Semirelativistic potential model for three-gluon glueballs

Vincent Mathieu[*] and Claude Semay[+]

Groupe de Physique Nucléaire Théorique, Université de Mons-Hainaut, Académie universitaire Wallonie-Bruxelles, Place du Parc 20, B-7000 Mons, Belgium

Bernard Silvestre-Brac[‡]

LPSC Université Joseph Fourier, Grenoble 1, CNRS/IN2P3, Institut Polytechnique de Grenoble, Avenue des Martyrs 53, F-38026 Grenoble-Cedex, France

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The three-gluon glueball states are studied with the generalization of a semirelativistic potential model giving good results for two-gluon glueballs. The Hamiltonian depends only on 3 parameters fixed on two-gluon glueball spectra: the strong coupling constant, the string tension, and a gluon size which removes singularities in the potential. The Casimir scaling determines the structure of the confinement. Our results are in good agreement with other approaches and lattice calculation for the odderon trajectory but differ strongly from lattice in the J^{+-} sector. We propose a possible explanation for this problem.

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I. INTRODUCTION

The quantum chromodynamics (QCD) theory allows the existence of bound states of gluons, called glueballs, but no firm experimental discovery of such states has been obtained yet. Glueballs are important in the understanding of several mechanisms. Some authors proposed that glueballs characterize deconfinement [1] and that their coupling to a proton affects the gluonic contribution to proton spin [2].

An important difficulty is that glueball states might possibly mix strongly with nearby meson states [3]. Nevertheless, the computation of pure gluon glueballs remains an interesting task. This could guide experimental searches and provide some calibration for more realistic models of glueballs.

Lattice calculations are undoubtedly a powerful tool to investigate the structure of glueballs. A previous study [4] predicts the existence of a lot of resonances between 2 and 4 GeV. A recent update of this work [5] confirms the results already obtained. It is worth mentioning that recent lattice calculations confirm the hierarchy of the glueball spectrum [6].

The potential model, which is so successful to describe bound states of quarks, is also a possible approach to study glueballs [7–13]. In a recent paper [14], a semirelativistic Hamiltonian is used to compute two-gluon glueballs with masses in good agreement with those obtained by lattice calculations of Ref. [4]. This Hamiltonian, model III in Ref. [14], relies on the auxiliary field formalism [15,16] and on a one-gluon exchange (OGE) interaction proposed in Ref. [7]. It depends only on three parameters: the strong coupling constant α_S, the string tension a, and a gluon size γ which removes singularities in the short-range part of the potential. The constituent gluon mass is dynamically generated, and it is assumed that the Casimir scaling determines the color structure of the confinement. These two ingredients are actually necessary to obtain a good agreement between the results from a potential model and from lattice calculations.

In a previous paper [17], we generalized the model built for two-gluon systems in Ref. [14] for the low-lying ($L = 0$) spectrum three-gluon glueballs. The purpose of this paper is to extend the results presented in Ref. [17] to higher glueballs. Compared to previous models [9,10], our approach is characterized by some improved features: semirelativistic kinematics, more realistic confinement, dynamical definition of the gluon mass, and consistent treatment of the gluon size. These points will be detailed below. The masses of the lowest glueballs are computed with a great accuracy and compared with lattice calculations [4–6]. In Sec. II, the three-gluon Hamiltonian is built, and the structure of the studied glueballs is presented in Sec. III. The three-gluon glueball spectrum is presented with the two-gluon glueball spectrum from Ref. [14] and is discussed in Sec. IV. Some concluding remarks are given in Sec. V.

II. HAMILTONIAN

A. Parameters

In Ref. [14], two sets of parameters, denoted A and B, were presented for model III (see Table I). With set A, it is possible to obtain glueball masses in agreement with the results of some experimental works [18,19]: the lowest 2^{++} state near 2 GeV, the lowest 0^{++} state near 1.5 GeV, and the lowest 0^{-+} state near 2.1 GeV. The values of a and α_S are close to the ones used in some recent baryon calculations [20]. With set B, glueball masses were com-

[*]vincent.mathieu@umh.ac.be
[+]claude.semay@umh.ac.be
[‡]silvestre@lpsc.in2p3.fr

TABLE I. Parameters for models A and B ($\sigma = 3a/4$). For both models, the gluon current mass is zero and $f = 0.9515$.

	Model A	Model B
a	0.16 GeV^2	0.21 GeV^2
α_S	0.40	0.50
γ	0.504 GeV^{-1}	0.495 GeV^{-1}

puted in agreement with the results of the lattice calculations of Ref. [4]. If the absolute glueball masses found in Ref. [14] with both sets are strongly different, the relative spectra are nearly identical. As we use in this work a three-body generalization of the Hamiltonian model III of Ref. [14], the two sets will also be considered. Nevertheless, in the following, we mainly focus our attention to the results obtained with set B.

It is worth mentioning how the parameters have been determined in Ref. [14]. The mass of the lightest 2^{++} is nearly independent of the values of α_S and γ but depends strongly on a. So this last parameter has been determined with this 2^{++} state. The remaining parameters α_S and γ have then been computed in order to reproduce the lightest 0^{++} and 0^{-+} states. The three states 2^{++}, 0^{++}, and 0^{-+} have been chosen because they are possible experimental glueball candidates [18,19] and because they are computed with relatively small errors in lattice calculations [4,5].

B. Confinement potential

A good approximation of the confining interaction between a quark and an antiquark in a meson is given by the linear potential ar, where r is the distance between the two particles and where a is the string tension. In a baryon, lattice calculations and some theoretical considerations indicate that each quark generates a flux tube and that these flux tubes meet in a junction point $\boldsymbol{R}_0$ which minimizes the potential energy (the so-called Y junction). Following this hypothesis, the confinement in a baryon could be simulated by the three-body interaction

$$V_{qqq} = a \sum_{i=1}^{3} |\boldsymbol{r}_i - \boldsymbol{R}_0|. \tag{1}$$

For such a potential, the point $\boldsymbol{R}_0$ minimizes also the length of the three flux tubes and is identified with the Toricelli point [21].

The energy density λ_c of a flux tube (string tension) can depend on the color charge c which generates it. Lattice calculations [22] and effective models of QCD [23] predict that the Casimir scaling hypothesis is well verified in QCD, that is to say, that the energy density is proportional to the value of the quadratic Casimir operator $\hat{F}_c^2$ of the color source

$$\lambda_c = \hat{F}_c^2 \sigma. \tag{2}$$

We have then $\lambda_q = \lambda_{\bar{q}} = 4\sigma/3 = a$ and $\lambda_g = 3\sigma$. In this work we will assume that the confinement in a three-body color singlet is given by

$$V_{ggg} = \sigma \sum_{i=1}^{3} \hat{F}_i^2 |\boldsymbol{r}_i - \boldsymbol{R}_0|. \tag{3}$$

This potential can be considered as the three-body generalization of the confinement used in Ref. [14]. No constant potential is added, contrary to the usual Hamiltonians in mesons and baryons [7,24]. Let us note that, if the three color charges are not the same, $\boldsymbol{R}_0$ is no longer identified with the Toricelli point [25].

Interaction (3) is very difficult to use in a practical calculation. A good approximation can be obtained for three identical color charges by substituting $\boldsymbol{R}_0$ by the center-of-mass coordinate $\boldsymbol{R}_{\rm cm}$ and by renormalizing the potential by a factor f which depends on the three-body system [21]. For three identical particles, the best value is $f = 0.9515$. We will use this approximation in the following, which seems more realistic than a confinement obtained by the sum of two-body forces [9,10].

We include in our model the spin-orbit correction to the confinement potential. It is given by the Thomas precession of the particles and reads [26]

$$V_{\rm Conf}^{LS} = -\frac{1}{2\mu^2} \sum_{i=1}^{3} \frac{1}{r_i} \frac{dV_{\rm Conf}}{dr_i} \boldsymbol{L}_i \cdot \boldsymbol{S}_i. \tag{4}$$

This second order correction depends on the effective gluon mass μ. We review in Sec. II C the main feature of this effective mass. Let us note that a Y junction for the confinement leads to a three-body spin orbit since the Torricelli point is a function of the three particles. But with our approximation relying on the center of mass, the Thomas precession term is a sum of three one-body spin-orbit interactions.

In Refs. [7,9,10], the confinement potential saturates at large distances in order to simulate the breaking of the color flux tube between gluons due to color screening effects. An interaction of type (3) seems *a priori* inappropriate since the potential energy can grow without limit. But the phenomenon of flux tube breaking must contribute only to the masses of the highest glueball states. Moreover, it has been shown that the introduction of a saturation could not be the best procedure to simulate the breaking of a string joining two colored objects [27].

C. Dynamical constituent gluon mass

Within the auxiliary field formalism (also called einbein field formalism) [15], which can be considered as an approximate way to handle semirelativistic Hamiltonians [16,28], the effective QCD Hamiltonian has a kinetic part depending on the particle *current* masses m_i, and the interaction is dominated by the confinement. A *state-dependent constituent* mass $\mu_i = \langle \sqrt{\boldsymbol{p}_i^2 + m_i^2} \rangle$ can be de-

fined for each particle, and all relativistic corrections (spin, momentum, etc.) to the static potentials are then expanded in powers of $1/\mu_i$. This approach has been used in Ref. [14] to build the two-gluon Hamiltonian. So the same formalism will be applied also in this paper.

Taking into account the considerations of Sec. II B, the simplest generalization to a three-gluon system of the dominant part of model III two-gluon Hamiltonian of Ref. [14] is

$$H_0 = \sum_{i=1}^{3} \sqrt{\boldsymbol{p}_i^2} + \frac{3}{4} f a \sum_{i=1}^{3} \hat{F}_i^2 |\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm}|, \tag{5}$$

with the condition $\sum_{i=1}^{3} \boldsymbol{p}_i = 0$, since we work in the center of mass of the glueball. The gluons have vanishing current masses, and their color is such that $\langle \hat{F}_i^2 \rangle = 3$. Contrary to some previous works [7,9,10], our Hamiltonian is a semirelativistic one. In Ref. [14], it has been shown that it is an important ingredient to obtain correct two-gluon glueball spectra.

Using the technique of Ref. [29], it is possible to obtain an analytical approximate formula giving the glueball mass M_0 and the constituent gluon mass μ_0 (the three constituent gluon masses are the same since the wave function is completely symmetrized; see Sec. III)

$$M_0 \approx 6\mu_0, \quad \text{with} \quad \mu_0 \approx \sqrt{f\sigma(N+3)}. \tag{6}$$

$N = 0, 1, \ldots$ is the excitation number. With a value of the meson string tension $a = 4\sigma/3$ around 0.2 GeV^2, the smallest gluon constituent mass is around 650 MeV. It is then relevant to use an expansion in powers of $1/\mu_0$. Such a value of the gluon mass is in agreement with the values used in Refs. [7,9,10], but here the constituent mass is dynamically generated.

Instead of using the auxiliary field formalism, it is possible to consider relativistic corrections which are expanded in powers of $1/E_i(\boldsymbol{p}_i)$, where $E_i(\boldsymbol{p}_i) = \sqrt{\boldsymbol{p}_i^2 + m_i^2}$ (see, for instance, Ref. [30]). But this leads to very complicated nonlocal potentials which are difficult to handle.

D. Short-range potential

The Hamiltonian H_0 (5) gives the main features of the three-gluon glueball spectra, but the introduction of a short-range potential is necessary to achieve a detailed study. In Ref. [14], a OGE interaction between two gluons, coming from Ref. [7], has been considered. It is not possible to use it directly for a three-gluon glueball because the color structure of the interaction is different. So we use here the last version of a OGE interaction between two gluons developed specifically for three-gluon glueballs [9,10]. Its explicit form, which is very similar to the form of the OGE interaction for two-gluon glueballs, is given below.

This interaction contains a tensor part and a spin-orbit part. In our previous study of the low-lying spectrum [17], we neglected these terms since we worked on $L = 0$ states. It has been shown that the tensor interaction between two gluons is small in two-gluon glueballs [14]. Hence, we add only a two-body spin-orbit interaction between each pair of gluons.

The OGE two-gluon potential has *a priori* a very serious flaw: Depending on the spin state, the short-range singular part of the potential may be attractive and lead to a Hamiltonian unbounded from below [10]. This problem is solved, as in Ref. [14], by giving a finite size to the gluon (see Sec. II E).

The OGE potential depends on the gluon constituent mass. To determine it, we follow the procedure proposed in Ref. [14]. For a given set of quantum numbers $\{\alpha\}$, the eigenstate $|\phi_\alpha\rangle$ of the Hamiltonian H_0 is computed. With this state, a constituent gluon mass is computed $\mu_\alpha = \langle \phi_\alpha | \sqrt{\boldsymbol{p}_1^2} | \phi_\alpha \rangle$. This value of μ_α is then used in the complete Hamiltonian (see Sec. II F) to compute its eigenstate with quantum numbers $\{\alpha\}$. It is worth noting that, with this procedure, two states which differ only by the radial quantum number are not orthogonal since they are eigenstates of two different Hamiltonians which differ by the value of μ. It is shown in Ref. [16] that this problem is not serious, the overlap of these states being generally weak.

E. Gluon size

In potential models, the gluon is considered as an effective degree of freedom with a constituent mass. Within this framework, it is natural to assume that a gluon is not a pure pointlike particle but an object dressed by a gluon and quark-antiquark pair cloud. Such an hypothesis for quarks leads to very good results in meson [31] and baryon [32] sectors. As in Ref. [14], we assume here a Yukawa color charge density for the gluon

$$\rho(\boldsymbol{u}) = \frac{1}{4\pi\gamma^2} \frac{e^{-u/\gamma}}{u}, \tag{7}$$

where γ is the gluon size parameter. The interactions between gluons are then modified by this density, a bare potential being transformed into a dressed one.

The main purpose of the gluon dressing is to remove all singularities in the short-range part of the interaction [32]. But, for consistency, the same regularization is applied to the confinement potential, although no singularity is present in this case. We think that the definition of a gluon size, which has a clear physical meaning, is preferable to the use of a smearing function only for potentials with singularity [8,10].

A one-body potential, such as $|\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm}|$, is dressed by a simple convolution over the density of the interacting gluon and the potential

$$V(\boldsymbol{r})^{*} = \int d\boldsymbol{r}' V(\boldsymbol{r}')\rho(\boldsymbol{r}-\boldsymbol{r}'). \tag{8}$$

A dressed two-body potential, depending on $|\boldsymbol{r}_i - \boldsymbol{r}_j|$, is obtained by a double convolution. This procedure is equivalent to the following calculation [33]:

$$V(\boldsymbol{r})^{**} = \int d\boldsymbol{r}' V(\boldsymbol{r}')\Gamma(\boldsymbol{r}-\boldsymbol{r}') \quad \text{, with}$$
$$\Gamma(\boldsymbol{u}) = \frac{1}{8\pi\gamma^3} e^{-u/\gamma}. \tag{9}$$

Note that for the spatial dependencies of the spin-orbit interactions, which depend on the derivative of the first order potentials, the convolution must be performed before taking the derivative.

For a nonvanishing value of γ, the value of the confinement potential at origin increases. This shifts the whole spectra to higher masses. Moreover, the strength of the Coulomb interaction is reduced. This also implies an increase of the glueball masses.

F. Total Hamiltonian

To obtain the total Hamiltonian for three-gluon glueballs which is the simplest generalization of the Hamiltonian for two-gluon glueballs from Ref. [14], we take the Hamiltonian H_0 given by relation (5) and its spin-orbit correction (4); we add the OGE interactions coming from Ref. [10] (without the tensor part); and we dress all of the potentials with the gluon color density (7). This gives the following Hamiltonian ($\hat{F}_i^2 = 3$, $\hat{F}_i \cdot \hat{F}_j = -3/2$):

$$H = \sum_{i=1}^{3} \sqrt{\boldsymbol{p}_i^2} + V_{\mathrm{OGE}}^{**} + V_{\mathrm{Conf}}^{*} + V_{\mathrm{Conf}}^{LS*} + V_{\mathrm{OGE}}^{LS**}, \quad \text{with} \tag{10a}$$

$$V_{\mathrm{OGE}}^{**} = -\frac{3}{2}\alpha_S \sum_{i<j=1}^{3} \left[\left(\frac{1}{4} + \frac{1}{3}\vec{S}_{ij}^2 \right) U(r_{ij})^{**} - \frac{\pi}{\mu^2}\delta(\boldsymbol{r}_{ij})^{**} \left(\beta + \frac{5}{6}\vec{S}_{ij}^2 \right) \right], \tag{10b}$$

$$U(r)^{**} = \frac{1}{(\mu^2\gamma^2-1)^2}\left(\frac{e^{-\mu r}}{r} - \frac{e^{-r/\gamma}}{r} \right) + \frac{e^{-r/\gamma}}{2\gamma(\mu^2\gamma^2-1)}, \quad \text{with} \quad U(r) = \frac{e^{-\mu r}}{r}, \tag{10c}$$

$$\delta(\boldsymbol{r})^{**} = \frac{1}{8\pi\gamma^3} e^{-r/\gamma}, \tag{10d}$$

$$V_{\mathrm{Conf}}^{*} = \frac{9}{4} f a \sum_{i=1}^{3} y_i^{*}, \quad \text{with} \quad y_i = |\boldsymbol{r}_i - \boldsymbol{R}_{\mathrm{cm}}| \quad \text{and} \quad r^{*} = r + 2\gamma^2 \frac{1 - e^{-r/\gamma}}{r}, \tag{10e}$$

$$V_{\mathrm{Conf}}^{LS*} = -\frac{9fa}{8\mu^2} \sum_{i=1}^{3} \boldsymbol{L}_i \cdot \boldsymbol{S}_i \frac{1}{y_i}\frac{d}{dy_i} y_i^{*}, \tag{10f}$$

$$V_{\mathrm{OGE}}^{LS**} = -\frac{9\alpha_S}{4\mu^2} \sum_{i<j=1}^{3} \boldsymbol{L}_{ij} \cdot \boldsymbol{S}_{ij} \frac{1}{r_{ij}}\frac{d}{dr_{ij}} U(r_{ij})^{**}, \tag{10g}$$

where $\sum_{i=1}^{3} \boldsymbol{p}_i = 0$ and $\vec{S}_{ij} = \vec{S}_i + \vec{S}_j$. $\beta = +1(-1)$ for a gluon pair in color octet antisymmetrical (symmetrical) state. The constituent state-dependent gluon mass μ is computed in advance with a solution of the Hamiltonian H_0.

III. WAVE FUNCTIONS

A gluon is a $I(J^P) = 0(1^-)$ color octet state. Two different three-gluon color singlet states exist [9], which are completely symmetrical or completely antisymmetrical. The total isospin state of a glueball is an isosinglet and is completely symmetrical. Different total spin states are allowed with different symmetry properties. They are presented in Table II. As gluons are bosons, the total wave function must be completely symmetrical. Its parity is the opposite of the spatial parity, and its C parity is positive for a color antisymmetrical state and negative for a color symmetrical state. Let us note that a two-gluon glueball has always a positive C parity.

In our previous work [17], we mainly considered glueballs with the lowest masses. These states are characterized by a vanishing total orbital angular momentum $L = 0$ and by a spatial wave function completely symmetrical with a

TABLE II. Characteristics of three-gluon spin functions with total spin S, intermediate couplings S_{int}, and symmetry properties which can be obtained by coupling (A: antisymmetrical, S: symmetrical, MS: mixed symmetry). The multiplicity of each symmetry type is indicated.

S	S_{int}	Symmetry
0	1	1A
1	0, 1, 2	1S, 2MS
2	1, 2	2MS
3	2	1S

positive parity. This immediately implies that the lowest glueballs are states with J^{PC} equal to 0^{-+}, 1^{--}, and 3^{--} [9,10]. No firm conclusion could be drawn for the 0^{-+} state because it appears in both two-gluon and three-gluon spectra. Our results for the 1^{--} and 3^{--} glueballs were in good agreement with lattice masses. We also presented the mass of the 2^{--} even though it was found with a mass higher than the one predicted by the lattice calculations. We could understand this bad result as a first hint that possibly our approach could not be the best way to handle pure gauge spectra.

We now extend our study to states with negative C parity. Indeed, at least 3 gluons are contained in those glueballs. Lattice QCD recently computed masses of J^{+-} glueballs: $0^{+-}(4780)$, $1^{+-}(2980)$, $2^{+-}(4230)$, and $3^{+-}(3600)$ in Ref. [5] and $1^{+-}(2670)$, $3^{+-}(3270)$, and $5^{+-}(4110)$ in Ref. [6]. The positive parity requests an odd angular momentum.

In order to explain the high energy behavior of the proton-antiproton scattering, the existence of a trajectory carrying vacuum quantum numbers was postulated: the pomeron [34]. The matching of the two-gluon spectrum and this trajectory is an important success of the theory [6,35]. The odderon is the negative C-parity counterpart of the pomeron [36]. One often argues that the odd spin glueballs lie on its trajectory. The first two states (1^{--} and 3^{--}) on its trajectory were already computed in Ref. [17]. In this paper, we also computed the 5^{--} mass to check this hypothesis. Although lattices do not report any result, we will compare our mass to a Coulomb gauge approach for the odderon [37]. The oddballs J^{--} masses for $J = 1, 2, 3, 5$, and 7 as well as the 0^{-+} are computed within the Coulomb gauge approach in this reference.

In order to reach a good accuracy, the trial spatial wave functions are expanded in the large Gaussian function basis [38]. Recently, we found the Fourier transform of a Gaussian function and applied it to give the matrix elements for the semirelativistic kinetic energy [39]. The interested reader may also find the matrix elements for spin-dependent operators in this base in Ref. [40], where applications for three-body systems were presented. With more than 10 Gaussian functions for each color/isospin/spin channel, we have checked that the numerical errors on masses presented are around or less than 1 MeV.

Using the a value from our previous models A and B [14,17], eigenvalues of the Hamiltonian H_0 (5) are presented in Table III for various J^{PC} quantum numbers. For each state, the corresponding constituent gluon mass μ_0 is indicated. It is used to define the complete Hamiltonian H (10). Let us note that, even if the Hamiltonian H_0 does not contain a spin-dependent potential, the spin of the wave function can strongly influence the mass through the symmetry of its spin part. With our numerical procedure, the $J^{+\pm}$ glueballs are characterized by $L \geq 1$ spatial wave functions with L odd. In Table III, one can see that the four lowest J^{+-} glueballs are degenerate ($L = 1$, $S = 1$) as long as no spin-dependent term is included in the potentials.

IV. RESULTS

We present here the three-gluon glueball masses obtained with the complete Hamiltonian H (10) together with the two-gluon glueball masses computed in Ref. [14] (see Table IV and Fig. 1). These masses are compared with results obtained by the lattice calculations of Ref. [5]. This last work is an update of a previous study [4]. So states not computed in Ref. [5] but presented in Ref. [4] are also considered here. The $5^{\pm-}$ masses are also computed and compared to the result of Ref. [6].

This work is an extension of our previous study [17] in which we concluded that our 1^{--} and 3^{--} states were in good agreement with lattice results. However, our 2^{--} state is clearly higher than its lattice counterpart. The lattice results predict a 2^{--} state at 4010 MeV near the 1^{--} and 3^{--} states. With our Hamiltonian, a mass more than 1 GeV above is computed. It is unavoidable in our model, since a spin 2 function has a mixed symmetry, which implies a mixed symmetry for the space function

TABLE III. Masses M_0 of the Hamiltonian H_0 (5) as a function of the J^{PC} quantum numbers. The (L, S) quantum numbers are indicated with the corresponding constituent gluon masses μ_0. Values in MeV are computed with the value of a from models A/B. The lowest masses are printed in italic.

J^{PC}	(L, S)	M_0	μ_0	J^{PC}	(L, S)	M_0	μ_0
0^{--}	(0, 0)	5574/6385	929/1064	0^{-+}	(0, 0)	*3211/3679*	535/613
1^{--}	(0, 1)	*3211/3679*	535/613	1^{-+}	(0, 1)	4156/4761	693/794
2^{--}	(0, 2)	4156/4761	693/794	2^{-+}	(0, 2)	4156/4761	693/794
3^{--}	(0, 3)	*3211/3679*	535/613	3^{-+}	(0, 3)	5574/6385	929/1064
5^{--}	(2, 3)	4182/4791	697/795				
0^{+-}	(1, 1)	3752/4298	625/717				
1^{+-}	(1, 1)	3752/4298	625/717				
2^{+-}	(1, 1)	3752/4298	625/717				
3^{+-}	(1, 2)	3752/4298	625/717				
5^{+-}	(3, 2)	4596/5265	766/878				

TABLE IV. Glueball masses in MeV, in parentheses, and glueball mass ratios normalized to lightest 2^{++}. The two-gluon masses are taken from Ref. [14]. The error bars for lattice mass ratios are computed without the normalization error on the masses. The lightest 0^{++}, 2^{++}, and 0^{-+} states are taken as inputs to fix the parameters. The first column indicates the valence gluon content as predicted by our model.

	J^{PC}	Lattice [Ref.]	Model A	Model B
gg	0^{++}	$1710 \pm 50 \pm 80$ (0.72 ± 0.03) [5]	1604 (0.78)	1855 (0.78)
		$2670 \pm 180 \pm 130$ (1.12 ± 0.09) [4]	2592 (1.26)	2992 (1.26)
	2^{++}	$2390 \pm 30 \pm 120$ (1.00 ± 0.03) [5]	2051 (1.00)	2384 (1.00)
	0^{-+}	$2560 \pm 35 \pm 120$ (1.07 ± 0.03) [5]	2172 (1.06)	2492 (1.05)
		$3640 \pm 60 \pm 180$ (1.52 ± 0.04) [4]	3228 (1.57)	3714 (1.56)
	2^{-+}	$3040 \pm 40 \pm 150$ (1.27 ± 0.03) [5]	2573 (1.25)	2984 (1.25)
		$3890 \pm 40 \pm 190$ (1.63 ± 0.04) [4]	3345 (1.63)	3862 (1.62)
	3^{++}	$3670 \pm 50 \pm 180$ (1.54 ± 0.04) [5]	3132 (1.53)	3611 (1.51)
ggg	1^{--}	$3830 \pm 40 \pm 190$ (1.60 ± 0.04) [5]	3433 (1.67)	3999 (1.68)
	2^{--}	$4010 \pm 45 \pm 200$ (1.68 ± 0.04) [5]	4422 (2.16)	5133 (2.15)
	3^{--}	$4200 \pm 45 \pm 200$ (1.76 ± 0.04) [5]	3569 (1.74)	4167 (1.75)
	0^{-+}		3688 (1.80)	4325 (1.81)
	0^{+-}	$4780 \pm 60 \pm 230$ (2.00 ± 0.05) [5]	4043 (1.97)	4656 (1.95)
	1^{+-}	$2980 \pm 30 \pm 140$ (1.25 ± 0.03) [5]	3992 (1.95)	4626 (1.94)
	2^{+-}	$4230 \pm 50 \pm 200$ (1.78 ± 0.04) [5]	3907 (1.90)	4542 (1.91)
	3^{+-}	$3600 \pm 40 \pm 170$ (1.51 ± 0.04) [5]	4033 (1.97)	4568 (1.92)
	5^{+-}	$4110 \pm 170 \pm 190$ [6]	4571 (2.23)	5317 (2.23)
	5^{--}		4521 (2.20)	5263 (2.21)

and then a greater mass for the corresponding glueball, in agreement with the results of Refs. [9,10]. It has been checked that the spatial wave function of the 2^{--} state is dominated by a configuration in which each internal variable is characterized by one unit of angular momentum. However, in Ref. [37], the 2^{--} state lies between the 1^{--} and 3^{--}. Actually, they computed the spin-dependent corrections in perturbation with a wrong wave function. When the correct wave function is used, the 2^{--} goes higher in agreement with our model [41].

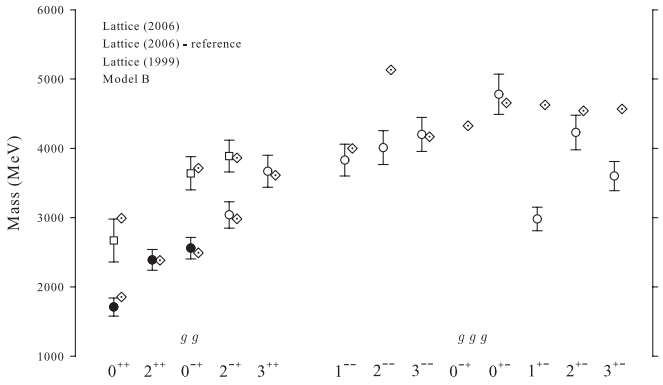

FIG. 1. Glueball masses given in MeV. Dotted diamonds: Results from model B (two-gluon masses are taken from Ref. [14] and three-gluon $J^{-\pm}$ masses are taken from Ref. [17]). Black and white circles: Lattice results from Ref. [5]. White squares: Lattice results from Ref. [4]. Black circles indicate the reference states taken as inputs to fix the parameters. The error bars for lattice results are computed by summing the two uncertainties (see Table IV).

Moreover, in our model, as long as we do not include spin-orbit forces, the 0^{+-}, 1^{+-}, 2^{+-}, and 3^{+-} glueballs are degenerate $L = 1$ states with the corresponding mixed symmetry for spin $S = 1$ or 2 (see Table II). We included spin-orbit interactions (coming from the one-gluon exchange and Thomas precession) to split the degeneracy. But first, the splitting between these states is not sufficient. Second, the hierarchy is not correct. Indeed, we checked that the two spin-orbit contributions canceled roughly each other. The one coming from the OGE decreases the masses by 100–200 MeV when the one coming from the confinement increases the masses by around the same amount. This fact is well known in baryons [42], and hence it is not very surprising in a glueball potential model. Let us note that this cancellation between two spin-orbit contributions was already noted for two-gluon glueballs [11,14], but the effect coming from OGE was always stronger. In lattice calculations, however, the situation is strongly different: The splitting between 1^{+-} and 0^{+-} is about 1.8 GeV. Such a splitting cannot be reproduced in our simple model.

The values of parameters a and α_S are quite well determined with lattice calculations and Regge phenomenology. The situation is very different for the gluon size γ. In this work, it is assumed that γ has the same value in two-gluon and three-gluon glueballs. We have checked that variations of the gluon size for three-gluon glueballs do not change the hierarchy of states, but the masses are globally shifted. As expected, the masses increase with γ.

It is worth noting that the 1^{+-} and 3^{+-} three-gluon glueballs have masses similar to $J = 2$ and $J = 3$ two-

gluon glueballs. Such low masses are difficult to explain. It is possible that these states are actually two-glueball bound states [4]. Let us also note that 1^{+-} and 3^{+-} glueballs have low masses in the closed flux tube model [43].

We have no firm explanation for such discrepancies. These problems could arise because the gluon has a constituent mass within our formalism. So it possesses a spin as any massive particle, that is to say, three states of polarization, and we use a basis where each state is labeled by a couple (L, S). In the two-gluon sector, the (L, S) basis leads to more states than the ones predicted by the lattice and also to $J = 1$ states forbidden by Yang's theorem. A solution to cancel the extra states is to deal with the so-called helicity formalism [44]. Recently, we reviewed and applied this formalism to two-gluon glueballs [45]. We showed that a simple Cornell potential together with the helicity formalism reproduces the correct hierarchy given by the lattice. An instanton-induced force was also needed to raise the degeneracy between the scalar and pseudoscalar glueballs. In lattice calculations, the gluon is a massless particle with a definite helicity and then only two states of polarization. We suspect that the implementation of the helicity formalism for three-body systems would solve the hierarchy problem in the J^{+-} sector but also for the 2^{--}. This difficult task is out of the scope of this paper, and we leave it for future work. Nevertheless, a attempt to generalization of the helicity formalism for a three-body system is presented in Refs. [46,47].

It has been suggested that a three-body interaction can inverse the state ordering in the gluelump sector [48]. It is not clear that such a force acts also in glueballs and could solve the hierarchy problem. Actually, the dominant three-body force in a glueball is the confinement, supplemented by its spin-orbit correction. In this work, as explained above, the confinement and its spin-orbit correction are simulated with a sum of one-body interactions.

The odderon trajectory carries the quantum numbers J^{--} with odd J. This trajectory was investigated in Ref. [37] within a Coulomb gauge Hamiltonian formalism. In this work, states are built with well-defined (L, S) quantum numbers. The spectrum is in good agreement with our results (see Table V). Let us note that the Coulomb gauge Hamiltonian is more complicated than ours. For instance, the annihilation diagram for two gluons is taken into account, whereas it vanishes in a potential model [50]. It is not surprising that both approaches give rather the same results since they used the same formalism [a (L, S) basis]. We found that the two first oddballs 1^{--} and 3^{--} lie in lattice error bars. This fact can be surprising at first glance. Indeed, we invoked the helicity formalism as a possibility to solve the mass problem of the J^{+-}. Hence, the same problem should arise for the J^{--}. In the helicity formalism, a given J^{PC} is a particular combination of (L, S) couples [44,45]. We suspect that the oddballs are largely dominated by the component $(L = J - 3, S = 3)$. It would be interesting to have more information about those states, but unfortunately lattices have difficulties identifying higher excited states.

TABLE V. Odderon quantum numbers and masses in MeV. L and S assignments agree in our model B and in Ref. [37].

J^{PC}	1^{--}	3^{--}	5^{--}	7^{--}
S	1	3	3	3
L	0	0	2	4
Model B	3999	4167	5263	
Coulomb gauge [37]	3950	4150	5050	5900
Lattice [5]	3830	4200		
Lattice [6]	3100	4150		
Wilson loops [49]	3490	4030		

V. CONCLUSION

The masses of pure three-gluon glueballs have been studied with the generalization of a semirelativistic potential model for pure two-gluon glueballs [14]. The short-range part of the potential is the sum of two-body OGE interactions. For the confinement, a potential simulating a genuine Y junction is used, and it is assumed that the Casimir scaling hypothesis is well verified. The gluon is massless, but the OGE interaction is expressed in terms of a state-dependent constituent mass. The Hamiltonian depends only on 3 parameters fixed in Ref. [14]: the strong coupling constant α_S, the string tension a, and a gluon size γ. All masses have been accurately computed with an expansion of trial states in Gaussian functions [38–40].

From our previous paper [14,17], we know that $J^{\pm+}$ two-gluon glueball spectra are in good agreement with lattice calculations [4,5], but extra states not seen in lattice calculations are predicted. The $J^{-\pm}$ three-gluon glueballs are also in quite good agreement with these lattice results, except for the 2^{--} state, which is computed with a very high mass in our model. In this work, the 5^{--} and some J^{+-} three-gluon glueballs are finally computed. The J^{--} candidates with J odd for the odderon trajectory are in agreement with some other works. But, if the 0^{+-} and 2^{+-} states are predicted in quite good agreement with lattices results, it is not the case for the 1^{+-} and 3^{+-} states computed with too high masses in our model. One could interpret such a discrepancy as due to the fact that the spin-orbit forces are too feeble in our model to raise the degeneracy between J^{+-} states. But we do not believe that a physical process, ignored here, is able to produce a strong enough spin-orbit force giving rise to a more than 1 GeV energy gap. We think that the strong discrepancies between our results and lattice computations are due to the fact that gluons do have constituent nonvanishing masses in our approach. They are then characterized by a spin and not by a helicity as could be expected for particles with a vanishing current mass. We have shown that working

with the helicity formalism could cure the problem of extra states in two-gluon glueballs [45]. We think that this formalism applied to three-gluon glueballs could also improve the predictions of a potential model. Such work is in progress.

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[1] E. V. Shuryak, Phys. Lett. B **515**, 359 (2001); V. Vento, Phys. Rev. D **75**, 055012 (2007).
[2] N. Kochelev and D. P. Min, Phys. Lett. B **633**, 283 (2006).
[3] N. Kochelev and D. P. Min, Phys. Rev. D **72**, 097502 (2005).
[4] C. J. Morningstar and M. J. Peardon, Phys. Rev. D **60**, 034509 (1999).
[5] Y. Chen *et al.*, Phys. Rev. D **73**, 014516 (2006).
[6] H. B. Meyer, arXiv:hep-lat/0508002; H. B. Meyer and M. J. Teper, Phys. Lett. B **605**, 344 (2005).
[7] J. M. Cornwall and A. Soni, Phys. Lett. B **120**, 431 (1983).
[8] F. Brau and C. Semay, Phys. Rev. D **72**, 078501 (2005).
[9] W. S. Hou and A. Soni, Phys. Rev. D **29**, 101 (1984).
[10] W. S. Hou, C. S. Luo, and G. G. Wong, Phys. Rev. D **64**, 014028 (2001).
[11] A. B. Kaidalov and Yu. A. Simonov, Phys. Lett. B **636**, 101 (2006).
[12] E. Abreu and P. Bicudo, J. Phys. G **34**, 195 (2007).
[13] F. Buisseret, Phys. Rev. C **76**, 025206 (2007).
[14] F. Brau and C. Semay, Phys. Rev. D **70**, 014017 (2004).
[15] V. L. Morgunov, A. V. Nefediev, and Yu. A. Simonov, Phys. Lett. B **459**, 653 (1999).
[16] C. Semay, B. Silvestre-Brac, and I. M. Narodetskii, Phys. Rev. D **69**, 014003 (2004).
[17] V. Mathieu, C. Semay, and B. Silvestre-Brac, Phys. Rev. D **74**, 054002 (2006).
[18] B. S. Zou, Nucl. Phys. **A655**, c41 (1999).
[19] D. V. Bugg, M. J. Peardon, and B. S. Zou, Phys. Lett. B **486**, 49 (2000).
[20] I. M. Narodetskii and M. A. Trusov, Yad. Fiz. **65**, 949 (2002) [Phys. At. Nucl. **65**, 917 (2002)]; Yad. Fiz. **67**, 783 (2004) [Phys. At. Nucl. **67**, 762 (2004)].
[21] B. Silvestre-Brac, C. Semay, I. M. Narodetskii, and A. I. Veselov, Eur. Phys. J. C **32**, 385 (2004).
[22] S. Deldar, Phys. Rev. D **62**, 034509 (2000); G. S. Bali, Phys. Rev. D **62**, 114503 (2000).
[23] C. Semay, Eur. Phys. J. A **22**, 353 (2004); T. H. Hanson, Phys. Lett. **166B**, 343 (1986).
[24] Yu. A. Simonov, Phys. Lett. B **515**, 137 (2001).
[25] V. Mathieu, C. Semay, and F. Brau, Eur. Phys. J. A **27**, 225 (2006).
[26] F. Buisseret and C. Semay, Phys. Rev. D **76**, 017501 (2007).
[27] E. S. Swanson, J. Phys. G **31**, 845 (2005).
[28] F. Buisseret and C. Semay, Phys. Rev. D **70**, 077501 (2004).
[29] C. Semay, F. Buisseret, N. Matagne, and Fl. Stancu, Phys. Rev. D **75**, 096001 (2007).
[30] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[31] F. Brau and C. Semay, Phys. Rev. D **58**, 034015 (1998).
[32] F. Brau, C. Semay, and B. Silvestre-Brac, Phys. Rev. C **66**, 055202 (2002).
[33] B. Silvestre-Brac, F. Brau, and C. Semay, J. Phys. G **29**, 2685 (2003).
[34] S. Donnachie, G. Dosch, O. Nachtmann, and P. Landshoff, Cambridge Monogr. Part. Phys., Nucl. Phys., Cosmol. **19**, 1 (2002).
[35] H. B. Meyer and M. J. Teper, Nucl. Phys. B, Proc. Suppl. **129**, 200 (2004); F. J. Llanes-Estrada, S. R. Cotanch, P. J. de A. Bicudo, J. E. F. Ribeiro, and A. P. Szczepaniak, Nucl. Phys. **A710**, 45 (2002); P. Bicudo, arXiv:hep-ph/0405223.
[36] L. Lukaszuk and B. Nicolescu, Lett. Nuovo Cimento Soc. Ital. Fis. **8**, 405 (1973).
[37] F. J. Llanes-Estrada, P. Bicudo, and S. R. Cotanch, Phys. Rev. Lett. **96**, 081601 (2006).
[38] Y. Suzuki and K. Varga, *Stochastic Variational Approach to Quantum Mechanical Few-Body Problems* (Springer-Verlag, Berlin, 1998).
[39] B. Silvestre-Brac and V. Mathieu, Phys. Rev. E **76**, 046702 (2007).
[40] B. Silvestre-Brac and V. Mathieu, Phys. Rev. E **77**, 036706 (2008).
[41] P. Bicudo and F. J. Llanes-Estrada (private communication).
[42] N. Isgur and G. Karl, Phys. Rev. D **18**, 4187 (1978).
[43] N. Isgur and J. Paton, Phys. Rev. D **31**, 2910 (1985).
[44] M. Jacob and G. C. Wick, Ann. Phys. (Paris) **7**, 404 (1959); Ann. Phys. (N.Y.) **281**, 774 (2000).
[45] V. Mathieu, F. Buisseret, and C. Semay, arXiv:0802.0088 [Phys. Rev. D (to be published)].
[46] D. R. Giebink, Phys. Rev. C **32**, 502 (1985).
[47] A. Stadler, F. Gross, and M. Frank, Phys. Rev. C **56**, 2396 (1997).
[48] P. Guo, A. P. Szczepaniak, G. Galata, A. Vassallo, and E. Santopinto, Phys. Rev. D **77**, 056005 (2008); A. P. Szczepaniak and P. Krupinski, Phys. Rev. D **73**, 034022 (2006).
[49] A. B. Kaidalov and Yu. A. Simonov, Phys. Lett. B **477**, 163 (2000).
[50] V. Mathieu and F. Buisseret, J. Phys. G **35**, 025006 (2008).