The $\eta(2225)$ observed by the BES Collaboration

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In the framework of the $^3P_0$ meson decay model, the strong decays of the 3^1S_0 and 4^1S_0 $s\bar{s}$ states are investigated. It is found that in the presence of the initial state mass being 2.24 GeV, the total widths of the 3^1S_0 and 4^1S_0 $s\bar{s}$ states are about 438 MeV and 125 MeV, respectively. Also, when the initial state mass varies from 2220 to 2400 MeV, the total width of the 4^1S_0 $s\bar{s}$ state varies from about 100 to 132 MeV, while the total width of the 3^1S_0 $s\bar{s}$ state varies from about 400 to 594 MeV. A comparison of the predicted widths and the experimental result of $(0.19 \pm 0.03^{+0.04}_{-0.06})$ GeV, the width of the $\eta(2225)$ with a mass of $(2.24^{+0.03+0.03}_{-0.02-0.02})$ GeV recently observed by the BES Collaboration in the radiative decay $J/\psi \to \gamma\phi\phi \to \gamma K^+K^-K^0_SK^0_L$, suggests that it would be very difficult to identify the $\eta(2225)$ as the 3^1S_0 $s\bar{s}$ state, and the $\eta(2225)$ seams a good candidate for the 4^1S_0 $s\bar{s}$ state.

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I. INTRODUCTION

Experimentally, a low-mass enhancement in J/ψ radiative decays $J/\psi \to \gamma\phi\phi$ at 2.25 GeV with a clear pseudoscalar assignment was first reported by the DM2 Collaboration [1]. Subsequently, the DM2 Collaboration [2] and the MARK III Collaboration [3] gave the evidence of a resonant $\phi\phi$ production around 2.2 GeV, preferably pseudoscalar, also in $J/\psi \to \gamma\phi\phi$. A fit to the $\phi\phi$ invariant-mass spectrum gave a mass of $(2230 \pm 25 \pm 15)$ MeV and a width of $(150^{+300}_{-60} \pm 60)$ MeV [3]. An angular analysis of the $\phi\phi$ signal found it to be consistent with a 0^{-+} [$\eta(2225)$] assignment. The nature of the $\eta(2225)$ is unclear. Possibilities of the nature of the $\eta(2225)$ include the second and third radial excitations of the pseudoscalar meson η', hybrid, glueball, or multiquark state. However, the large uncertainty of the width of the $\eta(2225)$ leads to that the theoretical interpretations perhaps remain open.

Recently, based on the 5.8×10^7 J/ψ events collected in the BESII detector, the radiative decay $J/\psi \to \gamma\phi\phi \to \gamma K^+K^-K^0_SK^0_L$ was analyzed by the BES Collaboration, and a near-threshold enhancement was found in the $\phi\phi$ invariant-mass distribution at 2.24 GeV with a statistical significance larger than 10σ. A partial wave analysis shows that this structure is dominated by a 0^{-+} [$\eta(2225)$] with a mass of $(2.24^{+0.03+0.03}_{-0.02-0.02})$ GeV and a width of $(0.19 \pm 0.03^{+0.04}_{-0.06})$ GeV, and the production branching fraction is $\mathrm{Br}(J/\psi \to \gamma\eta(2225))\mathrm{Br}(\eta(2225) \to \phi\phi) = (4.4 \pm 0.04 \pm 0.8) \times 10^{-4}$ [4]. The improved measurements of the $\eta(2225)$ performed by the BES Collaboration maybe open a window for revealing the nature of the $\eta(2225)$.

It is very important to exhaust possible conventional $q\bar{q}$ description of the $\eta(2225)$ before resorting to more exotic interpretations such as a hybrid, glueball, or multiquark state as mentioned above. In the present work, we shall focus on the possibility of the $\eta(2225)$ being the ordinary pseudoscalar $q\bar{q}$ state. From PDG2006 [5], the 1^1S_0 meson nonet (π, η, η', and K) as well as the 2^1S_0 members [$\pi(1300)$, $\eta(1295)$, and $\eta(1475)$] have been well established. In our previous work [6], we suggested that the $\pi(1800)$ and $K(1830)$, together with the $X(1835)$ and $\eta(1760)$ observed by the BES Collaboration [7,8], constitute the 3^1S_0 meson nonet. Theoretically, both the second [9] and third [10] radial excitations of the η' are predicted to lie in the mass range of the $\eta(2225)$. The main purpose of this work is to evaluate the widths of the 3^1S_0 and 4^1S_0 $s\bar{s}$ states in the $^3P_0$ meson decay model, and then check which of these two pictures can reasonably account for the total width of the $\eta(2225)$.

The organization of this paper is as follows. In Sec. II, the brief review of the $^3P_0$ decay model is given (for the detailed review see e.g. Refs. [11–14].) In Sec. III, the decay widths of the 3^1S_0 and 4^1S_0 $s\bar{s}$ states are presented, and the summary and conclusion are given in Sec. IV.

II. THE $^3P_0$ MESON DECAY MODEL

The $^3P_0$ decay model, also known as the quark-pair creation model, was originally introduced by Micu [15] and further developed by Le Yaouanc *et al.* [11]. The $^3P_0$ decay model has been widely used to evaluate the strong decays of hadrons [16–25], since it gives a good description of many of the observed decay amplitudes and partial widths of the hadrons. The main assumption of the $^3P_0$ decay model is that strong decays take place via the creation of a $^3P_0$ quark-antiquark pair from the vacuum. The new produced quark-antiquark pair, together with the $q\bar{q}$ within the initial meson regroups into two outgoing mesons in all possible quark rearrangement ways, which corresponds to the two decay diagrams as shown in Fig. 1 for the meson decay process $A \to B + C$.

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The transition operator T of the decay $A \to BC$ in the $^3P_0$ model is given by

$$T = -3\gamma \sum_m \langle 1m1-m|00\rangle \int d^3\vec{p}_3 d^3\vec{p}_4 \delta^3(\vec{p}_3+\vec{p}_4) \times \mathcal{Y}_1^m\left(\frac{\vec{p}_3-\vec{p}_4}{2}\right)\chi^{34}_{1-m}\phi_0^{34}\omega_0^{34} b_3^\dagger(\vec{p}_3) d_4^\dagger(\vec{p}_4), \quad (1)$$

where γ is a dimensionless parameter representing the probability of the quark-antiquark pair $q_3\bar{q}_4$ with $J^{PC} = 0^{++}$ creation from the vacuum, $\vec{p}_3$ and $\vec{p}_4$ are the momenta of the created quark q_3 and antiquark $\bar{q}_4$, respectively. ϕ_0^{34}, ω_0^{34}, and $\chi^{34}_{1,-m}$ are the flavor, color, and spin wave functions of the $q_3\bar{q}_4$, respectively. The solid harmonic polynomial $\mathcal{Y}_1^m(\vec{p}) \equiv |p|^1 Y_1^m(\theta_p, \phi_p)$ reflects the momentum-space distribution of the $q_3\bar{q}_4$.

For the meson wave function, we adopt the mock meson $|A(n_A^{2S_A+1}L_{AJ_AM_{J_A}})(\vec{P}_A)\rangle$ defined by [26]

$$\begin{aligned} |A(n_A^{2S_A+1}L_{AJ_AM_{J_A}})(\vec{P}_A)\rangle &\equiv \sqrt{2E_A}\sum_{M_{L_A},M_{S_A}} \langle L_A M_{L_A} S_A M_{S_A}|J_A M_{J_A}\rangle \\ &\times \int d^3\vec{p}_A \psi_{n_A L_A M_{L_A}}(\vec{p}_A)\chi^{12}_{S_A M_{S_A}}\phi_A^{12}\omega_A^{12} \\ &\times \left| q_1\left(\frac{m_1}{m_1+m_2}\vec{P}_A + \vec{p}_A\right)\bar{q}_2\left(\frac{m_2}{m_1+m_2}\vec{P}_A - \vec{p}_A\right)\right\rangle, \end{aligned} \quad (2)$$

where m_1 and m_2 are the masses of the quark q_1 with a momentum of $\vec{p}_1$ and the antiquark $\bar{q}_2$ with a momentum of $\vec{p}_2$, respectively. n_A is the radial quantum number of the meson A composed of $q_1\bar{q}_2$. $\vec{S}_A = \vec{s}_{q_1} + \vec{s}_{q_2}$, $\vec{J}_A = \vec{L}_A + \vec{S}_A$, $\vec{s}_{q_1}$ ($\vec{s}_{q_2}$) is the spin of q_1 (q_2), $\vec{L}_A$ is the relative orbital angular momentum between q_1 and q_2. $\vec{P}_A = \vec{p}_1 + \vec{p}_2$, $\vec{p}_A = \frac{m_1\vec{p}_1 - m_1\vec{p}_2}{m_1+m_2}$. $\langle L_A M_{L_A} S_A M_{S_A}|J_A M_{J_A}\rangle$ is a Clebsch-Gordan coefficient, and E_A is the total energy of the meson A. $\chi^{12}_{S_A M_{S_A}}$, ϕ_A^{12}, ω_A^{12}, and $\psi_{n_A L_A M_{L_A}}(\vec{p}_A)$ are the spin, flavor, color, and space wave functions of the meson A, respectively. The mock meson satisfies the normalization condition

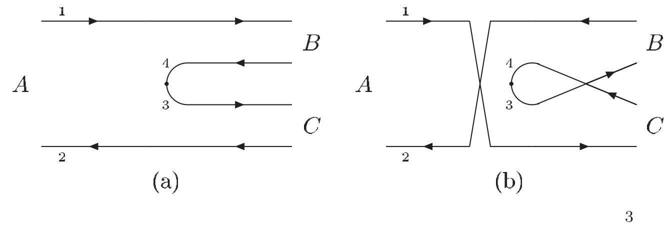

FIG. 1. The two possible diagrams contributing to $A \to B + C$ in the $^3P_0$ model.

$$\langle A(n_A^{2S_A+1}L_{AJ_AM_{J_A}})(\vec{P}_A)|A(n_A^{2S_A+1}L_{AJ_AM_{J_A}})(\vec{P}'_A)\rangle = 2E_A\delta^3(\vec{P}_A - \vec{P}'_A). \quad (3)$$

The S-matrix of the process $A \to BC$ is defined by

$$\langle BC|S|A\rangle = I - 2\pi i\delta(E_A - E_B - E_C)\langle BC|T|A\rangle, \quad (4)$$

with

$$\langle BC|T|A\rangle = \delta^3(\vec{P}_A - \vec{P}_B - \vec{P}_C)\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}, \quad (5)$$

where $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$ is the helicity amplitude of $A \to BC$. In the center of mass frame of meson A, $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$ can be written as

$$\begin{aligned} \mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\vec{P}) = \gamma\sqrt{8E_AE_BE_C} &\sum_{M_{L_A},M_{S_A},M_{L_B},M_{S_B},M_{L_C},M_{S_C},m} \langle L_A M_{L_A} S_A M_{S_A}|J_A M_{J_A}\rangle\langle L_B M_{L_B} S_B M_{S_B}|J_B M_{J_B}\rangle \\ &\times \langle L_C M_{L_C} S_C M_{S_C}|J_C M_{J_C}\rangle\langle 1m1-m|00\rangle\langle\chi^{14}_{S_BM_{S_B}}\chi^{32}_{S_CM_{S_C}}|\chi^{12}_{S_AM_{S_A}}\chi^{34}_{1-m}\rangle[f_1 I(\vec{P}, m_1, m_2, m_3) \\ &+ (-1)^{1+S_A+S_B+S_C} f_2 I(-\vec{P}, m_2, m_1, m_3)], \end{aligned} \quad (6)$$

with $f_1 = \langle\phi_B^{14}\phi_C^{32}|\phi_A^{12}\phi_0^{34}\rangle$ and $f_2 = \langle\phi_B^{32}\phi_C^{14}|\phi_A^{12}\phi_0^{34}\rangle$, corresponding to the contributions from Figs. 1(a) and 1(b), respectively, and

$$I(\vec{P}, m_1, m_2, m_3) = \int d^3\vec{p}\,\psi^*_{n_BL_BM_{L_B}}\left(\frac{m_3}{m_1+m_2}\vec{P}_B + \vec{p}\right)\psi^*_{n_CL_CM_{L_C}}\left(\frac{m_3}{m_2+m_3}\vec{P}_B + \vec{p}\right)\psi_{n_AL_AM_{L_A}}(\vec{P}_B + \vec{p})\mathcal{Y}_1^m(\vec{p}), \quad (7)$$

where $\vec{P} = \vec{P}_B = -\vec{P}_C$, $\vec{p} = \vec{p}_3$, m_3 is the mass of the created quark q_3.

The spin overlap in terms of Winger's $9j$ symbol can be given by

$$\begin{aligned} \langle\chi^{14}_{S_BM_{S_B}}\chi^{32}_{S_CM_{S_C}}|\chi^{12}_{S_AM_{S_A}}\chi^{34}_{1-m}\rangle = &\sum_{S,M_S}\langle S_BM_{S_B}S_CM_{S_C}|SM_S\rangle\langle S_AM_{S_A}1-m|SM_S\rangle(-1)^{S_C+1} \\ &\times\sqrt{3(2S_A+1)(2S_B+1)(2S_C+1)}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & S_A \\ \frac{1}{2} & \frac{1}{2} & 1 \\ S_B & S_C & S \end{Bmatrix}. \end{aligned} \quad (8)$$

In order to compare with the experiment conventionally, $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\vec{P})$ can be converted into the partial amplitude by a recoupling calculation [27]

$$\mathcal{M}^{LS}(\vec{P}) = \sum_{M_{J_B},M_{J_C},M_S,M_L} \langle LM_L SM_S|J_A M_{J_A}\rangle \times \langle J_B M_{J_B} J_C M_{J_C}|SM_S\rangle \times \int d\Omega Y^*_{LM_L} \mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\vec{P}). \tag{9}$$

If we consider the relativistic phase space, the decay width $\Gamma(A \to BC)$ in terms of the partial wave amplitudes is

$$\Gamma(A \to BC) = \frac{\pi P}{4M_A^2}\sum_{LS}|\mathcal{M}^{LS}|^2. \tag{10}$$

Here $P = |\vec{P}| = \frac{\sqrt{[M_A^2-(M_B+M_C)^2][M_A^2-(M_B-M_C)^2]}}{2M_A}$, M_A, M_B, and M_C are the masses of the meson A, B, and C, respectively.

The decay width can be derived analytically if the simple harmonic oscillator (SHO) approximation for the meson space wave functions is used. In momentum-space, the SHO wave function is

$$\psi_{nLM_L}(\vec{p}) = R^{\rm SHO}_{nL}(p)Y_{LM_L}(\Omega_p), \tag{11}$$

where the radial wave function is given by

$$R^{\rm SHO}_{nL} = \frac{(-1)^n(-i)^L}{\beta^{3/2}} \times \sqrt{\frac{2n!}{\Gamma(n+L+\frac{3}{2})}}\left(\frac{p}{\beta}\right)^L e^{-(p^2/2\beta^2)} L_n^{L+(1/2)}\left(\frac{p^2}{\beta^2}\right). \tag{12}$$

Here β is the SHO wave function scale parameter, and $L_n^{L+(1/2)}(\frac{p^2}{\beta^2})$ is an associated Laguerre polynomial.

The SHO wave functions cannot be regarded as realistic, however, they are a *de facto* standard for many nonrelativistic quark model calculations. Moreover, the more realistic space wave functions such as those obtained from Coulomb, plus the linear potential model do not always result in systematic improvements due to the inherent uncertainties of the $^3P_0$ decay model itself [17,18,20]. The SHO wave function approximation is commonly employed in the $^3P_0$ decay model in literature. In the present work, the SHO wave function approximation for the meson space wave functions is taken.

III. DECAYS OF THE 3^1S_0 AND 4^1S_0 $s\bar{s}$ STATES IN THE $^3P_0$ MODEL

Under the SHO wave function approximation, the parameters used in the $^3P_0$ decay model involve the $q\bar{q}$ pair production strength parameter γ, the SHO wave function scale parameter β, and the masses of the constituent quarks. In the present work, we take $\gamma = 6.95$ and $\beta = 0.4$ GeV, the typical values used to evaluate the light meson decays [18–23],[1] and $m_u = m_d = 0.33$ GeV, $m_s = 0.55$ GeV [24]. Based on the partial wave amplitudes listed in Appendixes A and B, and the flavor and charge multiplicity factors shown in Table I, from (10), the numerical values of the partial decay widths of the 4^1S_0 and 3^1S_0 $s\bar{s}$ states are listed in Table II. The initial state mass is set to 2.24 GeV and masses of the final mesons are taken from PDG2006 [5] except for the $K(2^3S_1)$ mass.[2]

Table II indicates that the total width of the 4^1S_0 $s\bar{s}$ state with a mass of 2.24 GeV predicted by the $^3P_0$ decay model is about 125.1 MeV, consistent with the experimental result of $\Gamma_{\eta(2225)} = (0.19 \pm 0.03^{+0.04}_{-0.06})$ GeV within errors, but the total width of the 3^1S_0 $s\bar{s}$ state with a mass of 2.24 GeV is predicted to be about 438.7 MeV, incompatible with the measured width of the $\eta(2225)$. Also, in order to check the dependence of the predicted results on the initial state mass, the variation of the total widths of the 4^1S_0 and 3^1S_0 $s\bar{s}$ states with the initial state mass is shown in Fig. 2. From Fig. 2, we can see that when the initial state mass varies from 2220 to 2400 MeV, the total width of the 4^1S_0 $s\bar{s}$ state varies from about 100 to 132 MeV, lying in the width range of the $\eta(2225)$, while the total width of the 3^1S_0 $s\bar{s}$ state varies from about 400 to 594 MeV, far more than the width of the $\eta(2225)$. Therefore, it would be very difficult to identify the $\eta(2225)$ as the 3^1S_0 $s\bar{s}$ state, but the assignment of the $\eta(2225)$ as the 4^1S_0 $s\bar{s}$ state seams reasonable for accounting for the total width of the $\eta(2225)$, assuming the $^3P_0$ meson decay model is accurate.

The variation of the partial decay widths of the 4^1S_0 and 3^1S_0 $s\bar{s}$ states with the initial state mass is also shown in Fig. 3. For both 4^1S_0 and 3^1S_0 $s\bar{s}$ states, the partial widths of the modes $\phi\phi$ and KK^* depend weakly on the initial state mass, while the partial widths of the modes K^*K^* and $KK_0^*(1430)$ vary dramatically with the initial state mass, and the $KK_2^*(1430)$ and $KK^*(1580)$ modes always have a sizable branch ratio in the mass region of the $\eta(2225)$. It is interesting to note that, in the mass region $2.22 \sim 2.40$ GeV, $\Gamma(3^1S_0 \to KK^*(1680))$ is almost 0 MeV, while $\Gamma(4^1S_0 \to KK^*(1680))$ varies from 0.15 MeV to 5.8 MeV.

[1]Our value of γ is higher than that used by other groups such as [20–23] by a factor of $\sqrt{96\pi}$ due to different field conventions, constant factor in T, etc. The calculated results of the widths are, of course, unaffected.

[2]The assignment of the $K^*(1410)$ as the 2^3S_1 kaon is problematic [23,28]. The quark model [29] and other phenomenological approaches [30] consistently suggest the 2^3S_1 kaon has a mass about 1580 MeV; here we take 1580 MeV as the mass of the 2^3S_1 kaon.

TABLE I. Flavor and charge multiplicity factors.

General decay	Subprocess	f_1	f_2	$\mathcal{F}$
$s\bar{s} \to K^*K$	$s\bar{s} \to K^{*+}K^-$	0	$-\frac{1}{\sqrt{3}}$	4
$s\bar{s} \to K_0^*(1430)K$	$s\bar{s} \to K_0^{*+}(1430)K^-$	0	$-\frac{1}{\sqrt{3}}$	4
$s\bar{s} \to K_2^*(1430)K$	$s\bar{s} \to K_2^{*+}(1430)K^-$	0	$-\frac{1}{\sqrt{3}}$	4
$s\bar{s} \to K^*(1580)K$	$s\bar{s} \to K^{*+}(1580)K^-$	0	$-\frac{1}{\sqrt{3}}$	4
$s\bar{s} \to K^*(1680)K$	$s\bar{s} \to K^{*+}(1680)K^-$	0	$-\frac{1}{\sqrt{3}}$	4
$s\bar{s} \to K^*K^*$	$s\bar{s} \to K^{*+}K^{*-}$	0	$-\frac{1}{\sqrt{3}}$	2
$s\bar{s} \to \phi\phi$	$s\bar{s} \to \phi\phi$	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	$\frac{1}{2}$

TABLE II. Decays of the 4^1S_0 and 3^1S_0 $s\bar{s}$ states in the $^3P_0$ model (in MeV). The initial state mass is set to 2240 MeV.

Mode	KK^*	K^*K^*	$KK_0^*(1430)$	$KK_2^*(1430)$	$KK^*(1580)$	$KK^*(1680)$	$\phi\phi$
$\Gamma_i(4^1S_0)$	9.1	0.5	1.5	43.5	56.9	1.0	12.6
$\Gamma_i(3^1S_0)$	26.4	8.1	0.1	173.3	138.2	0.0	92.6

$\Gamma(4^1S_0) = 125.1$, $\Gamma(3^1S_0) = 438.7$, $\Gamma_{\eta(2225)} = (190 \pm 30^{+40}_{-60})$

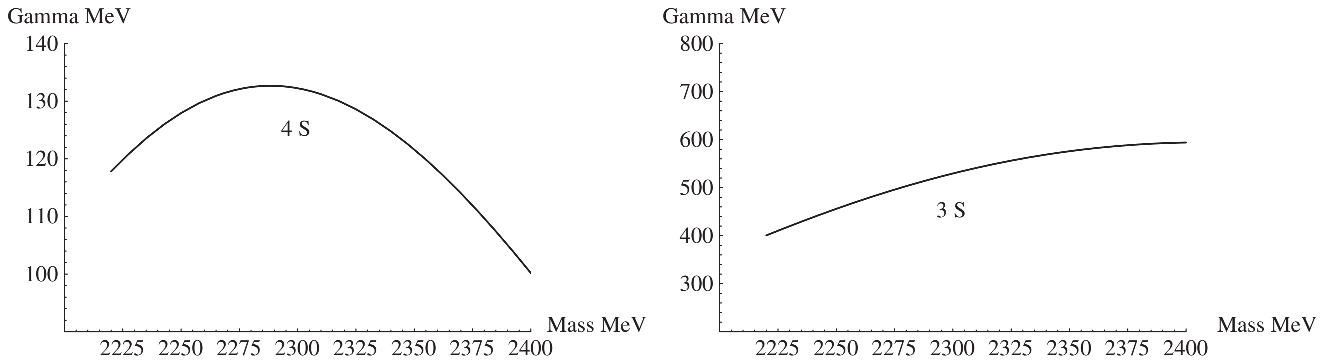

FIG. 2. The total widths of the 4^1S_0 and 3^1S_0 $s\bar{s}$ states dependence on the initial state mass in the $^3P_0$ decay model.

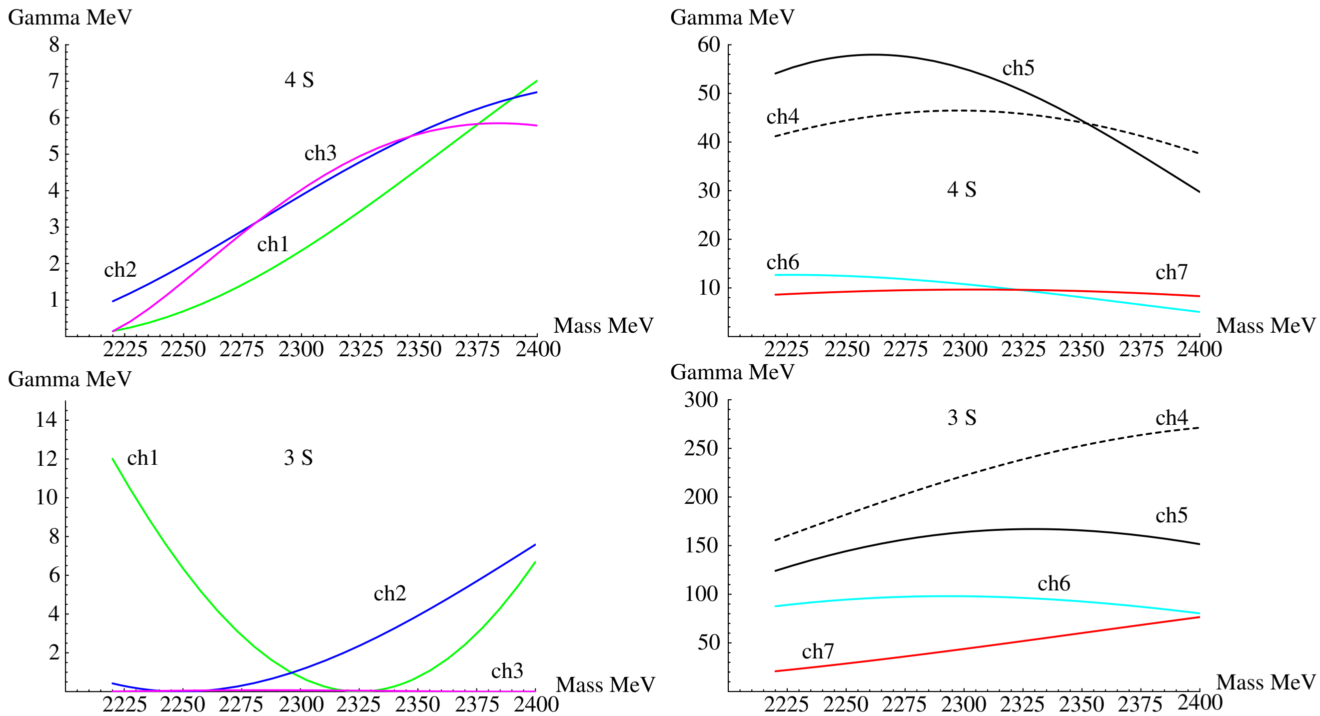

FIG. 3 (color online). The partial widths of the 4^1S_0 and 3^1S_0 $s\bar{s}$ states dependence on the initial state mass in the $^3P_0$ decay model. ch1 = K^*K^*, ch2 = $KK_0^*(1430)$, ch3 = $KK^*(1680)K$, ch4 = $KK_2^*(1430)$, ch5 = $KK^*(1580)$, ch6 = $\phi\phi$, and ch7 = KK^*.

IV. SUMMARY AND CONCLUSION

The strong decays of the 3^1S_0 and 4^1S_0 $s\bar{s}$ states in the $^3P_0$ meson decay model indicates that if the initial state mass is set to 2.24 GeV, the central value of the $\eta(2225)$ mass measured by the BES Collaboration [4], the total widths of the 3^1S_0 and 4^1S_0 $s\bar{s}$ states are predicted to be about 438 MeV and 125 MeV, respectively. Also, the variation of the total widths of the 4^1S_0 and 3^1S_0 $s\bar{s}$ states with the initial state mass shows that, in the mass region of the $\eta(2225)$, the total width of the 4^1S_0 $s\bar{s}$ state lies in the range about $100 \sim 132$ MeV, while the total width of the 3^1S_0 $s\bar{s}$ state lies in the range about $400 \sim 594$ MeV. A comparison of the $^3P_0$ model predictions and $\Gamma_{\eta(2225)} = (0.19 \pm 0.03^{+0.04}_{-0.06})$ GeV reported by the BES Collaboration [4] indicates that it would be very difficult to identify the $\eta(2225)$ as the 3^1S_0 $s\bar{s}$ state, while the assignment of the $\eta(2225)$ as the 4^1S_0 $s\bar{s}$ state seems reasonable to reproduce the total width of the $\eta(2225)$. We therefore tend to conclude that the $\eta(2225)$ may be a good candidate for the 4^1S_0 $s\bar{s}$ state.

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APPENDIX A: THE AMPLITUDES FOR THE 4^1S_0 $q\bar{q}$ DECAY IN THE $^3P_0$ MODEL

$$\begin{aligned}
&\mathcal{M}^{LS}(4^1S_0 \to 1^3S_1 + 1^3S_1) \\
&= \gamma e^{-([m_1m_2(m_2-m_3)m_3+m_2^2m_3^2+m_1^2(m_2^2+m_2m_3+m_3^2)]P^2)/(3\beta^2(m_1+m_3)^2(m_2+m_3)^2)}\sqrt{E_aE_bE_c}\frac{1}{\pi^{3/4}}(f_1+f_2)P[8505\beta^6(m_1+m_3)^6(m_2+m_3)^6 \\
&\quad\times(3m_1m_2+2m_1m_3+2m_2m_3+m_3^2)-1134\beta^4(m_1+m_3)^4(m_2+m_3)^4(7m_1m_2+6m_1m_3+6m_2m_3+5m_3^2)(m_2m_3+2m_1m_2 \\
&\quad+m_1m_3)^2P^2+108\beta^2(m_1+m_3)^2(m_2+m_3)^2(5m_1m_2+6m_1m_3+6m_2m_3+7m_3^2)(m_2m_3+2m_1m_2+m_1m_3)^4P^4 \\
&\quad-8(m_2m_3+2m_1m_2+m_1m_3)^6(m_1m_2+2m_1m_3+2m_2m_3+3m_3^2)P^6]\frac{\sqrt{2}}{19683\sqrt{105}\beta^{15/2}}\frac{1}{(m_1+m_3)^7(m_2+m_3)^7},
\end{aligned} \tag{A1}$$

$$\begin{aligned}
&\mathcal{M}^{LS}(4^1S_0 \to 1^3S_1 + 1^1S_0) \\
&= \gamma e^{-([m_1m_2(m_2-m_3)m_3+m_2^2m_3^2+m_1^2(m_2^2+m_2m_3+m_3^2)]P^2)/(3\beta^2(m_1+m_3)^2(m_2+m_3)^2)}\sqrt{E_aE_bE_c}\frac{1}{\pi^{3/4}}(f_1-f_2)P[8505\beta^6(m_1+m_3)^6(m_2+m_3)^6 \\
&\quad\times(3m_1m_2+2m_1m_3+2m_2m_3+m_3^2)-1134\beta^4(m_1+m_3)^4(m_2+m_3)^4(7m_1m_2+6m_1m_3+6m_2m_3+5m_3^2)(m_2m_3+2m_1m_2 \\
&\quad+m_1m_3)^2P^2+108\beta^2(m_1+m_3)^2(m_2+m_3)^2(5m_1m_2+6m_1m_3+6m_2m_3+7m_3^2)(m_2m_3+2m_1m_2+m_1m_3)^4P^4 \\
&\quad-8(m_2m_3+2m_1m_2+m_1m_3)^6(m_1m_2+2m_1m_3+2m_2m_3+3m_3^2)P^6]\frac{1}{19683\sqrt{105}\beta^{15/2}}\frac{1}{(m_1+m_3)^7(m_2+m_3)^7},
\end{aligned} \tag{A2}$$

$$\begin{aligned}
&\mathcal{M}^{LS}(4^1S_0 \to 1^3P_0 + 1^1S_0) \\
&= i\gamma e^{(-[(m_1m_2(m_2-m_3)m_3+m_2^2m_3^2+m_1^2(m_2^2+m_2m_3+m_3^2)]P^2)/(3\beta^2(m_1+m_3)^2(m_2+m_3)^2)}\sqrt{E_aE_bE_c}\frac{1}{\pi^{3/4}}\Big\{(f_1+f_2)\frac{\sqrt{70}}{81\beta^{1/2}}+[(10m_2^2m_3^2 \\
&\quad+6m_2m_3^3+19m_1^2m_2^2+19m_1^2m_2m_3+4m_1^2m_3^2+28m_1m_2^2m_3+23m_1m_2m_3^2+3m_1m_3^3)f_2+(4m_2^2m_3^2+3m_2m_3^3 \\
&\quad+19m_1m_2^2m_3+23m_1m_2m_3^2+6m_1m_3^3+19m_1^2m_2^2+28m_1^2m_2m_3+10m_1^2m_3^2)f_1]\frac{-\sqrt{70}}{729\beta^{5/2}}\frac{1}{(m_1+m_3)^2(m_2+m_3)^2}P^2 \\
&\quad+[(m_2+m_3)(5m_1^2m_2+6m_2m_3^2+12m_1m_2m_3+m_1m_3^2)f_2+(m_1+m_3)(m_2m_3^2+5m_1m_2^2+12m_1m_2m_3+6m_1m_3^2)f_1] \\
&\quad\times(m_2m_3+2m_1m_2+m_1m_3)^2\frac{2\sqrt{14}}{729\sqrt{5}\beta^{9/2}}\frac{1}{(m_1+m_3)^4(m_2+m_3)^4}P^4+[(14m_2^2m_3^2+18m_2m_3^3+5m_1^2m_2^2+5m_1^2m_2m_3 \\
&\quad-4m_1^2m_3^2+20m_1m_2^2m_3+25m_1m_2m_3^2-3m_1m_3^3)f_2+(-4m_2^2m_3^2-3m_2m_3^3+5m_1^2m_2^2+20m_1^2m_2m_3+14m_1^2m_3^2 \\
&\quad+5m_1m_2^2m_3+25m_1m_2m_3^2+18m_1m_3^3)f_1](m_2m_3+2m_1m_2+m_1m_3)^4\frac{-4\sqrt{2}}{6561\sqrt{35}\beta^{13/2}}\frac{1}{(m_1+m_3)^6(m_2+m_3)^6}P^6 \\
&\quad+[(m_1m_2-m_1m_3+2m_2m_3)f_2+(m_1m_2+2m_1m_3-m_2m_3)f_1](m_2m_3+2m_1m_2+m_1m_3)^6 \\
&\quad\times(m_1m_2+2m_1m_3+2m_2m_3+3m_3^2)\frac{8\sqrt{2}}{177147\sqrt{35}\beta^{17/2}}\frac{1}{(m_1+m_3)^8(m_2+m_3)^8}P^8\Big\},
\end{aligned} \tag{A3}$$

$$\mathcal{M}^{LS}(4^1S_0 \to 1^3P_2 + 1^1S_0) = i\gamma e^{(-[(m_1m_2(m_2-m_3)m_3+m_2^2m_3^2+m_1^2(m_2^2+m_2m_3+m_3^2)]P^2)/(3\beta^2(m_1+m_3)^2(m_2+m_3)^2)}\sqrt{E_aE_bE_c}\frac{1}{\pi^{3/4}}$$
$$\times\Bigg\{4032(f_1+f_2)\beta^{18}\left(\frac{m_1}{m_1+m_3}+\frac{m_2}{m_2+m_3}\right)^2P^2\frac{1}{19\,683\sqrt{35}\beta^{41/2}}+[(2m_2m_3^2(10m_2+9m_3)$$
$$+m_1^2(17m_2^2+17m_2m_3+2m_3^2)+m_1m_3(44m_2^2+49m_2m_3+9m_3^2))35\beta^2f_2+(m_2m_3^2(2m_2+9m_3)$$
$$+m_1m_3(17m_2^2+49m_2m_3+18m_3^2)+m_1^2(17m_2^2+44m_2m_3+20m_3^2))35\beta^2f_1-128(f_1+f_2)m_1^2m_2^2P^2]$$
$$\times\frac{2}{2187\sqrt{35}\beta^{9/2}}\frac{1}{(m_1+m_3)^2(m_2+m_3)^2}P^2+[(2m_2m_3^2(26m_2+27m_3)+m_1^2(37m_2^2+37m_2m_3-2m_3^2)$$
$$+m_1m_3(100m_2^2+113m_2m_3+9m_3^2))f_2+(m_2m_3^2(9m_3-2m_2)+m_1^2(37m_2^2+100m_2m_3+52m_3^2)$$
$$+m_1m_3(37m_2^2+113m_2m_3+54m_3^2))f_1](m_2m_3+2m_1m_2+m_1m_3)^2\frac{-4\sqrt{7}}{6561\sqrt{5}\beta^{9/2}}\frac{1}{(m_1+m_3)^4(m_2+m_3)^4}P^4$$
$$+[((2m_2m_3^2(22m_2+27m_3)+m_1^2(23m_2^2+23m_2m_3-10m_3^2)+m_1m_3(68m_2^2+79m_2m_3-9m_3^2))f_2$$
$$+((m_1^2(23m_2^2+68m_2m_3+44m_3^2)-m_2m_3^2(10m_2+9m_3)+m_1m_3(23m_2^2+79m_2m_3+54m_3^2))f_1]$$
$$\times(m_2m_3+2m_1m_2+m_1m_3)^4\frac{8}{19\,683\sqrt{35}\beta^{13/2}}\frac{1}{(m_1+m_3)^6(m_2+m_3)^6}P^6+[(m_1m_2-m_1m_3+2m_2m_3)f_2$$
$$+(m_1m_2+2m_1m_3-m_2m_3)f_1](m_2m_3+2m_1m_2+m_1m_3)^6(m_1m_2+2m_1m_3+2m_2m_3+3m_3^2)$$
$$\times\frac{-16}{177\,147\sqrt{35}\beta^{17/2}}\frac{1}{(m_1+m_3)^8(m_2+m_3)^8}P^8\Bigg\}, \tag{A4}$$

$$\mathcal{M}^{LS}(4^1S_0 \to 2^3S_1 + 1^1S_0)$$
$$= \gamma e^{(-[(m_1m_2(m_2-m_3)m_3+m_2^2m_3^2+m_1^2(m_2^2+m_2m_3+m_3^2)]P^2)/(3\beta^2(m_1+m_3)^2(m_2+m_3)^2)}\sqrt{E_aE_bE_c}\frac{1}{\pi^{3/4}}P(f_2-f_1)$$
$$\times\{[25515\beta^8(m_1+m_3)^8(m_2+m_3)^8(53m_1m_2+46m_1m_3+34m_2m_3+27m_3^2)-3402\beta^6(m_1+m_3)^6$$
$$\times(m_2+m_3)^6(20m_2^2m_3^3(2m_2+3m_3)+61m_1m_2m_3^2(46m_2^2+91m_2m_3+35m_3^2))+3m_1^2m_3(208m_2^3+497m_2^2m_3$$
$$+334m_2m_3^2+65m_3^3)+m_1^3(419m_2^3+1098m_2^2m_3+885m_2m_3^2+226m_3^3)]P^2+[2m_2m_3^2(4m_2+21m_3)$$
$$+m_1^2(95m_2^2+263m_2m_3+134m_3^2)+m_1m_3(116m_2^2+331m_2m_3+147m_3^2)]927\beta^4m_1(m_1+m_3)^4$$
$$\times(m_2+m_3)^5(m_2m_3+2m_1m_2+m_1m_3)^2P^4-[16m_2^3m_3^3-6m_1m_2m_3^2(16m_2^2+51m_2m_3+27m_3^2)$$
$$+3m_1^2m_3(24m_2^3+139m_2^2m_3+266m_2m_3^2+135m_3^3)+m_1^3(89m_2^3+438m_2^2m_3+675m_2m_3^2+310m_3^3)]$$
$$\times24\beta^2(m_1+m_3)^2(m_2+m_3)^2(m_2m_3+2m_1m_2+m_1m_3)^4P^6+16(m_2m_3+2m_1m_2+m_1m_3)^6$$
$$\times(m_2m_3-m_1m_2-2m_1m_3)^2(m_1m_2+2m_1m_3+2m_2m_3+3m_3^2)P^8\}\frac{1}{531\,441\sqrt{70}\beta^{19/2}}\frac{1}{(m_1+m_3)^9(m_2+m_3)^9}, \tag{A5}$$

$$\mathcal{M}^{LS}(4^1S_0\to 1^3D_1+1^1S_0)$$
$$=\gamma e^{(-[(m_1m_2(m_2-m_3)m_3+m_2^2m_3^2+m_1^2(m_2^2+m_2m_3+m_3^2)]P^2)/(3\beta^2(m_1+m_3)^2(m_2+m_3)^2)}\sqrt{E_aE_bE_c}\frac{1}{\pi^{3/4}}$$
$$\times\bigg\{\bigg[\bigg[\bigg(119070-\frac{11907m_2(m_2+4m_3)P^2}{\beta^2(m_2+m_3)^2}-\frac{162m_2^3(43m_2+18m_3)P^4}{\beta^4(m_2+m_3)^4}+\frac{4m_2^5(89m_2+216m_3)P^6}{\beta^6(m_2+m_3)^6}+\frac{8m_2^7(m_2+2m_3)P^8}{\beta^8(m_2+m_3)^8}\bigg)f_1$$
$$+\bigg(8505+\frac{3402m_2(15m_2+14m_3)P^2}{\beta^2(m_2+m_3)^2}-\frac{324m_2^3(97m_2+138m_3)P^4}{\beta^4(m_2+m_3)^4}+\frac{8m_2^5(443m_2+702m_3)P^6}{\beta^6(m_2+m_3)^6}$$
$$-\frac{32m_2^7(3m_2+5m_3)P^8}{\beta^8(m_2+m_3)^8}\bigg)f_2\bigg]\frac{-2\sqrt{2}}{885735\sqrt{7}\beta^{3/2}}\frac{m_1P}{m_1+m_3}+[2(-1701\beta^6(m_2+m_3)^6(23m_2+22m_3)+81\beta^4m_2^2(m_2+m_3)^4$$
$$\times(32m_2+141m_3)P^2+12\beta^2m_2^4(m_2+m_3)^2(37m_2+63m_3)P^4-4m_2^6(4m_2+5m_3)P^6)f_1+(-1701\beta^6(m_2+m_3)^6$$
$$\times(22m_2+m_3)+486\beta^4m_2^2(2m_2-47m_3)(m_2+m_3)^4P^2+12\beta^2m_2^4(m_2+m_3)^2(214m_2+549m_3)P^4$$
$$-8m_2^6(18m_2+37m_3)P^6)f_2]\frac{-2\sqrt{2}}{885735\sqrt{7}\beta^{19/2}}\frac{m_1^2P^3}{(m_2+m_3)^7(m_1+m_3)^2}+\bigg[-567\beta^6(40f_1+29f_2)$$
$$+\frac{162\beta^4m_2(129m_2f_1+276m_3f_1+127m_2f_2+18m_3f_2)P^2}{(m_2+m_3)^2}-\frac{4\beta^2m_2^3(218m_2f_1+468m_3f_1+427m_2f_2-468m_3f_2)P^4}{(m_2+m_3)^4}$$
$$-\frac{8m_2^5(9m_2f_1+16m_3f_1+5m_2f_2+26m_3f_2)P^6}{(m_2+m_3)^6}\bigg]\frac{-2\sqrt{2}}{885735\sqrt{7}\beta^{19/2}}\frac{m_1^3P^3}{(m_1+m_3)^3}+[81\beta^4(m_2+m_3)^4(50m_2f_1+132m_3f_1$$
$$+40m_2f_2+15m_3f_2)-2\beta^2m_2^2(m_2+m_3)^2(752m_2f_1+1647m_3f_1+628m_2f_2+378m_3f_2)P^2+4m_2^4(2m_2f_1-5m_3f_1$$
$$+12m_2f_2+5m_3f_2)P^4]\frac{-4\sqrt{2}}{885735\sqrt{7}\beta^{19/2}}\frac{m_1^4P^5}{(m_2+m_3)^5(m_1+m_3)^4}+[81\beta^4(16f_1+3f_2)(m_2+m_3)^4+6\beta^2m_2(m_2+m_3)^2$$
$$\times(133m_2f_1+468m_3f_1+20m_2f_2+72m_3f_2)P^2-4m_2^3(19m_2f_1+26m_3f_1+9m_2f_2+16m_3f_2)P^4]$$
$$\times\frac{4\sqrt{2}}{885735\sqrt{7}\beta^{19/2}}\frac{m_1^5P^5}{(m_2+m_3)^4(m_1+m_3)^5}+[\beta^2(m_2+m_3)^2(122m_2f_1-396m_3f_1+118m_2f_2-9m_3f_2)$$
$$+2m_2^2(16m_2f_1+37m_3f_1-2m_2f_2+5m_3f_2)P^2\frac{-8\sqrt{2}}{885735\sqrt{7}\beta^{19/2}}\frac{m_1^6P^7}{(m_2+m_3)^3(m_1+m_3)^6}+[\beta^2(104f_1+25f_2)(m_2+m_3)^2$$
$$+2m_2(m_2f_1+20m_3f_1-3m_2f_2-2m_3f_2)P^2]\frac{-8\sqrt{2}}{885735\sqrt{7}\beta^{19/2}}$$
$$\times\frac{m_1^7P^7}{(m_2+m_3)^2(m_1+m_3)^7}+[4(m_2-m_3)f_1+m_3f_2]\frac{16\sqrt{2}}{885735\sqrt{7}\beta^{19/2}}\frac{m_1^8P^9}{(m_2+m_3)(m_1+m_3)^8}+(4f_1-f_2)\frac{16\sqrt{2}}{2657205\sqrt{7}\beta^{19/2}}$$
$$\times\frac{m_1^9P^9}{(m_1+m_3)^9}+\bigg[\bigg(-25515(m_2+m_3)^8+\frac{49329m_2^2(m_2+m_3)^6P^2+5103m_2(m_2+m_3)^7P^2}{\beta^2}$$
$$+\frac{1458m_2^4(m_2+m_3)^4P^4-7290m_2^3(m_2+m_3)^5P^4}{\beta^4}+\frac{108m_2^5(m_2+m_3)^3P^6-300m_2^6(m_2+m_3)^2P^6}{\beta^6}$$
$$+\frac{24m_2^7(m_2+m_3)P^8-8m_2^8P^8}{\beta^8}\bigg)m_2f_1+\bigg(-178605(m_2+m_3)^8+\frac{34020m_2^2(m_2+m_3)^6P^2+112266m_2(m_2+m_3)^7P^2}{\beta^2}$$
$$+\frac{3888m_2^4(m_2+m_3)^4P^4-32076m_2^3(m_2+m_3)^5P^4}{\beta^4}+\frac{2376m_2^5(m_2+m_3)^3P^6-624m_2^6(m_2+m_3)^2P^6}{\beta^6}$$
$$+\frac{16m_2^8P^8-48m_2^7(m_2+m_3)P^8}{\beta^8}\bigg)2m_2f_2\bigg]\frac{-2\sqrt{2}}{2657205\sqrt{7}\beta^{3/2}}\frac{P}{(m_2+m_3)^9}\bigg\}. \quad \text{(A6)}$$

APPENDIX B: THE AMPLITUDES FOR THE 3^1S_0 $q\bar{q}$ DECAY IN THE $^3P_0$ MODEL

$$\begin{aligned}
&\mathcal{M}^{LS}(3^1S_0 \to 2^3S_1 + 1^1S_0) \\
&= \gamma e^{(-[(m_1m_2(m_2-m_3)m_3+m_2^2m_3^2+m_1^2(m_2^2+m_2m_3+m_3^2)]P^2)/(3\beta^2(m_1+m_3)^2(m_2+m_3)^2)}\sqrt{E_aE_bE_c}\frac{1}{\pi^{3/4}}(f_1-f_2)P\{[405\beta^6(m_1+m_3)^6 \\
&\times(m_2+m_3)^6(63m_1m_2+70m_1m_3+50m_2m_3+57m_3^2)-54\beta^4(m_1+m_3)^4(m_2+m_3)^4(22m_2^3m_3^3+45m_2^2m_3^4 \\
&+m_1m_2m_3^2(163m_2^2+472m_2m_3+240m_3^2)+m_1^3(285m_2^3+1022m_2^2m_3+1022m_2m_3^2+308m_3^3) \\
&+m_1^2m_3(448m_2^3+1515m_2^2m_3+1328m_2m_3^2+330m_3^3))]P^2+[36\beta^2(m_1+m_3)^2(m_2+m_3)^2 \\
&\times(m_2m_3+2m_1m_2+m_1m_3)^2(m_2^2(2m_2-3m_3)m_3^3-m_1m_2m_3^2(13m_2^2+46m_2m_3+18m_3^2)+m_1^3(18m_2^3+88m_2^2m_3 \\
&+133m_2m_3^2+58m_3^3)+m_1^2m_3(20m_2^3+96m_2^2m_3+166m_2m_3^2+75m_3^3))]P^4-8(m_2m_3+2m_1m_2+m_1m_3)^4 \\
&\times(m_2m_3-m_1m_2-2m_1m_3)^2(m_1m_2+2m_1m_3+2m_2m_3+3m_3^2)P^6\}\frac{1}{19683\sqrt{15}\beta^{15/2}}\frac{1}{(m_1+m_3)^7(m_2+m_3)^7},
\end{aligned} \tag{B1}$$

$$\begin{aligned}
&\mathcal{M}^{LS}(3^1S_0 \to 1^3D_1 + 1^1S_0) \\
&= 4\gamma e^{(-[(m_1m_2(m_2-m_3)m_3+m_2^2m_3^2+m_1^2(m_2^2+m_2m_3+m_3^2)]P^2)/(3\beta^2(m_1+m_3)^2(m_2+m_3)^2)}\sqrt{E_aE_bE_c}\frac{1}{\pi^{3/4}}P\{[(405\beta^6(m_1+m_3)^6 \\
&\times(m_2+m_3)^6(3m_1m_2+14m_1m_3-11m_2m_3)+27\beta^4(m_1+m_3)^4(m_2+m_3)^4(50m_2^3m_3^3+33m_2^2m_3^4) \\
&+m_1m_2m_3^2(203m_2^2+68m_2m_3-84m_3^2)+m_1^3(27m_2^3-374m_2^2m_3-608m_2m_3^2-224m_3^3) \\
&-m_1^2m_3(-152m_2^3+423m_2^2m_3+776m_2m_3^2+252m_3^3))P^2+36\beta^2(m_1+m_3)^2(m_2+m_3)^2(m_2m_3+2m_1m_2 \\
&+m_1m_3)^2(m_2^2(m_2-3m_3)m_3^3-4m_1m_2m_3^2(2m_2^2+8m_2m_3+3m_3^2)+m_1^3(3m_2^3+26m_2^2m_3+53m_2m_3^2+26m_3^3) \\
&+m_1^2m_3(4m_2^3+27m_2^2m_3+71m_2m_3^2+36m_3^3))P^4-(4(m_2m_3+2m_1m_2+m_1m_3)^4(m_2m_3-m_1m_2-2m_1m_3)^2 \\
&\times(m_1m_2+2m_1m_3+2m_2m_3+3m_3^2))P^6]f_1+[(405\beta^6(m_1+m_3)^6(m_2+m_3)^6(-3m_1m_2-14m_2m_3 \\
&+11m_1m_3)+27\beta^4(m_1+m_3)^4(m_2+m_3)^4(224m_2^3m_3^3+252m_2^2m_3^4)+4m_1m_2m_3^2(152m_2^2+194m_2m_3+21m_3^2) \\
&-m_1^3(27m_2^3+152m_2^2m_3+203m_2m_3^2+50m_3^3)+m_1^2m_3(374m_2^3+423m_2^2m_3-68m_2m_3^2-33m_3^3))P^2 \\
&-36\beta^2(m_1+m_3)^2(m_2+m_3)^2(m_2m_3+2m_1m_2+m_1m_3)^2(2m_2^2m_3^3(13m_2+18m_3) \\
&+m_1m_2m_3^2(53m_2^2+71m_2m_3-12m_3^2)+m_1^3(3m_2^3+4m_2^2m_3-8m_2m_3^2+m_3^3)+m_1^2m_3(26m_2^3+27m_2^2m_3 \\
&-32m_2m_3^2-3m_3^3))P^4+(4(m_1m_2-m_1m_3+2m_2m_3)^2(m_2m_3+2m_1m_2+m_1m_3)^4(m_1m_2+2m_1m_3 \\
&+2m_2m_3+3m_3^2))P^6]f_2\}\frac{1}{98\,415\sqrt{3}\beta^{15/2}}\frac{1}{(m_1+m_3)^7(m_2+m_3)^7}.
\end{aligned} \tag{B2}$$

The amplitudes of $3^1S_0 \to 1^3S_1 + 1^3S_1$, $1^3S_1 + 1^1S_0$, $1^3P_0 + 1^1S_0$ and $1^3P_2 + 1^1S_0$ are taken from Appendix A of Ref. [6].

APPENDIX C: FLAVOR AND CHARGE MULTIPLICITY FACTORS

The flavor factors f_1 and f_2 can be calculated using the matrix notation introduced in Ref. [13] with the meson flavor wave functions following the conventions of Ref. [29] for the special process with definite charges like $s\bar{s} \to K^{*+}K^-$. In order to obtain the general (i.e. charge independent) width of decays like $s\bar{s} \to K^*K$, one should multiply the width $\Gamma(s\bar{s} \to K^{*+}K^-)$ by a charge multiplicity factor $\mathcal{F}$. The f_1, f_2, and $\mathcal{F}$ for all the processes considered in this work are given in Table I.

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