Consequences of approximate S_3 symmetry of the neutrino mass matrix

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Assuming that the neutrino mass matrix is dominated by a term with the permutation symmetry S_3 it is possible to explain neutrino data only if the masses are quasidegenerate. A subdominant term with an approximate $\mu - \tau$ symmetry leads to an approximate tribimaximal form. Experimental consequences are discussed.

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Early solar neutrino data suggested that one neutrino eigenstate could be

$$S = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau). \tag{1}$$

This led to the consideration of an S_3 symmetry [1]. Today the Mikheyev-Smirnov-Wolfenstein (MSW) solution to the solar neutrino problem has the higher-energy neutrinos emerging from the Sun in a state given to a good approximation by S. Here we consider the possibility that the neutrino mass matrix is dominated by a term with S_3 symmetry leading to S as an eigenstate. We then consider possible perturbations that violate the symmetry.

Our assumption is that neutrino mass is due to new physics not directly related to the origin of the masses of other particles. A large number of papers [2] have presented detailed models based on S_3 symmetry. Here we do not consider a model but simply try to relate possible symmetries of the new physics to observations. The most general Majorana mass matrix invariant under S_3 is

$$M_0 = \begin{pmatrix} A & B & B \\ B & A & B \\ B & B & A \end{pmatrix}. \tag{2}$$

The eigenstates are necessarily [1] a singlet given by S and a degenerate doublet D which can be chosen as

$$D_a = \frac{\nu_\mu - \nu_\tau}{\sqrt{2}}, \tag{3a}$$

$$D_b = \sqrt{\frac{2}{3}}\nu_e - \sqrt{\frac{1}{6}}(\nu_\mu + \nu_\tau). \tag{3b}$$

The masses are

$$m_S = A + 2B, \tag{4a}$$

$$m_D = A - B. \tag{4b}$$

The eigenstates in Eqs. (1) and (3) are those of the tribimaximal form of the mixing matrix discussed in many papers [3] as a fit to neutrino oscillation data. However, in the fit, the largest mass splitting is that between D_a and D_b responsible for the atmospheric neutrino oscillation with smaller splitting between S and D_b associated with the solar neutrino oscillation. We assume that the breaking of the degeneracy is due to the perturbation that breaks S_3. In order that the S_3 term dominate we require that all three masses start out approximately equal by choosing

$$B = -2A + b, \tag{5a}$$

with $b \ll B$ so that

$$m_D \approx -m_S \approx 3A. \tag{5b}$$

The minus sign means that the state S has the opposite CP eigenvalue from that of D. We have assumed here for simplicity that A and B are real; otherwise D and S would have a relative Majorana phase. The subdominant mass matrix M_1 that breaks S_3 has the result of raising the mass of one D state above m_S and leaving the mass of the other slightly below m_S. These mass values then correspond to what is called the "quasidegenerate" case for neutrino masses.

We now assume that the perturbing matrix M_1, which is added to M_0, breaks S_3 but retains a S_2 symmetry between ν_μ and ν_τ.

$$M_1 = \begin{pmatrix} e & f & f \\ f & t & \epsilon \\ f & \epsilon & t \end{pmatrix}. \tag{6}$$

As a result of the symmetry D_a remains an eigenstate and the parameter known as θ_{13} vanishes. In addition to providing the mass splitting between D_a and D_b, M_1 causes a small mixing of D_b with S. The parameters e and f can be absorbed into A and B and so they are set to zero in what follows. Matrices of the form $M_0 + M_1$, with four parameters are discussed in many papers [4].

We now identify the states which start out as (D_a, S, D_b) as (3, 2, 1). The mass m_2 is understood to be positive although m_S is negative (assuming A is positive). The mass differences are

$$m_3 - m_1 = \tfrac{2}{3}(t - 2\epsilon), \tag{7a}$$

$$m_2 - m_1 = -(b + t + \epsilon). \tag{7b}$$

The small value of $m_2 - m_1$ required to fit the data involves the fine-tuning of the value of b. The resulting deviation of the factor $\frac{1}{\sqrt{3}}$ for ν_e in S is given approximately by

$$\Delta = \frac{2}{3\sqrt{3}}\left(\frac{t+\epsilon}{6A}\right) = \frac{k}{\sqrt{3}}\left(\frac{m_3 - m_1}{2m_2}\right),$$
$$k = \frac{t+\epsilon}{t-2\epsilon}, \qquad \sin^2\theta_{12} = \left(\frac{1}{\sqrt{3}} + \Delta\right)^2. \tag{8}$$

Since by our assumption of a quasidegenerate neutrino mass spectrum, the mass ratio in Eq. (8) is small so that Δ is predicted to be small. To obtain the doublet mass splitting without large parameters we choose $\frac{\epsilon}{t}$ to be negative. As $\frac{\epsilon}{t}$ varies from 0 to a large negative value k varies from 1 to $-\frac{1}{2}$; for $\frac{\epsilon}{t} = -1$, $\Delta = 0$, and we obtain the tribimaximal form. Choosing values for the mass splittings fitted from oscillation data [5]

$$m_3^2 - m_2^2 = 2.6 \times 10^{-3}\ \mathrm{eV}^2,$$
$$m_2^2 - m_1^2 = 8 \times 10^{-5}\ \mathrm{eV}^2, \tag{9}$$

we give in Table I three sets of mass values. The largest values (like set 1) are limited by cosmology [6] whereas the smallest values (like set 3) are limited by the requirement that the magnitude of M_1 is smaller than M_0. For each of these we show in Fig. 1 the solar neutrino survival $\sin^2\theta_{12}$ for the higher-energy neutrinos for the MSW solution as a function of $\frac{\epsilon}{t}$. Note that the sign of the deviation from $\frac{1}{3}$ can be either positive or negative. We have shown the case of the "normal hierarchy" with $(m_3 - m_1)$ positive. In the case of the inverse hierarchy the curves are flipped about the $\sin^2\theta_{12} = \frac{1}{3}$ axis. Assuming negligible Majorana phases, the mass that enters the double beta-decay formula is

$$m_{ee} = -\sin^2\theta_{12} m_2 + \cos^2\theta_{12} m_1 \approx (1 - 2\sin^2\theta_{12}) m_2, \tag{10}$$

given the small difference between m_2 and m_1.

We finally consider a possible small violation of $\mu - \tau$ symmetry by changing the 22 element in Eq. (6) to $t + \frac{\delta}{2}$ and the 33 element to $t - \frac{\delta}{2}$. The main effect is to mix D_a and D_b or the states now labeled 3 and 1. There is also a small mixing of 2 and 1 but this is suppressed by the "mass difference" 6A. The important result is a nonzero value of θ_{13}, the ν_e amplitude in state 3. Directly correlated with θ_{13} there is a deviation of θ_{23}, the ν_μ amplitude in state 3, from $\frac{\pi}{4}$.

Starting with the tribimaximal mixing, corresponding to the limit $\frac{\epsilon}{t} = -1$, this correlation is given by

TABLE I. Three sets of mass values.

	m_1 (eV)	m_2 (eV)	m_3 (eV)
1	0.1845	0.1847	0.1913
2	0.1247	0.1250	0.1350
3	0.0512	0.0520	0.0729

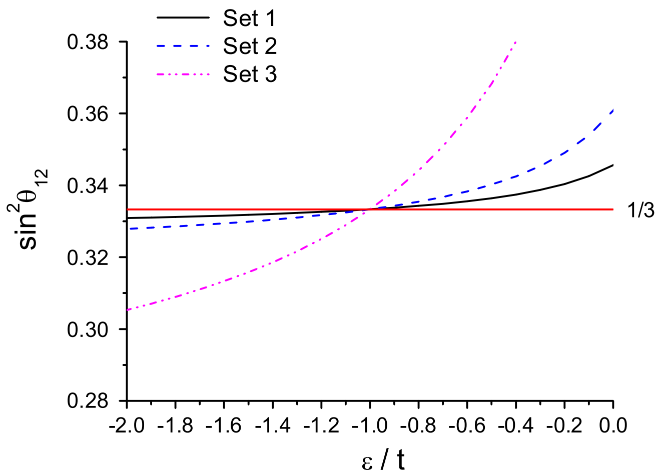

FIG. 1 (color online). The solar neutrino survival $\sin^2\theta_{12}$ for the higher-energy neutrinos for the MSW solution as a function of $\frac{\epsilon}{t}$.

$$\tan^2\theta_{23} = 1 - 2\sqrt{2}X + 4X^2,$$
$$X = \sin\theta_{13}\left(\frac{1+2\lambda}{1-\lambda}\right), \qquad \lambda = \frac{m_3 - m_1}{m_2 + m_1}, \tag{11}$$

to order X^2. In Fig. 2, we show $(\tan^2\theta_{23} - 1)$ as a function of $\sin\theta_{13}$. Different values of $\frac{\epsilon}{t}$ makes only small changes since they are proportional to $\lambda\Delta$. Given the small value of θ_{13} from experiment, there is less than as 1% contribution of θ_{13} to Eq. (10) for m_{ee}.

In this paper, we have looked at possible experimental signatures of the assumption that the physics yielding the neutrino mass matrix has a predominant S_3 symmetry. We further assume a subdominant term which breaks S_3 but has an S_2 $\mu - \tau$ symmetry. This leads to

(1) The neutrino masses must be quasidegenerate. Therefore this theory would be ruled out if cosmological analysis convincingly gave a very low limit on the sum of the masses. Considering case 3 in

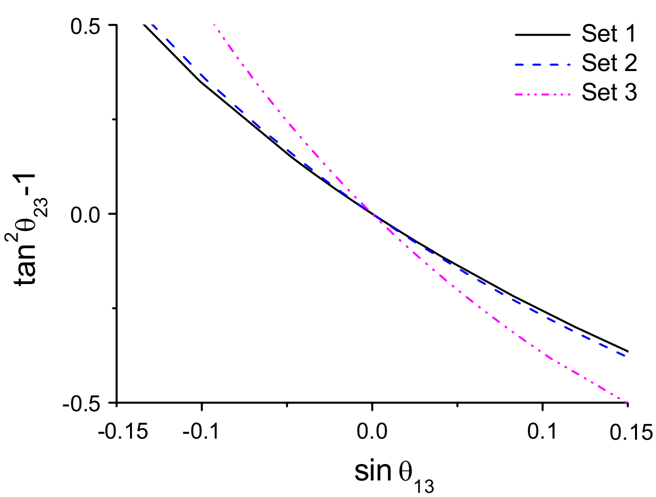

FIG. 2 (color online). Diagram with $\tan^2\theta_{23} - 1$ vs $\sin\theta_{13}$.

Table I as barely quasidegenerate, the sum of the masses should not be less than 0.17 eV.

(2) θ_{13}, the ν_e component in the atmospheric mixing, vanishes and the mixing is maximal.

(3) The high-energy solar neutrino survival, governed by the MSW solution, deviates only a little from $\frac{1}{3}$ as illustrated in Fig. 1.

(4) In the absence of significant Majorana phases, the mass m_{ee} governing double beta decay is approximately equal to $\frac{m_2}{3}$. Thus the theory would be ruled out if the value of m_{ee} was found to be too large. For example, if the cosmological limit on the sum of the masses was 0.4 eV consistent with case 2 in Table I the value of m_{ee} must be less than 0.05 eV.

If we further allow a small term involving only ν_μ and ν_τ that violates the S_2 symmetry, then there is a nonzero θ_{13}. In this case, the atmospheric mixing angle is no longer maximum and its value is directly correlated with θ_{13} as shown in Eq. (11) and Fig. 2.

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