Baryons with $D5$-brane vertex and k-quark states

Kazuo Ghoroku[1,*] and Masafumi Ishihara[2,†]
[1]*Fukuoka Institute of Technology, Wajiro, Higashi-ku Fukuoka 811-0295, Japan*
[2]*Department of Physics, Kyushu University, Hakozaki, Higashi-ku Fukuoka 812-8581, Japan*
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We study baryons in $SU(N)$ gauge theories, according to the gauge/string correspondence based on IIB string theory. The $D5$-brane, in which N fundamental strings are dissolved as a color singlet, is introduced as the baryon vertex, and its configurations are studied. We find a point- and split-type of vertex. In the latter case, two cusps appears, and they are connected by a flux composed of dissolved fundamental strings with a definite tension. In both cases, N fundamental quarks are attached on the cusp(s) of the vertex to cancel the surface term. In the confining phase, we find that the quarks in the baryon feel the potential increasing linearly with the distance from the vertex. At finite temperature and in the deconfining phase, we find stable k-quark "baryons", which are constructed of an arbitrary number of $k(<N)$ quarks.

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I. INTRODUCTION

In the context of string/gauge theory correspondence [1–3], the large-N dynamics of baryons can be related to the $D5$-branes embedded in anti–de Sitter (AdS) space [4,5]. This point, in the context of the nonconfining $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, has been studied in [6,7] using the Born-Infeld approach for constructing strings out of D-branes [8,9]. The fundamental strings (F strings) are dissolved in the $D5$-brane in these approaches, and they could flow out as separated strings from the singular point(s) appearing on the surface of the $D5$-brane. So we can consider the system of the $D5$-brane and F strings, which are out of the $D5$-brane, as baryon. The approach in this direction has been applied to the confining gauge theories [10–12]. In this case, the no-force condition, a balance of the tensions of the $D5$-brane and the F strings, is imposed at their connected point(s) [13,14]. However, in [13,14], the configuration of $D5$-branes with dissolved F strings has not been considered. In [10], the $D5$ configuration has been taken into account, but the no-force condition has been used only in a restricted direction, so then the structure of the F strings is neglected. However, taking into account the structure of both the $D5$-brane and F strings is important to obtain possible baryon configurations as a system of both objects.

Here, we study the plausible baryon configurations by embedding these objects in two simple bulk backgrounds which correspond to confining and nonconfining $SU(N)$ gauge theories. In a confining theory, we can see two kinds of tensions in the baryon. One is the string tension of the F strings, which works on the quarks in the baryon. And the other is observed as the tension of the bundle composed of many F strings. The latter is found in a special configuration where the $D5$-brane is stretched.

In the deconfining and high temperature phase, which is expressed by the AdS Schwarzschild background, we could find the color nonsinglet baryon configuration constructed of the quarks with the number $k < N$. This configuration could be generated due to the fact that the $N - k$ F strings of the baryon could disappear into the horizon. As a result, the baryon is separated to free $N - k$ quarks and the remaining k quarks connected to the $D5$ baryon vertex as k-quark baryon.

The AdS_5 background expresses also the deconfining phase, so the k-string state has been found also in this theory with a constraint for the number k as $5N/8 < k < N$ in [13,14]. In these approaches, any configuration of the $D5$ vertex has not been considered as a solution of the brane action. In our case, however, such structures are taken into account; then the lower bound for k is removed as a result. Because of the geometrical freedom of the brane and the F strings, any k-quark baryon is possible.

In Sec. II we give our model and $D5$-brane action with a nontrivial $U(1)$ gauge field which represents the dissolved F strings. And the equations of motion for $D5$-branes are given. In Sec. III, we give a point vertex solution and study possible baryon configurations in the confining phase. We show the linear relation of the baryon mass and the distance of a quark in the baryon from the vertex. In Sec. IV, the split vertex solutions are studied. This solution is constructed by the bundle of F-string flux with a definite tension between two cusps on $\mathbf{S}^5$. And we estimate the tension of this flux. In Sec. V, we study the baryon in the deconfinement phase at finite temperature, and stable color nonsinglet baryons are shown. And in the final section, we summarize our results and discuss future directions.

II. SET OF THE MODEL

We derive the equations for a $D5$-brane embedded in the appropriate ten-dimensional background. As a specific supergravity background, we consider the following ten-

*gouroku@dontaku.fit.ac.jp
†masafumi@higgs.phys.kyushu-u.ac.jp

dimensional background in string frame given by a nontrivial dilaton Φ and axion χ [15,16],

$$ds_{10}^2 = e^{\Phi/2}\left(\frac{r^2}{R^2}A^2(r)\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2\right). \tag{1}$$

First, we consider the supersymmetric solution

$$A = 1, \qquad e^\Phi = 1 + \frac{q}{r^4}, \qquad \chi = -e^{-\Phi} + \chi_0, \tag{2}$$

with self-dual Ramond-Ramond field strength

$$G_{(5)} \equiv dC_{(4)} = 4R^4\left(\mathrm{vol}(S^5)d\theta_1 \wedge \ldots \wedge d\theta_5 - \frac{r^3}{R^8}dt \wedge \ldots \wedge dx_3 \wedge dr\right), \tag{3}$$

where $\mathrm{vol}(S^5) \equiv \sin^4\theta_1 \mathrm{vol}(S^4) \equiv \sin^4\theta_1 \sin^3\theta_2 \sin^2\theta_3 \sin\theta_4$. The solution (2) is dual to $N = 2$ super Yang-Mills theory with gauge condensate q and is chirally symmetric.

Here the baryon is constructed from the vertex and N fundamental strings. In the string theory, the vertex is considered as the $D5$-brane wrapped on an $\mathbf{S}^5$ on which N fundamental strings terminate and they are dissolved in it [4,5]. Then the $D5$-brane action is written by the Born-Infeld plus Chern-Simons term

$$S_{D5} = -T_5 \int d^6\xi e^{-\Phi}\sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab})} + T_5 \int (2\pi\alpha' F_{(2)} \wedge c_{(4)})_{0\ldots5}, \tag{4}$$
$$g_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu},$$
$$c_{a_1\ldots a_4} \equiv \partial_{a_1}X^{\mu_1}\ldots\partial_{a_4}X^{\mu_4}C_{\mu_1\ldots\mu_4},$$

where $T_5 = 1/(g_s(2\pi)^5 l_s{}^6)$ is the brane tension. The Born-Infeld term involves the induced metric g and the $U(1)$ world volume field strength $F_{(2)} = dA_{(1)}$. The second term is the Wess-Zumino coupling of the world volume gauge field, and it is also written as

$$S = -T_5 \int d^6\xi e^{-\Phi}\sqrt{-\det(g + F)} + T_5 \int A_{(1)} \wedge G_{(5)},$$

in terms of (the pullback of) the background five-form field strength $G_{(5)}$, which effectively endows the five-brane with a $U(1)$ charge proportional to the $\mathbf{S}^5$ solid angle that it spans. Namely,

$$\int_{S^4} d^4\theta \frac{\partial L}{\partial F_{(2)}} = \int_{S^4} d^4\theta \frac{\partial L}{\partial B_{(2)}} = kT_F, \tag{5}$$

where $T_F = 1/(2\pi\alpha')$ and $k \subset Z$.

Then we give the embedded configuration of the $D5$-brane in the given ten-dimensional background. At first, we fix its world volume as $\xi^a = (t, \theta, \theta_2, \ldots, \theta_5)$. For simplicity we restrict our attention to $SO(5)$ symmetric configurations of the form $r(\theta)$, $x(\theta)$, and $A_t(\theta)$ (with all other fields set to zero), where θ is the polar angle in spherical coordinates. The action then simplifies to

$$S = T_5\Omega_4 R^4 \int dt d\theta \times \sin^4\theta\{-\sqrt{e^\Phi(r^2 + r'^2 + (r/R)^4 x'^2) - F_{\theta t}^2} + 4A_t\}, \tag{6}$$

where $\Omega_4 = 8\pi^2/3$ is the volume of the unit four-sphere.

The gauge field equation of motion following from this action reads

$$\partial_\theta D = -4\sin^4\theta,$$

where the dimensionless displacement is defined as the variation of the action with respect to $E = F_{t\theta}$, namely $D = \delta\tilde{S}/\delta F_{t\theta}$ and $\tilde{S} = S/T_5\Omega_4 R^4$. The solution to this equation is

$$D \equiv D(\nu, \theta) = [\tfrac{3}{2}(\nu\pi - \theta) + \tfrac{3}{2}\sin\theta\cos\theta + \sin^3\theta\cos\theta]. \tag{7}$$

Here, the integration constant ν is expressed as $0 \le \nu = k/N \le 1$, where k denotes the number of Born-Infeld strings emerging from one of the poles of the $\mathbf{S}^5$. Next, it is convenient to eliminate the gauge field in favor of D and Legendre transform the original Lagrangian to obtain an energy functional of the embedding coordinate only:

$$U = \frac{N}{3\pi^2\alpha'}\int d\theta e^{\Phi/2}\sqrt{r^2 + r'^2 + (r/R)^4 x'^2}\sqrt{V_\nu(\theta)}, \tag{8}$$

$$V_\nu(\theta) = D(\nu, \theta)^2 + \sin^8\theta, \tag{9}$$

where we used $T_5\Omega_4 R^4 = N/(3\pi^2\alpha')$. Using this expression (8), we solve the $D5$-brane configuration in the following.

III. THE POINT VERTEX

In this section we study solutions which correspond to a baryon localized at a particular point in our four-dimensional (4D) space-time. To localize the vertex in x, we set $x' = 0$, and the equation of motion for $r(\theta)$ is obtained from (8) as

$$\partial_\theta\left(\frac{r'}{\sqrt{r^2 + (r')^2}}\sqrt{V_\nu(\theta)}\right) - \left(1 + \frac{r}{2}\partial_r\Phi\right) \times \frac{r}{\sqrt{r^2 + (r')^2}}\sqrt{V_\nu(\theta)} = 0. \tag{10}$$

For our background solution (2), it is rewritten as

$$\partial_\theta\left(\frac{r'}{\sqrt{r^2 + (r')^2}}\sqrt{V_\nu(\theta)}\right) - \frac{1 - q/r^4}{1 + q/r^4}\frac{r}{\sqrt{r^2 + (r')^2}}\sqrt{V_\nu(\theta)} = 0. \tag{11}$$

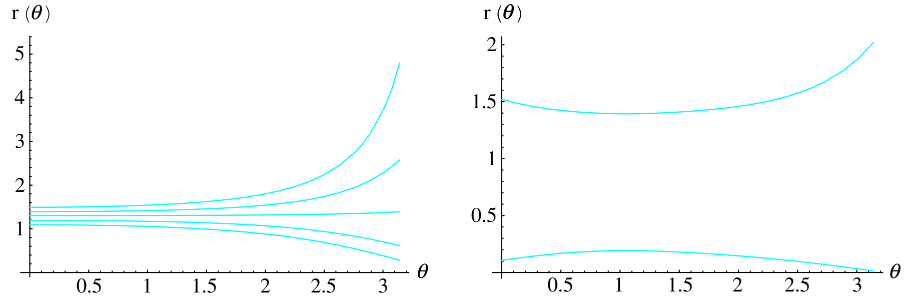

FIG. 1 (color online). Family of solutions for $q = 2.8$ and $r(0) = q^{1/4} + \epsilon$ with various ϵ, $\epsilon = 0.2, 0.1, 0.01, -0.2, -0.3$, for the five curves from the upper to the lower one, respectively. The right-hand side shows the solutions for $\nu = 0.2$ and $r(\theta_c) = q^{1/4} + 0.1$ (the upper) and $q^{1/4} - 1.1$ (the lower).

In the case of $q = 0$, the background reduces to $AdS_5 \times S^5$, where the equation is analytically solved and well studied in [7]. In this case, however, the quarks are not confined. In order to see the baryon configuration, we should consider the case of non-Bogomol'nyi-Prasad-Sommerfield monopoles confinement phase, which is realized in our model for nonzero $q > 0$. The parameter q represents the gauge condensate in the 4D Yang-Mills theory and provides the tension of the meson states. So we could expect to be able to extract the characteristic properties of the baryon in the confinement phase.

However, for $q \neq 0$, it is difficult to find an analytical solution, so we try to solve the above equation numerically. The "potential" $V(\theta)$ has three extremum points. In solving the equation, we impose the boundary condition at one of these points, $\theta = \theta_c$, which is the minimum of $V_\nu(\theta)$ and is given by the solution of

$$\pi\nu = \theta_c - \sin\theta_c \cos\theta_c. \tag{12}$$

Then the boundary conditions are set as

$$r(\theta_c) = r_0, \qquad \frac{\partial r(\theta_c)}{\partial\theta} = 0, \tag{13}$$

where r_0 is a parameter which determines the configuration of the $D5$-brane. The typical solutions are shown in Fig. 1. These solutions have cusps at the pole points, $\theta = \pi$ and $\theta = 0$ for $\nu = 0.2$ and only at $\theta = \pi$ for $\nu = 0$.

For simplicity, we consider here the case of $\nu = 0$. In this case, $\theta_c = 0$ and the solution $r(\theta)$ is smooth at $\theta = 0$ but it has a cusp at $\theta = \pi$ on S^5. The solutions depend on q and r_0 through the ratio $\zeta = r_0/q^{1/4}$, and they are classified to two types, (A) and (B), by ζ as follows:

(A) $\zeta \geq 1$:

(i) For $\zeta \gg 1$, the solution is near the Bogomol'nyi-Prasad-Sommerfield monopoles "tube" solution [7], which arrives at $\theta = \pi$ only at $r = \infty$.

(ii) When ζ decreases toward $\zeta = 1$, the solution begins to tilt and crosses the symmetry axis, $\theta = \pi$, at a finite value of $r = r_c$. And $r'(\pi)$ is finite; then it forms a cusp.

(iii) And we notice that $r(\theta) = q^{1/4}$ at $\zeta = 1$, and this constant value of r is an exact solution to the equations of motion. Its shape is completely spherical. This is the only one allowable constant solution.
For this solution, we obtain from (8) the following:

$$U = \frac{N}{3\pi^2\alpha'} 2^{1/2} q^{1/4} j(0), \tag{14}$$

where

$$j(\nu) = \int_0^\pi \sqrt{V_\nu(\theta)}$$

and it is estimated as $j(0) = 10.67$ for $\nu = 0$. This represents the vertex energy for the constant solution of r. A similar solution has been given in [13,14] for the AdS_5 background. In our case, however, this solution corresponds to a special one in our nonconformal background. Actually, this solution disappears into the horizon in the AdS_5 limit of $q \to 0$.

(B) $0 < \zeta < 1$: In this case, the term $1 - q/r^4$ in the second term of Eq. (11) turns to negative, and then r decreases with θ. This implies that the north and south poles on S^5 are opposite to the one of the solution for $\zeta > 1$, and then the direction of the force coming from the $D5$ tension at the cusp is also reversed. Further, the minimum, $r(\pi)$, approaches zero for $r_0 \to 0$, but the brane could not arrive at $r = 0$ since the action diverges there despite how small r_0 is. In other words, an infinite energy is necessary to realize $r(\pi) = 0$. In other words, the $D5$ vertex is never absorbed in the horizon due to the confinement force, and ζ is restricted as $\zeta > 0$.

A. Baryon configuration and no-force condition

The schematic configurations of the baryons formed from the two types of vertex, (A) and (B), are shown in Fig. 2. In the present case, the fundamental strings, which are dissolved in the $D5$-brane, should flow out from the cusp at $\theta = \pi$. The $D5$-brane configuration is singular at this point. This singularity is cancelled out by the boundary

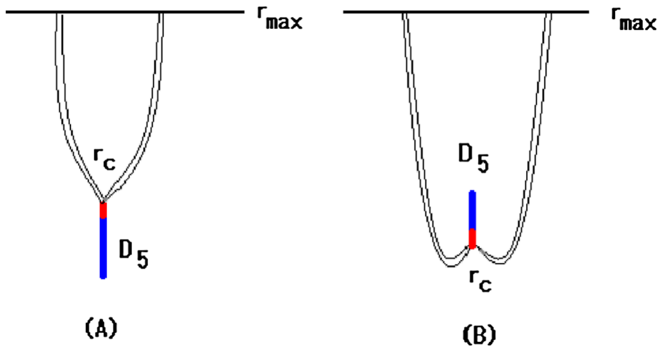

FIG. 2 (color online). Point vertex baryon. For the solutions of (A) $r_c > r_0$ and (B) $r_c < r_0$.

term of the fundamental string stretching from this point. This cancellation is equivalent to the so called no-force condition, namely, the cancellation of the tension forces among the fundamental strings and the $D5$-brane at the cusp point. It is possible to consider various configurations of the $D5$-brane and strings which satisfy the no-force condition. They are supposed as the various possible flowings of the fundamental strings from the $D5$-brane.

Then the fundamental strings coming out from the $D5$-brane could stretch separately to any direction when they are allowed dynamically. The dynamical conditions to be considered are separated into two parts, (a) the equations of motion of F strings in the given background and (b) the no-force conditions at the cusp point.

As for the condition (b), when the stretching direction of the F strings is restricted to the r direction only as considered in [10], then the resultant baryon configuration is reduced to the supersymmetric flux tube of $D5$ since the tension force of the F strings is stronger than the one of the $D5$ vertex. However, we could consider other configurations of F strings which spread also to the x direction. As a result, the force in the r direction is weakened, and the configurations given in Fig. 2 become possible.

Such a string configuration has been considered in [13], but the authors in [13] did not take into account the structure of the $D5$-brane except for the constant r configuration as mentioned above. In the present model, more general configurations than the one considered in [13] are studied.

Here we suppose that the two end points of the F string are connected to the cusp $r = r_c$ of the $D5$ vertex and the other probe brane, for example, the $D7$ probe brane, which corresponds to the brane providing the flavor quarks and is put at an appropriate position, $r = r_{\max}(>r_c)$. Here $r_{\max}$ plays a role in the cutoff of r, and we do not give a $D7$-brane configuration with the attached strings. This problem is postponed to future works. The quark mass in this case is given approximately by $T_F(r_{\max} - r_c)$ for the AdS_5 background.

Now we turn to the no-force condition at $r = r_c$. In the r direction, the tensions of the fundamental strings and the one of the $D5$-brane should be balanced. As for the x direction, the balance should be realized by the F strings themselves. For the $D5$-brane, the tension in the r direction is estimated in terms of U given by (8) under the variation $r_c \to r_c + \delta r_c$ [10]. It can be seen from (8), by using the Euler-Lagrange equation, that the energy of the brane changes only by a surface term at r_c,

$$\frac{\partial U}{\partial r_c} = N T_F e^{\Phi/2} \frac{r'_c}{\sqrt{r'^2_c + r^2_c}}, \tag{15}$$

where $r'_c = \partial_\theta r|_{\theta=\pi}$. From this equation, we can see that the direction of the force is reversed at the point $\zeta = 1$ when ζ changes from (A) $\zeta > 1$ to (B) $\zeta < 1$ since the sign of r'_c changes. This behavior is important in our model as found below.

As for the fundamental string, which extends to the x direction, its action is written as

$$S_F = -\frac{1}{2\pi\alpha'} \int dt dx e^{\Phi/2} \sqrt{r_x^2 + (r/R)^4}, \tag{16}$$

where $r_x = \partial r/\partial x$ and the world sheet coordinates are set as (t, x). Then, its energy and r-directed tension at the point $r = r_c$ are obtained as follows:

$$U_F = T_F \int dx e^{\Phi/2} \sqrt{r_x^2 + (r/R)^4}, \tag{17}$$

$$\frac{\partial U_F}{\partial r_c} = T_F e^{\Phi/2} \frac{r_x}{\sqrt{r_x^2 + (r_c/R)^4}}. \tag{18}$$

For the remaining $N - 1$ strings, it is possible to take their world sheet coordinates in a different directions from the x direction. Here, for simplicity, we assume that they extend in the same plane, for example, the xy plane, and their forces make a valence with the same tension in this plane. The quarks are put on a circle whose center is the $D5$ vertex. Then, the forces of the strings in the r direction are added up, and the value is N times of (18), and it should keep a valence with the one of the $D5$-brane. We notice that the direction of this tension force is also reversed at $\zeta = 1$.

Thus we find the following no-force condition as discussed in [13,14] with a slightly different setting,

$$r_x^{(c)} = r'_c \frac{r_c}{R^2}, \tag{19}$$

at $r = r_c$, where $r_x^{(c)}$ denotes the value of r_x at $r = r_c$. Here the sign of $r_x^{(c)}$ and r'_c should be the same.

B. Baryon mass and distance between quark and vertex

Thus we find the baryon configuration as depicted schematically in Fig. 2. For small baryon mass, its configuration is expressed by figure (A), and the configuration changes to (B) when the mass increases. In configuration (B), the fundamental string could become long with increasing baryon mass. As a result, any large baryon mass is realized by configuration (B).

The energy (or the mass) of the baryon is therefore given as follows:

$$E = NE_F + E_J^{(S)}, \quad (20)$$

where E_F is the F-string part and $E_J^{(S)}$ represents the $D5$-brane part, i.e. the baryon vertex. The latter is obtained from (8) by setting as $x' = 0$,

$$E_J^{(S)} = \frac{N}{3\pi^2\alpha'}\int_0^\pi d\theta e^{\Phi/2}(r^2 + r'^2)^{1/2}\sqrt{V_{\nu=0}(\theta)}. \quad (21)$$

While the baryon vertex is seen as a point in our 4D spacetime, it has a structure in inner space and has a finite value of $E_J^{(S)}$.

Although r_0 is not equal to $q^{1/4}$ in general, the rough estimation of this energy is given analytically in this limit of $r_0 = q^{1/4}$ or $\zeta = 1$. The $D5$-brane is squashed, at this point, to a point in the r direction. Then we obtain

$$E_J^{(S)}|_{r_0=q^{1/4}} = \frac{N}{3\pi^2\alpha'}2^{1/2}q^{1/4}j(0), \quad (22)$$

where $j(0) = 10.67$ as given above. Since q is written as $q = \lambda\langle F_{\mu\nu}^2\rangle\alpha'^4$ [15], this vertex energy increases with the 't Hooft coupling like $\lambda^{1/4}$ when $\langle F_{\mu\nu}^2\rangle$ is fixed. On the other hand, we find that the F-string tension is independent of λ as seen below. Then, the vertex energy is expected to become large and the main part of the baryon mass at large 't Hooft coupling.

Next, we turn to the energy of the F-string part. From (16), we can set the following constant h:

$$e^{\Phi/2}\frac{r^4}{R^4\sqrt{r_x^2 + (r/R)^4}} = h. \quad (23)$$

Then, by eliminating r_x in terms of the above equation with a constant h, we get the formula of the distance L between the quark and the vertex and the string energy E as

$$L_{q-v} = R^2\int_{r_c}^{r_{\max}} dr\frac{1}{r^2\sqrt{e^\Phi r^4/(h^2R^4) - 1}}, \quad (24)$$

$$E_F = T_F\int_{r_c}^{r_{\max}} dr\frac{e^{\Phi/2}}{\sqrt{1 - h^2R^4/(e^\Phi r^4)}}. \quad (25)$$

These formulas are equivalent to the one of the mesons made of quark and antiquark except for the lower bound of the r integration, which is given here as the $D5$ cusp point r_c. And the no-force condition (19) is imposed at this point.

Eqs. (24) and (25) are evaluated for the solutions (A) and (B) separately since the string shapes are different in the two cases. First, we consider the solution (B). In this case, r_x is negative at r_c; then r decreases with increasing $|x - x(r_c)|$ after it departs from the cusp point $x(r_c)$, but the string can never reach the horizon $r = 0$ since the action diverges at this point. Thus r reaches the minimum $r(\equiv r_{\min})$ at some $x = x_0$, where $r_x = 0$; then it begins to increase toward $r_{\max}$ (see Fig. 2). The shape of this F string is determined as the extremum of U_F; namely, it is given as a solution of the equation of motion derived from (17). The total configuration of the baryon made of F strings and the $D5$ vertex is controlled by the one parameter r_0. For a given r_0, all the values of $r'(\pi)$, r_c, $r_{\min}$, and $r_x|_{r_c}$ are determined, so the energy of the baryon is also determined.

Therefore, when a value of $r_0(<q^{1/4})$ is fixed, it is convenient to separate the F string to the region of (i) $r_{\min} < r < r_{\max}$ and (ii) $r_{\min} < r < r_c$. Then the energy can be written as

$$E_F^{(\mathrm{B})} = E_F^{(\mathrm{B}_i)} + E_F^{(\mathrm{B}_{ii})}. \quad (26)$$

The first term corresponds to the half of the meson configuration made of quark and antiquark. When the energy becomes large or the F string grows long, $E_F^{(\mathrm{B}_i)}$ is approximated by the following formula [16]:

$$E_F^{(\mathrm{B}_i)} = m_q^{\mathrm{eff}} + \tau_M\frac{L_{q-\bar{q}}}{2},$$

where

$$\tau_M = T_F\sqrt{\frac{q}{R^4}}, \quad (27)$$

$$m_q^{\mathrm{eff}} = T_F\int_{r_{\min}}^{r_{\max}} dre^{\Phi/2}. \quad (28)$$

Here m_q^{eff} expresses effective quark mass in the thermal medium. In this calculation, the constant h is taken as

$$h = e^{\Phi(r_{\min})/2}\left(\frac{r_{\min}}{R}\right)^2 = e^{\Phi(r_c)/2}\left(\frac{r_c}{R}\right)^2\frac{1}{\sqrt{(r'(r_c)/r_c)^2 + 1}}. \quad (29)$$

Because of this boundary condition, $r_{\min}$ is determined by using r_c, then by r_0 since r_c is determined by r_0 as mentioned above. The remaining part of the F string is obtained as

$$E_F^{(\mathrm{B}_{ii})} = T_F\int_{r_{\min}}^{r_c} dr\frac{e^{\Phi/2}}{\sqrt{1 - h^2R^4/(e^\Phi r^4)}}. \quad (30)$$

Next, we turn to the solution (A), whose configuration is shown in (A) in Fig. 2. In this case, since $r'(\pi) > 0$, then the r of the F string increases from r_c toward $r_{\max}$ monotonically. There is no point of $r_x = 0$. So the configuration of the F string is obtained in terms of (24) and (25) by the setting of the lower bound of r integration as r_c and with the boundary condition at this point,

$$h = e^{\Phi(r_c)/2}\left(\frac{r_c}{R}\right)^2\frac{1}{\sqrt{(r'(r_c)/r_c)^2 + 1}}. \quad (31)$$

In this case, the energy of the F string is given as

$$E_F^{(A)} = T_F \int_{r_c}^{r_{\max}} dr \frac{e^{\Phi/2}}{\sqrt{1 - h^2R^4/(e^\Phi r^4)}}. \tag{32}$$

The above h could take its minimum, $e^{\Phi(r_c)/2}(\frac{r_c}{R})^2$, at $r'(r_c) = 0$, which is realized at $r_0 = q^{1/4}$, and the distance L_{q-v} becomes the maximum in the configuration (A). Actually, we can estimate the maximum of L_{q-v} as

$$L_{q-v}^{\max} = R^2\left(\frac{4\pi^2}{q}\right)^{1/4}\frac{\Gamma(3/4)}{\Gamma(1/4)}, \tag{33}$$

where we take $r_{\max} = \infty$. So the baryon is expressed by the solution (A) for $0 < L_{q-v} \leq L_{q-v}^{\max}$ and by the (B) for $L_{q-v}^{\max} < L_{q-v}$.

Now we can calculate the total energy of the baryon E versus L_{q-v}. The energy is given as

$$E = NE_F + E_{D5}, \tag{34}$$

where $E_F = E_F^{(A)}$ or $E_F^{(B)}$ for short or long L_{q-v}, respectively. Assuming that all fundamental quarks are put at the same distance from the vertex, E is numerically estimated as a function of L_{q-v}. An example is shown in Fig. 3 for $r_{\max} = 20$.

In this figure, the value of E at $L = 0$ shows the value of $E_J^{(S)}$ with $r_c = r_{\max}$, namely, the pure $D5$-brane energy. When L begins to increase, $E_J^{(S)}$ decreases rapidly with L and approaches a constant, and E_F becomes dominant. It is expressed by $E_F = E_F^{(A)}$ for small L, in the region of $L \leq 0.7$, and by $E_F = E_F^{(B)}$ for $L \geq 0.7$ in the present case. We can see for large L that E increases with L_{q-v} linearly with the tension,

$$N\tau_M/2, \tag{35}$$

where τ_M is given in (27). This tension is equal to the sum of the one of the independent F strings.

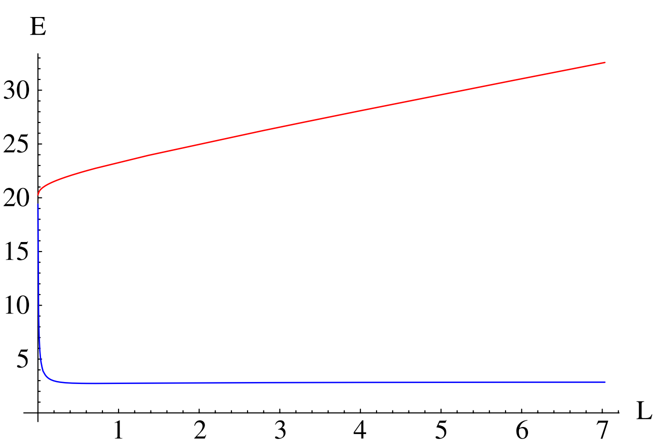

FIG. 3 (color online). EL_{q-v} plots for $q = 2.0$, $R = 1$, $r_{\max} = 20$. The upper (lower) curve shows the baryon energy E (the vertex energy E_{D5}).

IV. SPLIT VERTICES

The equations of motion for $D5$ embedding are solved here by adding the freedom of $x(\theta)$ without the restriction $x' = 0$. In this case, it is convenient to use a parametric Hamiltonian formalism as in [10].

First we rewrite the energy (8) in terms of a general world volume parameter s defined by functions $\theta = \theta(s)$, $r = r(s)$, $x = x(s)$ as

$$U = \frac{N}{3\pi^2\alpha'}\int ds e^{\Phi/2}\sqrt{r^2\dot{\theta}^2 + \dot{r}^2 + (r/R)^4\dot{x}^2}\sqrt{V_\nu(\theta)}, \tag{36}$$

where dots denote derivatives with respect to s. The momenta conjugate to r, θ, and x are

$$p_r = \dot{r}\Delta, \qquad p_\theta = r^2\dot{\theta}\Delta, \qquad p_x = (r/R)^4\dot{x}\Delta,$$
$$\Delta = e^{\Phi/2}\frac{\sqrt{V_\nu(\theta)}}{\sqrt{r^2\dot{\theta}^2 + \dot{r}^2 + (r/R)^4\dot{x}^2}}, \tag{37}$$

since the Hamiltonian that follows from the action (36) vanishes identically due to reparametrization invariance in s. Then we consider the following identity

$$2\tilde{H} = p_r^2 + \frac{p_\theta^2}{r^2} + \frac{R^4}{r^4}p_x^2 - (V_\nu(\theta))e^\Phi = 0. \tag{38}$$

This constraint can be taken as the new Hamiltonian in order to get the following canonical equations of motion:

$$\dot{r} = p_r, \qquad \dot{p}_r = \frac{2}{r^5}p_x^2R^4 + \frac{p_\theta^2}{r^3} + \frac{1}{2}(V_\nu(\theta))e^\Phi\partial_r\Phi, \tag{39}$$

$$\dot{\theta} = \frac{p_\theta}{r^2}, \qquad \dot{p}_\theta = -6\sin^4\theta(\pi\nu - \theta + \sin\theta\cos\theta)e^\Phi, \tag{40}$$

$$\dot{x} = \frac{R^4}{r^4}p_x, \qquad \dot{p}_x = 0. \tag{41}$$

In solving these equations, the initial conditions should be chosen such that $\tilde{H} = 0$.

We now solve these equations numerically to obtain a configuration spreading in the x direction with $p_x \neq 0$ (i.e. $x' \neq 0$). We find two types of configurations of the $D5$-brane, the U shaped and the cap ($\bigcap$) shaped one. Their example solutions are shown in Fig. 4. The U shaped one is obtained for large $r(0)$ or small mass baryon, and the cup shaped is obtained when $r(0)$ or baryon mass increases.

Since r and $\partial r/\partial\theta$ are finite at the end points of these configurations, the end points of both sides are the cusps. Then the baryon configurations given here are split into two distinct cusps, which are connected to νN and $(1 - \nu)N$ quarks, respectively, for the case of a given value of ν. We can see that the cusps of the U shaped configuration are the type (A) which is given in the previous section, and

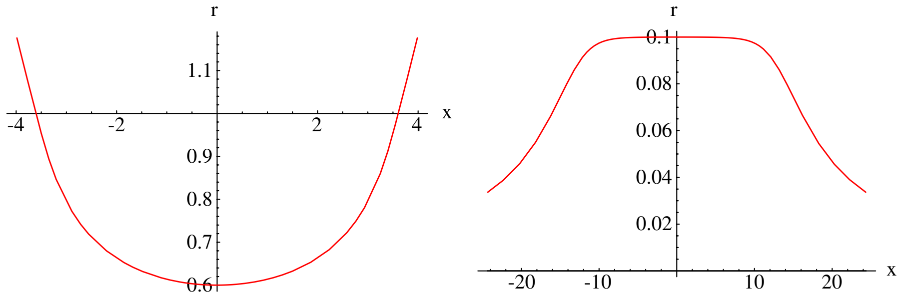

FIG. 4 (color online). The solution with cusps for $R = 1$, $q = 2$, and $\nu = 0.5$. The left figure is for $r(\theta = \pi/2) = 0.6$ and $r(\pi) = r(0) = 0.490\,507$. The right one is for $r(\pi/2) = 0.1$ and $r(\pi) = r(0) = 0.033\,682\,2$.

type (B) cusps are seen for the cap shaped one. Then the quarks are attached as depicted in Fig. 5 by considering the no-force condition.

In both cases, the two cusps are connected by a confining flux tube of the gauge theory. We estimate the tension of this flux tube for a tuned configuration shown in Fig. 6 as an example for the U shaped solution.

This tuned U shaped $D5$-brane is very similar to the quark and antiquark meson state configuration which is obtained by the fundamental string action. However the present case is for the $D5$-brane tube, so the tension of this tube is not equal to the one of the meson (27). In Fig. 6 the tube sits almost at a constant $\theta \simeq \theta_c$ where $\dot{p}_\theta \simeq 0$. This behavior is understood from the fact that θ_c, which is given above by the solution (12), is the extremum of the potential $V_0(\theta)$.

Then we can estimate the tension (τ_v) of the flux tube between the two cusp points as follows. From the above solutions for $r(x)$ and $x(\theta)$, we can approximate $\theta = \theta_c$ and $x'(\theta_c) \gg 1$ in the flux region (see the right-hand side of Fig. 6). Then the $D5$-brane energy (8) can be approximated as follows:

$$U_{\rm flux} = \frac{N}{3\pi^2\alpha'}\sin^3\theta_c \int dx e^{\Phi/2}\sqrt{r_x^2 + (r/R)^4}, \tag{42}$$

$$= \frac{2N}{3\pi}\sin^3\theta_c U_F, \tag{43}$$

where U_F is the energy of the fundamental string given by (17). This implies the linear relation of the energy of the flux and its length L_{vv}. According to the method given for the meson case, we obtain

$$\tau_v = \frac{2N}{3\pi}\sin^3\theta_c \tau_M, \tag{44}$$

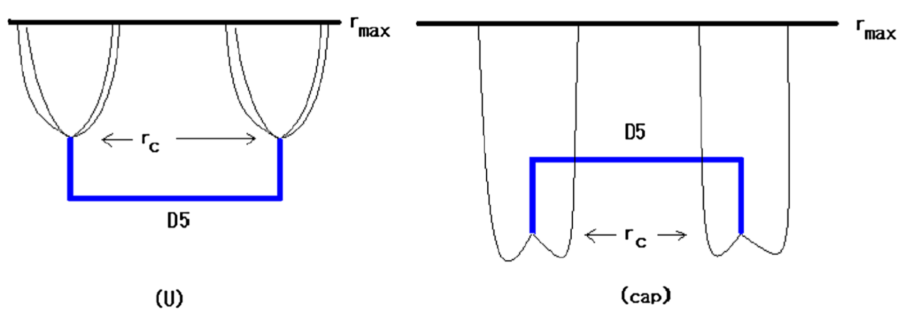

FIG. 5 (color online). Split vertex baryon. For the solutions of U shaped $r_c > r_0$ and cap shaped $r_c < r_0$.

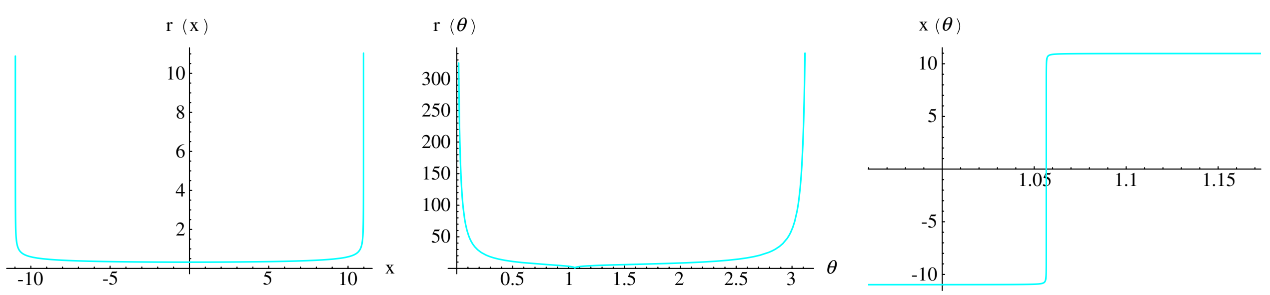

FIG. 6 (color online). The U shaped solution of the $D5$-brane for $R = 1$, $r_0 = 0.32$, $q = 3.5$, and $\nu = 0.2$.

where $\tau_M = T_F\sqrt{\frac{q}{R^4}}$ denotes the tension between the quark and antiquark which is given above by (27). The dependence of the flux tube tension on ν is seen from the factor $\sin^3\theta_c$. From Eq. (12), we find $\sin^3\theta_c \simeq 3\pi\nu/2$ for small ν. This implies that $\tau_v(\nu = 1/N)$ reduces to the same tension as the one of the meson state formed by quark and antiquark [16]. This result has a natural gauge theory interpretation. When one quark is pulled out from the $SU(N)$ baryon (a color singlet), the remaining $N - 1$ must be in the antifundamental representation of the gauge group. The flux tube extending between this bundle and a single quark should then have the same properties as the standard QCD string which connects a quark and an antiquark. On the other hand, for $\nu N \equiv k > 1$, the k quarks are bounded by the confining force since $\tau_v < k\tau_M$.

Here we comment on a similar formula of the flux tension, which has been obtained in another confining 4D Yang-Mills theory represented by the AdS black hole background [10,12]. It is written as

$$\sigma_B = \frac{8\pi}{27}N(g_{YM_4}^2 N)T^2\nu(1-\nu). \tag{45}$$

This has a similar ν dependence to our τ_v in (44), but σ_B differs from τ_v in two points. (i) While σ_B/N increases with the 't Hooft coupling λ linearly, our τ_v/N is independent of λ. (ii) The scale of our τ_v is determined by $\langle F_{\mu\nu}^2\rangle$ which is independent of the compact space scale, but σ_B is determined by the compact $U(1)$ scale T.

It would be useful to write the baryon energy or the mass formula for the split baryon by separating it into three parts as follows:

$$E_B^{\rm sp} = N(\nu E_F(r_{c_1}) + (1-\nu)E_F(r_{c_2})) + \tau_5 L_{vv} + E_J^{(\nu)} + E_J^{(1-\nu)}, \tag{46}$$

where L_{vv} is the flux length and $E_F(r_{c_i})$ are given by the same form as(25) for the two different cusp points r_{c_i}: $i = 1$ and 2. The last two terms describe the $D5$-brane parts between the cusps and the $D5$ flux; we call them "junction" here. They might be given as

$$E_J^{(\nu)} \simeq \frac{N}{3\pi^2\alpha'}\int_0^{\theta_c} d\theta e^{\Phi/2}(r^2 + r'^2)^{1/2}\sqrt{V_\nu(\theta)}, \tag{47}$$

$$E_J^{(1-\nu)} \simeq \frac{N}{3\pi^2\alpha'}\int_{\theta_c}^{\pi} d\theta e^{\Phi/2}(r^2 + r'^2)^{1/2}\sqrt{V_\nu(\theta)}. \tag{48}$$

In the symmetric case of $\nu = 1/2$, we can set $r_{c_1} = r_{c_2} = r_c$; then the above formulas are simplified as

$$E = NE_F(r_c) + \tau_5 L + 2E_J^{(1/2)}. \tag{49}$$

It is an interesting problem to compare the junction energy $2E_J^{(1/2)}$ with the point vertex case at $L = 0$ and at the same r_c. From the above formula, we obtain at r_c

$$2E_J^{(1/2)} = 2\frac{N}{3\pi^2\alpha'}(4q)^{1/4}j^{1/2}(\pi/2), \tag{50}$$

where

$$j^{1/2}(\pi/2) = \int_0^{\pi/2}\sqrt{V_{0.5}(\theta)} = 4.214. \tag{51}$$

Then the resultant junction energy is compared with $E_J^{(S)}$ given by (22), and we find

$$2E_J^{(1/2)} = \frac{2j^{1/2}(\pi/2)}{j(0)}E_J^{(S)} = 0.79E_J^{(S)}. \tag{52}$$

This implies that the split vertex is energetically favorable rather than the point vertex when the flux length L_{vv} is negligible.

It would be an interesting problem to consider a $D5$-brane configuration which splits to more numbers of fractions of string fluxes. An approach in this direction has been given by Imamura [11,12], and he has proposed the junction as the vertex part of the split flux of the fundamental strings. Imamura has estimated this part by a numerical calculation under appropriate assumptions.

In the present model, however, it means studying the three or more numbers of split $D5$ vertices. It would be a difficult task to obtain such a solution as a smooth numerical solution. In this sense, this problem is the open one here.

V. FINITE TEMPERATURE AND k-QUARK BARYON

Here, we consider the baryon configurations in the nonconfining Yang-Mills theory. Such a model is given by the AdS black hole solution, which represents the high temperature gauge theory. In our theory with dilaton, the corresponding background solution is given as [17]

$$ds_{10}^2 = e^{\Phi/2}\left(\frac{r^2}{R^2}[-f^2dt^2 + (dx^i)^2] + \frac{R^2}{r^2f^2}dr^2 + R^2d\Omega_5^2\right), \tag{53}$$

$$f = \sqrt{1 - \left(\frac{r_T}{r}\right)^4}, \qquad e^\Phi = 1 + \frac{q}{r_T^4}\log\left(\frac{1}{f^2}\right), \qquad \chi = -e^{-\Phi} + \chi_0. \tag{54}$$

The temperature (T) is denoted by $T = r_T/(\pi R^2)$. The world volume action of the $D5$-brane is rewritten by eliminating the $U(1)$ flux in terms of its equation of motion as above; then we get its energy as

$$U = \frac{N}{3\pi^2\alpha'}\int d\theta e^{\Phi/2}f\sqrt{r^2 + r'^2/f^2 + (r/R)^4x'^2}\sqrt{V_\nu(\theta)}, \tag{55}$$

where V is the same form as (9).

For simplicity, we concentrate on the point vertex configuration. Then, we set $x' = 0$ as above, and the equation of motion of $r(\theta)$ is obtained as follows:

$$\partial_\theta\left(\frac{r'}{\sqrt{r^2 f^2 + (r')^2}}\sqrt{V_\nu(\theta)}\right) - \frac{g}{\sqrt{r^2 f^2 + (r')^2}}\sqrt{V_\nu(\theta)} = 0, \tag{56}$$

$$g = \tfrac{1}{2e^\Phi}\partial_r(e^\Phi r^2 f^2). \tag{57}$$

In this case also, we find the two types of solutions (A) and (B) which have been given in Sec. III for the $T = 0$ confinement phase. However, in the present finite T case, the theory is in the quark deconfinement phase. So free F strings could exist, and these F strings touch on the horizon r_T. As a result, $N - k$ quarks disappear, and we find the k-quark baryon. This implies that we could find the color nonsinglet baryons as depicted in Fig. 7. This baryon is therefore constructed from $k(<N_c)$ quarks and the color singlet $D5$ vertex.

For fixed k, the solution (A) is obtained at small baryon mass or small r_0 where g is positive. When the mass becomes large, then g changes to negative value, and we find the solution (B). From Eq. (57), we can estimate the temperature (T_{c_1}) where the solution changes from (A) to (B) as

$$T < \frac{\gamma}{\pi R^2} q^{1/4} \equiv T_{c_1}, \qquad \gamma = 0.579, \tag{58}$$

where γ is obtained through a numerical analysis. In any case, we would find $k(<N_c)$-quark baryon at finite temperature before it resolves to independent quarks completely at high enough temperature. This point is different from the meson, which is broken from the meson to the quark and antiquark at high temperature and there is no middle state as in the case of baryons.

Next, we consider the no-force condition at the cusp of the k-quark baryon for the configurations given in Fig. 7. The tension of the $D5$-brane at r_c is given as

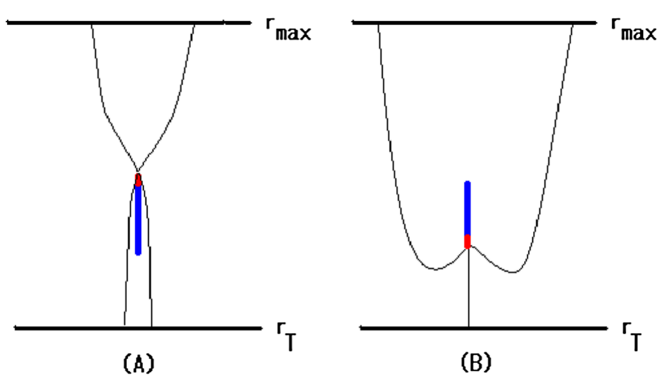

FIG. 7 (color online). k-quark baryons at finite temperature.

$$\frac{\partial U}{\partial r_c} = N T_F e^{\Phi/2} \frac{r_c'}{\sqrt{r_c'^2 + r_c^2 f(r_c)^2}}, \tag{59}$$

and for F strings as

$$\frac{\partial U_F}{\partial r_c} = T_F e^{\Phi/2} \frac{r_x^{(c)}}{\sqrt{r_x^{(c)2} + (r_c/R)^4 f(r_c)^2}}. \tag{60}$$

Here, for the $(N - k)$ strings ending on the horizon in (A) in Fig. 7, we consider the limit of the vertical lines as the one in (B). Then the no-force condition is obtained as

$$N \frac{r_c'}{\sqrt{r_c'^2 + r_c^2 f(r_c)^2}} + (N - k) = k \frac{r_x^{(c)}}{\sqrt{r_x^{(c)2} + (r_c/R)^4 f(r_c)^2}}. \tag{61}$$

Notice that $r_x^{(c)}$ and r_c' are positive for the solution (A) and negative for (B), respectively. Then, from $0 < k < N$, the no-force condition Eq. (61) is rewritten as

$$r_x^{(c)} > \frac{r_c}{R^2} r_c'. \tag{62}$$

When this is compared with the no-force condition (19) for the confinement phase, we can understand the above condition (62) as a reasonable one. The force of the remaining k quarks must cover the lacked part of $(N - k)$ quarks, so it must become larger.

Although other complicated configurations are possible, these baryon configurations would be observed just after the deconfinement transition occurred at high temperature, $T = T_c^{(\mathrm{dec})}$. The temperature increases above $T_c^{(\mathrm{dec})}$ and nears $q^{1/4}$, and then the type (B) k-string baryons will disappear first, and the type (A) configurations remain. When the temperature increases further, the cusp point arrives at r_{max} at the temperature T_{melt}. Since $r_c < r_{\mathrm{max}}$, all the k-quark (including N quarks) baryons should be collapsed to the free quarks in the medium of quark gluon plasma for $T > T_{\mathrm{melt}}$. In other words, k-quark baryons are observed in a range of the temperature,

$$T_c^{(\mathrm{dec})} < T < T_{\mathrm{melt}}.$$

We need some qualitative and phenomenological analyses to estimate this temperature range. This point is very interesting, but it is not discussed further here.

On the other hand, similar k-quark baryon configurations have been proposed with the restriction, $k \geq 5N/8$, for $\mathcal{N} = 4$ supersymmetric Yang-Mills theory [13,14]. In this theroy, the k-quark baryon is possible since the quarks are not confined. But, the authors in [13,14] have not considered the inner structure of the $D5$-brane with dissolved F strings. In our high temperature model, instead, the k-quark baryon is possible for any number of k with $k < N$. This difference between the condition given in [13,14] and ours could be reduced to the fact that the $D5$-brane structure is considered or not in deriving the no-force condition.

Actually, the no-force condition (61) is rewritten as

$$N(1 + Q_5) = k(1 + Q_F), \tag{63}$$

$$Q_5 = \frac{r_c'}{\sqrt{r_c'^2 + r_c^2 f(r_c)^2}}, \quad Q_F = \frac{r_x^{(c)}}{\sqrt{r_x^{(c)2} + (r_c/R)^4 f(r_c)^2}}. \tag{64}$$

Since $|Q_F| \leq 1$, we obtain

$$|Q_F| = \left|\frac{N(1 + Q_5) - k}{k}\right| \leq 1. \tag{65}$$

From this, we obtain the lower bound of k as

$$\frac{N}{2}(1 + Q_5) \leq k. \tag{66}$$

In the case of the structureless $D5$-brane, we obtain $Q_5 = 1/4$ [13,14]; then we find $5N/8 \leq k$. However, in our model, Q_5 could approach -1 since r_c could approach the horizon r_T. This implies the lower bound of k is zero.

But we must notice that we need infinite energy to realize the limit of $k = 0$ since the vertex energy U approaches infinity in this limit. When the energy of the baryon increases, its energy is used mainly to extend the length of the F strings, namely, the value of L_{q-v}. However, at finite temperature and in the deconfinement phase, L_{q-v} has its maximum value due to the color screening. In this sense, the lower bound of k would be small but finite for $T > T^{\mathrm{dec}}$. So we expect $k = 0$ in the limit of the $T \to T^{\mathrm{dec}}$. In our model, however, the deconfinement temperature is $T = 0$, so we could find $k = 0$ in the limit of $T \to 0$. In other words, the small-k state is seen just above T^{dec}, and the lower bound of k increases with T. So at a high enough temperature, which would be below T^{melt} mentioned above, on the other hand, we cannot see any $k(<N)$-quark baryon, and only the $k = N$ baryon is allowed there. So the lower bound of k is dependent on the temperature. It is important to ensure the details of this statement in a more realistic model with a finite T^{dec}. This point requires further study.

The estimations of the mass of these states should be the subject of further studies.

VI. SUMMARY AND DISCUSSION

The baryon configuration is studied based on the type IIB string theory by embedding the $D5$-brane as a probe in the background corresponding to two kinds of large-N Yang-Mills theories, the confining and the deconfining gauge theories. The $D5$-brane is needed as the baryon vertex of the quarks to make a color singlet, and the structure of the vertex is studied by solving the embedding equations for the fifth coordinate (r) and a direction (x) of our three-dimensional space.

As for the x direction, two typical configurations, the pointlike and the split vertex, are given. In the latter case, the quarks are separated into two clumps, and a color flux tube is running between them. As found in other confining models, the energy of such a configuration is proportional to the separation between the two quark bundles. And its tension (τ_v), the energy per unit length, is given by a similar formula given before for the confining theory. Estimating τ_v, we find a natural dependence on the color charges of the individual clumps and that this flux is interpreted as a bound state of a number of fundamental strings with a finite binding energy. The vertex energy is then given by the length of this flux times τ_v.

As for the r direction, we first show the r direction for the point vertex case. We find that the configurations are characterized by the relation of the position of the cusp(s) (r_c) and the extremum point of the $D5$-brane volume r_0, as (A) $r_c > r_0$ and (B) $r_c < r_0$. These configurations represent the same baryon at different energy (or mass) state. The configuration (A) corresponds to the low energy state, and (B) does for the high energy one. So the configuration of the fundamental strings in the rx plane changes when the baryon energy increases. Considering both configurations, the relation of the baryon energy (E), which is the sum of the $D5$-brane and fundamental strings, and the distance (L_{q-v}) between a fundamental quark in the baryon and the vertex is examined. And we find a linear relation of E and L_{q-v} at large L_{q-v}. The tension in this case is given by that of the meson times the quark number N since the vertex energy is almost constant at large L_{q-v}. In obtaining this relation, the two vertex configurations (A) and (B) mentioned above appear. The configuration (A) is dominant at small L_{q-v}, where the linear relation of E and L_{q-v} is not still seen. At an appropriate L_{q-v}, the vertex configuration changes from (A) to (B), and the linear relation appears.

In the case of the split vertex, the configuration should be characterized in the xr plane as the U shaped and the cap ($\bigcap$) shaped. The U shaped one has two type (A) cusps, and the cap shaped one has two type (B) cusps. The problem of adding the fundamental strings in this case could be solved by applying the results obtained in the point vertex case to each cusp of the split vertex. Then we would observe two kinds of the energy scale for the heavy baryon, τ_M and τ_v, for the split vertex baryon.

As an example of deconfining Yang-Mills theory, finite temperature theory is examined. At a high enough temperature, the baryon collapses to quarks completely. Before arriving at this temperature, we find that there is an intermediate temperature, where quarks are already not confined but they form a "baryon" state, which is composed of k quarks with $k < N$ and a color singlet vertex. So this is a color nonsinglet baryon. This baryon state would be made just after the phase transition of quark confinement and deconfinement. It is a very interesting question whether or not we could find this kind of baryon at a high temperature.

As the next step, we should introduce the probe brane of the quarks and solve its embedding problem consistently with our baryon configurations obtained here. Another important problem is to quantize the baryon system to obtain their mass spectrum. Some approaches in this direction have been seen [18–20]

ACKNOWLEDGMENTS

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