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Electromagnetic form factors of the pion and kaon from the instanton vacuum

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We investigate the pion and kaon (π^+, K^+, K^0) electromagnetic form factors in the spacelike region: $Q^2 \lesssim 1$ GeV, based on the gauged low-energy effective chiral action from the instanton vacuum in the large N_c limit. Explicit flavor SU(3) symmetry breaking is taken into account. The nonlocal contributions turn out to be crucial to reproduce the experimental data. While the pion electromagnetic form factor is in good agreement with the data, the kaon one seems underestimated. We also calculate the electromagnetic charge radii for the pion and kaon: $\langle r^2\rangle_{\pi^+} = 0.455$ fm^2, $\langle r^2\rangle_{K^+} = 0.534$ fm^2, and $\langle r^2\rangle_{K^0} = -0.060$ fm^2 without any adjustable free parameter except for the average instanton size and interinstanton distance, and they are compatible with the experimental data. The low-energy constant L_9 in the large N_c limit is estimated to be 8.42×10^{-3} from the pion charge radius.

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I. INTRODUCTION

Understanding the structure of the pion has been one of the most important issues in quantum chromodynamics (QCD), since it is the lightest hadron, so that it is identified as the Goldstone boson arising from spontaneous breaking of chiral symmetry. In particular, the electromagnetic (EM) form factor of the pion reveals its internal structure in terms of quark and gluon degrees of freedom. Theoretically, it can be described at high energy spacelike region of the momentum transfer by the factorization theorem [1–5], i.e. it can be explained by separating the hard perturbative amplitude from the soft-part pion wave function which contains nonperturbative information of QCD. At very small momentum transfers, chiral perturbation theory (χPT) provides a good framework to explain the pion EM form factor [6]. A recent work has calculated the pion and kaon form factors to next-to-next-to leading order in χPT [7] in the range of the momentum transfer $-0.25 \leq q^2 \leq 0$ GeV^2. Moreover, the pion EM form factor has been studied also in lattice QCD [8–10] with various different methods used. Experimentally, there has been a great deal of measurements of the pion EM form factor [11–23].

In the intermediate spacelike region of the momentum transfer, one has to use model approaches in order to investigate the pion EM form factor, since neither perturbative QCD nor χPT can be applied to this region. Actually, there has been a great amount of theoretical works to explain the pion EM form factor, various theoretical models being employed: For example, the vector dominance model [24], the Nambu-Jona-Lasinio (NJL) model [25–27], Bethe-Salpeter amplitudes [28,29], Instanton approach [30–34], Kroll-Lee-Zumino model [35], and so on.

In the present work, we want to investigate the pion and kaon EM form factors in the spacelike momentum transfer region ($0 \leq Q^2 \leq$ GeV), based on the gauged low-energy effective chiral action (EχA) from the instanton vacuum [36]. In order to describe the kaon EM form factor, we need to consider SU(3) symmetry breaking explicitly. Thus, we employ the extended effective chiral action developed in Refs. [37–40] in which the effects of the current quark mass have been explicitly considered in the instanton vacuum. Since the instanton vacuum realizes χSB naturally via quark zero modes, it may provide a good framework to study the EM form factor of the pion and kaon, i.e. of the pseudo-Goldstone bosons. Another virtue comes from the fact that there are only two parameters in this approach, namely, the average instanton size $\bar\rho \approx \frac{1}{3}$ fm and average interinstanton distance $\bar R \approx 1$ fm. The normalization point of this approach is determined by the average size of instantons and is approximately equal to $\rho^{-1} \approx 0.6$ GeV. The values of the $\bar\rho$ and $\bar R$ were estimated many years ago phenomenologically in Ref. [41] as well as theoretically in Ref. [42–44]. Furthermore, it was confirmed by various lattice simulations of the QCD vacuum [45–47]. Also lattice calculations of the quark propagator [48,49] are in a remarkable agreement with that of Ref. [42]. A recent lattice simulation with the interacting instanton liquid model obtains $\bar\rho \simeq 0.32$ fm and $\bar R \simeq 0.76$ fm with the finite current quark mass m taken into account [50].

The smallness of the packing parameter $\pi\bar\rho^4/\bar R^4 \approx 0.1$ makes it possible to average the determinant over collective coordinates of instantons with fermionic quasiparticles, i.e. constituent quarks ψ introduced. The averaged determinant turns out to be the light-quark partition function $Z[V, m]$ which is a functional of the electromagnetic field V and can be represented by a functional integral over the constituent quark fields with the gauged effective chiral action $S[\psi^\dagger, \psi, V]$. However, it is not trivial to make the

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action gauge-invariant due to the nonlocality of the quark-quark interactions generated by instantons. In the previous paper [40], it was demonstrated how to gauge the nonlocal effective chiral action in the presence of the external electromagnetic field and was shown that the low-energy theorem of the axial anomaly relevant to the process $G\tilde{G}\to\gamma\gamma$ is satisfied. Moreover, the gauged effective chiral action was shown to be very successful in describing various observable of mesons and vacuum properties [51–57].

In the present work, we want to show that the pion EM form factor is indeed in good agreement with experimental data. In addition, the results for the kaon EM form factor will be given. The EM charge radii of the mesons are shown to be: $\langle r^2\rangle_{\pi^+} = 0.455\ \mathrm{fm}^2$, $\langle r^2\rangle_{K^+} = 0.486\ \mathrm{fm}^2$ and $\langle r^2\rangle_{K^0} = -0.060\ \mathrm{fm}^2$ without any adjustable free parameter, which are all in good agreement with experimental data. The low-energy constant L_9 in the large N_c limit is found to be 8.42×10^{-3}.

We organize the present work as follows: In Sec. II, we briefly explain the general formalism relevant for studying the EM form factors. In Sec. III, we introduce the nonlocal chiral quark model from the instanton vacuum. In Sec. IV, the numerical results are discussed and compared with those of other works. The final section is devoted to summarizing the present work and to drawing conclusions.

II. ELECTROMAGNETIC FORM FACTORS FOR PION AND KAON

The electromagnetic (EM) form factor of a pseudoscalar meson $F_{\pi,K}$ can be defined by the following matrix element:

$$\langle\mathcal{M}(P_f)|j_\mu^{\mathrm{EM}}(0)|\mathcal{M}(P_i)\rangle = (P_i+P_f)_\mu F_{\mathcal{M}}(q^2), \tag{1}$$

where $|\mathcal{M}\rangle$ stands for the pseudoscalar meson state, for example, $|\pi^+(u\bar{d})\rangle$, $|K^0(d\bar{s})\rangle$ and $|K^+(u\bar{s})\rangle$. The P_i and P_f represent the initial and final on-shell momenta for the meson, respectively. Then the masses of the mesons can be obtained from $P_i^2 = P_f^2 = m_{\pi,K}^2$, and we use 140 MeV and 495 MeV for the pion and kaon masses, respectively. The momentum transfer is written as $q = P_f - P_i$, of which the square is given by $-q^2 = Q^2 > 0$ in the spacelike region. Here, the EM current in flavor SU(3) is defined as follows:

$$\begin{aligned} j_\mu^{\mathrm{EM}}(x) &= \sum_{\mathrm{u,d,s}} e_f\psi_f^\dagger(x)\gamma_\mu\psi_f(x) \\ &= \frac{2}{3}u^\dagger(x)\gamma_\mu u(x) - \frac{1}{3}d^\dagger(x)\gamma_\mu d(x) \\ &\quad -\frac{1}{3}s^\dagger(x)\gamma_\mu s(x), \end{aligned} \tag{2}$$

where e_f indicates the electric charge of the quark with flavor f. Note that all calculations are performed in Euclidean space, since we are interested in the form factor in the spacelike region. The EM form factors satisfy the normalization conditions at $Q^2 = 0$ by the gauge invariance:

$$F_{\pi^+}(0) = F_{K^+}(0) = 1, \qquad F_{K^0}(0) = 0. \tag{3}$$

The EM (vector) mean square charge radii for the mesons can be evaluated by the derivatives of the form factors at $Q^2 = 0$:

$$\langle r^2\rangle_{\mathcal{M}} = -6\left[\frac{\partial F_{\mathcal{M}}(Q^2)}{\partial Q^2}\right]_{Q^2=0}. \tag{4}$$

Moreover, the pion charge radius relates to the Gasser-Leutwyler low-energy constant (LEC) L_9 [6,7] in the large N_c limit:

$$\langle r^2\rangle_{\pi^+} = \frac{12L_9}{F_\pi^2}. \tag{5}$$

III. GAUGED EFFECTIVE CHIRAL ACTION IN THE PRESENCE OF THE ELECTROMAGNETIC FIELD

The light-quark partition function $Z[V,m]$ with the current quark mass m and in the presence of the external vector field V_μ is defined as

$$Z[V,m] = \int DA_\mu e^{-G^4/4}\mathrm{Det}[i\partial\!\!\!/ + A\!\!\!/ + V\!\!\!/ + im], \tag{6}$$

where A_μ denotes the gluon field and $G_{\mu\nu}$ is the gluon field strength tensor. The m stands for the current quark mass. In the instanton liquid model, it is assumed that the functional integral of Eq. (6) can be solved by expanding it around the classical vacuum. It was already shown in Refs. [36,42] how to solve the integral in Eq. (6) in the chiral limit and in the absence of the external fields. In the present work, we briefly review a method of how to derive a low-energy effective partition function to solve this integral in the presence of the external vector field, closely following Refs. [40,52].

The quark determinant in Eq. (6) can be decomposed into the low- and high-frequency parts: $\mathrm{Det} = \mathrm{Det}_{\mathrm{low}}\cdot\mathrm{Det}_{\mathrm{high}}$. While the high-frequency part is related to the perturbative renormalization, the low-frequency one, which we consider here, is relevant to the low-energy regime. Thus, we consider here the low-frequency part. We first consider the zero-mode approximation for the quark propagator interacting with the ith instanton:

$$S_i = \frac{1}{i\partial\!\!\!/ + A\!\!\!/_i + im} = \frac{1}{p\!\!\!/} + \frac{|\Phi_{i,0}\rangle\langle\Phi_{i,0}|}{im}, \tag{7}$$

where $|\Phi_{i,0}\rangle$ represents the quark zero-mode solution, satisfying

$$(i\partial\!\!\!/ + A\!\!\!/_i)|\Phi_{i,0}\rangle = 0. \tag{8}$$

Though this zero-mode approximation works well for the very small mass of the quark, we have to extend it in order to consider the finite current quark mass as follows:

$$S_i = S_0 + S_0 \not{p} \frac{|\Phi_{0i}\rangle\langle\Phi_{0i}|}{c_i} \not{p} S_0, \tag{9}$$

where the S_0 is the free quark propagator:

$$S_0 = \frac{1}{\not{p} + im} \tag{10}$$

and c_i is a matrix element defined as

$$\begin{aligned} c_i &= -\langle\Phi_{0i}|\not{p} S_0 \not{p}|\Phi_{0i}\rangle = im\langle\Phi_{0i}|S_0\not{p}|\Phi_{0i}\rangle \\ &= im\langle\Phi_{0i}|\not{p} S_0|\Phi_{0i}\rangle. \end{aligned} \tag{11}$$

The approximation given in Eq. (9) allows us to project S_i to the correct zero-modes with the finite m:

$$S_i|\Phi_{0i}\rangle = \frac{1}{im}|\Phi_{0i}\rangle, \qquad \langle\Phi_{0i}|S_i = \langle\Phi_{0i}|\frac{1}{im}. \tag{12}$$

Now, we introduce the external vector field, so that the quark propagator is modified as

$$\tilde{S} = \frac{1}{\not{p} + \not{A} + \not{V} + im}, \qquad \tilde{S}_i = \frac{1}{\not{p} + \not{A}_i + \not{V} + im}. \tag{13}$$

Here, we assume that the total instanton field A may be approximated as a sum of the single instanton fields, $A = \sum_{i=1}^{N} A_i$, which is justified in the dilute instanton liquid ($\bar{\rho} \simeq \frac{1}{3}$ fm and $\bar{R} \simeq 1$ fm). If we switch off the instanton fields, then we can express the quark propagator in the presence of the external fields as follows:

$$\tilde{S}_0 = \frac{1}{\not{p} + \not{V} + im}. \tag{14}$$

The total propagator $\tilde{S}$ can now be expanded with respect to a single instanton A_i:

$$\tilde{S} = \tilde{S}_0 + \sum_i(\tilde{S}_i - \tilde{S}_0) + \sum_{i\neq j}(\tilde{S}_i - \tilde{S}_0)\tilde{S}_0^{-1}(\tilde{S}_j - \tilde{S}_0) + \cdots. \tag{15}$$

Expanding $\tilde{S}$ with respect to the external field V, we are able to express $\tilde{S}_i$ in terms of S_i. Since we use the zero-mode approximation, the gauge invariance for the external vector field is broken. Thus, we have to restore it by introducing the following auxiliary field V' and gauge connection L_i:

$$\not{V}'_i = \bar{L}_i(\not{p} + \not{V})L_i - \not{p}, \tag{16}$$

where the gauge connection L_i is defined as a path-ordered exponent

$$\begin{aligned} L_i(x, z_i) &= \mathrm{P}\exp\left[i\int_{z_i}^{x} ds_\mu V_\mu(s)\right], \\ \bar{L}_i(x, z_i) &= \gamma_0 L_i^\dagger(x, z_i)\gamma_0 \end{aligned} \tag{17}$$

with an instanton coordinate z_i. The field $V'_i(x, z_i)$ under flavor rotation $\psi(x) \to U(x)\psi(x)$ is transformed as $V'_i(x, z_i) \to U(z_i)V'_i(x, z_i)U^{-1}(z_i)$. The propagators $\tilde{S}_i$ and $\tilde{S}_0$ then have the following form:

$$\begin{aligned} \tilde{S}_i &= L_i S'_i \bar{L}_i, \qquad S'_i = \frac{1}{\not{p} + \not{A}_i + \not{V}'_i + im}, \\ \tilde{S}_0 &= L_i S'_{0i} \bar{L}_i, \qquad S'_{0i} = \frac{1}{\not{p} + \not{V}'_i + im}, \end{aligned} \tag{18}$$

Expanding S'_i with respect to $\not{V}'_i$ and resumming it, we obtain the following expression:

$$S'_i = S_i\left[1 + \sum_n(-\hat{V}'_i S_i)^n\right] = S'_{0i} + S'_{0i}\not{p}\frac{|\Phi_{0i}\rangle\langle\Phi_{0i}|}{c_i - b_i}\not{p} S'_{0i}, \tag{19}$$

where

$$\begin{aligned} b_i &= \langle\Phi_{0i}|\not{p}(S'_{0i} - S_0)\not{p}|\Phi_{0i}\rangle, \\ c_i - b_i &= -\langle\Phi_{0i}|\not{p} S'_{0i}\not{p}|\Phi_{0i}\rangle. \end{aligned} \tag{20}$$

Rearranging Eq. (15) for the total propagator, we get

$$\begin{aligned} \tilde{S} &= \tilde{S}_0 + \tilde{S}_0\sum_{i,j}\bar{L}_i^{-1}\not{p}|\Phi_{i0}\rangle\left[\frac{1}{-\mathcal{D}} + \frac{1}{-\mathcal{D}}\mathcal{C}\frac{1}{-\mathcal{D}} + \cdots\right]_{ij} \\ &\quad \times\langle\Phi_{0j}|\not{p} L_j^{-1}\tilde{S}_0 \\ &= \tilde{S}_0 + \tilde{S}_0\sum_{i,j}\bar{L}_i^{-1}\not{p}|\Phi_{i0}\rangle\left[\frac{1}{-\mathcal{V} - \mathcal{T}}\right]_{ij}\langle\Phi_{0j}|\not{p} L_j^{-1}\tilde{S}_0, \end{aligned} \tag{21}$$

where

$$\begin{aligned} \mathcal{V}_{ij} &= \langle\Phi_{0i}|\not{p}(L_i^{-1}\tilde{S}_0\bar{L}_j^{-1})\not{p}|\Phi_{0j}\rangle - \langle\Phi_{0i}|\not{p} S_0 L_i^{-1}L_j\not{p}|\Phi_{0j}\rangle, \\ \mathcal{T}_{ij} &= (1 - \delta_{ij})\langle\Phi_{0i}|\not{p} S_0 L_i^{-1}L_j\not{p}|\Phi_{0j}\rangle, \\ \mathcal{D}_{ij} &= \delta_{ij}\mathcal{V}_{ij} \equiv (b_i - c_i)\delta_{ij}, \\ \mathcal{C}_{ij} &= (1 - \delta_{ij})\mathcal{V}_{ij}. \end{aligned} \tag{22}$$

We introduce now the modified zero-mode solution as follows:

$$|\phi_0\rangle = \frac{1}{\not{p}}L\not{p}|\Phi_0\rangle, \tag{23}$$

which has the same chiral properties as the zero-mode solution $|\Phi_0\rangle$. Then we get

$$\tilde{S} - \tilde{S}_0 = -\tilde{S}_0\sum_{i,j}\not{p}|\phi_{0i}\rangle\left\langle\phi_{0i}\left|\frac{1}{\mathcal{V} + \mathcal{T}}\right|\phi_{0j}\right\rangle\langle\phi_{0j}|\not{p}\tilde{S}_0 \tag{24}$$

with

$$\mathcal{V} + \mathcal{T} = \not{p}\tilde{S}_0\not{p}. \tag{25}$$

The final expression for Eq. (24) is obtained as

$$\mathrm{Tr}(\tilde{S} - \tilde{S}_0) = -\sum_{i,j}\langle\phi_{0,j}|\not{p}\tilde{S}_0^2\not{p}|\phi_{0,i}\rangle\left\langle\phi_{0,i}\left|\frac{1}{\not{p}\tilde{S}_0\not{p}}\right|\phi_{0,j}\right\rangle. \tag{26}$$

In order to derive the low-frequency part of the quark determinant, we now introduce a matrix operator $\tilde{B}(m)$ defined as follows:

$$\tilde{B}(m)_{ij} = \langle\phi_{0,i}|\not{p}\tilde{S}_0\not{p}|\phi_{0,j}\rangle = \langle\Phi_{0i}|\not{p}[L_i^{-1}\tilde{S}_0\bar{L}_j^{-1}]\not{p}|\Phi_{0j}\rangle, \tag{27}$$

where i, j are indices for the different instantons. Then, we can show that

$$\begin{aligned}\ln(\mathrm{Det}_{\mathrm{low}}) &= \mathrm{Tr}\ln\left[\frac{i\not{\partial} + \not{A} + \not{V} + im}{i\not{\partial} + \not{V} + im}\right]\\ &= i\,\mathrm{Tr}\int^m dm'[\tilde{S}(m') - \tilde{S}_0(m')]\\ &= \sum_{i,j}\int^m dm'\frac{d\tilde{B}(m')_{ij}}{dm'}[\tilde{B}(m')]^{-1}_{ji}\\ &= \mathrm{Tr}\ln\tilde{B}(m),\end{aligned}$$

where Tr denotes the trace over the subspace of the quark zero modes. Thus, we have

$$\mathrm{Det}_{\,\mathrm{low}}[V, m] \cong \mathrm{Det}\tilde{B}(m), \tag{28}$$

where $\tilde{B}$ is identified as an extension of the Lee-Bardeen matrix B [58] in the presence of the external vector field V. Taking m to be small and switching off the external fields, we can show that $\tilde{B}$ turns out to be the same as B to order $\mathcal{O}(m)$.

Averaging $\mathrm{Det}_{\mathrm{low}}$ over the instanton collective coordinates z, which provides the effective chiral partition function $Z[V, m]$, we can obtain the fermionized representation of the partition function in Eq. (28):

$$\begin{aligned}Z_{\mathrm{eff}}[V, m] &= \int D\psi D\psi^\dagger \exp\left[\int d^4x\psi^\dagger(i\not{D} + im)\psi\right]\\ &\quad\times\prod^{N_\pm}W_\pm[\psi^\dagger, \psi],\end{aligned} \tag{29}$$

where iD_μ is the covariant derivative defined as

$$iD_\mu = i\partial_\mu + V_\mu. \tag{30}$$

m represents the current quark mass: $m = \mathrm{diag}(m_{\mathrm{u}}, m_{\mathrm{d}}, m_{\mathrm{s}})$, and the quark-instanton interaction, $W_\pm$ is given by

$$\begin{aligned}W_\pm[\psi^\dagger, \psi] &= \int d^4z_\pm\prod_{N_f}\int d^4xd^4y[\psi^\dagger(x)\bar{L}^{-1}(x, z_\pm)\\ &\quad\times\not{p}\Phi_{\pm,0}(x; z_\pm)]\\ &\quad\times[\Phi^\dagger_{\pm,0}(y; z_\pm)(\not{p}L^{-1}(y, z_\pm)\psi(y)].\end{aligned} \tag{31}$$

The fermion fields $\psi^\dagger$ and ψ are interpreted as the dynamical quark fields or constituent quark fields induced by the zero modes of the instantons.

Note that the partition function of Eq. (29) is invariant under local flavor rotations due to the gauge connection L in the interaction term $W_\pm[\psi^\dagger, \psi]$. While we preserve the gauge invariance of the effective chiral action by introducing the gauge connection, we have to pay a price: The effective action depends on the path in the gauge connection L. We will choose the straight-line path as a trial for brevity, though there is in general no physical reason why other choices should be excluded.

The low-energy partition function given in Eq. (29) can be bosonized as shown in Refs. [42,43]. While the bosonization procedure is exact in the case of $N_f = 2$, it is nontrivial for $N_f \geq 3$. However, as pointed out in Refs. [42,43,59], one still can do it taking the limit of large N_c. Assuming that the external vector field is weak and taking the large N_c limit, we can express the quark-instanton interaction in Eq. (29) in the following simpler form:

$$\begin{aligned}W_\pm[\psi^\dagger, \psi] &= (-i)^{N_f}\left[\frac{(2\pi\bar{\rho})^2}{N_c}\right]^{N_f}\int\frac{d^4x}{V}\det{}_f[iJ_\pm(x)],\\ J_\pm(x) &= \int\frac{d^4kd^4p}{(2\pi)^8}e^{i(k-p)\cdot x}[2\pi\bar{\rho}F(K)][2\pi\bar{\rho}F(P)]\\ &\quad\times\left[\psi^\dagger(k)\frac{1\pm\gamma_5}{2}\psi(p)\right], \qquad N_c\to\infty,\end{aligned} \tag{32}$$

where we have dropped the color and flavor indices. Note that $J_\pm$ is identified by its chirality. The form factor $F(k)$ is the Fourier transform of the quark zero-mode solution:

$$\begin{aligned}F(k) &= 2t\left[I_0(t)K_1(t) - I_1(t)K_0(t) - \frac{1}{t}I_1(t)K_1(t)\right],\\ t &= \frac{k\bar{\rho}}{2}.\end{aligned} \tag{33}$$

The V represents the volume factor of Euclidean space. Note that the quark-instanton interaction given in Eq. (32) is simply the same form as the original one with the gauged form factor $F(K)$. Here, the K denotes the covariant momentum $k_\mu - V_\mu$. Thus, the low-energy effective partition function can be written as

$$\begin{aligned}Z_{\mathrm{eff}}[V, m] &= \int D\psi D\psi^\dagger\\ &\quad\times\exp\left(\int d^4x\sum_f\psi_f^\dagger(i\not{D} + im)\psi_f\right)W_+^{N_+}W_-^{N_-}.\end{aligned} \tag{34}$$

The detailed procedure of the bosonization is found in Refs. [59]. Thus, having bosonized Eq. (34), we arrive at the following expression for the low-energy effective partition function:

$$Z_{\mathrm{eff}}[\mathcal{M}^\alpha, V, m] = \int D\mathcal{M}^\alpha\exp(-\mathcal{S}_{\mathrm{eff}}[\mathcal{M}^\alpha, V, m]), \tag{35}$$

where $\mathcal{S}_{\mathrm{eff}}$ stands for the effective chiral action

$$\begin{aligned}\mathcal{S}_{\,\mathrm{eff}}[\mathcal{M}^\alpha, V, m] &= -\mathrm{Sp}_{c,f,\gamma}\ln[i\not{D} + im\\ &\quad + i\sqrt{M(iD)}U^{\gamma_5}\sqrt{M(iD)}].\end{aligned} \tag{36}$$

Here, the $\mathrm{Sp}_{c,f,\gamma}$ stands for the functional trace, i.e. $\int d^4x \mathrm{Tr}_{c,f,\gamma}\langle x|\cdots|x\rangle$, in which the subscripts c, f, and γ represent the traces over color, flavor, and Dirac spin spaces, respectively. The U^{γ_5} reads:

$$U^{\gamma_5} = U(x)\frac{1+\gamma_5}{2} + U^\dagger(x)\frac{1-\gamma_5}{2} = 1 + \frac{i}{F_\pi}\gamma_5(\mathcal{M}\cdot\lambda) - \frac{1}{2F_\pi^2}(\mathcal{M}\cdot\lambda)^2\cdots. \tag{37}$$

The pseudoscalar meson fields π^α are defined explicitly as

$$\mathcal{M}\cdot\lambda = \frac{1}{\sqrt{2}} \times \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \tag{38}$$

where λ^α stands for the Gell-Mann SU(3) flavor matrix, satisfying $[\lambda^\alpha, \lambda^\beta] = 2\delta^{\alpha\beta}$.

The dynamical quark mass in the square brackets in the r.h.s. of Eq. (35) can be written explicitly as follows:

$$M(iD) = M_0 F^2(iD) f(m) = M_0 F^2(iD)\left[\sqrt{1+\frac{m^2}{d^2}} - \frac{m}{d}\right], \tag{39}$$

where $F(iD)$ denotes the form factor arising from the Fourier transform of the quark zero-mode solution and consists of the modified Bessel functions of the second kind as shown in Eq. (33). However, in the present calculation, we take a simple dipole-type parameterization of $F(iD)$ for numerical simplicity:

$$F(iD) = \frac{2\Lambda^2}{2\Lambda^2 - D^2}, \tag{40}$$

where Λ is the cutoff mass being chosen to be $1/\bar{\rho} \approx 600$ MeV as usual. The current-quark mass (m) correction term, $f(m)$ in Eq. (39) is obtained as in Refs. [37,60]. The parameter d can be calculated uniquely as follows [38]:

$$d = \sqrt{\frac{0.08385}{2N_c}\frac{8\pi\bar{\rho}}{R^2}} \approx 0.198 \text{ GeV}. \tag{41}$$

The value of the constituent quark mass at zero momentum M_0 can be self-consistently determined by the saddle-point equation:

$$\frac{N}{V} = 4N_c \int \frac{d^4k}{(2\pi)^4}\frac{M^2(k)}{k^2+M^2(k)}, \tag{42}$$

which gives $M_0 \approx 350$ MeV. We select the up and down quark masses to be 5 MeV and the strange quark mass to be 150 MeV.

IV. ELECTROMAGNETIC FORMFACTORS FOR PION AND KAON FROM THE INSTANTON VACUUM

We are now in a position to calculate the matrix element for the EM form factor in Eq. (1) using the gauged effective chiral action, defined in Eq. (36). It is worth mentioning that all relevant diagrams for the EM form factor are obtained from the gauged effective chiral action straightforwardly. Moreover, the Ward-Takahashi identity is fully satisfied as well, that is, the pion EM form factor at $Q^2 = 0$ becomes exactly $F_\pi(0) = 1$. The meson matrix element in Eq. (1) can be evaluated via the functional derivative of the effective chiral action:

$$\left(\frac{\delta^3\mathcal{S}_{\mathrm{eff}}[\pi,V,m]}{\delta\pi^\alpha(y)\delta\pi^\beta(x)\delta V_\mu(0)}\right)_{V=0} = e_q N_c \mathrm{Tr}_{f,\gamma}\Bigg[\underbrace{\frac{1}{\mathcal{D}}X^{\alpha\beta}\frac{1}{\mathcal{D}}(\gamma_\mu + W_\mu - Z_\mu)}_{\mathrm{a,b}} - \underbrace{\frac{1}{\mathcal{D}}(W_\mu^{\alpha\beta} - Z_\mu^{\alpha\beta})}_{\mathrm{c}}\underbrace{\frac{1}{\mathcal{D}}X^\alpha\frac{1}{\mathcal{D}}X^\beta\frac{1}{\mathcal{D}}(\gamma_\mu + W_\mu - Z_\mu)\frac{1}{\mathcal{D}}X^\beta\frac{1}{\mathcal{D}}X^\alpha\frac{1}{\mathcal{D}}(\gamma_\mu + W_\mu - Z_\mu)}_{\mathrm{e,f}} + \underbrace{\frac{1}{\mathcal{D}}X^\alpha\frac{1}{\mathcal{D}}(W_\mu^\beta - Z_\mu^\beta)}_{\mathrm{g}} + \underbrace{\frac{1}{\mathcal{D}}X^\beta\frac{1}{\mathcal{D}}(W_\mu^\alpha - Z_\mu^\beta)}_{\mathrm{h}}\Bigg], \tag{43}$$

where we use the following abbreviations:

$$\mathcal{D} = i\not{\partial} + im + iM(i\partial), \qquad \frac{\delta X}{\delta\pi^\alpha} = X^\alpha, \qquad \frac{\delta^2 X}{\delta\pi^\alpha\delta\pi^\beta} = X^{\alpha\beta} \tag{44}$$

with

$$X = i\sqrt{M(i\partial)}U^{\gamma_5}\sqrt{M(i\partial)}, \tag{45}$$

and

$$W_\mu = i\left(\frac{\partial}{\partial p_\mu}\sqrt{M(i\partial)}\right)U^{\gamma_5}\sqrt{M(i\partial)}, \qquad Z_\mu = i\sqrt{M(i\partial)}U^{\gamma_5}\left(\frac{\partial}{\partial p_\mu}\sqrt{M(i\partial)}\right). \tag{46}$$

In Eq. (43) we group each term corresponding to Fig. 1, in which we show all relevant Feynman diagrams containing local (solid box) and nonlocal (solid circle) interacting vertices to the photon field. The solid and dashed lines represent the quark and meson fields, respectively. The terms with V and W represents those with the nonlocal interacting vertices. Note that, within the present model, diagram (d), which contains one-quark loop and a local interacting vertex, is not generated as shown in Eq. (43).

Using Eqs. (43) and (44) and carrying out a straightforward manipulation of the functional trace, we have seven contributions to the meson EM form factor as depicted schematically in Fig. 1 excluding diagram 1(d). Diagrams 1(a)–1(c) stand for the isosinglet contributions, whereas all other diagrams contain both the isosinglet and isotriplet ones. Note that diagram 1(e) arises from the three-point correlation function in the (local) impulse approximation. Diagram 1(c), however, becomes zero, since they contain a single quark propagator. We list the properties of all these diagrams in Table I as a summary.

Having performed all remaining calculations, we arrive at the final expression for the pion and kaon EM form factors:

$$F_{\pi,K}(Q^2) = \frac{1}{F^2_{\pi,K}} \sum_{\text{flavor}} \frac{8e_q N_c}{(2p_i \cdot q + m^2_{\pi,K})} \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} \times \sum_{i=\mathrm{a}}^{\mathrm{h}} \mathcal{F}_i(k, q), \tag{47}$$

where

$$\begin{aligned}
\mathcal{F}^{\mathrm{L}}_{\mathrm{a}} &= \frac{\sqrt{M_b M_c}(\bar{M}_c k_{bd} + \bar{M}_b k_{cd})}{2(k_b^2 + \bar{M}_b^2)(k_c^2 + \bar{M}_c^2)},\\
\mathcal{F}^{\mathrm{NL}}_{\mathrm{b}} &= \frac{\sqrt{M_b M_c}(\sqrt{M_c}\hat{M}_{bd} - \sqrt{M_b}\hat{M}_{cd})(k_{bc} - \bar{M}_b\bar{M}_c)}{(k_b^2 + \bar{M}_b^2)(k_c^2 + \bar{M}_c^2)}, \qquad \mathcal{F}^{\mathrm{NL}}_{\mathrm{c}} = 0, \qquad \mathcal{F}^{\mathrm{NL}}_{\mathrm{d}} = 0,\\
\mathcal{F}^{\mathrm{L}}_{\mathrm{e}} &= \frac{M_a\sqrt{M_b M_c}(k_{ab}k_{cd} + k_{ac}k_{bd} - k_{bc}k_{ad} + \bar{M}_a\bar{M}_c k_{bd} + \bar{M}_a\bar{M}_b k_{cd} - \bar{M}_b\bar{M}_c k_{ad})}{(k_a^2 + \bar{M}_a^2)(k_b^2 + \bar{M}_b^2)(k_c^2 + \bar{M}_c^2)},\\
\mathcal{F}^{\mathrm{NL}}_{\mathrm{f}} &= \frac{M_a\sqrt{M_b M_c}(\sqrt{M_b}\hat{M}_{cd} - \sqrt{M_c}\hat{M}_{bd})(\bar{M}_c k_{ab} + \bar{M}_b k_{ac} - \bar{M}_a k_{bc} + \bar{M}_a\bar{M}_b\bar{M}_c)}{(k_a^2 + \bar{M}_a^2)(k_b^2 + \bar{M}_b^2)(k_c^2 + \bar{M}_c^2)},\\
\mathcal{F}^{\mathrm{NL}}_{\mathrm{g}} &= \frac{\sqrt{M_a M_c}[\sqrt{M_b}\hat{M}_{ad} - \sqrt{M_a}\hat{M}_{bd}](k_{ac} + \bar{M}_a\bar{M}_c)}{2(k_a^2 + \bar{M}_a^2)(k_c^2 + \bar{M}_c^2)},\\
\mathcal{F}^{\mathrm{NL}}_{\mathrm{h}} &= \frac{\sqrt{M_a M_b}[\sqrt{M_c}\hat{M}_{ad} - \sqrt{M_a}\hat{M}_{cd}](k_{ab} + \bar{M}_a\bar{M}_b)}{2(k_a^2 + \bar{M}_a^2)(k_b^2 + \bar{M}_b^2)},
\end{aligned} \tag{48}$$

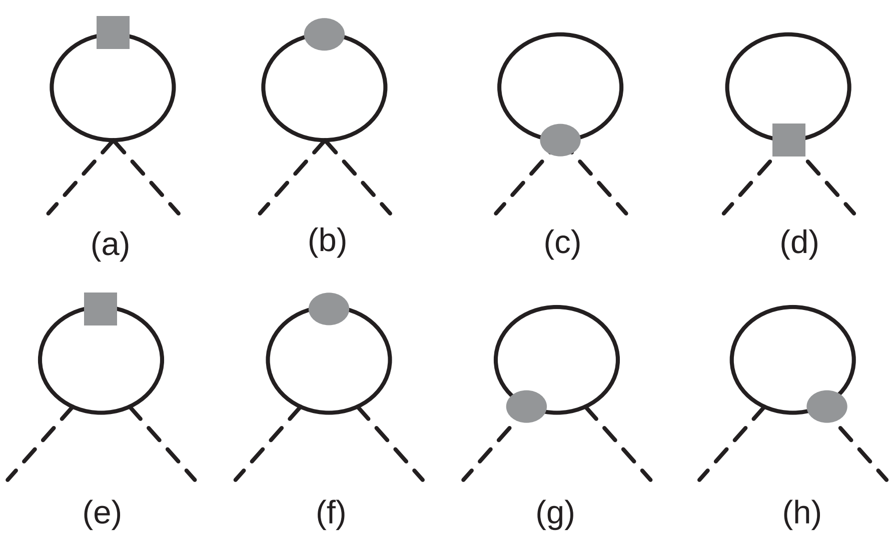

FIG. 1. Relevant Feynman diagrams for the pion (kaon) EM form factor. The solid and dashed lines denote the quark and pseudoscalar meson, respectively. The solid box and circle stand for the local- and nonlocal-interaction vertices, respectively, to which the external photon field is coupled.

TABLE I. Properties of the relevant diagrams. We use the following abbreviations: L (local), NL (nonlocal), S (singlet), and T (triplet).

Diagram	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Interaction	L	NL	NL	L	L	NL	NL	NL
Isospin	S	S	S	S	S,T	S,T	S,T	S,T
Contribution	∘	∘	×	×	∘	∘	∘	∘

where the subscripts $a \sim h$ correspond to the diagrams shown in Fig. 1. Note that all integrals are carried out in Euclidean space. The notations are defined as $k_a = k - P_i/2 - q/2$, $k_b = k + P_i/2 - q/2$, $k_c = k + P_i/2 + q/2$, and $k_d = P_i$. The $\bar{M}_\alpha$ is the sum of the current and momentum-dependent quark masses, $m + M(k_\alpha)$. We use also the following abbreviations: $k_{\alpha\beta} = k_\alpha \cdot k_\beta$, and $\hat{M}_{\alpha\beta}$ is defined as the derivative of the dynamical quark mass with respect to the momenta:

$$\hat{M}_{\alpha\beta} = \frac{\partial\sqrt{M_\alpha}}{\partial k_\alpha^\mu} k_{\beta\mu} = -\frac{4\sqrt{M}\Lambda^2}{(2\Lambda^2 + k_\alpha^2)^2}(k_\alpha \cdot k_\beta). \quad (49)$$

V. NUMERICAL RESULTS AND DISCUSSION

In this section, we discuss the numerical results for the pion and kaon EM form factors. Since our framework is fully relativistic, we choose the Breit-momentum frame for convenience as done in Refs. [26,57] (see Appendix). In the upper-left panel of Fig. 1, we draw each contribution from the diagrams depicted in Fig. 1. Note that the diagrams of 1(e), 1(g), and 1(h) are the major contribution to the EM form factor. This tendency was already observed in Ref. [25] as well as in Ref. [31]. However, in contrast, the contributions from diagrams 1(a), 1(b), and 1(f) are relatively small and zero at $Q^2 = 0$. Moreover, we observe that the charge radii ($\langle r^2\rangle_{\mathcal{M}}$) sensitively depend on them. In particular, the cancellation between the diagrams 1(b) and 1(f) plays an important role in producing the value of the pion EM charge radius compatible to the experimental one.

The results of the pion and kaon EM form factors $F_{\pi^+}(Q^2)$ and $F_{K^+}(Q^2)$ are drawn in the upper-right and lower-left panels, respectively, in Fig. 2. The F_{K^0} is not presented, since it is almost zero in the region: $(0 \lesssim Q^2 \lesssim 1$ GeV). The experimental data for $F_{\pi^+}(Q^2)$ are taken from Refs. [11,12,18,21]. We test the normalization conditions for the form factors given in Eq. (3). Using the model values for the pion and kaon weak decay constants, $F_\pi = 93$ MeV and $F_K = 108$ MeV [57], which are almost the

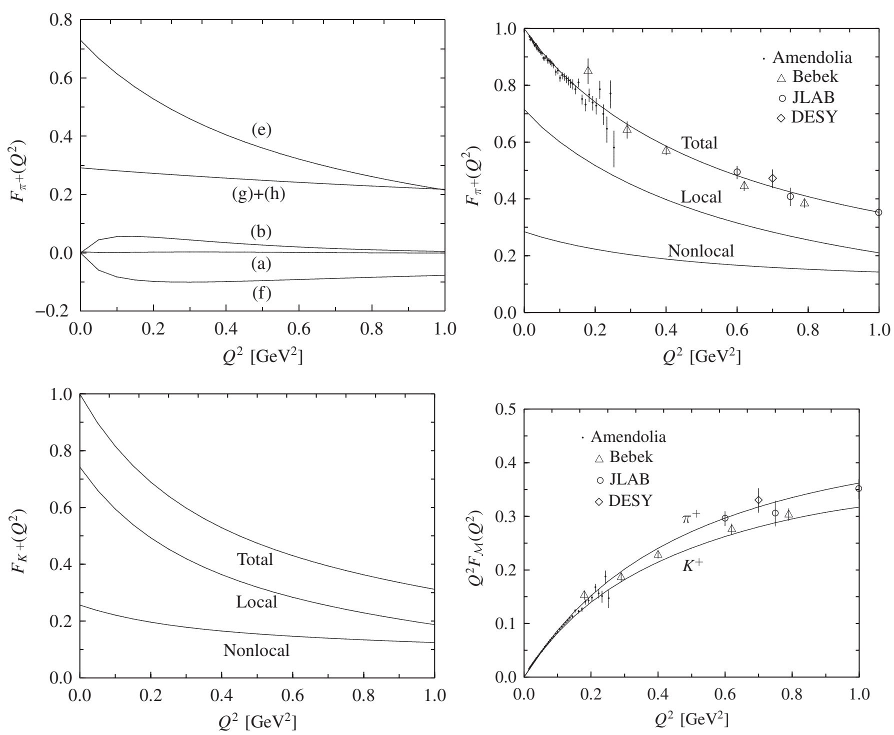

FIG. 2. Contribution from each diagram, shown in Fig. 1 for $F_{\pi^+}(Q^2)$ (upper left). $F_{\pi^+}(Q^2)$ (upper right) and $F_{K^+}(Q^2)$ (lower left) depicted with the local and nonlocal contributions separately. $Q^2F_{K^+}(Q^2)$ and $Q^2F_{K^+}(Q^2)$ are also given (lower right). The experimental data for $F_{\pi^+}(Q^2)$ are taken from Refs. [11,12] (△), [18] (·), [21] (∘) and [64] (◇).

same with the empirical values, we have $F_{\pi^+}(0) = F_{K^+}(0) = 1$ and $F_{K^0}(0) = 0$ without adjusting any parameter. It indicates that the current conservation is preserved in the present work. Here, we want to discuss the charge normalization condition for the form factor in detail. For definiteness, we consider diagrams 1(e), 1(g), and 1(h) which survive at $Q^2 = 0$ as shown in the upper-left panel of Fig. 2. Taking the limit $Q^2 \to 0$, we have $k_a \to k - P/2$ and $k_{b,c} \to k + P/2$ in which $P = P_i = P_f$ [57]. Using them, the contributions from diagrams 1(e), 1(g), and 1(h) in Eq. (47), the pion EM form factor at $Q^2 = 0$ can be rewritten in the chiral limit as follows:

$$\lim_{q\to 0} F_\pi(Q^2) \approx \frac{4N_c}{F_\pi^2}\int_{-\infty}^{\infty}\frac{d^4k}{(2\pi)^4}\left[\frac{M^2}{(k^2+M^2)^2}+\frac{2M\sqrt{M}\tilde{M}}{k^2+M^2}\right] + \mathcal{O}(k\cdot P, m_\pi^4), \tag{50}$$

where M' is defined as

$$\tilde{M} = -\frac{1}{|k|}\frac{\partial\sqrt{M}}{\partial|k|} = \frac{4\sqrt{M_0}\Lambda^2}{(2\Lambda^2+k^2)^2}. \tag{51}$$

In deriving Eq. (50), we have assumed $P \ll 1$ for simplicity. Since the pion decay constant F_π is expressed in the present model as follows:

$$F_\pi^2 \approx 4N_c\int_{-\infty}^{\infty}\frac{d^4k}{(2\pi)^4}\left[\frac{M^2}{(k^2+M^2)^2}+\frac{2M\sqrt{M}\tilde{M}}{k^2+M^2}\right] + \mathcal{O}(k\cdot P, m_{\mathcal{M}}^4), \tag{52}$$

we show that the pion form factor calculated in the present model satisfies correctly the following normalization condition:

$$\lim_{q\to 0} F_\pi(Q^2) = 1. \tag{53}$$

We again want to emphasize that the normalization condition of Eq. (3) is satisfied without any adjustment of parameters. In other words, once we choose the $1/\bar{\rho} \simeq$ 600 MeV (equivalently Λ), then the M_0 and F_π are uniquely determined within the model [by the saddle-point equation of Eq. (42) for instance].

We have verified that the charge normalization condition beyond the chiral limit [$F_K(0)$] is also satisfied by Eqs. (50) and (52), replacing M by $m + M$ and with the model value for the kaon decay constant F_K. However, we note that the value of the F_K in the present work, compared to the empirical value $F_K = 113$ MeV, turns out to be underestimated: $F_K \approx 108$ MeV. This can be understood, since we take the large N_c limit. The meson-loop corrections ($1/N_c$ corrections) are known to be essential in improving the value of the F_K. Thus, within the present framework, the charge normalization conditions are satisfied consistently for the kaon EM form factor as well as the pion one.

We draw the local and nonlocal contributions separately, in addition to the total one in the lower-left panel of Fig. 2. We see also from Fig. 2 that the $F_{\pi^+}(Q^2)$ is reproduced quantitatively well in comparison to the experimental data. It turns out that the nonlocal terms contribute to the form factors by about 30%. Thus, it is of great importance to consider the gauge invariance from the outset. Note that the overall shape and behavior of the kaon EM form factor F_{K^+} are rather similar to $F_{\pi^+}(Q^2)$ apart from the fact that it falls off slightly faster than that of the pion. In the lower right panel of Fig. 2, we plot $Q^2 F_{\pi^+,K^+}(Q^2)$.

We now examine the pion and kaon charge radii $\langle r^2\rangle_{\mathcal{M}}^{1/2}$, defined as Eq. (4). In Table II we list the results of the pion and kaon charge radii (π^+, K^+, and K^0) with each contribution separately. For the neutral kaon (K^0), we list the mean square charge radius. Note that these values are well compatible with those computed from the general QCD parametrization method [62]. As seen in the case of the EM form factors, we find that the nonlocal part contributes to the radii by about 30% again. The nonlocal contributions obtained in the present work are more significant, compared with those of Refs. [25,31], in which they are estimated to be only ~10%. On the contrary, the local contribution is rather similar to that of Ref. [31] (0.55 fm). Interestingly, we obtain the K^+ charge radius about 30% larger than the experimental data. As mentioned previously, the $1/N_c$ meson-loop corrections are known to be essential in describing the kaon EM form factors, which we have neglected in the present work. Actually, we have drawn a similar conclusion in the case of the kaon semileptonic form factors [57]. The mean square charge radius of the neutral kaon $\langle r^2\rangle_{K^0}$ is also slightly underestimated in the present work by about 10%, which is again due to the absence of the $1/N_c$ meson-loop corrections.

In Ref. [26], nonperturbative Dyson-Schwinger (DS) and Bether-Salpeter (BS) equations were employed with the confining quark propagator and dressed quark-photon vertex similar to our framework. However, being different from ours, the nonlocal-interaction has been taken into account explicitly in Ref. [26] in addition to the local ones, and parameters for the algebraic quark propagator were fitted with the LECs in the local impulse approximation. References [25,63] used the generalized NJL model in which the separable interaction was considered for the pion and kaon charge radii. Their values were well comparable

TABLE II. Each contribution to the pion and kaon EM charge radii. We present also the mean square charge radius for the neutral kaon (K^0). The abbreviation Exp. denotes the experimental data taken from Ref. [61].

	Local	Nonlocal	Total	Exp. [61]
$\langle r^2\rangle_{\pi^+}^{1/2}$ [fm]	0.594	0.319	0.675	0.672 ± 0.008
$\langle r^2\rangle_{K^+}^{1/2}$ [fm]	0.658	0.318	0.731	0.560 ± 0.031
$\langle r^2\rangle_{K^0}$ [fm^2]	-0.044	-0.016	-0.060	-0.077 ± 0.010

with the experimental data as well as with χPT results [7]. In those works, there were two parameters, namely, constituent quark mass at $q^2 = 0$ and the cutoff mass Λ, which were fitted to LECs, $F_{\pi,K}$ and its two-photon decay width. Note that in the present work the parameter M_0 is fixed by the saddle-point equation and the strange current quark mass m_s is fitted to the kaon mass. Thus, the model is much constrained from the instanton vacuum.

In χPT, the pion EM form factor has been already studied to order $\mathcal{O}(p^6)$ [7]. The pion EM charge radius is quite comparable with our results, whereas that for the K^+ is about 30% smaller than the present one. This discrepancy tells us again that the meson-loop corrections are essential in describing the kaon EM form factor.

Finally, we discuss the Gasser-Leutwyler LEC L_9 in the large N_c limit, which can be determined by the pion EM charge radius. From the present numerical value for $\langle r^2\rangle_{\pi^+}$, we obtain $L_9 = 8.42 \times 10^{-3}$. In χPT [7], the renormalized LEC L_9^r at $\mu = 0.77$ GeV has been determined by using the data for the pion and kaon EM form factors: $(5.93 \pm 0.43) \times 10^{-3}$. This difference turns out to be around 30%.

VI. SUMMARY AND CONCLUSION

In the present work, we have investigated the pion and kaon electromagnetic form factors in the range of $0 \le Q^2 \le 1$ GeV from the instanton vacuum with explicit SU (3) symmetry breaking effects taken into account. We have briefly shown how to construct the gauged effective chiral action following Refs. [40,52]. There are eight independent Feynman diagrams (local and nonlocal, isosinglet, and isotriplet) for the pion and kaon electromagnetic form factors. Among them, two diagrams including a single quark propagator vanish. It turned out that the nonlocal corrections contribute to the pion EM form factors by about 30%. We stress that all relevant diagrams for the pion and kaon EM form factors were generated without any phenomenological assumptions, the conservation of the electromagnetic current being satisfied. Moreover, all parameters, i.e. the constituent quark mass at the zero virtuality and the current quark mass are already fixed from the instanton vacuum, so that we do not have any additional free parameter to adjust. These features provide considerable merit to describe the pion and kaon electromagnetic form factors.

First, we have computed the pion and kaon form factors $[F_{\pi^+,K^+,K^0}(Q^2)]$. We have observed that the numerical results for the pion electromagnetic form factor $F_{\pi^+}(Q^2)$ are in good agreement with the experimental data. While the $F_{K^+}(Q^2)$ turned out to be similar to that of the pion in its overall shape, it falls off faster than the pion one as Q^2 increases. The $F_{K^0}(Q^2)$ is almost negligible for the considered region in comparison to those of π^+ and K^+: $|F_{K^0}(Q^2)| \lesssim 10^{-2}$.

The pion and kaon EM charge radii have been calculated as well. The numerical results are summarized as follows: $\langle r^2\rangle^{1/2}_{\pi^+} = 0.675$ fm, $\langle r^2\rangle^{1/2}_{K^+} = 0.731$ fm, and $\langle r^2\rangle_{K^0} = -0.060\ \mathrm{fm}^2$ without any adjustable free parameters again. Compared to the experimental data, the result of the pion electromagnetic charge radius is in very good agreement with the data, while that of the $\langle r^2\rangle_{K^+}$ is about 30% larger than the experimental one, which indicates that the $1/N_c$ meson-loop corrections are required to describe the kaon properties quantitatively [57]. From the pion charge radius, we were able to extract the low-energy constant $L_9 = 8.42 \times 10^{-3}$.

In conclusion, the nonlocal chiral quark model from the instanton vacuum with SU(3) symmetry breaking effects provides a good framework to study the pion and kaon EM form factors. Moreover, all the relevant contributions to the EM form factors were generated in a simple and systematic way. However, in order to investigate kaonic properties within the present framework, the $1/N_c$ meson-loop contributions [55,56] are essential for which we consider as our future work.

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APPENDIX

Definition of the momenta in the Breit-momentum frame:

$$
\begin{aligned}
p_i &= \left(0, 0, \frac{Q}{2}, i\sqrt{m_{\mathcal{M}}^2 + \frac{Q^2}{4}}\right), \\
q &= (0, 0, iQ, 0), \\
k &= (k_r \sin\phi \sin\psi \cos\theta, k_r \sin\phi \sin\psi \sin\theta, k_r \sin\phi \cos\psi, \\
&\quad k_r \cos\phi),
\end{aligned}
$$

where the angles (ϕ, ψ, θ) are defined in the four-dimensional Euclidean integral as follows:

$$
\int_{-\infty}^{\infty} d^4k = \int_0^\infty k_r^3 dk_r \int_0^\pi \sin^2\phi d\phi \int_0^\pi \sin\psi d\psi \int_0^{2\pi} d\theta.
$$

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