Hidden solution to the μ/B_μ problem in gauge mediation

Tuhin S. Roy* and Martin Schmaltz+

Physics Department, Boston University, Boston, Massachusetts 02215, USA
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We propose a solution to the μ/B_μ problem in gauge mediation. The novel feature of our solution is that it uses dynamics of the hidden sector, which is often present in models with dynamical supersymmetry breaking. We give an explicit example model of gauge mediation where a very simple messenger sector generates both μ and B_μ at one loop. The usual problem, that B_μ is then too large, is solved by strong renormalization effects from the hidden sector which suppress B_μ relative to μ. Our mechanism relies on an assumption about the signs of certain incalculable anomalous dimensions in the hidden sector. Making these assumptions not only allows us to solve the μ/B_μ problem but also leads to a characteristic superpartner spectrum which would be a smoking gun signal for our mechanism.

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Models with gauge mediated supersymmetry breaking [1–10] are attractive because they introduce no new flavor violation beyond the standard model.[1] However, gauge mediation is not free of problems. In this paper we are concerned with the μ/B_μ problem [19], which is particularly severe in gauge mediation. Solutions to the μ/B_μ problem usually involve an elaborate messenger sector or extra light particles, often requiring fine-tuning of parameters. In this paper we point out an alternative solution to the μ/B_μ problem, which does not require a complicated messenger sector or new particles at the weak scale.

The B_μ problem in gauge mediation is related to the μ problem, which is common to all supersymmetric models. The effective low-energy minimal supersymmetric standard model (MSSM) Lagrangian contains a supersymmetric Higgs mass term

$$\int d^2\theta \mu H_u H_d. \tag{1}$$

Natural electroweak symmetry breaking requires that the mass parameter μ is of the same size as superpartner masses. Relating the supersymmetry preserving μ parameter to the supersymmetry violating soft masses is the μ problem in supersymmetric theories. In supergravity, Giudice and Masiero proposed a simple solution [20]: the μ term stems from a higher-dimensional operator coupling the supersymmetry breaking field X to the MSSM Higgs fields, such that the μ term in Eq. (1) is generated when X is replaced by its supersymmetry breaking vacuum expectation value (vev) F_X,

$$\int d^4\theta k_\mu \frac{1}{M} X^\dagger H_u H_d \to \int d^2\theta \mu H_u H_d \quad \text{with} \quad \mu = k_\mu \frac{F_X}{M}. \tag{2}$$

Here M is the mediation scale and it is given by M_{Planck} in supergravity theories. The gaugino masses M_a (a labels the three SM gauge groups) come from similar higher-dimensional operators

$$\int d^2\theta w_a \frac{1}{M} X W^a W^a \to M_a = w_a \frac{F_X}{M}. \tag{3}$$

If all these operators are generated by supergravity we may assume that the coupling constants k_μ and w_a are of order 1 and we find $\mu \sim M_a$ as desired.

Another important part of the Higgs potential is the B_μ term. Natural electroweak symmetry breaking requires $B_\mu \sim \mu^2$. The B_μ term arises from a higher-dimensional operator (the B_μ operator)

$$\int d^4\theta k_B \frac{1}{M^2} X^\dagger X H_u H_d \to B_\mu = k_B \frac{|F_X|^2}{M^2}. \tag{4}$$

In many models, k_B is generated with a similar size as k_μ and w_a. If, in addition, $k_B \sim k_\mu \sim w_a \sim \mathcal{O}(1)$ as in minimal supergravity with the Giudice-Masiero mechanism, then we obtain the desired relations $B_\mu \sim \mu^2 \sim M_a^2$.

The situation is more complicated in models with gauge mediation. Here gaugino and scalar masses are generated from gauge loop diagrams involving messenger fields of mass M. The μ and B_μ terms cannot be generated by gauge loops because the operators Eqs. (2) and (4) are forbidden by a Peccei-Quinn symmetry. A simple way to generate a μ term is to break Peccei-Quinn symmetry by coupling the Higgs superfields to the messengers in the superpotential. Now μ and B_μ terms are generated by the one-loop diagrams shown in Fig. 1 and we expect $k_B \sim k_\mu \sim w_a \sim 1/16\pi^2$. Hence μ is naturally of the size of the gaugino masses which also arise at one loop. However, the fact that the B_μ operator is also generated at one loop implies that the mass-squared parameter B_μ is too large by a loop factor compared to μ^2 and the gaugino masses squared

*tuhin@bu.edu
+schmaltz@bu.edu

[1]For recent developments see, for example, [11–18].

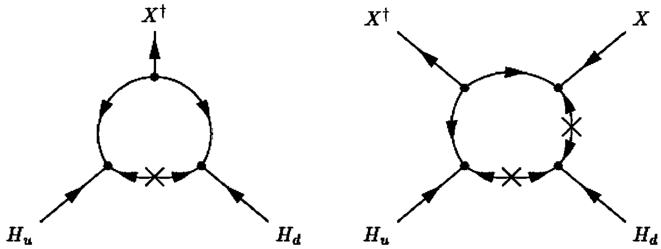

FIG. 1. Superfield diagrams which generate the μ and B_μ operators at one loop. The fields in the loop are messengers. A specific superpotential with couplings which generate such diagrams is given in Eq. (8).

$$B_\mu \sim 16\pi^2\mu^2 \sim 16\pi^2 M_a^2. \tag{5}$$

This is the B_μ problem in gauge mediation.

Solutions to the above problem are nontrivial. The basic point of the solution in [19] is to design the messenger superpotential such that at the leading order only the μ term is generated—the B_μ operator in Eq. (4) is generated at higher order. Models based on this scheme require extra heavy gauge singlets with carefully chosen masses and couplings. Another popular scheme is to introduce light scalars as in the next to minimal supersymmetric standard model (NMSSM). However, since it is difficult to obtain soft masses for gauge singlets in gauge mediation, one usually ends up either fine-tuning electroweak symmetry breaking or predicting unacceptably light particles. For a review of the B_μ problem in gauge mediation and references, see [10].

In this paper we propose a new solution to the B_μ problem: we start with a very simple messenger sector, requiring that it generates a μ term of the right size, but allowing that the B_μ term at the messenger scale is too large. We then argue that renormalization effects due to strong hidden sector interactions can sufficiently suppress the B_μ operator relative to the μ operator at low energies. This relative renormalization between μ and B_μ due to hidden sector interactions was also pointed out in [21].

Let us explain our mechanism in more detail. First, note that since the MSSM interactions are weak they cannot significantly suppress the B_μ term due to renormalization. This is why solutions to the B_μ problem generally require $B_\mu \lesssim \mu^2$ at the messenger scale even though phenomenology only requires this condition near the weak scale. On the other hand, in many models of dynamical supersymmetry breaking hidden sector interactions are strong and can induce large renormalization. For our mechanism we require a large positive anomalous dimension for the B_μ operator to suppress B_μ relative to μ^2 at low energies.

A formalism for renormalization which takes into account arbitrary hidden sector interactions as well as MSSM interactions was developed in [22,23]. We follow this formalism and notation and adopt a holomorphic basis in which no wave function renormalization is performed. We begin with the renormalization of the μ operator. It is well known that in the holomorphic basis the superpotential is not renormalized. This nonrenormalization theorem can be generalized to Kähler potential operators which factor into a product of a chiral visible sector operator times an antichiral hidden sector operator

$$\frac{1}{M^d}\mathcal{O}_h^*\mathcal{O}_v|_{\theta^4}. \tag{6}$$

The proof assumes that the dimension of the operator in question is low enough so that its renormalization can only involve single insertions of higher-dimensional hidden-visible interactions. Then its renormalization factorizes into separate visible and hidden sector contributions, and we can compute these contributions independently. To compute the visible sector running, we treat the hidden sector fields as background fields with an expectation value for their $\bar{\theta}^2$ components. This turns the operator in Eq. (6) into a chiral superpotential for the visible fields which is protected by the usual nonrenormalization theorem. By supersymmetry, this is also true for the full operator with arbitrary hidden sector fields. Similarly, as far as purely hidden sector interactions are concerned the operator in Eq. (6) is antichiral and therefore not renormalized.

Applying this result we see that the μ operator is not renormalized. Using a similar argument we can show that the B_μ operator is not renormalized by visible sector interactions because it is chiral in visible fields. However, hidden sectors interactions do renormalize B_μ because the operator is real in hidden sector fields and therefore not protected by nonrenormalization theorems. Schematically, these renormalizations are represented by the diagram in Fig. 2.

This implies the following general form for the renormalization group equations of the couplings k_μ and k_B (again in the holomorphic basis)

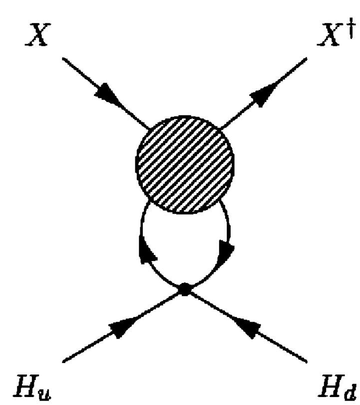

FIG. 2. Interactions of the hidden sector field X renormalize $X^\dagger X H_u H_d$.

$$\frac{d}{dt}k_\mu = 0 \Rightarrow k_\mu|_E = k_\mu|_M,$$
$$\frac{d}{dt}k_B = \gamma k_B \Rightarrow k_B|_E = \exp\left(-\int_t^0 ds\gamma(s)\right)k_B|_M \equiv G\left(\frac{E}{M}\right)k_B|_M, \quad (7)$$

where $t = \ln(E/M)$ and E is the renormalization scale. γ is the anomalous dimension of the operator in the holomorphic basis. For example, if γ is constant, then $G(E/M) = (E/M)^\gamma$.

We now see that if γ is positive and of order 1 then we can easily obtain large suppression factors $G \lesssim 1/16\pi^2$ for the B_μ operator at low energies. This then implies that $B_\mu \lesssim \mu^2$ at the scale where the hidden sector dynamics ends. For our mechanism to work the hidden sector must be strongly coupled over at least a couple of decades of energy scales. The most familiar such theories are strongly coupled conformal field theories.

Now we describe an explicit model which demonstrates our mechanism. The model has the following features: a very economical messenger sector which naturally predicts a μ parameter of the size of the gaugino masses (but B_μ is too large at the messenger scale). Below the messenger scale, our hidden sector fields have strong, approximately conformal interactions which lead to large anomalous dimensions for at least a couple of decades of running. Assuming that the anomalous dimensions governing the running of B_μ are positive we find that B_μ is suppressed to acceptably small values. Finally, at an even lower scale, supersymmetry is broken spontaneously in the hidden sector.

A simple messenger sector that suffices our purposes was described in [19]. The messengers R_1, R_2, $\bar{R}_1$, and $\bar{R}_2$ are vectorlike under the standard model gauge group and couple to the gauge singlet X and the MSSM Higgses in the superpotential

$$W_{\text{messenger}} = \bar{R}_1(M + X)R_1 + \bar{R}_2(M + X)R_2 + H_u\bar{R}_1R_2 + H_d\bar{R}_2R_1, \quad (8)$$

where all Yukawa couplings are assumed to be of order 1. At the scale M the messengers are integrated out and the higher-dimensional operators for μ, B_μ, as well as gaugino and scalar masses are generated. The operators in Eqs. (2)–(4), all appear at one loop which implies the relation $w_a|_M \sim k_\mu|_M \sim k_B|_M \sim 1/16\pi^2$ (i.e. the B_μ term is too large: $k_B \sim 16\pi^2 k_\mu^2$).

As our model of the hidden sector we take supersymmetric QCD with gauge group $SU(N)$ and F flavors each of $Q + \bar{Q}$ and $P + \bar{P}$ in the fundamental and antifundamental representations. The supersymmetry breaking field X is a gauge singlet and couples to Q and $\bar{Q}$ with the following hidden sector superpotential:

$$X(\Lambda^2 + \lambda Q\bar{Q}) + m(Q\bar{P} + P\bar{Q}) + \kappa(Q\bar{Q})^2. \quad (9)$$

Here m and Λ are mass parameters with $m > \Lambda$, λ is a Yukawa coupling, κ is a coupling to be discussed below, and all hidden sector flavor indices are contracted with δ_{ij}. For $\frac{3}{2}N < 2F < 3N$ and at energies above m the theory approaches an infrared attractive conformal fixed point for both gauge and Yukawa couplings [24]. During this running the suppression of scalar masses and B_μ takes place. At the scale m conformal symmetry is broken, the field X decouples from the gauge dynamics, and supersymmetry breaks at the scale Λ. While this story appears simple, there are subtleties which will require us to modify the model. We will now discuss these subtleties.

First, we study the renormalization of operators which couple hidden and visible fields in the energy regime $m < E < M$, where the hidden sector is governed by strong conformal dynamics. As already shown above, operators coupling chiral hidden sector operators to visible fields are not renormalized in the holomorphic basis which we employ. The gauge-invariant nonchiral operators with the lowest scaling dimensions of the hidden sector are presumably the bilinears $X^\dagger X$, $Q_i^\dagger Q_j$, $\bar{Q}_i^\dagger\bar{Q}_j$, $P_i^\dagger P_j$, and $\bar{P}_i^\dagger\bar{P}j$. In general, any of these operators may couple to the Higgs bilinear

$$\int d^4\theta \frac{1}{M^2}H_uH_d(k_X X^\dagger X + k_Q^{ij}Q_i^\dagger Q_j + \bar{k}_Q^{ij}\bar{Q}_i^\dagger\bar{Q}_j + k_P^{ij}P_i^\dagger P_j + \bar{k}_P^{ij}\bar{P}_i^\dagger\bar{P}_j). \quad (10)$$

Integrating out the messengers at scale M only generates k_X but hidden sector interactions can generate other couplings through operator mixing. At one loop, the operator $Q^\dagger Q + \bar{Q}^\dagger\bar{Q}$ is generated from the Yukawa interaction, and at two loops the operator $P^\dagger P + \bar{P}^\dagger\bar{P}$ is generated from gauge interactions. All other possible operators in Eq. (10) are not generated. It is easy to understand why they are not generated by considering the global symmetries of the model. Setting the coupling $\kappa = 0$ for the moment, only the dimensionless λ and the $SU(N)$ gauge coupling are relevant to the renormalization. Both preserve $SU(F)_Q \times SU(F)_P \times SU(F)_{\bar{P}}$ non-Abelian flavor symmetries and several $U(1)$ symmetries. The non-Abelian flavor symmetries together with the $U(1)$ symmetries forbid all operators except $X^\dagger X$, $Q^\dagger Q + \bar{Q}^\dagger\bar{Q}$, $Q^\dagger Q - \bar{Q}^\dagger\bar{Q}$, $P^\dagger P - \bar{P}^\dagger\bar{P}$, and $P^\dagger P + \bar{P}^\dagger\bar{P}$. The two operators with the minus signs contain Nöther currents corresponding to two baryon number symmetries acting separately on the Q, $\bar{Q}$ and P, $\bar{P}$. The baryon number symmetries are preserved by the interactions of the conformal field theory (CFT) and therefore the corresponding currents are conserved. Current conservation implies that these operators are not renormalized in the canonical basis for the fields. The holomorphic basis differs from the canonical one only by wave function renormalization. And since wave function renormalization is multiplicative new couplings to these currents cannot be generated in the holomorphic basis either. The current $P^\dagger P + \bar{P}^\dagger \bar{P}$ is broken by the axial anomaly but it is easy to see that diagrams proportional to the anomaly only enter its renormalization at two loops. This leaves us with three operators which mix in the running of B_μ to all orders

$$\int d^4\theta \frac{H_u H_d}{M^2}\left(k_X X^\dagger X + \frac{k_Q}{\sqrt{2NF}}(Q^\dagger Q + \bar{Q}^\dagger \bar{Q}) + \frac{k_P}{\sqrt{2NF}}(P^\dagger P + \bar{P}^\dagger \bar{P})\right). \quad (11)$$

The operator coefficients k satisfy the renormalization group equation

$$\frac{d}{dt}k = \gamma k, \quad (12)$$

where γ is a 3×3 dimensional matrix of anomalous dimensions. Assuming that our theory is approximately conformal, the anomalous dimensions are approximately constant. Diagonalizing γ and denoting its smallest eigenvalue by $\gamma_<$, we find that the operators in Eq. (11) are suppressed by a mixing angle times $(\frac{E}{M})^{\gamma_<}$ at low energies E. The suppression turns into an enhancement if $\gamma_<$ is negative. In order to achieve sufficient suppression of the B_μ operator we see that we must require $(\frac{E}{M})^{\gamma_<} \lesssim 1/16\pi^2$ or $\gamma_< \gtrsim \log(16\pi^2)/\log(M/E)$, where E is now the energy scale at which the hidden sector stops interacting. To put it simply, all eigenvalues of the anomalous dimension matrix must be of order 1 and positive.

Does our model satisfy this criterion? In the one-loop approximation k_P vanishes and the anomalous dimension matrix γ is only 2×2

$$\gamma = \frac{1}{16\pi^2}\begin{pmatrix} 0 & 2\lambda^2\sqrt{2NF} \\ 2\lambda^2\sqrt{2NF} & 2\lambda^2 + g^2(N^2-1)/N \end{pmatrix}. \quad (13)$$

This matrix has one positive and one negative eigenvalue which would be a problem. At strong coupling, the anomalous dimensions are not calculable, they are expected to be nonvanishing and of order unity but we do not know their sign.

We will now show that for hidden sector operators corresponding to conserved hidden sector currents of the CFT the sign can be determined and is negative. Thus they must be avoided in building suitable hidden sector models by either breaking the corresponding symmetry in the CFT or by preserving the symmetry also in the messenger sector so that the dangerous operators are never generated. The argument for why conserved current operators are dangerous is simple. In the canonical basis (which we will adopt for the argument in this paragraph) such a conserved current is not renormalized. Therefore the operator coupling the current to the MSSM fields scales as $1/M^2$ even in the presence of strong interactions. On the other hand, the μ term arises from a gauge-singlet antichiral operator of the hidden sector multiplying the MSSM Higgs fields. This operator has a positive anomalous dimension in the canonical basis. This follows from unitarity arguments which imply that the dimensions of chiral gauge-invariant operators must be greater than one. Thus in the presence of operators corresponding to conserved currents of the CFT B_μ actually increases compared to μ^2. Therefore exact currents of the CFT which are broken by messenger physics must be avoided for our mechanism to work.[2] Equivalently, in the holomorphic basis, the μ operator is not renormalized, but the conserved current operator now has a negative anomalous dimension leading to an unwanted enhancement of $B\mu$.

This is why we added the coupling $\kappa(Q\bar{Q})^2$ to the superpotential in Eq. (9). The CFT without this coupling has an exact nonanomalous $U(1)$ symmetry under which X carries charge 2, Q and $\bar{Q}$ carry charge -1, and P and $\bar{P}$ carry charge 1. This symmetry is broken by the messenger sector and therefore a coupling of the corresponding current $2X^\dagger X - Q^\dagger Q - \bar{Q}^\dagger \bar{Q} + P^\dagger P + \bar{P}^\dagger \bar{P}$ is generated by the messenger interactions. Since the current is conserved in the CFT, it does not scale away. To fix the problem we must break the $U(1)$ symmetry by strong CFT interactions. This is achieved with the coupling $(Q\bar{Q})^2$ which is relevant for $F < N$.

Having broken all dangerous $U(1)$ symmetries of the CFT we know that all three of k_X, k_Q, k_P renormalize strongly. Unfortunately, there is no known technique for determining their anomalous dimensions. Therefore we cannot determine if this specific model does indeed sequester the B_μ operator. This is a general feature of models which employ our mechanism: since anomalous dimensions must be large, they cannot be computed in perturbation theory and even their sign is unknown. Thus we cannot determine if any particular hidden sector model suppresses B_μ. But we expect that among the large number of possible hidden sector CFTs there are some which have only positive anomalous dimensions for the operators in question. In the appendix we discuss a simple perturbative example for a toy hidden sector (with no supersymmetry breaking) where the anomalous dimensions have the required signs for conformal sequestering of $B\mu$. We also show that conformal field theories with weakly coupled Banks-Zaks fixed points do not have the correct signs for all the relevant anomalous dimensions to be suitable as sequestering hidden sectors.

We now discuss supersymmetry breaking. At the scale m (times renormalization factors from the wave functions of the fields) the hidden sector quarks obtain masses and are integrated out of the theory. The remaining flavorless SQCD theory confines and generates a nonperturbative superpotential which depends on the determinant of the masses of the quarks through the matching of the holomor-

[2]This is similar to the constraints on currents of hidden sector CFTs for conformal sequestering [25].

phic gauge coupling. The particular choice of masses and Yukawa couplings in Eq. (9) ensures that this dynamical superpotential does not depend on X. Therefore the low-energy superpotential for X only involves the linear term $W = \Lambda^2 X$ and supersymmetry is broken spontaneously. The scalar expectation value for X is stabilized at the origin of field space by a nontrivial Kähler potential obtained from integrating out the hidden sector quarks.[3]

What is the resulting pattern of MSSM soft masses? We obtain the masses renormalized at the supersymmetry breaking scale by replacing X with its expectation value $\langle X|_F\rangle = F$. Note that this expectation value is for the holomorphic field X, it is not necessary to perform the wave function rescaling to switch to the canonical X. Soft masses which stem from operators linear in X are then given by $\sim \frac{g^2}{16\pi^2}\frac{F}{M}$. All squark and slepton masses as well as $B\mu$ arise from operators of the form $X^\dagger X$ times visible fields and are therefore negligibly small at the supersymmetry breaking scale. They are regenerated by MSSM renormalization group running from the intermediate scale down to the weak scale. Except for the Higgs soft masses, this spectrum is similar to the one of gaugino mediation [27–30] which has been studied in the literature [29–31]. Renormalization of the Higgs soft masses is different from the renormalization of other soft masses. The difference is that the one-loop diagram of Fig. 3 involving the μ operator of Eq. (2) along with hidden sector interactions generates the operator $\int d^4\theta k_H X^\dagger X H^\dagger H/M^2$, where H stands for either of the MSSM Higgses. The Higgs soft masses squared are then expected to be of the same order as the μ term at the intermediate scale because of this new additive contribution to their renormalization group equations. For details see [22,23].

We close with three comments about our mechanism for solving the μ/B_μ problem. First, our mechanism relies on strong hidden sector interactions which are quite generic in theories of dynamical supersymmetry breaking.

Second, the strong suppression of scalar masses at the intermediate scale also suppresses flavor violation which may have entered the scalar masses from high scale flavor physics. Thus our mechanism for solving the B_μ problem also makes the theory safer from flavor-changing-neutral-current (FCNC) constraints. Interestingly, this allows raising the messenger scale to near the grand unified theory (GUT) or string scale without having to worry about flavor violation from stringy physics.

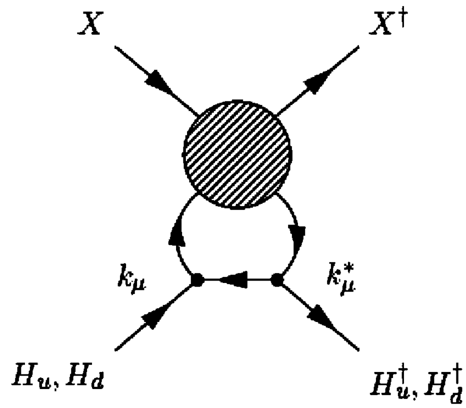

FIG. 3. The μ term renormalizes the Higgs soft masses.

Third, throughout this work we used the holomorphic basis, which is convenient because we are interested in the scaling of the B_μ term relative to the μ term. The soft masses of MSSM superpartners are obtained after replacing the holomorphic superfield X with its F-expectation value. In this basis the wave function renormalization factor Z_X does not enter MSSM superpartner masses. However the Z factor does appear in the relationship between MSSM superpartner masses and the gravitino mass [21,25,32,33]. Since we are relying on strong hidden sector interactions one expects that $Z_X \gg 1$ which makes the gravitino much heavier than the MSSM superpartners.

Summary.—conventional solutions to the μ/B_μ problem in gauge mediation rely either on complicated messenger sectors with tuned parameters and/or on extra light degrees of freedom. In this paper we propose a new solution. Our messenger sector is the simplest that generates μ of the same size as gaugino masses. Assuming a positive sign for the relevant anomalous dimensions, the strong hidden sector interactions suppress the B_μ operator such that $B_\mu \lesssim \mu^2$ at the intermediate scale where the hidden sector interactions end. MSSM interactions below the intermediate scale regenerate B_μ. We presented an explicit example for a hidden sector model which realizes our mechanism.

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Note added.—During completion of the manuscript we learned that Murayama, Nomura, and Poland are developing a similar solution to the μ/B_μ problem [34].

[3] A one-loop diagram with quarks in the loop generates the term $-(X^\dagger X)^2/m^2$ in the Kähler potential which stabilizes the X vev at the origin. However, the theory is strongly coupled at the scale m and we cannot be sure about the sign of the Kähler term beyond the one-loop approximation. A simple way to make this calculation reliable is to add N new flavors of quarks $T + \bar{T}$ and an $N \times N$ matrix of singlets Y with the superpotential $W_T = YT\bar{T} - V^2\,\mathrm{Tr}Y$. This potential forces a complete breaking of the gauge symmetry at the scale $V \gg m$, all newly added fields and the gauge bosons pick up masses of order V, and the low-energy theory of X, Q, $\bar{Q}$, P, $\bar{P}$ is weakly coupled at the scale m so that a perturbative calculation of the X Kähler potential is reliable. Note that we must reduce the number of flavors of Q and P so that the theory remains a strongly coupled CFT above the scale V and the superpotential term $(Q\bar{Q})^2$ remains relevant. Using a maximization [26] and checking the Seiberg dual of our theory we find that the desired fixed point exists for F near $N/2$.

APPENDIX

In this appendix we discuss two examples of perturbative field theories as toy models for the hidden sector and compute the anomalous dimensions which determine whether operators of the form $\frac{1}{M^2} k_B X^\dagger X H_u H_d$ sequester relative to $\frac{1}{M} k_\mu X^\dagger H_u H_d$.

Our first toy hidden sector has only a single chiral superfield X with the superpotential

$$W = \frac{\lambda}{3!} X^3. \tag{A1}$$

In the holomorphic basis k_μ is not renormalized but

$$\frac{d}{dt} k_B = \gamma k_B = \frac{2|\lambda|^2}{16\pi^2} k_B. \tag{A2}$$

Thus the anomalous dimension of k_B is positive as desired, and B_μ is sequestered relative to μ. Of course, this toy hidden sector is not suitable for our mechanism because (ii) Yukawa theories are not strongly coupled over a range of energy scales, thus any sequestering effects are necessarily small and (i) it is too simple to include supersymmetry breaking.

As our second example we consider a conformal theory with a perturbative Banks-Zaks (BZ) fixed point. We will show that BZ fixed point theories necessarily have operators of the form $\frac{1}{M^2} k_B X^\dagger X H_u H_d$ which do not sequester in the IR. Our proof of this statement will depend on the fact that in BZ fixed point theories anomalous dimensions are small so that (i) scaling dimensions of operators are close to free field values and (ii) one-loop anomalous dimensions dominate over higher loops. Neither of these properties apply to the case of strongly coupled CFTs which are needed to generate significant sequestering. Therefore our "no-go" result for BZ theories does not extend to the theories of interest, but BZ theories provide a nice laboratory to study the general mechanism of conformal sequestering.

To start, we note that our BZ theory must contain a gauge-invariant fundamental chiral superfield X so that the μ-term operator $\frac{1}{M} k_\mu X^\dagger H_u H_d$ is invariant and not suppressed by higher powers of the messenger scale M. Since we want operators involving $X^\dagger X$ to sequester, X must interact. Then its scaling dimension $D[X]$ is greater than 1 by unitarity (in the canonical basis). We adopt the canonical basis for the arguments of this example. At weak coupling in a conformal BZ theory all marginal superpotential operators are trilinear in the fields. Hence the only possibility for coupling X to the conformal dynamics is to introduce a superpotential $XQ\bar{Q}$ where Q and $\bar{Q}$ stand for any (not necessarily distinct) chiral superfields which are charged under the BZ gauge group. This coupling gives a positive anomalous dimension to X already at one loop. Therefore operators of the form $X^\dagger X$ must obtain even larger positive anomalous dimensions at one loop in order for them to sequester. We will now show that at least one operator involving $X^\dagger X$ has a vanishing anomalous dimension at one loop.

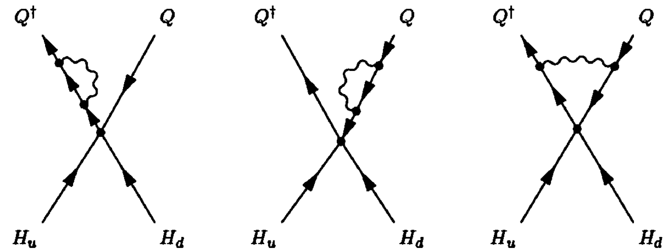

FIG. 4. Operators quadratic in hidden sector fields receive no corrections due to hidden sector gauge interactions at one loop. Current conservation implies that the diagrams above cancel.

First note that in the canonical basis operators quadratic in hidden sector fields (e.g. the B_μ operator) do not receive corrections from hidden sector gauge interactions at one loop. Current conservation at one loop implies that the diagrams in Fig. 4 cancel with each other. We therefore only need to focus on the renormalization due to Yukawa couplings.

If X is charged under a global $U(1)$ symmetry which is unbroken by the Yukawa couplings, then the corresponding current is protected by a nonrenormalization theorem. This current will involve the operator $X^\dagger X$ and therefore the anomalous dimension of the $B\mu$ operator vanishes (at one loop). Thus for sequestering to work we must introduce Yukawa couplings to break any global $U(1)$ symmetry under which X is charged. The absence of a global $U(1)$ symmetry implies that the conformal R symmetry which determines the scaling dimension of X must be uniquely determined from the superpotential of the CFT. But since the superpotential only contains trilinear terms the only possible solution is that the scaling dimensions of the fields

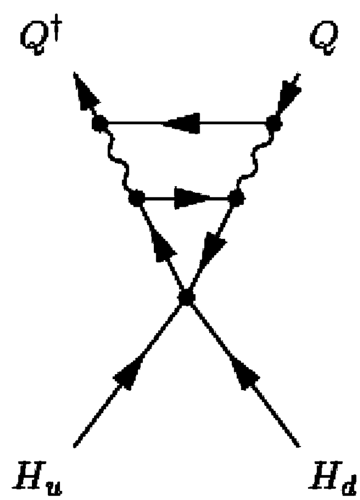

FIG. 5. Operators quadratic in hidden sector fields receive corrections due to hidden sector gauge interactions at two loops.

in these Yukawa couplings are all equal to 1. But then X is a free field by unitarity in contradiction to our initial assumptions.

We conclude that for weakly coupled theories we have a choice: if we insist on conformal symmetry we are forced into allowing global $U(1)$ symmetries of the superpotential leading to no sequestering (at one loop). Alternatively, we can give up conformal symmetry as in our first example.

Let us reiterate that this argument does not carry over to strongly interacting CFTs for two reasons. One is that for strongly coupled CFTs anomalous dimensions are large and superpotential couplings which are higher order in fields may be marginal. Furthermore, gauge interactions are now important to the renormalization. This is because the cancellation in the renormalization of classically conserved currents (Fig. 4) does not extend to higher loops when there are anomalies. For example, diagrams of the form shown in Fig. 5 do lead to sequestering of the bilinear $X^\dagger X$. It was shown in [25] that in strongly coupled IR attractive conformal field theories with no conserved global currents all operators of the form $X^\dagger X$ times MSSM fields have positive anomalous dimensions. For the purposes of solving the $\mu/B\mu$ problem as advocated in this paper we must further demand that these anomalous dimensions are greater than twice the anomalous dimension of X.

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