$|F_{12}^d| < |F_{13}^d|, |F_{23}^d| \tag{3}$$

automatically suppressing the flavor violations in the K sector relative to B mesons. A specific subclass of these models has the remarkable property that the FCNC couplings are determined completely in terms of the CKM matrix and the quark masses [24]. These models therefore provide yet another example of MFV in spite of the presence of FCNC. The models to be discussed were presented long ago [22–24] and the aim of the present paper is to update constraints on them in view of the substantial experimental information that has become available from the Tevatron and B factories.

The next section introduces the class of models we discuss and presents the structure of the FCNC couplings. Section III is devoted to the analytic and numerical studies of the consequences assuming that either the charged Higgs or a neutral Higgs dominates the $P^0 - \bar{P}^0$ ($P^0 = K^0, B^0_d, B^0_s$) mixing. The last section summarizes the salient features of the paper.

II. MODEL AND THE STRUCTURE OF FCNC

Consider an SU(2) \otimes U(1) model with two Higgs doublets ϕ_a , (a = 1, 2) and the following Yukawa couplings:

$$-\mathcal{L} = \bar{Q}_L' \Gamma_a^d \phi_a d_R' + \bar{Q}_L' \Gamma_a^u \tilde{\phi}_a u_R' + \text{H.c..}$$
(4)

 Q'_{iL} (*i* = 1, 2, 3) represents three generations of weak doublets and u'_{iR} , d'_{iR} are the corresponding singlets and $\tilde{\phi}_a \equiv i\tau_2 \phi_a^*$. Let us consider a class of models [22] represented by a specific choice of the matrices Γ_a^d and their permutations

$$\Gamma_1^d = \begin{pmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{pmatrix}; \qquad \Gamma_2^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{pmatrix}, \quad (5)$$

where *x* represents an entry which is allowed to be nonzero. We do not impose *CP* on Eq. (4) allowing elements in $\Gamma_{1,2}^d$ to be complex. The above forms of Γ_a^d are technically natural as they follow from imposition of discrete symmetries on Eq. (4), the simplest being a Z_2 symmetry under which only Q'_{3L} and ϕ_2 change sign.

The down quark mass matrix M_d follows from Eq. (5) when the Higgs fields obtain their vacuum expectation values: $\langle \phi_1^0 \rangle = v_1$ and $\langle \phi_2^0 \rangle = v_2 e^{i\alpha}$. Let $V_{L,R}^d$ be the unitary matrices connecting the mass (unprimed) and the weak basis $d'_{L,R} = V_{L,R}^d d_{L,R}$. Then

$$V_L^{d\dagger} M_d V_R^d = D_d, (6)$$

 D_d being a diagonal matrix of the down quark masses m_i . The conventional two Higgs doublet models with natural flavor conservation correspond to taking $\Gamma_2^d = 0$ in Eq. (5) and a replacement of Γ_1^d by an arbitrary complex matrix. The neutral Higgs couplings in this case are diagonal in the quark mass basis and there are no FCNC at tree level. In contrast, M_d here obtains contributions from two different Higgs fields leading to the Higgs induced FCNC in the down quark sector. Equations (4)–(6) are used to obtain

$$-\mathcal{L}_{\text{FCNC}} = \frac{(2\sqrt{2}G_F)^{1/2}}{\sin\theta\cos\theta} F_{ij}^d \bar{d}_{iL} d_{jR} \phi^0 + \text{H.c.}, \quad (7)$$

where $\tan \theta = \frac{v_2}{v_1}$ and

$$\phi^0 \equiv \cos\theta \phi_2^0 e^{-i\alpha} - \sin\theta \phi_1^0 \tag{8}$$

is a specific combination of $\phi_{1,2}$ with zero vacuum expectation value. The orthogonal combination plays the role of the standard model Higgs. The strength of FCNC current is determined in the fermion mass basis by [22]

$$F_{ij}^{d} \equiv (V_{L}^{d\dagger} \Gamma_{2}^{d} \upsilon_{2} e^{i\alpha} V_{R}^{d})_{ij} = (V_{L}^{d*})_{3i} (V_{L}^{d})_{3j} m_{j}.$$
 (9)

Note that the specific texture of $\Gamma_{1,2}^d$ allowed us to express F_{ij}^d in terms of the left-handed mixing and the down quark masses m_j and the dependence on the unphysical V_R^d disappeared. The F_{ij}^d depend on the left-handed mixing matrix V_L^d which is *a priori* unknown but would be correlated to the CKM matrix. One observes that

- (i) independent of the values of elements of V_L^d , the F_{ij}^d display hierarchy given in Eq. (3).
- (ii) all the FCNC couplings are suppressed if the offdiagonal elements of V_L^d are smaller than the diago-

nal ones. The model in this sense illustrates the principle of near flavor conservation [25]. This is a generic possibility in view of the strong mass hierarchy among quarks unless there are some special symmetries.

(iii) F_{ij}^d can be determined in terms of the CKM matrix elements for a specific structure of M_u [24] given as follows:

$$M_{u} = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{pmatrix}.$$
 (10)

The above postulated structures of $M_{u,d}$ follow from discrete symmetries [24] rather than being *ad hoc*. A particular example can be

$$(Q'_{1,2L}, \phi_1) \to \omega(Q'_{1,2L}, \phi_1), u'_{1,2R} \to \omega^2 u'_{1,2R}.$$
 (11)

Here ω , $\omega^2 \neq 1$ are complex numbers. The fields not

shown above remain unchanged under the symmetry. The particular form of M_u as given above implies that $(V_{dL})_{3i} = V_{3i}$ as a result of which the F_{ij}^d in Eq. (9) are completely determined in terms of the CKM matrix V

$$F_{ij}^d = V_{3i}^* V_{3j} m_j. (12)$$

As a consequence of Eq. (11), $(M_u)_{33}$ gets contribution from ϕ_2 while the first two generations from ϕ_1 with no mixing with the third one. As a result, there are no FCNC in the up quark sector while they are determined as in Eq. (12) in the down quark sector.

The tree level couplings of the charged Higgs $H^+ \equiv \cos\theta e^{-i\alpha}\phi_2^+ - \sin\theta\phi_1^+$ can be read off from Eq. (4) and are given by

$$- (2\sqrt{2}G_F)^{1/2}H^+ \left\{ \bar{u}_R \hat{D}_u V d_L + \bar{u}_L \times \left(V D_d \tan\theta - \frac{1}{\sin\theta\cos\theta} V F^d \right) d_R \right\} + \text{H.c.}, \quad (13)$$

where $\hat{D}_u \equiv \text{diag}(-m_u \tan\theta, -m_c \tan\theta, m_t \cot\theta)$.

It follows from Eqs. (7), (12), and (13) that all the Higgs fermion couplings are determined by the CKM matrix Vgiving rise to MFV. There can however be an additional source of CP violation in the model. This can arise if CP is violated in the Higgs sector through a scalar-pseudoscalar Higgs mixing. As noted in [24], the discrete symmetry of Eq. (11) prevents this mixing in the Higgs potential even if one allows for explicit CP violation and a bilinear soft symmetry breaking term $\mu(\phi_1^{\dagger}\phi_2)$ + H.c.. Thus, the minimal version of the model corresponds to the MFV scenario with no other CP violating phases present. CP violation in Higgs mixing can however be induced by adding a complex Higgs singlet field [24,26] along with the above soft discrete symmetry breaking term. In this case, there would be an additional phase which mixes the real and the imaginary parts of the Higgs ϕ^0 defined in Eq. (8).

An independent motivation for introducing the Higgs singlet is provided by the strong *CP* problem. It is known that the Peccei Quinn (PQ) solution [13] to this problem can be made phenomenologically viable by invoking a Higgs singlet. It would thus be natural to have singlet fields play a dual role of providing weak CP violation and solving the strong CP problem [26]. This can be done here by replacing the discrete symmetry in Eq. (11) with a continuous symmetry defined by $\omega \rightarrow e^{i\theta}$. This symmetry can play the role of the PQ symmetry and would also enforce the desired structures of the Yukawa couplings $\Gamma_{1,2}^q$. But the Higgs potential gets further restricted. Now a simple Higgs potential with two doublets and a singlet and the above PQ symmetry does not admit CP violation, but this can be done by adding one more singlet. Consider the following PQ symmetric couplings between singlets and doublets in the Higgs potential:

$$\begin{aligned} (\phi_1^{\dagger}\phi_2)(\sigma_1\eta_1^2 + \sigma_2\eta_2^{*2}) + \lambda_{12}(\eta_1\eta_2)^2 \\ + \mu_{12}\eta_1\eta_2 + \text{H.c.}, \end{aligned} \tag{14}$$

where $\sigma_{1,2}$, μ_{12} , and λ_{12} are complex parameters. $\eta_{1,2}$ are two singlets such that $\eta_1 \rightarrow e^{(i/2)\theta} \eta_1$, $\eta_2 \rightarrow e^{-(i/2)\theta} \eta_2$ under the PQ symmetry. Quark fields and ϕ_1 transform as in Eq. (11) with $\omega \equiv e^{i\theta}$ while ϕ_2 and the remaining fields are invariant. Minimization of the full potential including the above terms (but $\mu_{12} = 0$) is carried out in [26] where it is shown that the desired mixing between the scalar and pseudoscalar components of ϕ^0 in Eq. (7) indeed takes place.

Without committing to any of the above scenario, we will simply assume for phenomenological purpose that Higgs mixing contains an effective *CP* violating phase which could be generated through singlets as outlined above.

There is an important quantitative difference between the present scenario and the general MFV analysis following from the effective field theory approach [7]. There the effective dominant FCNC couplings between down quarks are given by

$$(\lambda_{\rm FC})_{ij} \approx \lambda_t^2 V_{3i}^* V_{3j},$$

where λ_t denotes the top Yukawa coupling. The same factor controls the loop induced contributions here but the tree level flavor violations are given by Eq. (12) which contains the same elements of V but involves the down quark masses linearly. Its contribution is still important or dominates over the top quark dependent terms because of its presence at the tree level.

One could consider variants of the above textures and symmetry obtained by permutations of flavor indices. These variants lead to different amount of FCNC. Labeling these variants by *a*, one has three models [24] with $F_{ij}^d(a) = V_{ai}^* V_{aj} m_j$, (a = 1, 2, 3). Alternatively, one could also consider equivalent models in which FCNC in

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the *d* quarks are absent while in the up quark sector they would be related to the CKM matrix elements and the up quark masses. The case a = 3 is special. It leads to the maximum suppression of FCNC in the 12 sector. We will mainly consider phenomenological implication of that case.

III. EXPERIMENTAL CONSTRAINTS AND THEIR IMPLICATIONS

A. Basic results

The strongest constraints on the model come from the $P^0 - \bar{P}^0$ ($P^0 = K^0, B_d^0, B_s^0$) mixing. In addition to the SM contribution, two other sources, namely, the charged Higgs induced box diagrams and the neutral Higgs ϕ^0 induced tree diagram contribute to this mixing.

The charged Higgs leads to new box diagrams which follow from Eq. (13). The last two terms of this equation are suppressed by the down quark masses (for modest $\tan\theta$) and the dominant contribution comes from the top quark. This term and hence the charged Higgs contributions remain the same as in 2HDM with NFC [12]. The contribution to the $B_q^0 - \bar{B}_q^0$ mixing is already given in Eq. (1). The contribution to ϵ is given [16] by

$$\epsilon^{H^+} = \frac{G_F^2 M_W^2 f_K^2 m_K B_K A^2 \lambda^6 \bar{\eta}}{6\sqrt{2}\pi^2 \Delta m_K} (f_1^H + f_2^H A^2 \lambda^4 (1-\bar{\rho})),$$
(15)

where functions $f_{1,2}^H$ can be read off from expressions given in [16]. λ , $\bar{\eta} \equiv \eta(1 - \frac{\lambda^2}{2})$, $\bar{\rho} \equiv \rho(1 - \frac{\lambda^2}{2})$ and A are the Wolfenstein parameters. f_K , B_K are the relevant decay constant and the bag parameter and Δm_K denotes the $K^0 - \bar{K}^0$ mass difference. Contribution of f_1^H to ϵ is practically negligible while the f_2^H can compete with the corresponding term in the SM expression [13]

$$\epsilon^{\text{SM}} = \frac{G_F^2 M_W^2 f_K^2 m_K B_K A^2 \lambda^6 \bar{\eta}}{6\sqrt{2}\pi^2 \Delta m_K} \times (f_1(x_t) + f_2(x_t) A^2 \lambda^4 (1-\bar{\rho}))$$
(16)

for moderate values of $tan\theta$.

The neutral Higgs contributions to the above observables follow from Eqs. (7) and (12). Define

$$\phi^0 \equiv \frac{R+iI}{\sqrt{2}} = \left(\frac{O_{R\alpha}+iO_{I\alpha}}{\sqrt{2}}\right) H^0_{\alpha} \equiv |C_{\alpha}| e^{i\eta_{\alpha}} H^0_{\alpha},$$

where H^0_{α} denotes the mass eigenstates with masses M_{α} . $\alpha = 1, 2, 3$ for the 2HDM while $\alpha = 1, ..., 5$ in the presence of a complex singlet introduced to induce the scalarpseudo scalar mixing leading to phases η_{α} in the Higgs mixing C_{α} . $O_{R\alpha,I\alpha}$ are elements of the mixing matrix. Using this definition and Eq. (12), the neutral Higgs contribution to M^0_{12} can be written as

$$(M_{12}^{q})^{H^{0}} = \frac{5\sqrt{2}G_{F}m_{b}^{2}m_{B_{q}}f_{B_{q}}^{2}B_{2q}}{12\mathrm{sin}^{2}2\theta M_{\alpha}^{2}} \times \left(\frac{m_{B_{q}}}{m_{b}+m_{q}}\right)^{2}C_{\alpha}^{2}(V_{3q}^{*}V_{33})^{2} + \mathcal{O}\left(\frac{m_{q}}{m_{b}}\right), \quad (17)$$

where we used the vacuum saturation approximation multiplied by the bag factor B_{2q}

$$\langle B_q^0 | (\bar{q}_L b_R)^2 | \bar{B}^0 \rangle = -\frac{5}{24} m_{B_q} f_{B_q}^2 B_{2q} \left(\frac{m_{B_q}}{m_b + m_q} \right)^2.$$
(18)

The $\mathcal{O}(\frac{m_q}{m_b})$ refers to contributions coming from the F_{3q}^{d*} terms in Eq. (7). Using the vacuum saturation approximation and Eq. (12), these terms are estimated to be only a few percent of the first term in Eq. (17) for q = s and much smaller for q = d. We do not display here the QCD corrections to $(M_{12})^{H^0}$. Such corrections can be significant and play important roles in the precise determination of the SM parameters. In contrast, the above expressions contain several unknowns of the Higgs sector because of which we prefer to simplify the analysis and retain only the leading terms as far as the Higgs contributions to various observables are concerned. The SM contribution is given by

$$(M_{12}^q)^{\rm SM} = \frac{G_F^2 m_W^2 m_{B_q} f_{B_q}^2 B_q \eta_B}{12\pi^2} (V_{3q}^* V_{33})^2 S_0(x_t), \quad (19)$$

with $S_0(x_t) \approx 2.3$ for $m_t \approx 161$ GeV and η_B represents the QCD corrections. Equations (17) and (19) together imply

$$\kappa_{q} \equiv \left| \frac{(M_{12}^{q})^{H^{0}}}{(M_{12}^{q})^{\text{SM}}} \right|$$

$$= \left(\frac{5\sqrt{2}\pi^{2}|C_{\alpha}|^{2}}{G_{F}M_{W}^{2}\sin^{2}2\theta S_{0}(x_{t})} \right) \left(\frac{m_{b}}{M_{\alpha}} \right)^{2} \frac{B_{2q}}{B_{d}\eta_{B}} \left(\frac{m_{B_{q}}}{m_{b} + m_{q}} \right)^{2}$$

$$+ \mathcal{O}\left(\frac{m_{q}}{m_{b}} \right). \tag{20}$$

The neutral Higgs contribution to ϵ is given by

$$\epsilon^{H^0} = \frac{5G_F m_K f_K^2 B_{2K}}{12 \sin^2 2\theta \Delta m_K M_{\alpha}^2} \left(\frac{m_K}{m_s + m_d}\right)^2 \text{Im}(F_{12}^d C_{\alpha})^2, \quad (21)$$

where B_{2K} is defined in analogy with Eq. (18). Using the expression of F_{12}^d from Eq. (12) and the Wolfenstein parametrization, one can rewrite the above equation as

$$\epsilon^{H^0} \approx \frac{5G_F m_s^2 m_K f_K^2 B_{2K}}{12 \sin^2 2\theta \Delta m_K M_\alpha^2} \left(\frac{m_K}{m_s + m_d}\right)^2 \times |C_\alpha|^2 A^4 \lambda^{10} [(1 - \bar{\rho})^2 + \bar{\eta}^2]^{1/2} \sin^2(\eta_\alpha - \beta), \quad (22)$$

where $\tan \beta = \frac{\bar{\eta}}{1-\bar{\rho}}$ is one of the angles of the unitarity triangle. The Higgs contribution to ϵ is suppressed here by the strange quark mass and ϵ^{H^0} is practically negligible compared to ϵ^{SM}

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$$\left| \frac{\epsilon^{H^0}}{\epsilon^{\text{SM}}} \right| \approx 3.810^{-4} \frac{B_{2K}}{B_K} \frac{|C_{\alpha}|^2}{\sin^2 2\theta} \left(\frac{100 \text{ GeV}}{M_{\alpha}} \right)^2 \\ \times \frac{\sin^2(\eta_{\alpha} - \beta)}{\cos\beta + 0.1 \sin\beta}.$$
(23)

The neutral Higgs contribution to the $K^0 - \bar{K}^0$ mass difference is even more suppressed compared to its experimental value.

B. Experimental inputs

Constraints on the present scheme come from several independent measurements. The complex amplitude M_{12}^d is known quite well. The magnitude is given in terms of the $B_d^0 - \bar{B}_d^0$ mass difference [27]

$$\Delta M^d \equiv 2|M_{12}^d| = (0.507 \pm 0.005) \text{ ps}^{-1}.$$
 (24)

The phase ϕ_d is measured through the mixing induced *CP* asymmetry in the $B_d^0 \rightarrow J/\psi K_S$ decay

$$\sin\phi_d = 0.668 \pm 0.028. \tag{25}$$

Likewise, the $B_s^0 - \bar{B}_s^0$ mass difference is quite well determined

$$\Delta M^s \equiv 2|M_{12}^s| = (17.77 \pm 0.12) \text{ ps}^{-1}.$$
 (26)

The corresponding phase ϕ_s is determined [28] by the D0 collaboration [29]

$$\phi_s = -0.70^{+0.47}_{-0.39} \tag{27}$$

by combining their measurements of (1) the light and the heavy B_s^0 width difference (2) the time dependent angular distribution in the $B_s^0 \rightarrow J/\psi\phi$ decay, and (3) the semileptonic charge asymmetries in the B^0 decays.

The SM predictions for the above quantities depend on the hadronic and the CKM matrix elements. The determination of $\bar{\rho}$, $\bar{\eta}$ is somewhat nontrivial when new physics is present. The conventional SM fits use the loop induced variables ϵ , ΔM^q , ϕ_d for determining $\bar{\rho}$, $\bar{\eta}$. These variables are susceptible to new physics contributions. This makes extraction of $\bar{\rho}$, $\bar{\eta}$ model dependent. It is still possible to determine these parameters and construct a universal unitarity triangle [30] for a unitary V by assuming that the tree level contributions in the SM are not significantly affected by new physics. In that case, one can use only the tree level measurements for determining $\bar{\rho}, \bar{\eta}$ [2]. Alternatively, one can allow for NP contributions [4–10] in the loop induced processes while determining elements of V. The tree level observables are the moduli of V and the unitarity angle γ [27]

$$\lambda = |V_{us}| = 0.2258 \pm 0.0014,$$

$$A = \frac{|V_{cb}|}{\lambda^2} = 0.82 \pm 0.014,$$

$$|V_{ub}|^{\text{excl}} = 0.0034 \pm 0.0004,$$

$$|V_{ub}|^{\text{incl}} = 0.0045 \pm 0.0003.$$
(28)

 γ is determined from purely tree level decay $B \rightarrow D^* K^*$. We will use the UTfit average value [5]

$$\gamma = (83 \pm 19)^{\circ}.$$
 (29)

In terms of the Wolfenstein parameters,

$$\bar{\rho} = R_b \cos\gamma, \qquad \bar{\eta} = R_b \sin\gamma,$$

$$R_b \equiv \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| = 0.46 \pm 0.03, \qquad (30)$$

$$= 0.35 \pm 0.04,$$

where the values given above are obtained using inclusive and exclusive determination, respectively. Equations (29)and (30) directly lead in the inclusive case to

$$\bar{\rho} = 0.06 \pm 0.15, \qquad \bar{\eta} = 0.46 \pm 0.03.$$
 (31)

These values solely based on the tree level observables are independent of any NP contributing to the loop induced variables. One could use the above values of $\bar{\rho}$, $\bar{\eta}$ to obtain predictions of ϵ and M_{12}^d in the SM. The errors involved are rather large, but it has the advantage of being independent of any new physics contributing to these observables. This approach has been used, for example, in [2–4] to argue that a nontrivial NP phase is required if $|V_{ub}|$ is close to its inclusive determination. We will use an alternative analysis which also leads to the same conclusion. The new physics contributions to the loop induced $\Delta F = 2$ observables is parametrized as follows:

$$M_{12}^{q} = (M_{12}^{q})^{\text{SM}}(1 + \kappa_{q}e^{i\sigma_{q}}) = \rho_{q}(M_{12}^{q})^{\text{SM}}e^{i\phi_{q}^{\text{NP}}},$$

$$\epsilon = \rho_{\epsilon}\epsilon_{\text{SM}}.$$
(32)

Model independent studies using the above or equivalent parametrization have been used to determine $\bar{\rho}$, $\bar{\eta}$, κ_q , σ_q , ρ_ϵ in a number of different works [2,4–6,10]. We will use the results from the UTfit group whenever appropriate.

In view of the several unknown Higgs parameters, we make a simplifying assumption that only one Higgs contributes dominantly. We distinguish two qualitatively different situations corresponding to the dominance of the charged Higgs H^+ or of a neutral Higgs.

C. Charged Higgs dominance

The effects of the charged Higgs on the $P^0 - \bar{P}^0$ mixing as well as on $\Delta F = 1$ processes such as $b \rightarrow s\gamma$ have been discussed at length in the literature [12,15,16,31]. The present case remains unchanged compared to the standard



FIG. 1. Left panel: The 2σ region in the $\tan\theta$, M_{H^+} plane allowed by ρ_d (solid lines) and ρ_{ϵ} (dotted lines) given in Eq. (34). Right panel: Allowed regions in $|C_H|^2$, M_H plane following from the inclusive determination of $|V_{ub}|$ for $\tan\theta = 3$ (solid lines) and 10 (dotted lines). The left (right) panel is based on the assumption that the charged Higgs (neutral Higgs) alone accounts for the required new physics contribution to M_{12}^q .

two Higgs doublet model of type II as long as the down quark mass dependent terms are neglected in Eq. (13). Just for illustrative purpose and completeness, we discuss some of the restrictions on the charged Higgs couplings and masses in this subsection before turning to our new results on the neutral Higgs contributions to flavor violations.

Rather than determining $\bar{\rho}$, $\bar{\eta}$ separately in the specific case of the charged Higgs, we may borrow the existing detailed fits in [4] to obtain

$$\bar{\rho} = 0.154 \pm 0.032, \qquad \bar{\eta} = 0.347 \pm 0.018.$$
 (33)

Unlike Eq. (31), the above fits use loop induced variables and an average value of $|V_{ub}|$, but allow NP contribution to be present in the former. However, it is assumed that the NP contributions display MFV and are thus related to each other [8]. These assumptions are valid in case of the charged Higgs contributions. Hence, the above fits can be used to constrain the Higgs parameters. We can substitute Eq. (33) in the SM expressions for ΔM^d and ϵ to obtain [27]

$$\rho_d \equiv \frac{\Delta M^d}{(\Delta M^d)^{\rm SM}} = 0.99 \pm 0.29,$$

$$\rho_\epsilon \equiv \frac{\epsilon}{\epsilon^{\rm SM}} = 0.94 \pm 0.09.$$
(34)

This can be translated into bounds on M_{H^+} and $\tan\theta$ using Eqs. (1), (15), and (16). The 2σ bounds following from Eq. (34) are shown in Fig. 1. The constraints from ϵ are stronger and allow the middle (dotted) strip in the M_{H^+} – $\tan\theta$ plane. These are illustrative bounds and we refer to the literature [12,15,16,31] for more detailed results which include QCD corrections. Generally, there is a sizable region in the $\tan\theta$, M_{H^+} plane (e.g. $\tan\theta \ge 2$ in Fig. 1)

for which the top induced charged Higgs contribution to $\rho_{d,\epsilon}$ is not important. But the neutral Higgs can contribute to these observables in these regions as we now discuss.

D. Neutral Higgs dominance

We label the dominating neutral Higgs field by $\alpha = H$ and retain only one term in Eq. (17). Unlike in the previous case, the neutral Higgs contribution to ϵ (and the $K^0 - \bar{K}^0$ mass difference) is very small. It can contribute significantly to $M_{12}^{d,s}$ but these contributions are strongly correlated. Using Eq. (17) and (20) one finds that

$$r = \frac{\kappa_s}{\kappa_d} = \frac{B_{2s}}{B_{2d}} \frac{B_d}{B_s} \left(\frac{m_{B_s}}{m_s + m_b}\right)^2 \left(\frac{m_d + m_b}{m_{B_d}}\right)^2,$$

$$\sigma_d = \sigma_s = 2\eta_H.$$
 (35)

This ratio does not involve most of the unknown parameters and is determined by masses and the bag parameters. The ratios of *B* parameter in Eq. (35) and hence *r* is very close to one. For example, the results in [32] for the bag parameters imply

$$r = 1.04 \pm 0.12.$$
 (36)

Assuming r = 1 leads to an important prediction

$$\frac{\Delta M^s}{\Delta M^d} = \left(\frac{\Delta M^s}{\Delta M^d}\right)^{\rm SM}$$

This prediction holds good in various MFV scenarios, e.g. supersymmetric MFV model at low tan β [8]. Here it remains true even in the presence of an extra phase η_H . The above prediction can be usefully exploited [30] for the determination of one of the sides of the unitarity triangle

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$$R_{t} \equiv \sqrt{(1-\bar{\rho})^{2} + \bar{\eta}^{2}} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|,$$
$$= \frac{\xi}{\lambda} \sqrt{\frac{m_{B_{s}}}{m_{B_{d}}}} \sqrt{\left| \frac{\Delta M^{d}}{\Delta M^{s}} \right|} \approx 0.93 \pm 0.05, \qquad (37)$$

where $\xi = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}} = 1.23 \pm 0.06$ [5]. We used the SM expression, Eq. (19) in the above equation and the approximation $|V_{ts}| = |V_{cb}|$.

The SM prediction for ΔM^s is independent of $\bar{\rho}$, $\bar{\eta}$. Using, $f_{B_s} \sqrt{B_s} = 0.262 \pm 0.035$ MeV [5] we obtain

$$\rho_s \equiv \left|\frac{\Delta M^s}{\Delta M^{s,SM}}\right| \approx 0.96 \pm 0.26. \tag{38}$$

The existing fits to the $\Delta F = 2$ processes in the presence of NP are carried out in the context of the MFV [4,5,7,8] or next to minimal flavor violation [10] scenario or in a model independent manner [4,5]. Most of these assume that NP contributes significantly to the $\Delta S = 2$ transition, particularly to ϵ . This is not the case here. On the other hand, the model independent fits neglect correlations between ΔM^d , ΔM^s as present here. In view of this, we performed our own but simplistic fits in the present case. We use ϕ_d , γ , R_b , R_t , ρ_s , and ϵ in the fits assuming all errors to be Gaussian. The expressions and the experimental values for these quantities are already given in respective equations. We use the standard model expression for ϵ . We have used r = 1 in Eq. (35) giving Eq. (37) and $\rho_d = \rho_s \equiv \tilde{\rho}$ and $\sigma_d = \sigma_s \equiv \sigma$. The above six observables are fitted in terms of the four unknowns $\bar{\rho}$, $\bar{\eta}$, $\tilde{\rho}$, ϕ_d^{NP} . The fitted values of the parameters are sensitive to $|V_{ub}|$. The accompanying Table I contains values of the fitted parameters and 1σ errors obtained in three cases which use (a) inclusive, (b) exclusive, and (c) average values of $|V_{\mu b}|$ as quoted in [33]. The predictions based on the average values agree within 1σ with the corresponding detailed model independent fits by the Utfit group [5]: $\bar{\rho} = 0.167 \pm 0.051$, $\bar{\eta} = 0.386 \pm 0.035$. The values of $\bar{\rho}$, $\bar{\eta}$ in the fit directly determine the phase of $(M_{12}^d)^{\text{SM}}$

$$\sin 2\beta_d = \frac{\bar{\eta}(1-\bar{\rho})}{\sqrt{\bar{\eta}^2 + (1-\bar{\rho})^2}}.$$

TABLE I. Determination of NP parameters and $\bar{\rho}$, $\bar{\eta}$ from detailed fits to predictions of the neutral Higgs induced FCNC. See the text for more details.

	$ V_{ub}^{\rm incl} $	$ V_{ub}^{\mathrm{excl}} $	$ V_{ub}^{\text{average}} $
$\bar{ ho}$	0.200 ± 0.039	0.121 ± 0.042	0.186 ± 0.039
$ar\eta$	0.391 ± 0.028	0.320 ± 0.026	0.378 ± 0.027
$ ho_{d,s}$	0.96 ± 0.26	0.96 ± 0.26	0.96 ± 0.26
$\sin \phi_d^{ m NP}$	-0.18 ± 0.08	0.03 ± 0.08	-0.14 ± 0.09

The phase ϕ_d as measured through $S(\psi K_S)$ is then given by

$$\phi_d = 2\beta_d + \phi_d^{\rm NP},$$

where ϕ_d^{NP} is defined in Eq. (32) and can also be written as

$$\tan\phi_q^{\rm NP} = \frac{\kappa_q \sin\sigma_q}{1 + \kappa_q \cos\sigma_q}.$$
 (39)

The results in Table I imply that if $|V_{ub}|$ is close to the exclusive value then the present results are consistent with SM. If $|V_{ub}|$ is large and close to the inclusive value then ϕ_d^{NP} is nonzero at the 2σ level. This conclusion is similar to observations made [2] on the basis of the use of the tree level observables R_b , γ alone but with somewhat different input values then used here. A nonzero ϕ_d^{NP} (and hence σ) has important qualitative implication for the model under consideration. Nonzero σ requires *CP* violating phase η_H from the scalar-pseudoscalar mixing. As already remarked, the minimal 2HDM with symmetry as in (11) cannot lead to such a phase and more general model with an additional singlet field will be required. Also the charged Higgs contribution by itself cannot account for such a phase.

At the quantitative level, $\tilde{\rho} \neq 1$ implies restrictions on the Higgs parameters, M_H , $|C_H|$, θ . These parameters are simply related to $\kappa \equiv |\tilde{\rho}e^{i\phi_d^{\rm NP}} - 1|$ which is related to the said parameters through Eq. (20). Results in the table imply $\kappa = 0.18 \pm 0.08$ if $|V_{ub}| = |V_{ub}^{\rm incl}|$. The values of M_H and $|C_H|^2$ which reproduce this κ within the 1σ range are shown in Fig. 1 for two illustrative values of $\tan \theta = 3$, 10. Both of these values of $\tan \theta$ are chosen to make the charged Higgs contribution to κ very small. Unlike general models with FCNC, relatively light Higgs is a possibility within the present scheme and there exists large ranges in θ and $|C_H|$ which allow this.

One major prediction of the model is equality of new physics contributions to the *CP* violation in the B_d and B_s system. If the top induced charged Higgs contribution dominates then this *CP* violation is zero. In the case of the neutral Higgs dominance, the phases σ_d and σ_s induced by the Higgs mixing are equal, see Eq. (35). Since the ratio *r* in this equation is nearly one, let us write $r = 1 + \delta_r$ with $\delta_r \approx \pm \mathcal{O}(0.1)$. Then ϕ_s^{NP} in Eq. (39) can be approximated as

$$\tan\phi_s^{\rm NP} \approx \tan\phi_d^{\rm NP} [1 + \delta_r (1 - \cot\sigma \tan\phi_d^{\rm NP})],$$
$$\approx (1 + \delta_r) \tan\phi_d^{\rm NP}. \tag{40}$$

This prediction is independent of the details of the Higgs parameters. Its importance follows from the fact that the CKM matrix induced *CP* phase in the B_s system is quite small, $\beta_s \sim -1.0^\circ$. Thus, observation of a relatively large $\phi_s = 2\beta_s + \phi_s^{\text{NP}}$ will signal new physics. The predicted values of $\tan \phi_s$ based on Eq. (40) and the numerical values given in the table give

$$\tan \phi_s \approx -0.18 \pm 0.08$$
 inclusive,
 $\approx .03 \pm 0.08$ exclusive,
 $\approx -0.14 \pm 0.09$ average. (41)

All of these values are at present consistent with the experimental determination Eq. (27), by the D0 collaboration [29]. Significant improvements in the errors is foreseen in the future at LHCb [34] and relatively large ϕ_s following from the inclusive $|V_{ub}|$ can be seen. The above predictions show correlation with $|V_{ub}|$ and also with the *CP* violating phase ϕ_d . So combined improved measurements of all three will significantly test the model. The predictions of ϕ_s in the present case are significantly different from several other new physics scenarios allowing larger values for ϕ_s [35].

IV. SUMMARY

The general two Higgs doublet models are theoretically disfavored because of the appearance of uncontrolled FCNC induced through Higgs exchanges at tree level. We have discussed here the phenomenological implications of a particular class of models in which FCNC are determined in terms of the elements of the CKM matrix. This feature makes these models predictive and we have worked out major predictions of the scheme. Salient aspects of the scheme discussed here are

(i) Many of the predictions of the scheme are similar to various other models [8] which display MFV. The tree level FCNC couplings are governed by the CKM elements and the down quark masses, while the dominant part of the charged Higgs couplings involves the same CKM factors but the top quark mass. Both contributions can be important and there exists regions of parameters ($\tan \theta \ge 2$) in which the former contribution dominates. Unlike general FCNC models, the neutral Higgs mass as light as the current experimental bound on the SM Higgs is consistent with the restrictions from the $P^0 - \bar{P}^0$ mixing, see Fig. 1.

- (ii) The neutral Higgs coupling to the ε parameter is suppressed in the model by the strange quark mass. This prediction differs from the general MFV models where the top quark contributes equally to the B⁰ B
 ⁰ mixing and ε. Detailed fits to experimental data are carried out which determine the CKM parameters p
 , η
 a displayed in Table I.
- (iii) The noteworthy and verifiable prediction of the model is the correlation (Eq. (40)) between the *CP* violation in $B_s \bar{B}_s$, $B_d \bar{B}_d$ systems and $|V_{ub}|$.
- (iv) We have restricted ourselves to the study of the $\Delta F = 2$ flavor violations in this paper. The tree level FCNC would give rise to additional contributions to $\Delta F = 1$ processes and to new processes such as flavor changing neutral Higgs decays [36]. Already existing information on the $\Delta F = 1$ and $\Delta F = 2$ processes can be very useful in identifying allowed parameter space and verifiable signatures of the model. Such a study will be taken up separately.

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