# $B \to K_1\gamma$ decays in the light-cone QCD sum rules

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(Received 17 March 2008; published 29 May 2008)

We present a detailed study of $B \to K_1(1270)\gamma$ and $B \to K_1(1400)\gamma$ decays. Using the light-cone sum rule technique, we calculate the $B \to K_{1A}(1^3P_1)$ and $B \to K_{1B}(1^1P_1)$ tensor form factors, $T_1^{K_{1A}}(0)$ and $T_1^{K_{1B}}(0)$, where the contributions are included up to the first order in $m_{K_1}/m_b$. We resolve the sign ambiguity of the $K_1(1270)$–$K_1(1400)$ mixing angle $\theta_{K_1}$ by defining the signs of decay constants, $f_{K_{1A}}$ and $f_{K_{1B}}^\perp$. From the comparison of the theoretical calculation and the data for decays $B \to K_1\gamma$ and $\tau^- \to K_1^-(1270)\nu_\tau$, we find that $\theta_{K_1} = -(34 \pm 13)^\circ$ is favored. In contrast to $B \to K^*\gamma$, the hard-spectator contribution suppresses the $B \to K_1(1270)\gamma$ and $B \to K_1(1400)\gamma$ branching ratios slightly. The predicted branching ratios are in agreement with the Belle measurement within the errors. We point out that a more precise measurement for the ratio $R_{K_1} = \mathcal{B}(B \to K_1(1400)\gamma)/\mathcal{B}(B \to K_1(1270)\gamma)$ can offer a better determination for the $\theta_{K_1}$, and consequently the theoretical uncertainties can be reduced.

DOI: 10.1103/PhysRevD.77.094023 PACS numbers: 13.20.He, 12.38.−t, 12.38.Lg, 14.40.Ev

## I. INTRODUCTION

$b \to s\gamma$ decays contain rich phenomenologies relevant to the standard model and new physics. Radiative $B$ decays involving a vector meson have been observed by CLEO, Belle, and *BABAR* [1–3]. Recently, the Belle Collaboration has measured the $B \to K_1\gamma$ decays for the first time [4]:

$$\mathcal{B}(B^- \to K_1^-(1270)\gamma) = (43 \pm 9 \pm 9) \times 10^{-6}, \quad (1.1)$$

$$\mathcal{B}(B^- \to K_1^-(1400)\gamma) < 15 \times 10^{-6}, \quad (1.2)$$

$$\mathcal{B}(\bar{B}^0 \to \bar{K}_1^0(1270)\gamma) < 58 \times 10^{-6}, \quad (1.3)$$

$$\mathcal{B}(\bar{B}^0 \to \bar{K}_1^0(1400)\gamma) < 15 \times 10^{-6}, \quad (1.4)$$

where $K_1$ is the orbitally excited (P-wave) axial-vector meson. The data indicate that $\mathcal{B}(B \to K_1(1270)\gamma) \sim \mathcal{B}(B \to K^*\gamma)$ and $\mathcal{B}(B \to K_1(1270)\gamma) \gg \mathcal{B}(B \to K_1(1400)\gamma)$. It is quite hard to explain the above-mentioned measurements using the existing theoretical calculations [5–10]. Therefore, these measurements represent a challenge for theory. The production of the axial-vector mesons has been seen in the two-body hadronic $D$ decays and in charmful $B$ decays [11]. As for charmless hadronic $B$ decays, $B^0 \to a_1^\pm(1260)\pi^\mp$ are the first modes measured by $B$ factories [12,13]. The *BABAR* Collaboration has recently reported the observation of the decays $\bar{B}^0 \to b_1^\pm\pi^\mp$, $b_1^+K^-$, $B^- \to b_1^0\pi^-$, $b_1^0K^-$, $a_1^0\pi^-$, $a_1^-\pi^0$ [14,15], and $\bar{B}^0 \to K_1^-(1270)\pi^+$, $K_1^-(1400)\pi^+$, $a_1^+K^-$, $B^- \to a_1^-\bar{K}^0$, $f_1(1285)K^-$, $f_1(1420)K^-$ [16]. The related phenomenologies have been studied in the literature [17–23].

In this paper, we will focus on the study of the $B \to K_1\gamma$ decays. The physical states $K_1(1270)$ and $K_1(1400)$ are the mixtures of $1^3P_1$ ($K_{1A}$) and $1^1P_1$ ($K_{1B}$) states. $K_{1A}$ and $K_{1B}$ are not mass eigenstates, and they can be mixed together due to the strange and nonstrange light quark mass difference. Following the convention given in Ref. [24], their relations can be written as

$$\begin{aligned} |\bar{K}_1(1270)\rangle &= |\bar{K}_{1A}\rangle \sin\theta_{K_1} + |\bar{K}_{1B}\rangle \cos\theta_{K_1}, \\ |\bar{K}_1(1400)\rangle &= |\bar{K}_{1A}\rangle \cos\theta_{K_1} - |\bar{K}_{1B}\rangle \sin\theta_{K_1}. \end{aligned} \quad (1.5)$$

In Ref. [24], two possible solutions with twofold ambiguity, $|\theta_{K_1}| \approx 33^\circ$ and $57^\circ$, were obtained. A similar constraint, $35^\circ \lesssim |\theta_{K_1}| \lesssim 55^\circ$, was found in Ref. [25]. From the data of $\tau \to K_1(1270)\nu_\tau$ and $K_1(1400)\nu_\tau$ decays, the mixing angle is extracted to be $\pm 37^\circ$ and $\pm 58^\circ$ in [26]. The sign ambiguity for $\theta_{K_1}$ is due to the fact that one can add arbitrary phases to $|\bar{K}_{1A}\rangle$ and $|\bar{K}_{1B}\rangle$. This sign ambiguity can be removed by fixing the signs for $f_{K_{1A}}$ and $f_{K_{1B}}^\perp$, which do not vanish in the SU(3) limit and are defined by

$$\langle 0|\bar{\psi}\gamma_\mu\gamma_5 s|\bar{K}_{1A}(P,\lambda)\rangle = -if_{K_{1A}} m_{K_{1A}} \epsilon_\mu^{(\lambda)}, \quad (1.6)$$

$$\langle 0|\bar{\psi}\sigma_{\mu\nu} s|\bar{K}_{1B}(P,\lambda)\rangle = if_{K_{1B}}^\perp \epsilon_{\mu\nu\alpha\beta}\epsilon_{(\lambda)}^\alpha P^\beta, \quad (1.7)$$

(with $\psi \equiv u$ or $d$) in the present paper. Following Ref. [27], we adopt the convention $f_{K_{1A}} > 0$, $f_{K_{1B}}^\perp > 0$, and $\epsilon^{0123} = -1$. Thus, the signs of the $\bar{B} \to \bar{K}_{1A,B}$ tensor form factors also depend on the definition mentioned above. See also the discussions after Eq. (5.2).

In the quark model calculation, it was argued that the radiative $B$ decay involving the $K_{1B}$, which is the pure $1^1P_1$ octet state, is forbidden because the effective operator $O_7$ is a spin-flip operator [5]. However, this is not true. Although, in the quark model, the $1^1P_1$ meson is represented as a constituent quark-antiquark pair with total spin $S = 0$ and angular momentum $L = 1$, a real hadron in QCD language should be described in terms of a set of Fock states, for which each state with the same quantum number as the hadron can be represented using light-cone distribution amplitudes (LCDAs). In terms of LCDAs, the leading-twist

 

LCDAs of the $\bar{K}_{1B}$ do not vanish, so that $\bar{B} \to \bar{K}_{1B}$ tensor form factors are not zero. As a matter of fact, due to the $G$ parity, the leading-twist LCDA $\Phi_\perp^{K_{1A}}$ ($\Phi_\parallel^{K_{1B}}$) of the $\bar{K}_{1A}$ ($\bar{K}_{1B}$) meson defined by the nonlocal tensor current (nonlocal axial-vector current) is antisymmetric under the exchange of *quark* and *antiquark* momentum fractions in the SU(3) limit, whereas the $\Phi_\parallel^{K_{1A}}$ ($\Phi_\perp^{K_{1B}}$) is symmetric [27,28]. The above properties were not well recognized in the previous light-cone (LC) sum rule calculation [7,29]. In Ref. [7], the author used only the "symmetrically" asymptotic form for leading-twist distribution amplitudes of the real states $K_1(1270)$ and $K_1(1400)$: $\Phi_\perp^{K_1(1270)}(u) = \Phi_\perp^{K_1(1400)}(u) = 6u\bar{u}$, in the LC sum rule calculation. In Ref. [29], only the $\bar{B} \to \bar{K}_{1B}$ tensor form factor $T_1^{K_{1B}}(0)$ [see Eq. (3.1) for the definition] is computed. The correct forms of LCDAs for the axial-vector mesons have been studied in detail in Ref. [27]. Using the LCDAs in Ref. [27], $B \to K_1\gamma$ decays have recently been investigated in the perturbative QCD (PQCD) approach [30].

In this paper, making use of the LCDAs for the $\bar{K}_{1A}$ and $\bar{K}_{1B}$ in Refs. [27,28], we study the $B \to K_1\gamma$ decays. We compute the relevant $\bar{B} \to \bar{K}_{1A}$ and $\bar{K}_{1B}$ tensor form factors in the LC sum rule approach. The method of LC sum rules has been widely used in the studies of nonperturbative processes, including weak baryon decays [31], heavy meson decays [32], and heavy to light transition form factors [33–35]. We find that the $B \to K_1\gamma$ data favor a negative $\theta_{K_1}$. The more precise estimate can be made through the analysis for the $\tau^- \to K_1^-(1270)\nu_\tau$ data. The predicted branching ratios for $B \to K_1(1270)\gamma$, $K_1(1400)\gamma$ are in agreement with the data within errors.

This paper is organized as follows. In Sec. II, the relevant effective Hamiltonian is given. In Sec. III, we provide the definition of $\bar{B} \to \bar{K}_1$ tensor form factors and then give the formula for the $B \to K_1\gamma$ branching ratios. In Sec. IV we derive the LC sum rules for the relevant tensor form factors, $T_{K_{1A}}$ and $T_{K_{1B}}$. The numerical results and detailed analyses are given in Sec. V. We conclude in Sec. VI. The relevant expressions for two-parton and three-parton LCDAs are collected in Appendixes A and B, respectively.

## II. THE EFFECTIVE HAMILTONIAN

Neglecting doubly Cabibbo-suppressed contributions, the weak effective Hamiltonian relevant to $b \to s\gamma$ is given by

$$\mathcal{H}_{\rm eff}(b \to s\gamma) = \frac{G_F}{\sqrt{2}}\Big\{V_{cb}V_{cs}^*(c_1(\mu)O_1^c(\mu) + c_2(\mu)O_2^c(\mu)) - V_{tb}V_{ts}^*\sum_{i=3}^{8} c_i(\mu)O_i(\mu)\Big\}, \tag{2.1}$$

where

$$\begin{aligned}
O_1^c &= (\bar{c}b)_{V-A}(\bar{s}c)_{V-A},\\
O_2^c &= (\bar{c}_\alpha b_\beta)_{V-A}(\bar{s}_\beta c_\alpha)_{V-A},\\
O_3 &= (\bar{s}b)_{V-A}\sum_q(\bar{q}q)_{V-A},\\
O_4 &= (\bar{s}_\alpha b_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V-A},\\
O_5 &= (\bar{s}b)_{V-A}\sum_q(\bar{q}q)_{V+A},\\
O_6 &= (\bar{s}_\alpha b_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V+A},\\
O_7 &= \frac{em_b}{8\pi^2}\bar{s}_\alpha\sigma^{\mu\nu}(1+\gamma_5)b_\alpha F_{\mu\nu},\\
O_8 &= \frac{g_s m_b}{8\pi^2}\bar{s}_\alpha\sigma^{\mu\nu}(1+\gamma_5)T^a_{\alpha\beta}b_\beta G^a_{\mu\nu},
\end{aligned} \tag{2.2}$$

where $\alpha$, $\beta$ are the $SU(3)$ color indices, $V \pm A$ correspond to $\gamma^\mu(1 \pm \gamma^5)$, and we have neglected corrections due to the $s$-quark mass. We will adopt the next-to-leading order (NLO) Wilson coefficients computed in Ref. [36].

## III. THE FORMULA FOR THE $B \to K_1\gamma$ BRANCHING RATIO

The *penguin* form factors for $B \to K_1$ are defined as follows:

$$\langle\bar{K}_1(p,\lambda)|\bar{s}\sigma_{\mu\nu}\gamma_5 q^\nu b|\bar{B}(p_B)\rangle = 2T_1^{K_1}(q^2)\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}_{(\lambda)}p_B^\rho p^\sigma, \tag{3.1}$$

$$\begin{aligned}
\langle\bar{K}_1(p,\lambda)|\bar{s}\sigma^{\mu\nu}q_\nu b|\bar{B}(p_B)\rangle = {}& -iT_2^{K_1}(q^2)[(m_B^2 - m_{K_1}^2)\epsilon^{*\mu}_{(\lambda)} - (\epsilon^*_{(\lambda)}q)(p+p_B)^\mu]\\
& - iT_3^{K_1}(q^2)(\epsilon^*_{(\lambda)}q)\\
& \times\Big[q^\mu - \frac{q^2}{m_B^2 - m_{K_1}^2}\\
& \times(p+p_B)^\mu\Big],
\end{aligned} \tag{3.2}$$

with

$$T_1^{K_1}(0) = T_2^{K_1}(0), \tag{3.3}$$

where $\bar{K}_1$ can be $\bar{K}_{1A}$ or $\bar{K}_{1B}$ [or $\bar{K}_1(1270)$, $\bar{K}_1(1400)$].

At the next-to-leading order of $\alpha_s$, the branching ratio can be expressed as [9,37,38]

$$\begin{aligned}
\mathcal{B}(B \to K_1\gamma) &= \tau_B\Gamma(B \to K_1\gamma)\\
&= \tau_B\frac{G_F^2\alpha|V_{tb}V_{ts}^*|^2}{32\pi^4}m_{b,\rm pole}^2 m_B^3(T_1^{K_1}(0))^2\\
&\quad\times\Big(1 - \frac{m_{K_1}^2}{m_B^2}\Big)^3|c_7^{(0)\rm eff} + A^{(1)}|^2,
\end{aligned} \tag{3.4}$$

where $m_{b,\rm pole}$ is the pole mass of the $b$ quark, and $\alpha$ is the electromagnetic fine structure constant. The effective coefficient $c_7^{(0)\mathrm{eff}}$ in the naive dimensional regularization (NDR) scheme is defined by $c_7^{(0)\mathrm{eff}} = c_7 - \frac{1}{3}c_5 - c_6$. $A^{(1)}$ can be decomposed as

$$A^{(1)}(\mu) = A^{(1)}_{C_7}(\mu) + A^{(1)}_{\mathrm{ver}}(\mu) + A^{(1)K_1}_{\mathrm{sp}}(\mu_{\mathrm{sp}}), \tag{3.5}$$

where $A^{(1)}_{c_7}$, $A^{(1)}_{\mathrm{ver}}$, which are the NLO corrections due to the Wilson coefficient $c_7^{(0)\mathrm{eff}}$ and in the $b \to s\gamma$ vertex, respectively, and $A^{(1)K_1}_{\mathrm{sp}}$, which is the hard-spectator correction, are given by

$$A^{(1)}_{c_7}(\mu) = \frac{\alpha_s(\mu)}{4\pi} c_7^{(1)\mathrm{eff}}(\mu), \tag{3.6}$$

$$A^{(1)}_{\mathrm{ver}}(\mu) = \frac{\alpha_s(\mu)}{4\pi}\left\{\frac{32}{81}[13c_1^{(0)}(\mu) - 9c_8^{(0)\mathrm{eff}}(\mu)]\ln\frac{\bar{m}_b}{\mu} + \frac{4}{27} \times (33 - 2\pi^2 + 6\pi i)c_8^{(0)\mathrm{eff}}(\mu) + r_2(z)c_1^{(0)}(\mu)\right\}, \tag{3.7}$$

$$A^{(1)K_1}_{\mathrm{sp}}(\mu_{\mathrm{sp}}) = \frac{\pi\alpha_s(\mu_{\mathrm{sp}})C_F}{3N_c}\frac{f_B f^{\perp}_{K_1}\lambda_B^{-1}}{m_B T_1^{K_1}(0)}\left\{c_8^{(0)\mathrm{eff}}(\mu_{\mathrm{sp}}) \times \langle u^{-1}\rangle^{(K_1)}_{\perp} - c_1^{(0)}(\mu_{\mathrm{sp}})\left\langle\frac{\Delta i_5(z_0^{(c)}, 0, 0)}{\bar{u}}\right\rangle_{\perp}\right\}. \tag{3.8}$$

Here $c_8^{\mathrm{eff}} = c_8 + c_5$, $m_B/\lambda_B$ describes the first negative moment of the $B$-meson distribution amplitude $\Phi_{B1}$ [38,39], and

$$\langle u^{-1}\rangle^{(K_1)}_{\perp} \equiv \int_0^1 du \frac{\Phi^{K_1}_{\perp}(u)}{u}, \tag{3.9}$$

$$\left\langle\frac{\Delta i_5(z_0^{(c)}, 0, 0)}{\bar{u}}\right\rangle_{\perp} \equiv \int_0^1 du \frac{\Delta i_5(z_0^{(c)}, 0, 0)}{\bar{u}}\Phi^{K_1}_{\perp}(u), \tag{3.10}$$

with $z = (\bar{m}_c/\bar{m}_b)^2$ and $z_0^{(c)} \simeq m_B^2\bar{u}/\bar{m}_c^2$, where $\bar{m}_c \equiv \bar{m}_c(\bar{m}_c)$ and $\bar{m}_b \equiv \bar{m}_b(\bar{m}_b)$ are the $\overline{\mathrm{MS}}$ $c$- and $b$-quark masses, respectively. The detailed definitions of the functions $r_2(z)$ and $\Delta i_5(z_0^{(c)}, 0, 0)$ can be found in Refs. [36,37]. In the numerical calculation, we set the scale for the vertex corrections to be $\mu = \bar{m}_b$ and the scale for the spectator interactions to be $\mu_{sp} = \sqrt{\Lambda_h \bar{m}_b}$, where $\Lambda_h \simeq 0.5$ GeV corresponds to the hadronic scale.

## IV. THE LIGHT-CONE SUM RULE FOR $T_1^{K_1}$

To calculate the form factor $T_1^{K_1}$, we consider the two-point correlation function, which is sandwiched between the vacuum and transverse polarized $K_1$ meson,

$$\begin{aligned} i\int d^4x e^{iqx}\langle\bar{K}_1(P,\perp)|T[\bar{s}(x)\sigma_{\mu\nu}b(x)j_B^{\dagger}(0)]|0\rangle &= -i\mathbb{A}(p_B^2, q^2)\{\epsilon^{*(\perp)}_{\mu}(2P+q)_{\nu} - \epsilon^{*(\perp)}_{\nu}(2P+q)_{\mu}\} \\ &+ i\mathbb{B}(p_B^2, q^2)\{\epsilon^{*(\perp)}_{\mu}q_{\nu} - \epsilon^{*(\perp)}_{\nu}q_{\mu}\} \\ &+ 2i\mathbb{C}(p_B^2, q^2)\frac{\epsilon^{*(\perp)}q}{m_B^2 - m_{K_1}^2}\{P_{\mu}q_{\nu} - q_{\mu}P_{\nu}\}, \end{aligned} \tag{4.1}$$

where $j_B = i\bar{\psi}\gamma_5 b$ (with $\psi \equiv u$ or $d$) is the interpolating current for the $B$ meson, $p_B^2 = (P+q)^2$, and $P$ is the momentum of the $K_1$ meson. Note that in this section $K_1 \equiv K_{1A}$ or $K_{1B}$. $\mathbb{A}$ is the only relevant term in the present study, and at the hadron level it can be written in the form

$$\mathbb{A}(p_B^2, q^2) = T_1^{K_1}(q^2)\cdot\frac{1}{m_B^2 - p_B^2}\cdot\frac{m_B^2 f_B}{m_b} + \cdots, \tag{4.2}$$

where the dots denote contributions that have poles $p_B^2 = m_{B^*}^2$ with $m_{B^*}$ being the masses of the higher resonance $B^*$ mesons. To obtain the result for $\mathbb{A}$, we have taken into account here the transverse polarized $K_1$, instead of its longitudinal component, because for the longitudinal $K_1$, $\mathbb{A}$ mixes with $\mathbb{B}$ and $\mathbb{C}$ for an energetic $K_1$.

In a region of sufficiently large virtualities, $m_b^2 - p_B^2 \gg \Lambda_{\mathrm{QCD}}m_b$, with a small $q^2 \geq 0$, the operator product expansion is applicable in Eq. (4.1), so that in QCD for an energetic $K_1$ meson the correlation function in Eq. (4.1) can be represented in terms of the LCDAs of the $K_1$ meson:

$$\begin{aligned} &i\int d^4x e^{iqx}\langle\bar{K}_1(P,\perp)|T[\bar{s}(x)\sigma_{\mu\nu}b(x)j_B^{\dagger}(0)]|0\rangle \\ &= \int_0^1 \frac{-i}{(q+k)^2 - m_b^2}\mathrm{Tr}[\sigma_{\mu\nu}(\not{q} + \not{k} + m_b)\gamma_5 M_{\perp}^{K_1}]|_{k=uEn_-}du \\ &+ \frac{1}{4}\int_0^1 dv\int_0^1 D\underline{\alpha} \\ &\times\frac{2vE^2(n_-q)(f^A_{3K_1}\mathcal{A}(\underline{\alpha}) + f^V_{3K_1}\mathcal{V}(\underline{\alpha}))\mathrm{Tr}(\sigma_{\mu\nu}\not{\epsilon}^*_{(\perp)}\not{n}_-)}{\{m_b^2 - [q + (\alpha_1 + \alpha_g v)En_-]^2\}^2} \\ &+ \mathcal{O}\left(\frac{m_{K_1}^2}{E^2}\right), \end{aligned} \tag{4.3}$$

where $f^A_{3K_1} \sim \mathcal{O}(f_{K_1}m_{K_1})$, $f^V_{3K_1} \sim \mathcal{O}(f_{K_1}m_{K_1})$, $E = |\vec{P}|$, $P^{\mu} = En_-^{\mu} + m_{K_1}^2 n_+^{\mu}/(4E) \simeq En_-^{\mu}$ with two lightlike vectors satisfying $n_-n_+ = 2$ and $n_-^2 = n_+^2 = 0$. Here $E \sim m_b$, and we have assigned the momentum of the $s$ quark in the $K_1$ meson to be

$$k^{\mu} = uEn_-^{\mu} + k_{\perp}^{\mu} + \frac{k_{\perp}^2}{4uE}n_+^{\mu}, \tag{4.4}$$

where $k_{\perp}$ is of order $\Lambda_{\mathrm{QCD}}$. In Eq. (4.3), in calculating contributions due to the two-parton LCDAs of the $\bar{K}_1$ in the momentum space, we have used the following substitution for the Fourier transform of $\langle\bar{K}_1(P,\perp)|\bar{s}_{\alpha}(x)\psi_{\delta}(0)|0\rangle$,

$$x^\mu \to -i\frac{\partial}{\partial k_\mu} \simeq -i\left(\frac{n_+^\mu}{2E}\frac{\partial}{\partial u} + \frac{\partial}{\partial k_{\perp\mu}}\right), \tag{4.5}$$

where the term of order $k_\perp^2$ is omitted. Thus, we can obtain the light-cone transverse projection operator $M_\perp^{K_1}$ of the $\bar{K}_1$ meson in the momentum space:

$$\begin{aligned} M_\perp^{K_1} = {}& i\frac{f_{K_1}^\perp}{4}E\not\epsilon_\perp^{*(\lambda)}\not n_-\gamma_5\Phi_\perp(u) - i\frac{f_{K_1}m_{K_1}}{4}\bigg\{\not\epsilon_\perp^{*(\lambda)}\gamma_5 g_\perp^{(a)}(u) \\ & - E\int_0^u dv\Phi_a(v)\not n_-\gamma_5\epsilon_{\perp\mu}^{*(\lambda)}\frac{\partial}{\partial k_{\perp\mu}} \\ & + i\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\epsilon_\perp^{*(\lambda)\nu}n_-^\rho\bigg[n_+^\sigma\frac{g_\perp^{(v)\prime}(u)}{8} \\ & - E\frac{g_\perp^{(v)}(u)}{4}\frac{\partial}{\partial k_{\perp\sigma}}\bigg]\bigg\}\bigg|_{k=up} + \mathcal{O}\left(\frac{m_{K_1}^2}{E^2}\right), \end{aligned} \tag{4.6}$$

where $\Phi_a \equiv \Phi_\parallel - g_\perp^{(a)}$ and the detailed definitions for the relevant two-parton LCDAs are collected in Appendix A. A similar discussion for the vector meson projection operators can be found in Ref. [40]. From the expansion of the transverse projection operator, one can find that contributions arising from $\Phi_a$, , $g_\perp^{(v)\prime}$, and $g_\perp^{(v)}$ are suppressed by $m_{K_1}/E$ as compared with that from $\Phi_\perp$. Note that in Eq. (4.3) the derivative with respect to the transverse momentum acts on the hard scattering amplitude before the collinear approximation is taken. The three-parton chiral-even distribution amplitudes of twist 3, $\mathcal{A}(\underline{\alpha})$ and $\mathcal{V}(\underline{\alpha})$, together with their decay constants, $f_{3K_1}^A$ and $f_{3K_1}^V$, are defined by

$$\begin{aligned} &\langle\bar{K}_1(P,\lambda)|\bar{s}(x)\gamma_\alpha\gamma_5 g_s G_{\mu\nu}(vx)\psi(0)|0\rangle \\ &\quad = p_\alpha[p_\nu\epsilon_{\perp\mu}^{*(\lambda)} - p_\mu\epsilon_{\perp\nu}^{*(\lambda)}]f_{3K_1}^A\mathcal{A}(v,-px) + \cdots, \end{aligned} \tag{4.7}$$

$$\begin{aligned} &\langle\bar{K}_1(P,\lambda)|\bar{s}(x)\gamma_\alpha g_s\tilde{G}_{\mu\nu}(vx)\psi(0)|0\rangle \\ &\quad = ip_\alpha[p_\mu\epsilon_{\perp\nu}^{*(\lambda)} - p_\nu\epsilon_{\perp\mu}^{*(\lambda)}]f_{3K_1}^V\mathcal{V}(v,-px) + \cdots, \end{aligned} \tag{4.8}$$

where we have set $p_\mu = P_\mu - m_{K_1}^2\bar{z}_\mu/(2P\bar{z})$ with

$$\bar{z}_\mu = x_\mu - \frac{P_\mu}{m_{K_1}^2}\{xP - [(xP)^2 - x^2m_{K_1}^2]^{1/2}\}.$$

Here the ellipses stand for terms of twist higher than 3, the following shorthand notations are used,

$$\mathcal{A}(v,-px) \equiv \int\mathcal{D}\underline{\alpha}e^{ipx(\alpha_1+v\alpha_g)}\mathcal{A}(\underline{\alpha}), \tag{4.9}$$

etc., and the integration measure is defined as

$$\int\mathcal{D}\underline{\alpha} \equiv \int_0^1 d\alpha_1\int_0^1 d\alpha_2\int_0^1 d\alpha_g\delta\left(1-\sum\alpha_i\right), \tag{4.10}$$

with $\alpha_1$, $\alpha_2$, $\alpha_g$ being the momentum fractions carried by the $s$ quark, $\bar{\psi}(\equiv\bar{u}$ or $\bar{d})$ quark, and gluon, respectively. At the quark-gluon level, after performing the integration of Eq. (4.3), the result for $\mathbb{A}^{\rm QCD}$ reads (with $\bar{u} = 1-u$)

$$\begin{aligned} \mathbb{A}^{\rm QCD} = {}& -\frac{m_b f_{K_1}^\perp}{2}\int_0^1 du\bigg\{\frac{1}{m_b^2 - up_B^2 - \bar{u}q^2}\bigg[\Phi^\perp(u) - \frac{m_{K_1}f_{K_1}}{m_b f_{K_1}^\perp}\left(ug_\perp^{(a)}(u) + \Phi_a(u) + \frac{g_\perp^{(v)}(u)}{4} - \frac{g_\perp^{(v)\prime}(u)}{4}\frac{p_B^2+q^2}{p_B^2-q^2}\right)\bigg] \\ & - \frac{m_{K_1}f_{K_1}}{4m_b f_{K_1}^\perp}\frac{(m_b^2+q^2)}{(m_b^2-up_B^2-\bar{u}q^2)^2}g_\perp^{(v)}(u)\bigg\} - \int_0^1 vdv\int_0^1 D\underline{\alpha}\frac{f_{3K_1}^A\mathcal{A}(\underline{\alpha}) + f_{3K_1}^V\mathcal{V}(\underline{\alpha})}{2(\alpha_1+v\alpha_g)} \\ & \times\bigg[\frac{1}{m_b^2-(\alpha_1+v\alpha_g)(p_B^2-q^2)-q^2} - \frac{m_b^2-q^2}{[m_b^2-(\alpha_1+v\alpha_g)(p_B^2-q^2)-q^2]^2}\bigg]. \end{aligned} \tag{4.11}$$

We have given the results of $\mathbb{A}$ from the hadron and quark-gluon points of view, respectively. Thus, the contribution due to the lowest-lying $K_1$ meson can be further approximated with the help of quark-hadron duality:

$$T_1^{K_1}(q^2)\cdot\frac{1}{m_B^2-p_B^2}\cdot\frac{m_B^2f_B}{m_b} = \frac{1}{\pi}\int_{m_b^2}^{s_0}\frac{{\rm Im}\mathbb{A}^{\rm QCD}(s,q^2)}{s-p_B^2}ds, \tag{4.12}$$

where $s_0$ is the excited state threshold. After applying the Borel transform $p_B^2 \to M^2$ to the above equation, we obtain

$$T_1^{K_1}(q^2) = \frac{m_b}{m_B^2f_B}e^{-m_B^2/M^2}\frac{1}{\pi}\int_{m_b^2}^{s_0}e^{s/M^2}{\rm Im}\mathbb{A}^{\rm QCD}(s,q^2)ds. \tag{4.13}$$

Finally, the light-cone sum rule for $T_1^{K_1}$ reads

$$
\begin{aligned}
T_1^{K_1}(q^2) = &-\frac{m_b^2 f_{K_1}^\perp}{2m_B^2 f_B} e^{(m_B^2-m_b^2)/M^2}\int_0^1 du\bigg\{\frac{1}{u}e^{\bar u(q^2-m_b^2)/(uM^2)}\theta[c(u,s_0)]\bigg[\Phi^\perp(u)-\frac{m_{K_1}f_{K_1}}{m_b f_{K_1}^\perp}\bigg(ug_\perp^{(a)}(u)+\Phi_a(u)+\frac{g_\perp^{(v)}(u)}{4}\\
&-\frac{g_\perp^{(v)\prime}(u)}{4}\frac{m_b^2+(u-\bar u)q^2}{m_b^2-q^2}\bigg)\bigg]-\frac{1}{u}e^{\bar u(q^2-m_b^2)/(uM^2)}\frac{1}{4}\frac{m_{K_1}f_{K_1}}{m_b f_{K_1}^\perp}(m_b^2+q^2)g_\perp^{(v)}(u)\bigg(\frac{\theta[c(u,s_0)]}{uM^2}+\delta[c(u,s_0)]\bigg)\\
&-\frac{m_{K_1}f_{K_1}}{m_b f_{K_1}^\perp}\frac{g_\perp^{(v)\prime}(u)}{2}\frac{q^2}{m_b^2-q^2}e^{(m_b^2-q^2)/M^2}\bigg\}\\
&-\frac{m_b}{2m_B^2 f_B}e^{(m_B^2-m_b^2)/M^2}\int_0^1 vdv\int_0^1 D\underline{\alpha}\frac{f_{3K_1}^A\mathcal{A}(\underline{\alpha})+f_{3K_1}^V\mathcal{V}(\underline{\alpha})}{(\alpha_1+v\alpha_g)^2}e^{(1-\alpha_1-v\alpha_g)(q^2-m_b^2)/[(\alpha_1+v\alpha_g)M^2]}\\
&\times\bigg\{\theta[c(\alpha_1+v\alpha_g,s_0)]-(m_b^2-q^2)\bigg(\frac{\theta[c(\alpha_1+v\alpha_g,s_0)]}{(\alpha_1+v\alpha_g)M^2}+\delta[c(\alpha_1+v\alpha_g,s_0)]\bigg)\bigg\},
\end{aligned}
\tag{4.14}
$$

where $c(u, s_0) = us_0 - m_b^2 + (1-u)q^2$ and $\theta[\cdots]$ is the step function. Note that here $f_{K_{1A}}^\perp$ is chosen to be $f_{K_{1A}}$, while $f_{K_{1B}}$ is adopted to be $f_{K_{1B}}^\perp(1\text{ GeV})$. [See Eq. (A4) and related discussions.]

## V. RESULTS

### A. $T_1^{K_{1A}}$ and $T_1^{K_{1B}}$ LCSR results and $B \to K_1\gamma$ branching ratios

Parameters relevant to the present study are collected in Table I. We first analyze the $T_1(0)$ sum rules numerically. The pole $b$ quark mass is adopted in the LC sum rule. The $f_{K_1}^\perp$ and parameters appearing in the distribution amplitudes are evaluated at the factorization scale $\mu_f = \sqrt{m_B^2 - m_{b,\text{pole}}^2}$. On the other hand, the form factor $T_1(0)$ depends on the renormalization scale of the effective Hamiltonian, for which the scale is set to be $\bar m_b(\bar m_b)$. The working Borel window is 7.0 GeV$^2 < M^2 <$ 13.0 GeV$^2$, where the correction originating from higher resonance states amounts to 15% to 35%. We do not include the contributions of the twist-4 LCDAs and three-parton twist-3 chiral-even LCDAs in the light-cone sum rule since these corrections to light-cone expansion series is of order $(m_{K_1}/m_b)^2$ and might be negligible. The excited state threshold $s_0$ can be determined when the most stable plateau of the LC sum rule result is obtained within the Borel window. We find that the corresponding threshold $s_0$ lies in the interval 32–36 GeV$^2$.

Two remarks are in order. First, we have consistently used $f_B = 190 \pm 10$ MeV in all numerical analyses. In the literature, it was *assumed* that the theoretical errors due to the radiative corrections in the form factor sum rules can be canceled if one adopts the $f_B$ sum rule result with the same order of $\alpha_s$ corrections in the calculation [34,35]. Nevertheless, the resulting sum rule result for $T_1^{BK^*}(0)$ seems to be significantly larger than the estimate extracted from the data [37], although the sum rule result can be improved by including $\alpha_s$ corrections [35]. We have checked that, using the physical value of $f_B$, that we adopt here, in the $T_1^{BK^*}(0)$ LC sum rule with the same order in $\alpha_s$ and $m_{K_1}/m_b$, we get $T_1^{BK^*}(0) \approx 0.25^{+0.03}_{-0.02}$ which is in good agreement with the result constrained by the data [37,41]. Extracting from the data, the current estimation is $T_1^{BK^*}(0) = 0.267 \pm 0.018$ [41]. The lattice QCD result is $T_1^{BK^*}(0) = 0.24 \pm 0.03^{+0.04}_{-0.01}$ [42]. Therefore, although the radiative corrections can be important in the form factor sum rule calculations, its effects are significantly reduced and may be negligible in the present analysis. Second, $a_1^{\parallel,K_{1A}}$, $a_0^{\perp,K_{1A}}$, $a_2^{\perp,K_{1A}}$, $a_0^{\parallel,K_{1B}}$, $a_2^{\parallel,K_{1B}}$, and $a_1^{\perp,K_{1B}}$ are $G$-parity violating Gegenbaur moments, which vanish in the SU(3) limit. Using the QCD sum rules, the relation $a_0^{\perp,K_{1A}} + (0.59 \pm 0.15)a_0^{\parallel,K_{1B}} = 0.17 \pm 0.11$ was obtained, instead of their individual values [27]. It will be seen later that, due to the data for $\mathcal{B}(B \to K_1(1270)\gamma) \gg \mathcal{B}(B \to K_1(1400)\gamma)$ and for $\tau^- \to K_1^-(1270)\nu_\tau$, $\theta_{K_1}$ and $a_0^{\parallel,K_{1B}}$ should be negative. Here we further make reasonable assumptions that $|a_0^{\parallel,K_{1B}} f_{K_{1B}}| \le 30\% \times f_{K_{1B}}^\perp$ and $|a_0^{\perp,K_{1A}} f_{K_{1A}}^\perp|(1\text{ GeV}) \le 30\% \times f_{K_{1A}}$ to account for the possible SU(3) breaking effect; i.e., we assume the $G$-parity correction is roughly less than 30%. [See Eqs. (5.3), (5.4), (5.5), and (5.6) for the detailed definitions of parameters.] Finally, we arrive at $a_0^{\parallel,K_{1B}} = -0.15 \pm 0.15$ and $a_0^{\perp,K_{1A}} = 0.26^{+0.04}_{-0.22}$. As shown in Table I, once these two parameters are determined, the remaining $G$-parity violating Gegenbaur moments are thus updated according to the relations given in Eq. (141) in Ref. [27].

To illustrate the qualities and uncertainties of the sum rules, we plot the results for $T_1^{K_{1A}}(0)$ and $T_1^{K_{1B}}(0)$ as functions of $M^2$ in Fig. 1. We obtain

$$
\begin{aligned}
T_1^{K_{1A}}(0) &= 0.31^{+0.06+0.01+0.06}_{-0.04-0.01-0.03},\\
T_1^{K_{1B}}(0) &= -(0.25^{+0.03+0.01+0.05}_{-0.02-0.01-0.07}),
\end{aligned}
\tag{5.1}
$$

where the first, second, and third error bars come from the variations of $m_{b,\text{pole}}$, $f_B$, and the remaining parameters, respectively. The third errors are mainly due to the $G$-parity violating Gegenbaur moments of the leading-

TABLE I. Summary of input parameters [11,27,39].

Running quark masses (GeV), pole $b$-quark mass (GeV), and couplings

| $\bar{m}_c(\bar{m}_c)$ | $m_s(2\text{ GeV})$ | $\bar{m}_b(\bar{m}_b)$ | $m_{b,\text{pole}}$ | $\alpha_s(m_Z)$ | $\alpha$ |
|---|---|---|---|---|---|
| $1.25 \pm 0.10$ | $0.09 \pm 0.01$ | $4.25 \pm 0.15$ | $4.90 \pm 0.05$ | $0.1176$ | $1/137$ |

CKM matrix elements and the moment of the $B$ distribution amplitude

| $\vert V_{cs}\vert$ | $\vert V_{cb}\vert$ | $\lambda_B$ |
|---|---|---|
| $0.957 \pm 0.095$ | $(41.6 \pm 0.6) \times 10^{-3}$ | $(0.35 \pm 0.15)$ GeV |

Masses (GeV) and decay constants (MeV) for mesons

| $m_{K_{1A}}$ | $m_{K_{1B}}$ | $f_{K_{1A}}$ | $f^{\perp}_{K_{1B}}(1\text{ GeV})$ | $f_B$ |
|---|---|---|---|---|
| $1.31 \pm 0.06$ | $1.34 \pm 0.08$ | $250 \pm 13$ | $190 \pm 10$ | $190 \pm 10$ |

Gegenbaur moments for the $K_{1A}$ meson at scales 1 GeV and 2.2 GeV (in parentheses)

| $a_1^{\parallel,K_{1A}}$ | $a_2^{\parallel,K_{1A}}$ | $a_0^{\perp,K_{1A}}$ | $a_1^{\perp,K_{1A}}$ | $a_2^{\perp,K_{1A}}$ |
|---|---|---|---|---|
| $-0.30^{+0.26}_{-0.00}$ | $-0.05 \pm 0.03$ | $0.26^{+0.03}_{-0.22}$ | $-1.08 \pm 0.48$ | $0.02 \pm 0.20$ |
| $(-0.24^{+0.21}_{-0.00})$ | $(-0.04 \pm 0.02)$ | $(0.24^{+0.03}_{-0.21})$ | $(-0.84 \pm 0.37)$ | $(0.01 \pm 0.15)$ |

Gegenbaur moments for the $K_{1B}$ meson at scales 1 GeV and 2.2 GeV (in parentheses)

| $a_0^{\parallel,K_{1B}}$ | $a_1^{\parallel,K_{1B}}$ | $a_2^{\parallel,K_{1B}}$ | $a_1^{\perp,K_{1B}}$ | $a_2^{\perp,K_{1B}}$ |
|---|---|---|---|---|
| $-0.15 \pm 0.15$ | $-1.95 \pm 0.45$ | $0.09^{+0.16}_{-0.18}$ | $0.30^{+0.00}_{-0.31}$ | $-0.02 \pm 0.22$ |
| $(-0.15 \pm 0.15)$ | $(-1.56 \pm 0.36)$ | $(0.06^{+0.11}_{-0.13})$ | $(0.25^{+0.00}_{-0.26})$ | $(-0.02 \pm 0.17)$ |

Parameters of twist-3 three-parton LCDAs of the $K_{1A}$ meson at the scale 2.2 GeV

| $f^V_{3,K_{1A}}$ (in GeV$^2$) | $\omega^V_{K_{1A}}$ | $\sigma^V_{K_{1A}}$ | $f^A_{3,K_{1A}}$ (in GeV$^2$) | $\lambda^A_{K_{1A}}$ | $\sigma^A_{K_{1A}}$ |
|---|---|---|---|---|---|
| $0.0034 \pm 0.0018$ | $-3.1 \pm 1.1$ | $-0.13 \pm 0.16$ | $0.0014 \pm 0.0007$ | $0.70 \pm 0.46$ | $2.4 \pm 2.0$ |

Parameters of twist-3 three-parton LCDAs of the $K_{1B}$ meson at the scale 2.2 GeV

| $f^V_{3,K_{1B}}$ (in GeV$^2$) | $\lambda^V_{K_{1B}}$ | $\sigma^V_{K_{1B}}$ | $f^A_{3,K_{1B}}$ (in GeV$^2$) | $\omega^A_{K_{1B}}$ | $\sigma^A_{K_{1B}}$ |
|---|---|---|---|---|---|
| $0.0029 \pm 0.0012$ | $0.09 \pm 0.24$ | $0.31 \pm 0.68$ | $-0.0041 \pm 0.0018$ | $-1.7 \pm 0.4$ | $-0.05 \pm 0.04$ |

twist LCDAs. Corrections arising from the three-parton LCDAs are less than 3%.

In calculating the $B \to K_1(1270)\gamma$ and $K_1(1400)\gamma$ branching ratios, $B \to K_1$ tensor form factors have the expressions

$$T_1^{K_1(1270)}(0) = T_1^{K_{1A}}(0)\sin\theta_{K_1} + T_1^{K_{1B}}(0)\cos\theta_{K_1},$$
$$T_1^{K_1(1400)}(0) = T_1^{K_{1A}}(0)\cos\theta_{K_1} - T_1^{K_{1B}}(0)\sin\theta_{K_1}. \quad (5.2)$$

From Eq. (4.14), we know that $T_1^{K_{1A}}$ and $T_1^{K_{1B}}$ depend on the

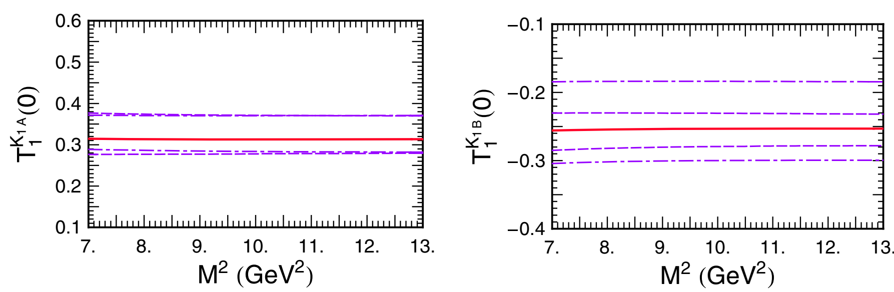


FIG. 1 (color online). $T_1^{K_{1A}}(0)$ and $T_1^{K_{1B}}(0)$ as functions of the Borel mass squared, where the central values of input parameters have been used in the solid curve. The dashed (dot-dashed) curves are for variation of the $m_{b,\text{pole}}$ (parameters for LCDAs) with the central values of the remaining theoretical parameters.

definition of the signs of $f_{K_{1A}}$ and $f^{\perp}_{K_{1B}}$, so that the resultant $\theta_{K_1}$ also depends on the signs of $f_{K_{1A}}$ and $f^{\perp}_{K_{1B}}$.

As for the relevant physical properties of $\bar{K}_1$ mesons, we have

$$\begin{aligned}\langle 0|\bar\psi\gamma_\mu\gamma_5 s|\bar K_1(1270)(P,\lambda)\rangle &= -if_{K_1(1270)}m_{K_1(1270)}\epsilon_\mu^{(\lambda)}\\ &= -i(f_{K_{1A}}m_{K_{1A}}\sin\theta_{K_1}\\ &\quad + f_{K_{1B}}m_{K_{1B}}a_0^{\parallel,K_{1B}}\cos\theta_{K_1})\epsilon_\mu^{(\lambda)},\end{aligned} \tag{5.3}$$

$$\begin{aligned}\langle 0|\bar\psi\gamma_\mu\gamma_5 s|\bar K_1(1400)(P,\lambda)\rangle &= -if_{K_1(1400)}m_{K_1(1400)}\epsilon_\mu^{(\lambda)}\\ &= -i(f_{K_{1A}}m_{K_{1A}}\cos\theta_{K_1}\\ &\quad - f_{K_{1B}}m_{K_{1B}}a_0^{\parallel,K_{1B}}\sin\theta_{K_1})\epsilon_\mu^{(\lambda)},\end{aligned} \tag{5.4}$$

$$\begin{aligned}\langle 0|\bar\psi\sigma_{\mu\nu} s|\bar K_1(1270)(P,\lambda)\rangle &= if^{\perp}_{K_1(1270)}\epsilon_{\mu\nu\alpha\beta}\epsilon^\alpha_{(\lambda)}P^\beta\\ &= i(f^{\perp}_{K_{1A}}a_0^{\perp,K_{1A}}\sin\theta_K\\ &\quad + f^{\perp}_{K_{1B}}\cos\theta_K)\epsilon_{\mu\nu\alpha\beta}\epsilon^\alpha_{(\lambda)}P^\beta,\end{aligned} \tag{5.5}$$

and

$$\begin{aligned}\langle 0|\bar\psi\sigma_{\mu\nu} s|\bar K_1(1400)(P,\lambda)\rangle &= if^{\perp}_{K_1(1400)}\epsilon_{\mu\nu\alpha\beta}\epsilon^\alpha_{(\lambda)}P^\beta\\ &= i(f^{\perp}_{K_{1A}}a_0^{\perp,K_{1A}}\cos\theta_K\\ &\quad - f^{\perp}_{K_{1B}}\sin\theta_K)\epsilon_{\mu\nu\alpha\beta}\epsilon^\alpha_{(\lambda)}P^\beta,\end{aligned} \tag{5.6}$$

where the values of $f_{K_{1A}}$, $f^{\perp}_{K_{1B}}$, $m_{K_{1A}}$, $m_{K_{1B}}$, $a_0^{\parallel,K_{1B}}$, and $a_0^{\perp,K_{1A}}$ are given in Table I, and use of $f_{K_{1B}} = f^{\perp}_{K_{1B}}(1\text{ GeV})$ and $f^{\perp}_{K_{1A}} = f^{\parallel}_{K_{1A}}$ is made in the present study. Following this definition, $a_0^{\parallel,K_{1B}}$ and $a_0^{\perp,K_{1A}}$ vanish in the SU (3) limit, and we have the relations

$$\Phi_\perp^{K_1(1270)}(u) = \frac{f^{\perp}_{K_{1A}}}{f^{\perp}_{K_1(1270)}}\Phi_\perp^{K_{1A}}(u)\sin\theta_{K_1} + \frac{f^{\perp}_{K_{1B}}}{f^{\perp}_{K_1(1270)}}\Phi_\perp^{K_{1B}}(u)\cos\theta_{K_1}, \tag{5.7}$$

$$\Phi_\perp^{K_1(1400)}(u) = \frac{f^{\perp}_{K_{1A}}}{f^{\perp}_{K_1(1400)}}\Phi_\perp^{K_{1A}}(u)\cos\theta_{K_1} - \frac{f^{\perp}_{K_{1B}}}{f^{\perp}_{K_1(1400)}}\Phi_\perp^{K_{1B}}(u)\sin\theta_{K_1}. \tag{5.8}$$

In Fig. 2 we plot the branching ratios of $B^- \to K_1^-(1270)\gamma$ and $B^- \to K_1^-(1400)\gamma$ as functions of $\theta_{K_1}$. The mixing angle dependence of the $K_1^-(1270)\gamma$ mode is opposite to that of the $K_1^-(1400)\gamma$ mode. To satisfy the observable $\mathcal{B}(B\to K_1(1270)\gamma) \gg \mathcal{B}(B\to K_1(1400)\gamma)$, we find that

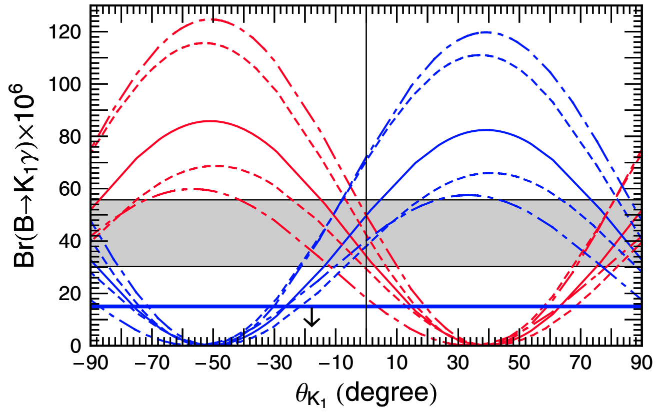


FIG. 2 (color online). Branching ratios as functions of the mixing angle $\theta_{K_1}$. The upper five (red) curves at $\theta_{K_1} = -50°$ are for the $K_1(1270)\gamma$ mode, and the lower five (blue) curves for the $K_1(1400)\gamma$ mode. The solid curves correspond to central values of the input parameters. The dot-dashed and dashed curves denote the theoretical uncertainties due to the parameters of LCDAs and $m_{b,\text{pole}}$, respectively. The horizontal line is the experimental limit on $B\to K_1(1400)\gamma$, and the horizontal band shows the experimental result for the $K_1(1270)\gamma$ mode with its $1\sigma$ error.

the sign of $\theta_{K_1}$ should be negative. The further constraint for $\theta_{K_1}$ can be obtained from the $\tau^- \to K_1^-(1270)\nu_\tau$ analysis.

### B. The constraint for $\theta_{K_1}$ from the $\tau^- \to K_1^-(1270)\nu_\tau$ data

The decay constant $f_{K_1(1270)}$ can be extracted from the measurement $\tau^- \to K_1^-(1270)\nu_\tau$ by ALEPH [43]: $\mathcal{B}(\tau^- \to K_1^-(1270)\nu_\tau) = (4.7 \pm 1.1)\times 10^{-3}$, where the formula for the decay rate is given by

$$\Gamma(\tau\to K_1\nu_\tau) = \frac{G_F^2}{16\pi}|V_{us}|^2 f^2_{K_1}\frac{(m_\tau^2 + 2m^2_{K_1})(m_\tau^2 - m^2_{K_1})^2}{m_\tau^3}. \tag{5.9}$$

It was obtained in Refs. [26,30] that

$$|f_{K_1(1270)}| = 169^{+19}_{-21}\text{ MeV}. \tag{5.10}$$

As obtained in the previous subsection, $\theta_{K_1}$ should be negative to account for the observable $\mathcal{B}(B\to K_1(1270)\gamma) \gg \mathcal{B}(B\to K_1(1400)\gamma)$. Using the values for $f_{K_{1A}}$ and $f_{K_{1B}}$ as given in Table I, the result for $f_{K_1(1270)}$ in Eq. (5.10), and the relation in Eq. (5.3), we find that $a_0^{\parallel,K_{1B}}$ should be negative. Further substituting $a_0^{\parallel,K_{1B}} = -0.15 \pm 0.15$ into Eq. (5.3), we obtain that $\theta_{K_1}$ lies in the interval $-21° - -47°$. We can use the obtained angle to predict the decay constants $f_{K_1(1270)}$ and $f_{K_1(1400)}$:

$$f_{K_1(1270)} = -(169^{+25+49}_{-25-40})\text{ MeV}, \tag{5.11}$$

$$f_{K_1(1400)} = 179^{+13+30}_{-13-39}\ \mathrm{MeV}, \qquad (5.12)$$

for $\theta_{K_1} = (-34 \pm 13)^\circ$, where the first error is due to the uncertainties of decay constants and $a_0^{\parallel,K_{1B}}$, and the second due to the variation of $\theta_{K_1}$. The first error is dominated by the variation of $a_0^{\parallel,K_{1B}}$. The predicted $\theta_{K_1} = (-34 \pm 13)^\circ$ is also consistent with the result given in Ref. [24], where $|\theta_{K_1}| \approx 33^\circ$ or $57^\circ$. We thus predict

$$\mathcal{B}(\tau^- \to K_1^-(1400)\nu_\tau) = (3.5^{+0.5+1.2}_{-0.5-1.5}) \times 10^{-3}, \qquad (5.13)$$

to be compared with the current data $\mathcal{B}(\tau^- \to K_1^-(1400)\nu_\tau) = (1.7 \pm 2.6) \times 10^{-3}$ [11] which has large experimental error. If a more precise measurement for $\mathcal{B}(\tau^- \to K_1^-(1400)\nu_\tau)$ can also be achieved, we can extract directly the values of $\theta_{K_1}$ and $a_0^{\parallel,K_{1B}}$. Consequently, we can have more precise predictions for the $\mathcal{B}(B \to K_1(1270)\gamma)$ and $\mathcal{B}(B \to K_1(1400)\gamma)$ branching ratios and $B \to K_1$ transition form factors.

### C. $B \to K_1\gamma$ branching ratios

Using $\bar{m}_c/\bar{m}_b = 1.25\ \mathrm{GeV}/4.25\ \mathrm{GeV}$, one finds

$$\mathcal{B}(B \to K_1\gamma) = \tau_B \frac{G_F^2\alpha|V_{tb}V_{ts}^*|^2}{32\pi^4} m_{b,\mathrm{pole}}^2 m_B^3\left(1 - \frac{m_{K_1}^2}{m_B^2}\right)^3 \times (T_1^{K_1}(0))^2|(-0.392 - i0.015) + A_{\mathrm{sp}}^{(1)K_1}(\mu_h)|^2, \qquad (5.14)$$

where $T_1^{K_1(1270)}(0)$ and $T_1^{K_1(1400)}(0)$, as given in Eq. (5.2), are $\theta_{K_1}$ dependent. For $\theta_{K_1} = -(34 \pm 13)^\circ$, we have

$$T_1^{K_1(1270)}(0) = -(0.38^{+0.06+0.08+0.02}_{-0.04-0.07-0.04}),$$
$$T_1^{K_1(1400)}(0) = 0.12^{+0.03+0.02+0.08}_{-0.02-0.00-0.09}, \qquad (5.15)$$

where the first uncertainty comes from the variation of $m_{b,\mathrm{pole}}$ and $f_B$ in the sum rules, the second from the parameters of LCDAs, and the third from $\theta_{K_1}$. To illustrate the contribution due to the hard-spectator correction, it is interesting to note that, using $\lambda_B = 0.35$ GeV, $\theta_{K_1} = -34^\circ$, $T_1^{K_{1A}}(0) = 0.31$, $T_1^{K_{1B}}(0) = -0.25$, and the center values of the remaining input parameters, we obtain

$$A_{\mathrm{sp}}^{(1)K_1(1270)}(\mu_h) = 0.016 + i0.013,$$
$$A_{\mathrm{sp}}^{(1)K_1(1400)}(\mu_h) = 0.017 - i0.047, \qquad (5.16)$$

which suppress the decay rates slightly by about 8%, in contrast to the $B \to K^*\gamma$ decay where the interference between the hard-spectator correction $A_{\mathrm{sp}}^{(1)K^*}(\mu_h) = -0.013 - i0.011$ and the remainder is constructive [37].

In Table II, we present a comparison of the resulting branching ratios in this work with the data. Our results are consistent with the Belle measurement [4] within errors. A much more precise determination of $\theta_{K_1}$ can be made by the measurement

TABLE II. Branching ratios for the radiative decays $B \to K_1(1270)\gamma$, $K_1(1400)\gamma$ (in units of $10^{-6}$) in this work and experiment [4]. The branching ratios correspond to $\theta_{K_1} = -(34^\circ \pm 13^\circ)$ in our work, where the first error comes from the variation of $m_{b,\mathrm{pole}}$ and $f_B$, the second from the parameters of LCDAs, the third from $\lambda_B$, and the fourth from $\theta_{K_1}$. The annihilation amplitudes are not included in the neutral $B$ decay modes.

| | $\mathcal{B}(B^- \to K_1^-(1270)\gamma)$ | $\mathcal{B}(B^- \to K_1^-(1400)\gamma)$ |
|---|---|---|
| Experiment | $43 \pm 13$ | $<15$ |
| This work | $79^{+25+36+2+7}_{-16-28-5-14}$ | $7.7^{+4.7+2.4+0.1+14.2}_{-2.6-0.0-0.2-7.1}$ |
| | $\mathcal{B}(\bar{B}^0 \to \bar{K}_1^0(1270)\gamma)$ | $\mathcal{B}(\bar{B}^0 \to \bar{K}_1^0(1400)\gamma)$ |
| Experiment | $<58$ | $<15$ |
| This work | $74^{+23+34+2+7}_{-15-26-5-13}$ | $7.2^{+4.4+2.2+0.1+13.3}_{-2.4-0.0-0.2-6.6}$ |

$$R_{K_1} = \frac{\mathcal{B}(B \to K(1400)\gamma)}{\mathcal{B}(B \to K(1270)\gamma)}. \qquad (5.17)$$

The current upper bound of this ratio is $R_{K_1} < 0.5$. It can be seen from Fig. 3 that $R_{K_1}$ weakly depends on the theoretical uncertainty. Thus, $R_{K_1}$ is a suitable quantity for measuring the mixing angle $\theta_{K_1}$. In the light-cone sum rule calculation, the physical quantities, including the branching ratios and transition form factors, receive large errors from the uncertainties of $G$-parity violating Gegenbaur moments. A more precise value for $\theta_{K_1}$ can be used to extract a better result of $a_0^{\parallel,K_{1B}}$ from the data for $\mathcal{B}(\tau^- \to K_1^-(1270)\nu_\tau)$; the remaining $G$-parity violating Gegenbaur moments can thus be determined using Eq. (141) in Ref. [27]. On the other hand, we can also obtain good estimates for $\theta_{K_1}$ and $a_0^{\parallel,K_{1B}}$ from the data $\mathcal{B}(\tau^- \to K_1^-(1270)\nu_\tau)$ and $\mathcal{B}(\tau^- \to K_1^-(1400)\nu_\tau)$ if we can improve the measurement for $\mathcal{B}(\tau^- \to K_1^-(1400)\nu_\tau)$. Consequently, theoretical uncertainties due to $G$-parity violating Gegenbaur moments

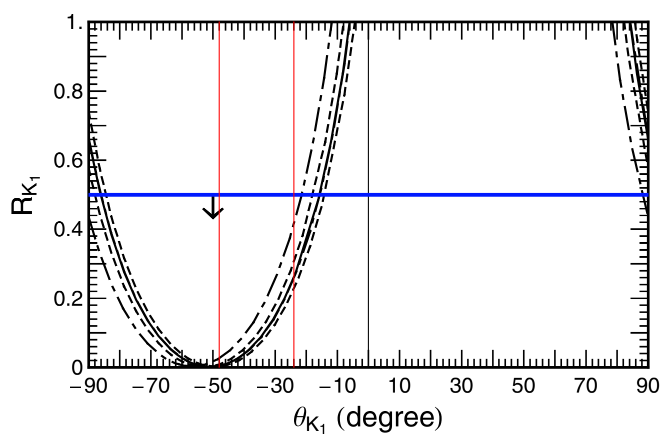


FIG. 3 (color online). Same as Fig. 2 except for the ratio $R_{K_1} = \mathcal{B}(B \to K_1(1400)\gamma)/\mathcal{B}(B \to K_1(1270)\gamma)$ as a function of the mixing angle $\theta_{K_1}$.

and $\theta_{K_1}$ can be reduced in the form factor and branching ratio calculations.

## VI. CONCLUSIONS

We have presented a detailed study of $B \to K_1(1270)\gamma$ and $B \to K_1(1400)\gamma$ decays. Our main results are as follows.

(i) Using the light-cone sum rule technique, we have evaluated the $B \to K_{1A}, K_{1B}$ tensor form factors, $T_1^{K_{1A}}(0)$ and $T_1^{K_{1B}}(0)$, where the contributions have been included up to the first order in $m_{K_1}/m_b$. We obtain $T_1^{K_{1A}}(0) = 0.31^{+0.06+0.01+0.06}_{-0.04-0.01-0.03}$ and $T_1^{K_{1B}}(0) = -(0.25^{+0.03+0.01+0.05}_{-0.02-0.01-0.07})$.

(ii) The sign ambiguity of the $K_1(1270)$–$K_1(1400)$ mixing angle $\theta_{K_1}$ can be resolved by defining $f_{K_{1A}}$ and $f^{\perp}_{K_{1B}}$ to be positive. Combining the analysis for the decays $B \to K_1\gamma$ and $\tau^- \to K_1^-(1270)\nu_\tau$, we find that the mixing angle $\theta_{K_1}$ should be negative, and its value lies in the interval $-(34 \pm 13)^\circ$. We obtain $f_{K_1(1270)} = -(169^{+25+49}_{-25-40})$ MeV and $f_{K_1(1400)} = 179^{+13+30}_{-13-39}$ MeV, and predict $\mathcal{B}(\tau^- \to K_1^-(1400)\nu_\tau) = (3.5^{+0.5+1.2}_{-0.5-1.5}) \times 10^{-3}$.

(iii) We find $T_1^{K_1(1270)}(0) = -(0.38^{+0.06+0.08+0.02}_{-0.04-0.07-0.04})$, $T_1^{K_1(1400)}(0) = 0.12^{+0.03+0.02+0.08}_{-0.02-0.00-0.09}$. The hard-spectator contribution suppresses the $B \to K_1(1270)\gamma$ and $B \to K_1(1400)\gamma$ decay rates slightly by about 8%, in contrast with the situation for $B \to K^*\gamma$. The predicted branching ratios for the decays $B \to K_1(1270)\gamma$ and $B \to K_1(1400)\gamma$ agree with the data within the errors.

(iv) We point out that better determinations of the $\theta_{K_1}$ and $G$-parity violating Gegenbaur moments of leading-twist light-cone distribution amplitudes can be obtained from a more precise measurement for the ratio $R_{K_1} = \mathcal{B}(B \to K_1(1400)\gamma)/\mathcal{B}(B \to K_1(1270)\gamma)$ or from an improved measurement for $\mathcal{B}(\tau^- \to K_1^-(1400)\nu_\tau)$ together with the $\mathcal{B}(\tau^- \to K_1^-(1270)\nu_\tau)$ data. Thus, the theoretical uncertainties can be further reduced.

## ACKNOWLEDGMENTS

This research was supported in part by the National Science Council of R.O.C. under Grant No. NSC96-2112-M-033-004-MY3 and No. NSC96-2811-M-033-004.

## APPENDIX A: TWO-PARTON DISTRIBUTION AMPLITUDES

In the calculation, the LCDAs of the axial meson appear in the following way:

$$\begin{aligned}\langle \bar{K}_1(P,\lambda)|\bar{s}_\alpha(y)\psi_\delta(x)|0\rangle = {}& -\frac{i}{4}\int_0^1 du e^{i(uPy+\bar{u}Px)}\bigg\{ f_{K_1} m_{K_1}\bigg[\not{P}\gamma_5 \frac{\epsilon^*_{(\lambda)}z}{Pz}\Phi_\parallel(u) + \bigg(\not{\epsilon}^* - \not{P}\frac{\epsilon^*_{(\lambda)}z}{Pz}\bigg)\gamma_5 g_\perp^{(a)}(u) \\ & - \not{z}\gamma_5 \frac{\epsilon^*_{(\lambda)}z}{2(Pz)^2} m_{K_1}^2 \bar{g}_3(u) + \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}_{(\lambda)} p^\rho z^\sigma \gamma^\mu \frac{g_\perp^{(v)}(u)}{4}\bigg] + f^\perp_{K_1}\bigg[\frac{1}{2}(\not{P}\not{\epsilon}^*_{(\lambda)} - \not{\epsilon}^*_{(\lambda)}\not{P})\gamma_5 \Phi_\perp(u) \\ & - \frac{1}{2}(\not{P}\not{z} - \not{z}\not{P})\gamma_5 \frac{\epsilon^*_{(\lambda)}z}{(Pz)^2} m_{K_1}^2 \bar{h}_\parallel^{(t)}(u) - \frac{1}{4}(\not{\epsilon}^*_{(\lambda)}\not{z} - \not{z}\not{\epsilon}^*_{(\lambda)})\gamma_5 \frac{m_{K_1}^2}{Pz}\bar{h}_3(u) + i(\epsilon^*_{(\lambda)}z) m_{K_1}^2 \gamma_5 \frac{h_\parallel^{(p)}(u)}{2}\bigg]\bigg\}_{\delta\alpha} \\ & + \mathcal{O}((x-y)^2), \end{aligned} \tag{A1}$$

where

$$\begin{aligned}\bar{g}_3(u) &= g_3(u) + \Phi_\parallel - 2g_\perp^{(a)}(u), \\ \bar{h}_\parallel^{(t)}(u) &= h_\parallel^{(t)}(u) - \tfrac{1}{2}\Phi_\perp(u) - \tfrac{1}{2}h_3(u), \\ \bar{h}_3(u) &= h_3(u) - \Phi_\perp(u), \end{aligned} \tag{A2}$$

$z^2 = (y-x)^2 \neq 0$, and $P^2 = m_{K_1}^2$. The detailed LCDAs are defined in Ref. [27]. Here $\Phi_\parallel$, $\Phi_\perp$ are of twist 2, $g_\perp^{(a)}$, $g_\perp^{(v)}$, $h_\parallel^{(t)}$, $h_\parallel^{(p)}$ of twist 3, and $g_3$, $h_3$ of twist 4. In the SU(3) limit, due to $G$ parity, $\Phi_\parallel$, $g_\perp^{(a)}$, $g_\perp^{(v)}$, and $g_3$ are symmetric (antisymmetric) under the replacement $u \leftrightarrow 1-u$ for the $1^3P_1$ ($1^1P_1$) states, whereas $\Phi_\perp$, $h_\parallel^{(t)}$, $h_\parallel^{(p)}$, and $h_3$ are antisymmetric (symmetric). For convenience, we normalize the distribution amplitudes of the $1^3P_1$ and $1^1P_1$ states to be subject to

$$\int_0^1 du \Phi_\parallel^{3P_1}(u) = 1, \qquad \int_0^1 du \Phi_\perp^{1P_1}(u) = 1. \tag{A3}$$

We take $f^\perp_{3P_1} = f_{3P_1}$ and $f_{1P_1} = f^\perp_{1P_1}(\mu = 1\ \text{GeV})$ in the study, such that we define

$$\begin{aligned}&\langle \bar{K}_{1A}(P,\lambda)|\bar{s}(0)\sigma_{\mu\nu}\gamma_5\psi(0)|0\rangle \\ &\quad = f^\perp_{K_{1A}} a_0^{\perp,K_{1A}}(\epsilon_\mu^{*(\lambda)}P_\nu - \epsilon_\nu^{*(\lambda)}P_\mu), \\ &\langle \bar{K}_{1B}(P,\lambda)|\bar{s}(0)\gamma_\mu\gamma_5\psi(0)|0\rangle = i f_{K_{1B}} a_0^{\parallel,K_{1B}} m_{K_{1B}}\epsilon_\mu^{*(\lambda)}, \end{aligned} \tag{A4}$$

where $a_0^{\perp,K_{1A}}$ and $a_0^{\parallel,K_{1B}}$ are the Gegenbaur zeroth moments, which vanish in the SU(3) limit.

We take into account the approximate forms of twist-2 distributions for the $\bar{K}_{1A}$ meson to be [27]

$$\Phi_\parallel(u) = 6u\bar{u}[1 + 3a_1^\parallel\xi + a_2^{\parallel}\tfrac{3}{2}(5\xi^2 - 1)], \tag{A5}$$

$$\Phi_\perp(u) = 6u\bar{u}[a_0^\perp + 3a_1^\perp\xi + a_2^\perp\tfrac{3}{2}(5\xi^2 - 1)], \qquad (A6)$$

and for the $\bar{K}_{1B}$ meson to be

$$\Phi_\parallel(u) = 6u\bar{u}[a_0^\parallel + 3a_1^\parallel\xi + a_2^\parallel\tfrac{3}{2}(5\xi^2 - 1)], \qquad (A7)$$

$$\Phi_\perp(u) = 6u\bar{u}[1 + 3a_1^\perp\xi + a_2^\perp\tfrac{3}{2}(5\xi^2 - 1)], \qquad (A.8)$$

where $\xi = 2u - 1$.

For the two-parton twist-3 chiral-even LCDAs, which are relevant here, we take the approximate expressions up to conformal spin 9/2 and $\mathcal{O}(m_s)$ [27]:

$$g_\perp^{(a)}(u) = \frac{3}{4}(1+\xi^2) + \frac{3}{2}a_1^\parallel\xi^3 + \left(\frac{3}{7}a_2^\parallel + 5\zeta^V_{3,K_{1A}}\right)(3\xi^2-1) + \left(\frac{9}{112}a_2^\parallel + \frac{105}{16}\zeta^A_{3,K_{1A}} - \frac{15}{64}\zeta^V_{3,K_{1A}}\omega^V_{K_{1A}}\right)(35\xi^4 - 30\xi^2 + 3) + 5\left[\frac{21}{4}\zeta^V_{3,K_{1A}}\sigma^V_{K_{1A}} + \zeta^A_{3,K_{1A}}\left(\lambda^A_{K_{1A}} - \frac{3}{16}\sigma^A_{K_{1A}}\right)\right]\xi(5\xi^2-3) - \frac{9}{2}\bar{a}_1^\perp\tilde{\delta}_+\left(\frac{3}{2} + \frac{3}{2}\xi^2 + \ln u + \ln\bar{u}\right) - \frac{9}{2}\bar{a}_1^\perp\tilde{\delta}_-(3\xi + \ln\bar{u} - \ln u), \qquad (A9)$$

$$g_\perp^{(v)}(u) = 6u\bar{u}\left\{1 + \left(a_1^\parallel + \frac{20}{3}\zeta^A_{3,K_{1A}}\lambda^A_{K_{1A}}\right)\xi + \left[\frac{1}{4}a_2^\parallel + \frac{5}{3}\zeta^V_{3,K_{1A}}\left(1 - \frac{3}{16}\omega^V_{K_{1A}}\right) + \frac{35}{4}\zeta^A_{3,K_{1A}}\right](5\xi^2-1) + \frac{35}{4}\left(\zeta^V_{3,K_{1A}}\sigma^V_{K_{1A}} - \frac{1}{28}\zeta^A_{3,K_{1A}}\sigma^A_{K_{1A}}\right)\xi(7\xi^2-3)\right\} - 18a_1^\perp\tilde{\delta}_+(3u\bar{u} + \bar{u}\ln\bar{u} + u\ln u) - 18a_1^\perp\tilde{\delta}_-(u\bar{u}\xi + \bar{u}\ln\bar{u} - u\ln u), \qquad (A10)$$

for the $\bar{K}_{1A}$ state, and

$$g_\perp^{(a)}(u) = \frac{3}{4}a_0^\parallel(1+\xi^2) + \frac{3}{2}a_1^\parallel\xi^3 + 5\left[\frac{21}{4}\zeta^V_{3,K_{1B}} + \zeta^A_{3,K_{1B}}\left(1 - \frac{3}{16}\omega^A_{K_{1B}}\right)\right]\xi(5\xi^2-3) + \frac{3}{16}a_2^\parallel(15\xi^4 - 6\xi^2 - 1) + 5\zeta^V_{3,K_{1B}}\lambda^V_{K_{1B}}(3\xi^2-1) + \frac{105}{16}\left(\zeta^A_{3,K_{1B}}\sigma^A_{K_{1B}} - \frac{1}{28}\zeta^V_{K_{1B}}\sigma^V_{K_{1B}}\right)(35\xi^4 - 30\xi^2 + 3) - 15\bar{a}_2^\perp\left[\tilde{\delta}_+\xi^3 + \frac{1}{2}\tilde{\delta}_-(3\xi^2-1)\right] - \frac{3}{2}[\tilde{\delta}_+(2\xi + \ln\bar{u} - \ln u) + \tilde{\delta}_-(2 + \ln u + \ln\bar{u})](1 + 6a_2^\perp), \qquad (A11)$$

$$g_\perp^{(v)}(u) = 6u\bar{u}\left\{a_0^\parallel + a_1^\parallel\xi + \left[\frac{1}{4}a_2^\parallel + \frac{5}{3}\zeta^V_{3,K_{1B}}\left(\lambda^V_{K_{1B}} - \frac{3}{16}\sigma^V_{K_{1B}}\right) + \frac{35}{4}\zeta^A_{3,K_{1B}}\sigma^A_{K_{1B}}\right](5\xi^2-1) + \frac{20}{3}\xi\left[\zeta^A_{3,K_{1B}} + \frac{21}{16}\left(\zeta^V_{3,K_{1B}} - \frac{1}{28}\zeta^A_{3,K_{1B}}\omega^A_{K_{1B}}\right)(7\xi^2-3)\right] - 5a_2^\perp[2\tilde{\delta}_+\xi + \tilde{\delta}_-(1+\xi^2)]\right\} - 6[\tilde{\delta}_+(\bar{u}\ln\bar{u} - u\ln u) + \tilde{\delta}_-(2u\bar{u} + \bar{u}\ln\bar{u} + u\ln u)](1 + 6a_2^\perp), \qquad (A12)$$

for the $\bar{K}_{1B}$ state, where

$$\tilde{\delta}_\pm = \pm\frac{f^\perp_{K_1}}{f_{K_1}}\frac{m_s}{m_{K_1}}, \qquad \zeta^{V,A}_{3,K_1} = \frac{f^{V,A}_{3K_1}}{f_{K_1}m_{K_1}}. \qquad (A13)$$

## APPENDIX B: THREE-PARTON CHIRAL-EVEN DISTRIBUTION AMPLITUDES OF TWIST 3

Taking into account the contributions up to terms of conformal spin 9/2 and considering the corrections of order $m_s$, the twist-3 three-parton chiral-even distribution amplitudes, defined in Eqs. (4.7) and (4.8), can be approximately written as [27]

$$\mathcal{A}(\underline{\alpha}) = 5040(\alpha_s - \alpha_\psi)\alpha_s\alpha_\psi\alpha_g^2 + 360\alpha_s\alpha_\psi\alpha_g^2[\lambda^A_{K_{1A}} + \sigma^A_{K_{1A}}\tfrac{1}{2}(7\alpha_g - 3)], \qquad (B1)$$

$$\mathcal{V}(\underline{\alpha}) = 360\alpha_s\alpha_\psi\alpha_g^2[1 + \omega^V_{K_{1A}}\tfrac{1}{2}(7\alpha_g - 3)] + 5040(\alpha_s - \alpha_\psi)\alpha_s\alpha_\psi\alpha_g^2\sigma^V_{K_{1A}}, \qquad (B2)$$

for the $\bar{K}_{1A}$ state, and

$$\mathcal{A}(\underline{\alpha}) = 360\alpha_s\alpha_\psi\alpha_g^2[1 + \omega^A_{K_{1B}}\tfrac{1}{2}(7\alpha_g - 3)] + 5040(\alpha_s - \alpha_\psi)\alpha_s\alpha_\psi\alpha_g^2\sigma^A_{K_{1B}}, \qquad (B3)$$

$$\mathcal{V}(\underline{\alpha}) = 5040(\alpha_s - \alpha_\psi)\alpha_s\alpha_\psi\alpha_g^2 + 360\alpha_s\alpha_\psi\alpha_g^2[\lambda^V_{K_{1B}} + \sigma^V_{K_{1B}}\tfrac{1}{2}(7\alpha_g - 3)], \qquad (B4)$$

for the $\bar{K}_{1B}$ state, where $\lambda$'s correspond to conformal spin 7/2, while $\omega$'s and $\sigma$'s are parameters with conformal spin 9/2. Note that as the SU(3)-symmetry (and $G$ parity) is restored, we have $\lambda$'s $= \sigma$'s $= 0$.

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