PHYSICAL REVIEW D **77,** 094001 (2008)

Renormalization, Wilson lines, and transverse-momentum-dependent parton-distribution functions

I. O. Cherednikov[1,2,*] and N. G. Stefanis[1,+]

[1]*Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*
[2]*Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia*
(Received 15 October 2007; published 1 May 2008)

We perform an analysis of transverse-momentum dependent parton-distribution functions, making use of their renormalization properties in terms of their leading-order anomalous dimensions. We show that the appropriate Wilson line in the light cone gauge, associated with such quantities, is a cusped one at light cone infinity. To cancel the ensuing cusp anomalous dimension, we include in the definition of the transverse-momentum dependent parton-distribution functions an additional soft counter term (gauge link) along that cusped transverse contour. We demonstrate that this is tantamount to an ''intrinsic (Coulomb) phase,'' which accumulates the full gauge history of the color-charged particle.

DOI: 10.1103/PhysRevD.77.094001 PACS numbers: 12.38.Bx, 11.10.Jj, 13.60.Hb

I. INTRODUCTION

A fundamental goal of QCD is to provide an accurate description of parton-distribution functions (PDFs) which contain the nonperturbative strong dynamics. While integrated PDFs can be defined in a gauge-invariant way that is compatible with factorization, ensuring multiplicative renormalizability and DGLAP evolution, the definition of unintegrated or, equivalently, transverse-momentum dependent (TMD), parton distributions poses severe problems (see, e.g., [1–3]): (a) Additional, so-called rapidity, divergences [4] appear, related to lightlike Wilson lines (or the use of the light cone gauge $A^+ = 0$) [5], that cannot be taken care of by ordinary ultraviolet (UV) renormalization alone. In the integrated case, these divergences also appear but they mutually cancel [4,6], allowing a probabilistic interpretation. (b) Moreover, in the light cone gauge, the result depends on the applied pole prescription in the gluon propagator. Only with the advanced boundary condition, which sets the transverse gauge link to unity, one recovers the results obtained in the Feynman gauge [7]. (c) The reduction to the integrated case is at least not straightforward [8]. (d) Universality is in general broken [9], an issue outside the scope of our analysis, given that we concentrate on unpolarized PDFs only. This point will be briefly addressed in the last section. Let us discuss these issues in more detail.

The first issue, i.e., the treatment of the rapidity divergences, is on the focus of our investigation and will be discussed in detail below. The second question has been addressed by Belitsky, Ji, and Yuan [7] (see also [10,11]), where a transverse gauge link was introduced in order to exhaust the gauge freedom of the TMD PDF. The third problem becomes trivial in our approach because it is avoided *ab initio* by the proposed definition of the TMD PDF. In particular, one may note the

(i) Collins-Soper (CS) approach [4] (or cutoff method) (see also [8]): These authors were the first to address issue (a) and to propose a solution of the problem by adopting either a nonlightlike axial gauge or by shifting the integration contour slightly off the light cone. This, however, entails the introduction of an additional rapidity parameter $\zeta = (p \cdot n)^2/n^2$ (with $n^2 \neq 0$) to encode the deviation from the light cone. To establish independence from this arbitrary variable, an additional evolution equation to the standard one has to be fulfilled causing the reduction to the integrated case questionable. Besides, factorization off the light cone also becomes problematic.

(ii) Collins-Hautmann approach [12] (or subtractive method): These authors suggest another way to circumvent problem (a). They restrict themselves to lightlike Wilson lines and remove the rapidity divergences by redefining the TMD PDF. The principal element in their approach is the introduction of a soft counter term that compensates these divergences, shown explicitly at the one-loop order and working in the Feynman gauge; a fresh look was given by Collins and Metz [13]. More recently, Hautmann [14] claimed that the reduction to the integrated case can also be performed within this method.

In our work, we will follow another strategy based on the renormalization properties of TMD PDFs in terms of their anomalous dimensions. The reason is that anomalous dimensions (within perturbative QCD) encode the key characteristics of Wilson lines in *local* form. In contrast, gauge contours are *global* objects and, hence, difficult to handle within a local-field theory framework. A properly defined TMD PDF should respect collinear factorization. But this turns out to be in conflict with the gauge link because the

*igorch@theor.jinr.ru
+stefanis@tp2.ruhr-uni-bochum.de

Wilson line contains not only longitudinal gluons that could be eliminated by imposing the light cone gauge, it also comprises transverse ones, with a distinct region of $\boldsymbol{k}_\perp$ that are accumulated after the quark has been struck by the hard current and changes its direction from x^+ to x^-. As a result, one cannot define a TMD PDF by introducing a straight lightlike line between the quarks (i.e., a "connector" [15]). The reason is that the two quark fields have a separation also in the transverse coordinate space and hence the gluons originating from this are not collinear to the struck quark (they mismatch in the gluon rapidity). The common assumption to avoid this problem is to use a combined contour which joins the quarks through light cone infinity. Our analysis shows that such a contour cannot be a smooth one, as usually tacitly assumed, but it has to contain some obstruction in the transverse direction (not specified yet) which will inevitably contribute to the total anomalous dimension of the TMD PDF. Therefore, in order to be able to reproduce the well-known result in the Feynman gauge, one has to define the TMD PDF in such a way as to cancel this unwanted anomalous-dimension term. To this end, we seek to recast the Wilson line in terms of the associated anomalous dimensions. We will show that this can be naturally achieved within a formalism which inherently respects gauge invariance by using manifestly gauge-invariant quark fields that account for the whole gauge "history" in the sense of Mandelstam [16]. Details will be given elsewhere [17]. The paper is organized as follows. In the next section, we first provide arguments for the necessity to insert a transverse gauge link. Then we continue with the calculation of the anomalous dimension of the TMD PDF and show that there is a contribution at light cone infinity in the transverse direction that can be associated with a cusp. In the same section we will supply a modified definition of the TMD PDF that provides the same anomalous dimension as the one in the Feynman gauge. Section III deals with the interpretation of the soft gauge-invariant counter term, introduced in Sec. II, as an "intrinsic Coulomb phase" in analogy to the QED case [18]. Some comments on universality and our conclusions are given in Sec. IV.

II. CALCULATION OF THE ANOMALOUS DIMENSION OF THE TMD PDF

A. Transverse gauge link

The standard definition of the TMD PDF [4], for a quark in a quark distribution supplemented by a transverse link [7], reads

$$\begin{aligned} f_{q/q}(x, \boldsymbol{k}_\perp) &= \frac{1}{2}\int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} e^{-ik^+\xi^- + i\boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp}\langle q(p)|\bar\psi(\xi^-, \xi_\perp)[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger \\ &\quad\times[\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \infty_\perp]^\dagger\gamma^+[\infty^-, \infty_\perp; \infty^-, \mathbf{0}_\perp][\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]\psi(0^-, \mathbf{0}_\perp)|q(p)\rangle|_{\xi^+=0}; \\ [\infty^-, \boldsymbol{z}_\perp; z^-, \boldsymbol{z}_\perp] &\equiv \mathcal{P}\exp\left[ig\int_0^\infty d\tau t^a n_\mu A_a^\mu(z+n\tau)\right], [\infty^-, \infty_\perp; \infty^-, \boldsymbol{\xi}_\perp] \equiv \mathcal{P}\exp\left[ig\int_0^\infty d\tau t^a \boldsymbol{l}\cdot\boldsymbol{A}_a(\boldsymbol{\xi}_\perp + \boldsymbol{l}\tau)\right], \end{aligned} \quad (1)$$

where $\boldsymbol{l}_i$ represents an arbitrary vector in the transverse direction and $\mathcal{P}$ denotes path ordering. The displayed gauge links $[\infty^-, \boldsymbol{z}_\perp; z^-, \boldsymbol{z}_\perp]$, and $[\infty^-, \infty_\perp; \infty^-, \boldsymbol{\xi}_\perp]$ involve gauge contours extending to light cone infinity in the lightlike and in the transverse direction, respectively. Analogous expressions hold for the other gauge links entering (1). Belitsky, Ji, and Yuan [7] have shown that the extra transverse gauge link is indispensable for the restoration of gauge invariance in the light cone gauge in which the gauge potential does not vanish asymptotically.

The necessity of the additional transverse gauge link in Eq. (1) can be most easily understood from the point of view of a complete gauge fixing in the axial light cone gauge. Using the spacetime picture of the interaction of a quark moving fast in the plus light cone direction with the hard spacelike photon, as depicted in Fig. 1, one can treat the "classical" current

$$j_\mu(y) = g\int dy'_\mu \delta^{(4)}(y - y'), \qquad y'_\mu = v_\mu\tau, \quad (2)$$

as a source of the gauge field. The gauge field related to such a current has the form

$$A^\mu(\xi) = \int d^4y D^{\mu\nu}(\xi - y)j_\nu(y), \quad (3)$$

where $D^{\mu\nu}$ is the gluon Green's function. Appealing to the spacetime structure of this process illustrated in Fig. 1(a), we recast the current in the form

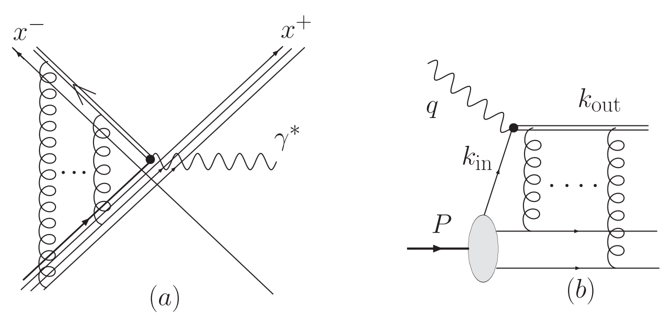

FIG. 1. Spacetime representation (a) and corresponding Feynman graph (b) of the collision of a quark with a hard photon in a deeply inelastic process. The struck quark (Wilson line) is denoted by a double line.

$$j_\mu(y) = g\delta^{(2)}(\boldsymbol{y}_\perp)\left[n_\mu^+\delta(y^-)\int\frac{dq^-}{2\pi}\frac{e^{-iq^-y^+}}{q^-+i0} - n_\mu^-\delta(y^+)\int\frac{dq^+}{2\pi}\frac{e^{-iq^+y^-}}{q^+-i0}\right], \quad (4)$$

which makes it clear that the first term in this expression corresponds to a gauge field created by a source moving from minus infinity to the origin in the plus light cone direction before being struck by the photon, whereas the second term corresponds to a gauge field being created by a source moving from the origin to plus infinity along the minus light cone ray after the collision. Then, using the gluon propagator in the light cone gauge

$$D^{\mu\nu}(z) = -\int\frac{d^4q}{(2\pi)^4}\frac{e^{-iqz}}{q^2-\lambda^2+i0} \times\left(g^{\mu\nu} - \frac{q^\mu(n^-)^\nu + q^\nu(n^-)^\mu}{[q^+]}\right), \quad (5)$$

one obtains

$$\boldsymbol{A}_\perp(\infty^-, \boldsymbol{\xi}_\perp) = \frac{g}{4\pi}C_\infty\boldsymbol{\nabla}\ln(\lambda|\boldsymbol{\xi}_\perp|), \quad (6)$$

where the numerical constant C_∞ depends on the pole prescription applied to regularize the light cone singularity

$$C_\infty = \begin{cases} 0, & \text{Adv: } [q^+] = q^+ - i0 \\ -1, & \text{Ret: } [q^+] = q^+ + i0 \\ -\frac{1}{2}, & \text{PV: } [q^+]^{-1} = \frac{1}{2}\left(\frac{1}{q^++i0} + \frac{1}{q^+-i0}\right) \end{cases}. \quad (7)$$

Obviously, the longitudinal components $A^\pm$ vanish. On the other hand, the components of the gauge field associated with the same source, but in a covariant gauge (labeled by a prime), read

$$\boldsymbol{A}'_\perp = 0, \qquad A'^- = 0,$$
$$A'^+(\xi) = -\frac{g}{4\pi}\delta(\xi^-)\ln(\lambda|\boldsymbol{\xi}_\perp|). \quad (8)$$

The (singular) gauge transformation, which connects these two field representations (i.e., the gauge-field components in the light cone gauge and those in a covariant gauge), is given by

$$A_\mu^{\rm LC} = A'_\mu + \partial_\mu\phi, \qquad \phi(\xi) = -\int_{-\infty}^{\xi^-}d\xi'^-A'^+(\xi'^-). \quad (9)$$

Equation (9) reflects exactly the gauge freedom remaining after fixing the light cone gauge $A^+ = 0$. We appreciate that a complete gauge fixing can only be achieved by inserting the additional singular gauge transformation

$$U_{\rm sing}(\infty^-, \boldsymbol{\xi}_\perp) = \left[1 - ig\int_{-\infty}^{\infty^-}dz^-A'^+_{\rm source}(z^-, \boldsymbol{z}_\perp) + O(g^2)\right], \quad (10)$$

which contains the cross talk effects of the struck parton with the light cone source. Therefore, the product of two (local) quark field operators in the completely fixed light cone gauge (marked below by a wide hat) differs from that in a covariant gauge by two phase factors and attains the form

$$[\bar\psi(\xi^-, \boldsymbol{\xi}_\perp)\gamma^+\psi(0^-, \boldsymbol{0}_\perp)]_{\widehat{\rm LC}} = \bar\psi_{\rm LC}(\xi)\mathcal{P}\exp\left[+ig\int_{\xi_\perp}^{\infty_\perp}d\boldsymbol{z}_\perp\boldsymbol{A}_{\rm source}^{\rm LC}(\infty^-, \boldsymbol{z}_\perp)\right]\gamma^+\mathcal{P}\exp\left[-ig\int_{0_\perp}^{\infty_\perp}d\boldsymbol{z}_\perp\boldsymbol{A}_{\rm source}^{\rm LC}(\infty^-, \boldsymbol{0}_\perp)\right]\psi_{\rm LC}(0) \quad (11)$$

in agreement with Eq. (1).

B. Anomalous dimensions

Within the CS approach, where $n^2 \neq 0$, the anomalous dimension associated with $f_{q/q}(x, \boldsymbol{k}_\perp)$ is

$$\gamma_{\rm CS} = \frac{1}{2}\mu\frac{d}{d\mu}\ln Z_f(\mu, \alpha_s; \epsilon) = \frac{3}{4}\frac{\alpha_s}{\pi}C_{\rm F} + O(\alpha_s^2) = \gamma_{\rm smooth}, \quad (12)$$

where Z_f is the renormalization constant of $f_{q/q}(x, \boldsymbol{k}_\perp)$ in the $\overline{\rm MS}$ scheme. As long as one assumes that the deformation of the Wilson line in the transverse direction off the light cone preserves the smoothness of the gauge contour, the associated anomalous dimension is only due to the endpoints and, therefore, equals that of the connector insertion [15]. Hence, as far as the renormalization of the pure gauge link with a finite contour is concerned, the straight lightlike line is enough to supply its anomalous dimension because other contour characteristics, e.g., its length, are irrelevant.

In general, in renormalizing the distribution $f_{q/q}$ of a quark in a quark, one faces UV divergences stemming from the momentum integration that can be renormalized in the usual way. But, using the light cone gauge $n^2 = 0$, extra UV divergences contribute due to the additional pole of the gluon propagator, as already mentioned. We calculate the UV divergences of the one-loop diagrams, shown in Fig. 2, which contribute to $f_{q/q}(x, \boldsymbol{k}_\perp)$ in the light cone gauge $(A\cdot n^-) = 0$, $(n^-)^2 = 0$, by using dimensional regularization. The poles $1/q^+$ of the gluon propagator

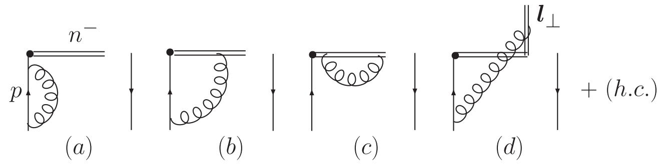

FIG. 2. One-loop gluon contributions to the UV divergences of the TMD PDF. Double lines denote gauge links. Diagrams (b) and (c) are absent in the light cone gauge.

$$D^{\rm LC}_{\mu\nu}(q) = \frac{1}{q^2}\left(g_{\mu\nu} - \frac{q_\mu n^-_\nu + q_\nu n^-_\mu}{[q^+]}\right), \tag{13}$$

are regularized according to

$$\frac{1}{[q^+]} = \frac{1}{q^+ \pm i\Delta}. \tag{14}$$

In what follows, we keep Δ small but finite.

The UV divergent part of diagrams 2(a) and 2(d) receives contributions owing to the p^+-dependent term

$$\Sigma^{\rm UV}_{\rm LC}(\alpha_s, \epsilon) = \frac{\alpha_s}{\pi} C_{\rm F} 2\left[\frac{1}{\epsilon}\left(\frac{3}{4} + \ln\frac{\Delta}{p^+}\right) - \gamma_E + \ln 4\pi\right] \tag{15}$$

in addition to those originating from the standard UV renormalization. In deriving expression (15), we find that the contribution associated with the transverse gauge link at infinity (diagram Fig. 2(d)) exactly cancels against the term entailed by the adopted pole prescription in the gluon propagator. This confirms the previous results by Belitsky, Ji, and Yuan, and establishes the dependence of the result on local quantities only. Therefore, the corresponding anomalous dimension is given by

$$\gamma_{\rm LC} = \frac{\alpha_s}{\pi} C_{\rm F}\left(\frac{3}{4} + \ln\frac{\Delta}{p^+}\right) = \gamma_{\rm smooth} - \delta\gamma. \tag{16}$$

The difference between $\gamma_{\rm smooth}$ and $\gamma_{\rm LC}$ is exactly that term induced by the additional divergence which has to be compensated by a suitable redefinition of the TMD PDF. Note that $p^+ = (p \cdot n^-) \sim \cosh\chi$ defines, in fact, an angle χ between the direction of the quark momentum p_μ and the lightlike vector n^-. In the large χ limit, $\ln p^+ \to \chi$, $\chi \to \infty$. Thus, we can conclude that the "defect" of the anomalous dimension, $\delta\gamma$, can be identified with the well-known cusp anomalous dimension [19]

$$\begin{aligned} \gamma_{\rm cusp}(\alpha_s, \chi) &= \frac{\alpha_s}{\pi} C_{\rm F}(\chi\coth\chi - 1), \\ \frac{d}{d\ln p^+}\delta\gamma &= \lim_{\chi\to\infty}\frac{d}{d\chi}\gamma_{\rm cusp}(\alpha_s, \chi) = \frac{\alpha_s}{\pi} C_{\rm F}. \end{aligned} \tag{17}$$

This provides formal support for our previous statement concerning the appropriate choice of the Wilson line in the definition of the TMD PDF.

As one knows from the renormalization of contour-dependent composite operators in QCD (see [20] and also [19,21,22] and earlier references cited therein), the standard UV renormalization procedure has to be generalized in order to be able to subtract angle-dependent singularities stemming from obstructions, like cusps or self-intersections. Having recourse to these techniques, we compute the extra renormalization constant associated with the soft counter term [12] and show that it can be expressed in terms of a vacuum expectation value of a specific gauge link. In order to cancel the anomalous-dimension defect $\delta\gamma$, we introduce the counter term

$$R \equiv \Phi(p^+, n^-|0)\Phi^\dagger(p^+, n^-|\xi), \tag{18}$$

where

$$\Phi(p^+, n^-|\xi) = \langle 0|\mathcal{P}\exp\left[ig\int_{\Gamma_{\rm cusp}} d\zeta^\mu t^a A^a_\mu(\xi + \zeta)\right]|0\rangle \tag{19}$$

and evaluate it along the nonsmooth, off-the-light cone integration contour $\Gamma_{\rm cusp}$, defined by

$$\begin{aligned} \Gamma_{\rm cusp}\colon \zeta_\mu = \{&[p^+_\mu s, -\infty < s < 0] \cup [n^-_\mu s', 0 < s' < \infty] \\ &\cup [\boldsymbol{l}_\perp \tau, 0 < \tau < \infty]\} \end{aligned} \tag{20}$$

with n^-_μ being the minus light cone vector illustrated in Fig. 3.

The one-loop gluon virtual corrections, contributing to the UV divergences of R, are shown in Fig. 4. For the UV divergent term we obtain

$$\Sigma^{\rm UV}_R = -\frac{\alpha_s}{\pi} C_{\rm F} 2\left(\frac{1}{\epsilon}\ln\frac{\Delta}{p^+} - \gamma_E + \ln 4\pi\right) \tag{21}$$

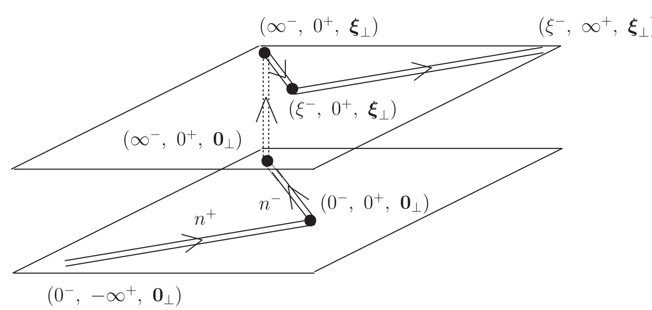

FIG. 3. The integration contour associated with the additional soft counter term.

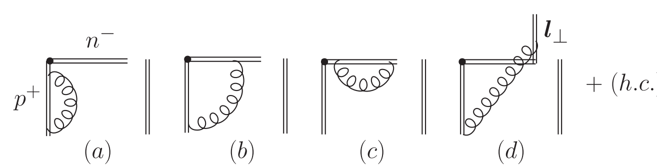

FIG. 4. Virtual gluon contributions to the UV divergences of the soft counter term (in analogy to Fig. 2).

and observe that this expression is equal, but with opposite sign, to the unwanted term in the UV singularity, related to the cusped contour, calculated before.

Hence, it is reasonable to redefine the conventional TMD PDF and absorb the soft counter term in its definition. Then we have

$$f_{q/q}^{\mathrm{mod}}(x, \boldsymbol{k}_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} e^{-ik_\perp\cdot\xi_\perp}\langle q(p)|\bar{\psi}(\xi^-, \xi_\perp)[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger[\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \infty_\perp]^\dagger\gamma^+[\infty^-, \infty_\perp; \infty^-, \mathbf{0}_\perp] \times [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]\psi(0^-, \mathbf{0}_\perp)|q(p)\rangle \cdot [\Phi(p^+, n^-|0^-, \mathbf{0}_\perp)\Phi^\dagger(p^+, n^-|\xi^-, \boldsymbol{\xi}_\perp)], \quad (22)$$

which is one of the main results of our work. For the renormalization of

$$f_{\mathrm{ren}}^{\mathrm{mod}}(x, \boldsymbol{k}_\perp) = Z_f^{\mathrm{mod}}(\alpha_s, \epsilon) f^{\mathrm{mod}}(x, \boldsymbol{k}_\perp, \epsilon) \quad (23)$$

the standard UV renormalization is sufficient. It yields the following renormalization constant

$$Z_f^{\mathrm{mod}} = 1 + \frac{\alpha_s}{4\pi} C_{\mathrm{F}} \frac{2}{\epsilon}\left(-3 - 4\ln\frac{\Delta}{p^+} + 4\ln\frac{\Delta}{p^+}\right) = 1 - \frac{3\alpha_s}{4\pi} C_{\mathrm{F}} \frac{2}{\epsilon}, \quad (24)$$

which in turn gives rise to the anomalous dimension

$$\gamma_f^{\mathrm{mod}} = \frac{1}{2}\mu\frac{d}{d\mu}\ln Z_f^{\mathrm{mod}}(\mu, \alpha_s, \epsilon) = \frac{3}{4}\frac{\alpha_s}{\pi} C_{\mathrm{F}} + O(\alpha_s^2). \quad (25)$$

It is obvious that (at least at the one-loop order) this expression coincides with γ_{smooth} given by Eq. (12).

III. INTRINSIC COULOMB PHASE

The physical meaning of the introduced soft counter term can be described as follows. Appealing to the exponentiation theorem for non-Abelian path-ordered exponentials [19], the vacuum average (19) can be recast in the form

$$\Phi(u, n^-) = \exp\left[\sum_{n=1}^{\infty} \alpha_s^n \Phi_n(u, n^-)\right], \quad (26)$$

where the functions Φ_n have, in general, a complicated structure. Nevertheless, the leading term in this series, Φ_1, is just a non-Abelian generalization of the Abelian expression

$$\Phi_1(u, n^-) = -4\pi C_{\mathrm{F}} \int_{\Gamma_{\mathrm{cusp}}} dx_\mu dy_\nu \theta(x - y) D^{\mu\nu}(x - y). \quad (27)$$

By virtue of the current

$$j_\nu^b(z) = t^b v_\nu \int_{\Gamma_{\mathrm{cusp}}} d\tau \delta^{(4)}(z - v\tau), \quad (28)$$

evaluated along the contour Γ_{cusp} [cf. Equation (20)] and where the velocity v_ν equals either u_ν, n^-, or $\boldsymbol{l}_\perp$ (depending on the segment of the contour along which the integration is performed), one can rewrite (27) as follows

$$\Phi_1(u, n^-) = -t^a 4\pi \int_{\Gamma_{\mathrm{cusp}}} dx_\mu \int d^4z \delta^{ab} D^{\mu\nu}(x - z) j_\nu^b(z). \quad (29)$$

This result proves that the additional soft counter term R can be treated within Mandelstam's manifestly gauge-invariant formalism and appears there as an "intrinsic Coulomb phase" [18] originating from the long-range interactions of a colored quark, created initially at the "point" $-\infty^+$ together with its oppositely color-charged counterpart, then travelling along the plus light cone ray to the origin, where it experiences a hard collision with the off-shell photon, subsequently changing its route and venturing along the minus ray to $+\infty^-$. Within such a context, the soft counter term can be conceived of as that part of the TMD PDF which accumulates the residual effects of the primordial separation of two oppositely color-charged particles, created at light cone infinity and being unrelated to the existence of external color sources.

IV. DISCUSSION AND CONCLUSIONS

The study presented above was performed for the semi-inclusive deep inelastic scattering (SIDIS). Before we conclude, it is appropriate to make some comments concerning the Drell-Yan lepton-pair production. In this case, it is known [9] that the direction of the integration contours in the gauge links should be reversed. In the light cone gauge, this corresponds to a change of sign of the additional regulator Δ [cf. Equation (14)]

$$\Delta_{\mathrm{SIDIS}} = -\Delta_{\mathrm{DY}}. \quad (30)$$

In the nonpolarized case, this affects only the imaginary parts, and, therefore, it does not contribute to the final expressions. In other words, the UV anomalous dimension of the nonpolarized TMD PDFs is universal as regards the SIDIS and the Drell-Yan processes. This, however, may not be true for the spin-dependent TMD PDFs, since in that case the imaginary parts play a crucial role and, thus, a sign change [expressed in (30)] might indeed affect the renormalization-group properties and the corresponding evolution equations. These issues will be considered elsewhere.

Let us summarize the cornerstones of our work:

(i) We performed an analysis of TMD PDFs based on anomalous dimensions that encapsulate the relevant Wilson-line characteristics in local form.

(ii) We showed by explicit calculation at the one-loop level that the appropriate Wilson contour in the light cone gauge is a cusped one, contributing an angle-dependent anomalous dimension to the TMD PDF, that has to be compensated in order to render it compatible with the collinear factorization. The validation of this cancellation in next-to-leading order is currently in progress.

(iii) We outlined how this new contribution can be included in the definition of the TMD PDF by means of a soft counter term, as proposed by Collins [9]. We found that this new term can be written as an "intrinsic Coulomb phase" that keeps track of the full gauge history of the colored quarks [18].

This phase may be given the following interpretation: Before the quark is being struck it is escorted only by longitudinal gluons that can be formally eliminated by imposing the light cone gauge. However, when it leaves the (hard) interaction region, it is not lying on the minus light cone direction and exchanges soft transverse gluons with the quark spectator. Hence, one cannot trivialize the interaction of the struck quark with the gauge field by imposing a single gauge choice on a lightlike ray, because the struck quark enters and leaves the hard-interaction vertex with different four velocities, as it becomes evident from Eq. (28). This complies with the interpretation given by Belitsky, Ji, and Yuan [7] (see also [11]) in terms of final-state interactions of the struck quark with the gluon field of the target spectators. As long as one disregards polarization effects, the direction of the Wilson line (expressed by means of the $i\epsilon$ prescription in the gluon propagator), appears only in intermediate steps of the calculation and cancels at the end because it is only a phase. In conclusion, our analysis may lead to a deeper insight of the dynamics of TMD PDFs and have wide-range phenomenological applications.

ACKNOWLEDGMENTS

We thank A. P. Bakulev, N. A. Kivel, A. I. Karanikas, S. V. Mikhailov, P. V. Pobylitsa, and O. V. Teryaev for discussions. I. O. C. is supported by the Alexander von Humboldt Foundation. This work was supported in part by the Deutsche Forschungsgemeinschaft under Grant No. 436 RUS 113/881/0, Russian Federation President's Grant No. 1450-2003-2, and the Heisenberg-Landau Program 2007 and 2008.

[1] A. V. Belitsky and A. V. Radyushkin, Phys. Rep. **418**, 1 (2005).
[2] J. C. Collins, Acta Phys. Pol. B **34**, 3103 (2003).
[3] F. Hautmann and H. Jung, arXiv:0712.0568.
[4] J. C. Collins and D. E. Soper, Nucl. Phys. **B193**, 381 (1981); **B213**, 545 (E) (1983).
[5] J. C. Collins, T. C. Rogers, and A. M. Stasto, Phys. Rev. D **77**, 085009 (2008).
[6] A. Bassetto, G. Heinrich, Z. Kunszt, and W. Vogelsang, Phys. Rev. D **58**, 094020 (1998).
[7] A. V. Belitsky, X. Ji, and F. Yuan, Nucl. Phys. **B656**, 165 (2003).
[8] X. Ji, J. Ma, and F. Yuan, Phys. Rev. D **71**, 034005 (2005).
[9] J. C. Collins, Phys. Lett. B **536**, 43 (2002).
[10] D. Boer, P. J. Mulders, and F. Pijlman, Nucl. Phys. **B667**, 201 (2003).
[11] X. d. Ji and F. Yuan, Phys. Lett. B **543**, 66 (2002).
[12] J. C. Collins and F. Hautmann, Phys. Lett. B **472**, 129 (2000).
[13] J. C. Collins and A. Metz, Phys. Rev. Lett. **93**, 252001 (2004).
[14] F. Hautmann, Phys. Lett. B **655**, 26 (2007).
[15] N. G. Stefanis, Nuovo Cimento Soc. Ital. Fis. A **83**, 205 (1984).
[16] S. Mandelstam, Ann. Phys. (N.Y.) **19**, 1 (1962).
[17] I. O. Cherednikov and N. G. Stefanis, arXiv:0802.2821.
[18] R. Jakob and N. G. Stefanis, Ann. Phys. (N.Y.) **210**, 112 (1991).
[19] G. P. Korchemsky and A. V. Radyushkin, Nucl. Phys. B **283**, 342 (1987).
[20] A. M. Polyakov, Nucl. Phys. **B164**, 171 (1980).
[21] N. S. Craigie and H. Dorn, Nucl. Phys. **B185**, 204 (1981).
[22] S. Aoyama, Nucl. Phys. **B194**, 513 (1982).