Photon polarization in radiative $B \to \varphi K \gamma$ decay

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The photon polarization in radiative decays $B \to Y\gamma$ is known to be a subtle probe of the effective Lagrangian structure and possible New Physics effects. We discuss exclusive decay mode $B^- \to \varphi K^- \gamma$ where the experimentally distinct final state makes analysis especially promising. The possibility to extract information on the photon polarization out of the data entirely depends on the partial waves' interference pattern in the φK^- system.

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I. INTRODUCTION

The main efforts of modern high energy physics are devoted to the searches of phenomena beyond the standard model (SM) and correspondingly to constraining different SM extensions. Flavor physics is an important area of this activity: B, D, and K-meson decay studies have brought a lot of information about different aspects of the Cabibbo-Kobayashi-Maskawa paradigm and suggest promising places to look for New Physics (NP). The unique experimental opportunities for the b-physics part of this research program have been related to *BABAR* and Belle experiments, while the main hope for now is concentrated on the LHCb experiment at CERN with prospects for the Super-B factory as a possible future project.

Among a wide variety of rare decays radiative B-meson decays $B \to Y\gamma$ are especially distinctive (and sometimes even called "the standard candles" of flavor physics [1]). The first obvious reason is that the electromagnetic part of these decays is under full theoretical control, while from an experimental point of view the energetic photon serves as a clean and unambiguous decay signal. This allowed to develop an effective theory of such decays and also to obtain impressive experimental data on the corresponding branching ratios (see, e.g., [2] and references therein).

Unfortunately comparison of experimentally measured branching ratios with theoretical predictions is plagued by hadron uncertainties of the latter. This motivates constant interest in theoretical and experimental studies of "gold-plated" observables, unaffected by hadron uncertainties. Radiative decays provide a polarization pattern of emitted photons (corresponding to angular correlations in the final hadron state) as a good example of such an observable.

Moreover, it was argued in [3] that measurements of the photon polarization in the final state turn out to be an effective tool for the NP searches. The point is that photons, emitted in the B^- and $\bar{B}^0$-meson decays are predominantly left-handed (and right-handed for the B^+ and B^0 decays) in the SM, while the admixture of photons with "wrong" polarization may be rather large in some SM extensions like, e.g., left-right symmetric model or minimal supersymmetric standard model. The information one can get in this way is extremely interesting since it provides a typical example of what is known as the "null tests" of the SM [4]. It probes the internal Lorentz structure of the photon emission vertex and hence essential features of the effective Hamiltonian structure.

Several ways have been suggested to look for signals beyond the SM through the photon helicity tests. In particular, the admixture of right-handed photons may be found via the time-dependent CP-asymmetry in $B^0(t) \to f^{CP}\gamma$ decays, where $f^{CP} = K^{*0} \to K_S \pi^0$:

$$\mathcal{A}(t) = \frac{\Gamma(B^0(t) \to f^{CP}) - \Gamma(\bar{B}^0(t) \to f^{CP})}{\Gamma(B^0(t) \to f^{CP}) + \Gamma(\bar{B}^0(t) \to f^{CP})} = S \sin(\Delta m_B t) - C \cos(\Delta m_B t). \tag{1}$$

The mixing-induced asymmetry S is proportional to the A_R/A_L ratio of the polarization amplitudes, which corresponds to right- and left-handed photon emission and is expected to be less than a few percent (see below) in the SM [3–5].

Another method makes use of the photons from the $B \to (K^* \to K\pi)\gamma$ decay, converting in the detector material into the electron-positron pair (see recent paper [6] in this respect). For these processes the distribution in the angle ϕ between e^+e^- and $K\pi$ planes should be isotropic for purely circular polarization, the deviations from this isotropy depend on the same parameter A_R/A_L, indicating the presence of right-handed photons [7–10]. So, the angular distribution for real photons is given by

$$\frac{d\sigma}{d\phi} \propto 1 + \eta \frac{A_L A_R}{A_L^2 + A_R^2} \cos(2\phi + \delta'), \tag{2}$$

where η and δ' are some hadronic parameters of no importance for us here.

Alternatively, one can study baryon decays $\Lambda_b \to \Lambda\gamma \to p\pi\gamma$ and measure the photon polarization directly. It is proportional here to the forward-backward asymmetry of the proton with respect to Λ_b in the rest frame of Λ or related to Λ_b polarization and forward-backward asymmetry of Λ momentum for the polarized Λ_b's [11–13].

In this paper we follow the standard method, which makes use of angular correlations among the three-body decay products in $B \to P_1 P_2 P_3 \gamma$, where P_i are either pions or kaons. This technique was suggested in [14,15] and used for the decay $B \to K\pi\pi\gamma$ with the manifest summation over intermediate hadron resonances. We consider the radiative decay mode $B \to (\varphi \to K^+ K^-)K\gamma$ in the present paper. The mode $B \to \varphi K \gamma$ is rather distinctive with many desirable features from the experimental point of view: the finite state is a photon plus only charged mesons (for charged B-mesons), the fact that φ is narrow reduces the effects of intermediate resonances interference, etc. The branching fraction for this decay mode was measured by *BABAR* and Belle Collaborations [16,17]:

$$\mathcal{B}(B^- \to \varphi K^- \gamma) = (3.5 \pm 0.6) \times 10^{-6}, \tag{3}$$

and this decay channel is currently being studied under LHCb rare decays program.

The general qualitative physical picture behind the photon polarization measurement procedure discussed in the present paper can be explained as follows. The b-quark belonging to the initial pseudoscalar B meson decays due to the weak penguin process into a photon γ and s-quark. The latter forms the hadron system Y (together with the spectator), which is characterized by total angular momentum $J \geq 1$ and its projection λ. Strong dynamics causes consequent decay of Y into a pseudoscalar P_3 (where the spectator quark goes) and a vector or tensor T (where the s-quark goes).

$$Y(J^P, \lambda) \to P_3[T \to P_1 P_2]. \tag{4}$$

We have $\vec{J} = \vec{j}_T + \vec{l}$ where $\vec{l}$ is the relative orbital momentum of the states T and P_3. The tensor helicity λ_T carries information about the s-quark helicity, which in turn is correlated with the photon polarization. The partial wave amplitude takes the form:

$$A_{l\lambda} \propto \sum_{\lambda_T = -j_T}^{j_T} (l, 0; j_T, \lambda_T | J, \lambda_T) \cdot \bar{A}_{\lambda_T \lambda}, \tag{5}$$

where $(l, 0; j_T, \lambda_T | J, \lambda_T)$ are Clebsch-Gordan coefficients. If relative angular momentum between P_3 and T is zero we have no way to uncover this information since for $l = 0$ all polarization states of T enter on equal footing and the amplitude $A_{0\lambda}$ has no sensitivity to λ. But it is not the case if $l \neq 0$ and then nontrivial asymmetric interference pattern

$$|A_R|^2 - |A_L|^2 \propto \vec{p}_\gamma \cdot [\vec{p}_1 \times \vec{p}_2] \tag{6}$$

starts to show up. This picture is applicable to both resonant amplitudes in the $B \to K\pi\pi\gamma$ channel and nonresonant $B \to K\varphi\gamma$ channel (where φ plays the role of T).

It is convenient to define the total decay amplitude as a convolution of weak radiative amplitude $c_{L,R} = A(B \to Y\gamma_{L,R})$ and strong polarization amplitude $A_{L,R} = A(Y \to [\varphi \to KK]K)$ for the consequent decay corresponding to the left- and right-polarized resonance Y, respectively (including all necessary form-factors and Breit-Wigner forms). The photon polarization parameter $\lambda_\gamma^{(i)}$ defined in terms of amplitude ratio for the decay $B \to Y^{(i)}\gamma_{(L,R)}$ could depend on the final state $Y^{(i)}$ quantum numbers. However, due to parity conservation by the strong interactions it does not [14]. Moreover, since we consider in what follows the $[\varphi K]$ system in a state with the fixed quantum numbers, we can define the photon polarization parameter simply as

$$\lambda_\gamma = \frac{|c_R|^2 - |c_L|^2}{|c_R|^2 + |c_L|^2}. \tag{7}$$

Another general comment is worth making. According to the standard quantum mechanics, the expression for the partial branching ratio contains a sum over final states, which in our case is a state of hadronic system plus a photon of definite helicity. From general principles it is clear however that the amplitudes, corresponding to emission of left-handed and right-handed photons, do not interfere since they correspond to different final states and, as a matter of principle, the photon helicity can be measured independently (for example, in a gedanken way by measuring the angular momentum of the detector). As a result, general expression for the partial decay width takes the following general form: [14,15]:

$$\frac{d\Gamma}{d\Phi} \propto |c_L A_L|^2 + |c_R A_R|^2, \tag{8}$$

where $d\Phi$ is the final particles' phase-space (see exact form after Eq. (20)) and the polarization amplitudes A_R (A_L) correspond to the left-(right)-handed photon emission. There are no interference terms $\sim |A_L^* A_R|$ in the expression (9), and this fact is completely independent of the structure of the amplitudes (i.e., whether they are real or complex, presence or absence of NP effects, etc.). This is in contrast with the results of the recent paper [18], where the same decay mode $B \to K\varphi\gamma$ is considered.

Taking into account the definition ([12]) one has for the partial decay width

$$\frac{d\Gamma}{d\Phi} \propto |A_R|^2 + |A_L|^2 + \lambda_\gamma(|A_R|^2 - |A_L|^2). \tag{9}$$

To find λ_γ one has to extract from the branching ratio (9) the angular part (6), sensitive to the discussed asymmetry.

II. PHOTON POLARIZATION IN THE STANDARD MODEL AND BEYOND

In the SM the radiative decay of the b-quark is governed by the lowest-order effective Hamiltonian:

$$H_{\mathrm{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^{*} (C_{7R} O_{7R} + C_{7L} O_{7L})$$
$$O_{7L,R} = \frac{em_b}{16\pi^2} F_{\mu\nu} \bar{s} \sigma^{\mu\nu} \frac{1 \pm \gamma_5}{2} b. \tag{10}$$

Here $C_{7L,R}$ are the Wilson coefficients corresponding to the amplitude for emission of left- or right-handed photons in the $b_R \to q_L \gamma_L (b_L \to q_R \gamma_R)$ decays. This can be seen by representing the electromagnetic field tensor for left-(right-)polarized photons: $F_{\mu\nu}^{L,R} = \frac{1}{2}(F_{\mu\nu} \pm i\tilde{F}_{\mu\nu})$, where $\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\sigma\rho}F^{\sigma\rho}$. Using the identity $\sigma_{\mu\nu}\gamma_5 = \frac{i}{2}\varepsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$ one can see that only $F_{\mu\nu}^{L}$ survives in the first term of (10) and only $F_{\mu\nu}^{R}$ in the second one. The ratio measuring the part of "wrong" helicity photons $|C_{7R}/C_{7L}|$ is proportional to the mass ratio m_s/m_b, because only left-handed components of external fermions couple to W-boson in the SM. However, besides the kinematical corrections controlled by the mass ratio m_s/m_b there are also QCD corrections—perturbative and nonperturbative. They were estimated as sufficiently large—about 10%—in the papers [19,20]. More detailed calculations, taking into account effects due to hard gluon emission, estimate the corrections at the 3–4% level [21]. The nonperturbative corrections resulting from the soft gluon emission by the c-quark loop in the effective operator O_2 turn out to be about 1%, while nonperturbative contributions from the annihilation diagrams and other operators are of the same order or smaller, as was estimated in the detailed light-cone sum rule method calculations [22]. Thus the total deviation of the right-to-left photons ratio from zero not exceeding 5% in the SM seems to be based on rather solid theoretical grounds. Larger values, if observed, have to be interpreted as a manifestation of NP.

The decay process $B \to Y\gamma$ receives, besides short-distance contributions described by (10), also long-distance contributions. The structure and relative role of the latter is rather complex and was analyzed in details in [23]. There are two outcomes of this analysis to be mentioned here. First, the short-distance term is always leading, despite the fact that the relative magnitude of the long-distance contributions can be sizeable. Second, and this is of prime importance for us, the dominant left-(right)-handedness of the emitted photon is not affected by the long-distance terms; in other words, the long-distance amplitudes for emission of the photon with the "wrong" polarization obey the same hierarchy with respect to the "right" ones as short-distance terms do. Since we concentrate in what follows on angular distributions and do not pretend to compute the absolute values of the branching ratios, we can safely assume that our strong amplitudes include both short- and long-distance contributions.

Summarizing the discussion above by performing the fitting procedure for particular partial decay width of the B^- meson with (8) one has to obtain $\lambda_\gamma = -1 + \xi^2$ in the SM, where the factor ξ not exceeding 3–5% takes into account all possible SM corrections for the right-handed photons admixture. On the other hand, there are NP scenarios where the suppression of the "wrong" helicity photon emission is absent. A good example are left-right symmetric models, in which the enhancement of the right-handed photons fraction is due to the $W_L - W_R$ mixing, and the chirality flip along the internal t-quark line in the loop leads to large factor m_t/m_b in the amplitude for producing right-handed photons. The predictions for the mixing-induced CP asymmetry in $B^0 \to K^*\gamma$ under the assumption that the radiative decay rates agree with the SM expectations are [3]

$$A(t) \approx \mp 2 \cdot (120\zeta) \cdot \sqrt{1 - (120\zeta)^2} \cos(2\beta) \sin(\Delta m_B t), \tag{11}$$

where $10° < \beta < 35°$ and the mixing parameter ζ is constrained by experimental observations $\zeta \leq 3 \cdot 10^{-3}$, so the asymmetry can be as large as 50%. It was shown that within the unconstrained minimal supersymmetric SM strong enhancement of order $m_{\tilde{g}}/m_b$ is possible due to chirality flip along the gluino line and left-right squark mixing. In this case the parameter λ_γ can take any value between -1 and 1 [24]. The model with anomalous right-handed top couplings [25] predicts sizeable contributions in A_R, resulting in the polarization parameter $-1 < \lambda_\gamma \lesssim -0.12$. In models with nonsupersymmetric extra dimensions there are also no reasons for a right-handed photon to be suppressed with respect to the left-handed one, so that λ_γ is close to zero and mixing-induced CP asymmetries are of the order of 1 [26].

III. ANGULAR DISTRIBUTION IN THE $B \to [\varphi K]^1 \gamma$ DECAY

As is well known in studies of many-body sequential decays, one can use either "helicity" or "tensor" formalism. Since our interest is focused on angular dependencies, the former approach is most suitable [27], see [28] for introduction and further references. An amplitude for the two-body decay $Y \to 1 + 2$ of the resonance of spin-parity J^P with the z-component M into particles 1 and 2 with spins and helicities s_1, λ_1 and s_2, λ_2, respectively, is given in terms of finite rotation of the z-axis to the axis of Y-decay:

$$A(Y \to 1 + 2) = N_J A^J_{\lambda_1\lambda_2} D^{J*}_{M\lambda}(\phi, \theta, 0), \tag{12}$$

where $\lambda = \lambda_1 - \lambda_2$, and the spherical angles (θ, ϕ) define the direction of the particle 1 momentum relative to the z-axis. All angular dependence is concentrated in the standard rotation matrix $D^j_{mm'}(\alpha, \beta, \gamma)$

$$D^j_{mm'}(\alpha, \beta, \gamma) = e^{-im\alpha} d^j_{mm'}(\beta) e^{-im\gamma}$$
$$d^j_{mm'}(\beta) = \langle jm| e^{-i\beta J_y} |jm'\rangle. \tag{13}$$

Let us define the coordinate systems and angles, related to the decay of interest. The z axis in the $[\varphi K]$ rest frame is antiparallel to the photon momentum: $\mathbf{p}_\gamma/|\mathbf{p}_\gamma| = -\mathbf{e}_z$. There is a plane defined by the 3-momenta of final state kaons

$$B \to [\varphi(-\mathbf{p}_3)K(\mathbf{p}_3)]\gamma \to K(\mathbf{p}_1)K(\mathbf{p}_2)K(\mathbf{p}_3)\gamma \quad (14)$$

in the $[\varphi K]$ rest frame $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = \mathbf{0}$. We define the z' axis as being orthogonal to this plane, while the y' axis is directed along $\mathbf{p}_3$:

$$\mathbf{e}_{z'} = [\mathbf{p}_1 \times \mathbf{p}_2]/|[\mathbf{p}_1 \times \mathbf{p}_2]|; \qquad \mathbf{e}_{y'} = \mathbf{p}_3/|\mathbf{p}_3| \quad (15)$$

and $\mathbf{e}_{x'} = [\mathbf{e}_{y'} \times \mathbf{e}_{z'}]$. Consequently, it is convenient to define the corresponding angles. First, we define the angle θ between the z and z' axes, i.e., between the photon momentum and normal to the $[\varphi K]$ decay plane. Second, there are polar and azimuthal angles (η, ϕ) for the vector $\mathbf{p}_3$ in the (x, y, z) frame. Note that these angles are defined in the $[\varphi K]$ rest frame and the angle ϕ is unobservable. In an analogous way in the φ rest frame one has $\varphi(\mathbf{p} = \mathbf{0}) \to K(\mathbf{p}_1^*)K(-\mathbf{p}_1^*)$, and the polar angle θ^* of the vector $\mathbf{p}_1^*$ is defined with respect to the y' axis, while the azimuthal angle ϕ^* measures the rotations of $\mathbf{p}_1^*$ around this axis. It is not independent and can be simply expressed as a function of η and θ. To summarize, we have

$$\begin{aligned} \cos\theta &= (\mathbf{e}_z \cdot \mathbf{e}_{z'}) \qquad & \cos\eta &= (\mathbf{e}_z \cdot \mathbf{e}_{y'}) \\ \cos\theta^* &= (\mathbf{e}_{y'} \cdot \mathbf{p}_1^*)/|\mathbf{p}_1^*| \qquad & \sin\phi^* &= \cos\theta/\sin\eta \end{aligned} \quad (16)$$

where $\mathbf{p}_1^*$ is the momentum of the first (taken as, e.g., the fastest) kaon resulting from φ decay in the φ rest frame. Figure 1 represents our momenta and angle conventions.

The amplitude A_M (where $M = 1$ corresponds to the right-handed photon and $M = -1$ to the left-handed one) for the sequential decay $Y \equiv [\varphi K] \to \{\varphi \to K(\mathbf{p}_1)K(\mathbf{p}_2)\}K(\mathbf{p}_3)$ is proportional to the standard convolution:

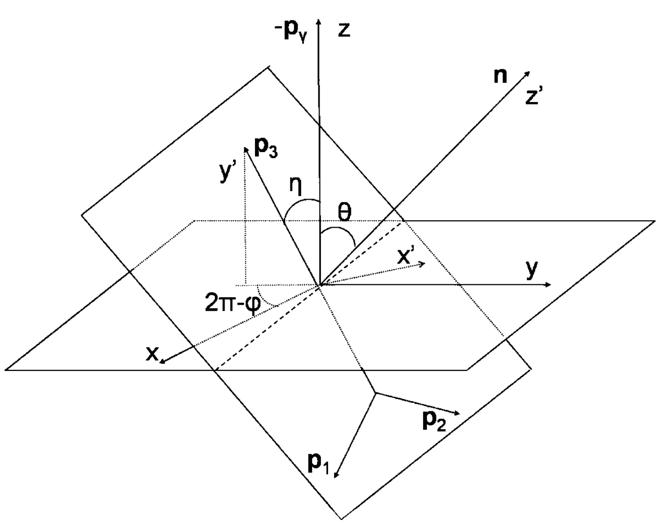

FIG. 1. Angle conventions for the decay $B^- \to [\varphi \to K^+(\mathbf{p}_1)K^-(\mathbf{p}_2)]K^-(\mathbf{p}_3)\gamma(\mathbf{p}_\gamma)$.

$$A_M \propto \sum_{\substack{J=1,2,\ldots \\ \lambda_\varphi=0,\pm1}} \langle K^+(\mathbf{p}_1)K^-(\mathbf{p}_2)|\Delta H_\varphi|\varphi(-\mathbf{p}_3, \lambda_\varphi)\rangle \times \langle \varphi(-\mathbf{p}_3, \lambda_\varphi)K^-(\mathbf{p}_3)|\Delta H_Y|Y^-(\mathbf{0}; J^P M)\rangle. \quad (17)$$

The first factor is the standard expression for p-wave decay of the vector φ-resonance into two K-mesons:

$$\langle K^+(\mathbf{p}_1)K^-(\mathbf{p}_2)|\Delta H_\varphi|\varphi(-\mathbf{p}_3, \lambda_\varphi)\rangle = \bar{a}_p \cdot D^{1*}_{\lambda_\varphi 0}(\phi^*, \pi - \theta^*, 0). \quad (18)$$

The second factor in the right-hand side of (17) can be expanded into the sum over the partial waves with each partial wave amplitude a_l entering with the factor

$$(2l + 1)^{1/2} \cdot (l, 0, 1, \lambda_\varphi|J, \lambda_\varphi) \cdot D^{J*}_{M\lambda_\varphi}(\phi, \pi - \eta, 0). \quad (19)$$

It describes the transition of the initial hadronic system at rest Y of spin J with z-component M, created after the photon emission in $B \to Y\gamma$ decays into a system of K and φ mesons with definite momenta $\mathbf{p}_3$ and $-\mathbf{p}_3$, respectively, and helicity $\lambda_\varphi = 0, \pm 1$ for φ-resonance.

It is not known *a priori* how many contributions are important in the sum over J in (17). Neither is it known how the partial waves' expansion saturates the sum (19). Contrary to the case of the $K\pi\pi$ channel studied in [15], we have here no independent information about the relative partial waves' phases. However, one has no reasons to expect strong coupling with the closest physical state in $[\varphi K]$ channel above the threshold $K_2(1770)J^P = 2^-$ since the latter dominantly couples to the $K\pi\pi$ mode. Because of the experimental kinematical cut on the maximum photon transverse momentum one is confined to the region of not-too-large invariant masses of Y. Therefore it seems reasonable to consider as the first approximation the simplest case with the only $J = 1$ term kept [29] in the sum (17). We sum over both parities of the intermediate state Y, which correspond to inclusion of $s-$ and $d-$ waves for the $J^P = 1^+$ state and $p-$ wave for the $J^P = 1^-$ state. Then summing over the intermediate φ-resonance polarizations and using the explicit expressions for D-functions, we obtain the differential decay rate in the following form:

$$\begin{aligned} \frac{d\Gamma}{d\Phi} \propto \Big[& c_1 \sin^2\theta^*(\cos^2\eta + \cos^2\phi^*\sin^2\eta) \\ & + c_2\sin^2\theta^*(\cos^2\eta + \sin^2\phi^*\sin^2\eta) \\ & + c_3\cos^2\theta^*\sin^2\eta + c_4 \sin 2\phi^*\sin^2\theta^*\sin^2\eta \\ & + \frac{1}{2}\sin 2\theta^* \sin 2\eta(c_5 \cos\phi^* + c_6 \sin\phi^*) \\ & + \lambda_\gamma(c_7\sin^2\theta^* \cos\eta + \sin 2\theta^* \sin\eta(c_8 \cos\phi^* \\ & + c_9 \sin\phi^*))\Big], \end{aligned} \quad (20)$$

where the phase-space volume is determined by the integration over the $m_{12}^2 = (p_1 + p_2)^2$ and four angles θ^*, ϕ^*, η,and unobservable angle ϕ: $d\Phi = dm_{12}^2 d\cos\eta d\phi d\cos\theta^* d\phi^*$. The terms proportional to $\cos m\eta$, $\sin m\eta$ for $m = 0$ and $m = 2$ have no sensitivity to the sign of λ_γ, while those for $m = 1$ do, and the contribution of these asymmetric terms to $d\Gamma$ is controlled by the hadron parameters c_7, c_8, c_9.

The notation goes as follows. The partial amplitude ratios are given by $a_1/a_0 = r_1 \exp(i\delta_1)$, $a_2/a_0 = r_2 \exp(i\delta_2)$. The coefficients read:

$$c_1 = r_1^2 \qquad c_2 = 1 + \frac{r_2^2}{2} + \sqrt{2} r_2 \cos\delta_2$$

$$c_3 = 1 + 2r_2^2 - 2\sqrt{2} r_2 \cos\delta \qquad c_4 = -\sqrt{\frac{3}{2}} r_1 \bar{\zeta}_+$$

$$c_5 = 1 - r_2^2 - \frac{r_2}{\sqrt{2}} \cos\delta_2 \qquad c_6 = \sqrt{\frac{3}{2}} r_1 \bar{\zeta}_-$$

$$c_7 = \sqrt{6} r_1 \zeta_+ \qquad c_8 = \sqrt{6} r_1 \zeta_- \qquad c_9 = \frac{3r_2}{\sqrt{2}} \sin\delta_2$$

with

$$\zeta_\pm = \cos\delta_1 \pm (\sqrt{2})^{\mp 1} r_2 \cos(\delta_1 - \delta_2)$$
$$\bar{\zeta}_\pm = \sin\delta_1 \pm (\sqrt{2})^{\mp 1} r_2 \sin(\delta_1 - \delta_2).$$

If we confine ourselves by the contribution of $J^P = 1^+$ states only, the result simplifies considerably, since $c_{1,4,6,7,8} = 0$ in this case and the only remaining asymmetric term takes the form:

$$|A_R|^2 - |A_L|^2 \propto c_9 \sin 2\theta^* \cos\theta. \qquad (21)$$

On the other hand, the p-wave contribution alone does not produce any asymmetry as can be deduced from general P-parity arguments and directly seen from (20), having no sensitivity to the photon polarization in this case.

The expression (20) is the main result of this paper. In principle one has nine independent angular structures and five unknowns for analysis ($r_{1,2}$, $\delta_{1,2}$ λ_γ). As a matter of principle it is perhaps more advantageous to fit unknown strong parameters $r_{1,2}$ and $\delta_{1,2}$ from the first six symmetric terms and then use the results to extract λ_γ from the last term. An alternative practical way is to perform integration $\int d\Gamma$ over some region of the Dalitz plot, as suggested in [15]. It is seen however that the possibility to proceed this way strongly depends on the actual value of the corresponding parameters. In particular, if the p-wave contribution is small and also $r_2 \sin\delta_2 \ll 1$, the discussed asymmetry will escape the detection.

Alternatively, one can try to fit the full differential rate over the maximal available part of the final state phase-space, using as a cross-check different constraints in the form of sum rules that the coefficients $c_1, .., c_9$ have to obey. Strong violation of such sum rules would indicate importance of higher-momenta terms in (17).

IV. CONCLUSION

We have applied the general method [15] of photon polarization parameter λ_γ measurement to the radiative $3 + 1$-body decay $B^- \to (\varphi \to K^+K^-)K^-\gamma$. Such measurement is among the hot topics of the LHCb rare decays physics program, and the detailed sensitivity studies are now in progress. The only chance for this method to provide sound experimental information on the photon polarization pattern is strong interference between the partial waves in the $[\varphi K]$-system with the latter being in the vector state. In fact, it is straightforward to proceed with more general calculations, taking into account higher momenta in the $[\varphi K]$ system. Leaving aside the cumbersome form of the results obtained, the interference pattern becomes so complicated in this case that any reasonable fitting procedure will certainly be impossible if higher momenta are indeed important in the decay of interest. Thus the method is rather restrictive from the parameter space point of view. On the other hand, if the approximations used will happen to be correct, the corresponding strong parameters can be determined using the same decay mode after the LHCb data will become available. The branching fraction for this mode is measured by the *BABAR* and Belle Collaborations, and this decay channel seems to be very promising for LHCb—it is expected, that one full year of LHC operation will give about 7000 selected $B \to \varphi K\gamma$ events [30], to be compared with only ~230 events obtained by Belle by the end 2008.

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