Glueball decay in holographic QCD

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Using holographic QCD based on D4-branes and D8-anti-D8-branes, we have computed couplings of glueballs to light mesons. We describe glueball decay by explicitly calculating its decay widths and branching ratios. Interestingly, while glueballs remain less well understood both theoretically and experimentally, our results are found to be consistent with the experimental data for the scalar glueball candidate $f_0(1500)$. More generally, holographic QCD predicts that decay of any glueball to $4\pi^0$ is suppressed, and that mixing of the lightest glueball with $q\bar{q}$ mesons is small.

I. INTRODUCTION

Glueballs, as excitations of gauge-invariant composite operators in Yang-Mills theories, remain illusive. Although the existence of the glueballs (of various types, such as scalar glueballs and tensor glueballs) is expected, their experimental identification in the hadron spectra remains difficult.[1] This difficulty is largely due to the inability to compute reliably couplings of glueballs to ordinary mesons in strongly coupled QCD. Lattice QCD predicts for the mass of the lightest scalar glueball to be around 1600–1700 MeV [3,4], but it does not yet provide enough information on the glueball couplings and decay products/widths, which are indispensable for their identification.

In this paper, we explicitly compute the couplings between light glueballs and light $q\bar{q}$ mesons, by using holographic QCD. Anti-de Sitter/conformal field theory (AdS/CFT) (gauge/gravity) correspondence (duality) [5,6] is one of the most important developments in string theory, and holographic QCD refers to the application of AdS/CFT to QCD studies. The basic claim of the AdS/CFT correspondence is that correlation functions of gauge-invariant composite operators in large N_c gauge theories at strong 't Hooft coupling correspond to classical gravitational computations in higher-dimensional gravity theories in curved backgrounds. The correspondence has been applied to (i) the computation of glueball spectrum in large N_c pure Yang-Mills theory and to (ii) $q\bar{q}$ meson spectra/dynamics in large N_c QCD, which we review briefly below. These efforts have been quite successful in reproducing lattice and experimental data of hadrons, even though the real QCD is recovered in the "CFT" side only when one incorporates various corrections in the large N_c and large

't Hooft coupling expansion. Here we combine these two efforts, (i) and (ii), in order to calculate couplings between the glueballs and the $q\bar{q}$ mesons, in the large N_c QCD.

The key merit of using holographic QCD is the fact that one not only can calculate the hadron spectra, but also can compute explicitly their *couplings*. It provides a more powerful method for constraining these couplings than the chiral perturbation technique. In particular, since glueballs are expected to be heavier than 1 GeV, derivative expansion in chiral perturbation becomes unreliable. Furthermore, current lattice calculations are not well suited for computing dynamical quantities such as decays and couplings. Our paper represents a first principle calculation for glueball decays, though in the approximation where the holographic duality is valid.

Let us briefly review here the holographic study of the two sectors (i) and (ii). Glueball studies began at the early stage of AdS/CFT correspondence, since they should exist in pure Yang-Mills theories whose supersymmetric version was the basic building block of the correspondence. Witten was the first to suggest a reliable way in breaking the supersymmetries thus allowing one to treat the bosonic Yang-Mills theory [7]. The gravity dual is the near horizon limit of a classical solution of 10-dimensional type IIA supergravity representing N_c D4-branes wrapping an S^1 with an antiperiodic boundary condition for fermions. After various developments along this direction [8–11], a complete spectrum of scalar/tensor glueballs in four-dimensional Yang-Mills theory was given in [12] where they appeared as graviton/dilaton/tensor fluctuations in the Witten's gravity background. The calculated glueball spectrum is consistent with the lattice computations, and in this paper, we compute glueball decays, based on this spectrum [12]. The lightest glueball is a scalar state with quantum numbers $J^{PC} = 0^{++}$. Since this state should be the easiest one to identify in hadronic data, we concentrate here on this lightest scalar glueball for explicit computations. The gravity dual of this lightest scalar glueball corresponds to a specific combination of metric fluctuations.

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[1]For details, we refer readers to the section of "Non-$q\bar{q}$ candidates" in Meson Particle Listings in [1], and to [2] for recent discussions.

The quark sector, (ii), is obtained in AdS/CFT correspondence by the introduction of flavor D-branes [13] intersecting with color N_c D4-branes. Various D-brane configurations (and also phenomenological holographic models) describing flavor/chiral physics have been proposed (see [14–18] for a partial list). Among these, we shall use the Sakai-Sugimoto model [17,18], which has been quite successful in reproducing various facets of low energy QCD dynamics while maintaining its string-theoretical origin. The Sakai-Sugimoto model uses N_f D8-branes and N_f anti-D8-branes as the flavor D-branes, and their intersection with color N_c D4-branes gives rise to string excitations corresponding to the quarks. Among various ways of introducing flavor D-branes, the Sakai-Sugimoto model beautifully realizes spontaneous chiral symmetry breaking and chiral dynamics in QCD.[2] The resulting low energy theory for the quark sector is dual to the probe D8-brane world volume theory (higher-dimensional Yang-Mills theory) in the Witten's supergravity background. In particular, the $q\bar{q}$ mesons are described as Kaluza-Klein (KK) decomposed massless fields on the probe D8-brane.

We would like to compute the couplings between the glueballs and the $q\bar{q}$ mesons in this setting. In the dual description through the AdS/CFT, they correspond to the supergravity fluctuations and the Yang-Mills fluctuations on the D8-branes, respectively. These two sectors are coupled in the combined system of supergravity plus D8-branes. We substitute the fluctuations (wave functions) of the supergravity fields (corresponding to the glueball) and the D8-brane massless fields (mesons) into the D8-brane action and integrate over the extra dimensions, to obtain the desired couplings. Combining sectors (i) and (ii) is important not only due to its phenomenological impact but also because this represents the first computation in holographic QCD of the couplings between the supergravity fluctuations and the fields on the probe D-branes.

Once the couplings are obtained, we can compute the decay widths for various decay channels of a glueball, and study its possible mixings. Because the whole $q\bar{q}$ meson sector is combined into the D8-brane action, which is a higher-dimensional Yang-Mills Lagrangian, several interesting mesonic features follow.[3] For example, at the leading order (in the expansion of the large 't Hooft coupling), glueball decay to $4\pi_0$ is prohibited. There is no direct $4\pi_0$ coupling to the glueball, and, furthermore, glueball-ρ meson coupling also does not allow the $4\pi_0$ decay mode. As for the mixing, we can show that the lightest glueball has no mixing with $q\bar{q}$ mesons at the leading order of our

expansion. These are our main predictions based on the holographic QCD.[4]

The organization of this paper is as follows. In Sec. II, we explicitly compute the glueball couplings in the holographic QCD, and obtain the interaction Lagrangian. We study generic features of the glueball decay following from the holographic QCD. In Sec. III, we compute the decay widths based on these interactions. We list possible decay products, and obtain widths for various allowed decay modes. We next compare these with the experimental data. It has been argued that $f_0(1500)$ is the most plausible candidate for the lightest scalar glueball [1], and we find that our results are consistent with the hadronic data for $f_0(1500)$. We reproduce the narrow width of the $f_0(1500)$, and also the decay products/branching ratio, qualitatively. In Sec. IV, we provide a summary, discussions, and a list of directions for future studies.

II. GLUEBALL INTERACTION

Holographic QCD, in particular, the Sakai-Sugimoto model, has provided a novel unified view of the mesons in a multiflavored QCD. All the mesons appear just as KK decomposed massless fields living in higher dimensions. As a consequence, there exist many interesting relations among couplings between the mesons. The Skyrm term is one example. Here our concern is with the glueballs, which live in a different sector in the dual side, i.e. in the supergravity fluctuations, not on the flavor D-branes. But the holographic features found in the meson sector are inherited also for the glueball-meson couplings. This is because these couplings are also controlled by the flavor D-brane action, and share the same flavor structure (commutator structure of the non-Abelian massless fields on the D8-branes). In this section, after reviewing the dual descriptions of the glueballs and $q\bar{q}$ mesons, we describe the generic features for glueball decays dictated by the holography. Finally, using the holographic QCD, we explicitly derive the couplings between the lightest scalar glueball and light $q\bar{q}$ mesons.

A. Brief review of holographic QCD: glueballs and mesons

1. Glueball sector

Glueballs are gauge-invariant composite states in Yang-Mills theory, and their duals are fluctuations in near horizon geometry of black-brane solutions. There are numerous ways to break supersymmetries by deforming the $AdS_5 \times S^5$ solution with which the original AdS/CFT correspondence was derived. Among them, the Witten's

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[2]Although the quarks of the model are massless (and so the pions are massless), it does not present a problem for our purpose. For a recent discussion on obtaining massive pions, see [19].

[3]One is the reproduction of the vector-meson dominance as shown in [18].

[4]We explicitly obtain numbers for the decay widths of the glueball, but these numbers should be understood as just an order estimation, because our evaluation in the holographic QCD is only at the leading order of the large coupling and the large N_c expansion.

background [7] is "reliable" in the sense that it knows how the supersymmetries are broken in the Yang-Mills side: antiperiodic boundary condition for the fermions on the D4-brane world volume.

Let us review briefly the description of the gravity dual for the lightest glueball in the four-dimensional QCD. It corresponds to supergravity fluctuations in the Witten's classical background in 10 dimensions. The type IIA supergravity solutions can be written conveniently in the notation of the 11-dimensional supergravity, and in that notation the Witten's solution is a doubly Wick-rotated AdS$_7$ blackhole,

$$ds^2 = \frac{r^2}{L^2}(f(r)d\tau^2 + \eta_{\mu\nu}dx^\mu dx^\nu) + \frac{L^2}{r^2}(f(r))^{-1}dr^2 + \frac{1}{4}L^2 d\Omega_4^2,\tag{2.1}$$

where $f(r) \equiv 1 - R^6/r^6$. L and R are the parameters of the solution. μ, ν run from 0 to 4, and the x^4 direction is the 11th dimension (the M theory circle). The τ direction is compactified to a circle, and its radius is fixed as $L^2/(3R)$ so that the background is nonsingular, and the manifold is smooth, around the "end" of the spacetime solution (at $r = R$).

The S^4 part is not necessary in the following discussion, so we integrate that part (and also the flux) to obtain the M theory supergravity action reduced to 7 dimensions:

$$S = \frac{-1}{2\kappa_{11}^2}\frac{L^4}{16}V_4 \int d^7x\sqrt{-\det G_{MN}}\left(R(G_{MN}) + \frac{30}{L^2}\right).\tag{2.2}$$

Here $V_4 \equiv 8\pi^2/3$ is the volume of a unit S^4, and we have followed the notation of [10].[5]

In [12], a complete bosonic spectrum was given, and the lightest state has quantum numbers $J^{PC} = 0^{++}$ in terms of the $x^0, \cdots, x^3$ spacetime. The metric fluctuations for this lightest state in the action (2.2) were explicitly obtained in [10][6][7] as

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[5]Further integration of the τ and the x^4 (M theory circle) gives a five-dimensional AdS gravity action,

$$S = \frac{-L^4 V_4}{48(2\pi)^6 l_s^8 g_s^2 R} \int dr d^4x r^2 \sqrt{f}\sqrt{-\det G_{MN}}\left(R(G_{MN}) + \frac{30}{L^2}\right).\tag{2.3}$$

Here M, N run through $(0, 1, 2, 3, r)$, and we have used the M theory $\leftrightarrow$ type IIA relations $R_{11} = g_s l_s$ and $2\kappa_{11}^2 = (2\pi)^8 l_s^9 g_s^3$.

[6]For the analogue state in three-dimensional QCD, see [11].

[7]In [10] there is a typo in Eqs. (38) and (40) (the sign of the functions c and b is opposite).

$$h_{rr} = -\frac{L^2}{r^2}f^{-1}\frac{3R^6}{5r^6 - 2R^6}H(r)G(x),$$

$$h_{44} = \frac{r^2}{L^2}\frac{1}{4}H(r)G(x),$$

$$h_{\mu\nu} = \frac{r^2}{L^2}\left(\eta_{\mu\nu}\frac{1}{4}H(r) - \left(\frac{1}{4} + \frac{3R^6}{5r^6 - 2R^6}H(r)\right)\frac{\partial_\mu \partial_\nu}{M^2}\right)G(x),$$

$$h_{r\mu} = \frac{360r^7 R^6}{M^2 L^2(10r^6 - 4R^6)^2}H(r)\partial_\mu G(x),$$

$$h_{\tau\tau} = -\frac{r^2}{L^2}fH(r)G(x).\tag{2.4}$$

Here $G(x^0, \cdots, x^3)$ is the glueball field in the real $1 + 3$-dimensional spacetime, and M is the mass of the glueball.[8]

The mass squared for this glueball state was found in [12] to be $M^2 = 7.308R^2/L^4$, by solving the eigenequation following from the equation of motion of the seven-dimensional AdS supergravity (2.2) given by the mass-shell condition of the glueball field $(\Box - M^2)G = 0$,

$$-\frac{d}{dr}(r^7 - rR^6)\frac{d}{dr}H(r)$$
$$-\left(L^4 M^2 r^3 + \frac{432r^5 R^{12}}{(5r^6 - 2R^6)^2}\right)H(r) = 0.\tag{2.5}$$

For our later purpose, it is useful to change the coordinate to a dimensionless Z defined by

$$r/R = K^{1/6}, \qquad K \equiv 1 + Z^2.\tag{2.6}$$

$Z = 0$ corresponds to the bottom of the background $r = R$, and the branch $Z(\geq 0)$ is smoothly connected to the branch $Z(\leq 0)$ [17]. In this Z coordinate, the eigenequation becomes

$$-\frac{3}{Z}\frac{d}{dZ}\left(3Z(1 + Z^2)\frac{d}{dZ}H(Z)\right)$$
$$-\left(\frac{L^4 M^2}{R^2}(1 + Z^2)^{-(1/3)} + \frac{432}{(5Z^2 + 3)^2}\right)H(Z) = 0.\tag{2.7}$$

The appropriate boundary condition for solving this is

$$\frac{d}{dZ}H|_{Z=0} = 0, \qquad H(Z = 0) \neq 0, \qquad H(Z = \infty) = 0.\tag{2.8}$$

Other fluctuations (such as the state corresponding to the 2^{++} glueball) can be constructed in the same manner [10,12].

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[8]The excitation tower for these graviton-dilaton fluctuations is denoted as S_4 in [12].

2. $q\bar{q}$ meson sector

The dual of the quark sector is the probe flavor D-branes intersecting with the color D-branes. To make sure that we are not simply constructing a phenomenological model but "deriving" the hadronic interactions from the first principle, we need to follow the so-called top-down approach from string theory. The Sakai-Sugimoto model [17] is the best known top-down construction for multiflavor quarks.

The $q\bar{q}$ mesons are described in the Sakai-Sugimoto model by the flavor D8-brane action in the Witten's background (written with type IIA string metric),

$$S_{\mathrm{D8}} = -(2\pi\alpha')^2 \mathcal{T}_{\mathrm{D8}}\,\mathrm{Tr}\int d^9x e^{-\Phi}\sqrt{-\det\tilde{g}}$$

$$\times\frac{1}{4}\tilde{g}^{PR}\tilde{g}^{QS}F_{PQ}F_{RS} + S_{\mathrm{Chern\text{-}Simons}}. \quad (2.9)$$

Here $\tilde{g}_{PQ}$ is the metric induced on the D8-brane world volume spanning the directions 0, 1, 2, 3, r, S^4 and we have already expanded the Dirac-Born-Infeld (DBI) action to the second order in the Yang-Mills field strength.[9] The normalization of the generators of the gauge group is chosen as $\mathrm{Tr}T_aT_b = \delta_{ab}$. The parameters in the background metric (2.1) are related to the notation of the Sakai-Sugimoto model by $L = 2R_{\mathrm{SS}}$, $R = 2\sqrt{R_{\mathrm{SS}}U_{\mathrm{KK}}}$, and (2.6), where R_{SS} denotes "R" in the original papers of Sakai and Sugimoto [17,18]. The typical mass scale appearing in all the computations in [17,18] is $M_{\mathrm{KK}} \equiv (3/2)U_{\mathrm{KK}}^{1/2}R_{\mathrm{SS}}^{-3/2}$.

Again, we integrate out the irrelevant S^4 part, leading to

$$S_{\mathrm{D8}} = \frac{-\mathcal{T}_{\mathrm{D8}}(2\pi\alpha')^2 V_4}{4g_s}\,\mathrm{Tr}\int d^4xdz\Big[3R_{\mathrm{SS}}^{3/2}U_{\mathrm{KK}}^{5/2}K\eta^{\mu\nu}$$

$$\times F_{\mu z}F_{\nu z} + \frac{2}{3}K^{-1/3}R_{\mathrm{SS}}^{9/2}U_{\mathrm{KK}}^{-1/2}\eta^{\mu\rho}\eta^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}\Big],$$

$$(2.10)$$

where $z \equiv U_{\mathrm{KK}}Z$ and μ, $\nu = 0$, 1, 2, 3. (The Chern-Simons term in (2.9) will be irrelevant to our discussion of the glueball decay; see Sec. III.) The KK decomposition along z (equivalently r, Z) in [17,18] is

$$A_z = \phi_0(z)\pi(x^\nu), \qquad A_\mu = \psi_1(z)\rho_\mu(x^\nu), \quad (2.11)$$

where we suppress all the other higher components, since we are interested in the decay of the glueball to light mesons. The eigenfunction ψ_1 should satisfy the eigenequation which follows from the action (2.10),

$$-K^{1/3}\partial_Z(K\partial_Z\psi_1) = \lambda_1\psi_1, \quad (2.12)$$

where $\lambda_1 = 0.669$ for normalizable ψ_1, leading to the mass

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[9]There is a tadpole for closed string modes, but in this paper we neglect the backreaction to the metric due to the presence of the D8-brane because it will be small in the large N_c limit.

squared for the ρ meson, $m_\rho^2 = \lambda_1 M_{KK}^2$. The pion is massless, with its eigenfunction $\phi_0(z) \propto 1/K$.

The trace in (2.10) is for the matrix-valued mesons. For N_f flavors, the pions and the ρ mesons are $N_f \times N_f$ matrices. The overall trace part of the pion should yield a mass from the chiral anomaly (see [17,18] for the description of the supergravity counterpart) but it is not included here. In this paper we take $N_f = 2$, hence the overall trace part of the pion is η (or η') meson, while that of the ρ meson is ω. Except when adjoint indices are explicitly written, we include η (η') and ω in the matrix notation of the fields π and ρ.

B. Generic features of holographic glueball decay

Our strategy to compute the interaction between the glueballs and the $q\bar{q}$ mesons is very simple. Since we know how all these hadrons are described in the dual side (as in (2.4) and (2.11)), we substitute them into the D8-brane action (2.9) and integrate it over the extra dimensions. In the original Sakai-Sugimoto model, the induced metric $\tilde{g}$ in (2.9) was just the background metric, but now the glueball appears as a fluctuation in the induced metric and the dilaton in the D8-brane action.

Since the appearance of the glueball does not break the non-Abelian structure of the D8-brane action, we can expect that some generic features of the glueball coupling may be read in the D8-brane action. As an obvious check, the glueballs should be flavor-blind; this can be seen as the fact that the supergravity fields are gauge invariant with respect to the gauge transformation on the D8-brane. As a consequence, couplings of the glueballs to $q\bar{q}$ mesons are universal against flavors.

Gauge invariance in higher dimensions also constrains the meson interactions. For example, as Sakai and Sugimoto have shown, the Skyrm term in the pion self-interactions is encoded in the structure of the higher-dimensional Yang-Mills Lagrangian. In our case, even though glueballs (= gravity and dilaton fluctuations) are now included, this flavor structure is almost unaltered. We note the following interesting features:

(a) There are no glueball interactions involving more than two pions.

(b) For glueball coupling to ρ and pion, the ρ meson couples to the pion as if the pion were charged under the ρ meson gauge field.

(c) Direct coupling of a glueball, G, with more than five mesons are suppressed by large 't Hooft coupling.

(a) is easily seen by noting the fact that π appears in A_z but there are no $(A_z)^n$ terms, with $n > 2$ in the D8-brane action (2.9). (b) follows from the fact that $F_{\mu z} \sim [A_\mu, A_z]$ can be decomposed as $[\rho_\mu, \partial_\mu\pi]$. These two features are precisely what were observed in the Sakai-Sugimoto model for the pure $q\bar{q}$ meson sector, and now inherited to the glueball couplings. Finally, (c) is due to the fact that the higher-dimensional Yang-Mills action (2.9) does not have

an A^5 term. For (a) and (c), DBI corrections give F^4 terms but they are suppressed by α' or equivalently the large 't Hooft coupling.

To be more explicit, we can list the couplings which appear in (2.9):

$$G\,\text{Tr}(\pi^2),\ \ G\,\text{Tr}(\pi, [\pi, \rho]),\ \ G\,\text{Tr}([\pi, \rho]^2),\ \ G\,\text{Tr}(\rho^2),$$

$$G\,\text{Tr}(\rho[\rho, \rho]),\ \ G\,\text{Tr}([\rho, \rho]^2). \quad (2.13)$$

Here we omit the derivatives and also possible indices. There are in fact no other couplings, and this is a generic result from the holographic QCD for interactions involving a single glueball. Even for multiglueball vertices, this flavor structure is maintained.

A direct consequence of this flavor structure is the fact that glueballs cannot decay to $4\pi_0$. The decay channel to $4\pi_0$ appears in the $F_{\mu z}^4$ term which is in the higher DBI corrections and thus suppressed by the large 't Hooft coupling. So, the holographic QCD predicts that, among the decay products of the glueballs, $4\pi_0$ is suppressed.

It is important to note that we work here in a "holographic gauge" (2.11) of the D8-brane action, in which interactions are seen in the simplest and the most transparent way. On the other hand, in the $A_z = 0$ gauge [17] the broken chiral symmetry is manifest because the pions fields appear in the action as $U = \exp(i\pi/f_\pi)$. Thus this gauge is appropriate for comparison with the chiral perturbation theory, but the interactions are complicated. A different gauge choice in the D8-brane action leads to a different field definition of the four-dimensional fields and, of course, this does not change the physics. Therefore, there should be some hidden structure in the QCD effective action, at least in the large N_c limit, because of the higher-dimensional gauge symmetry. For example, the vector-meson dominance is its consequence [18]. The suppression of the $4\pi_0$ in the glueball decay is also a consequence of this hidden structure which is not manifest in a gauge choice other than the "holographic gauge" (2.11).

For explicit computations, we concentrate on the couplings of the lightest scalar glueball, because of its phenomenological interest. But it is obvious that couplings of other glueball excitations can be computed in the same manner. One of the phenomenologically interesting excitations is the 2^{++} state. Fortunately, the supergravity fluctuation for this 2^{++} state is simple (see, for example, [10] whose notation we follow), in particular, it consists of only the fluctuation of the metric components of μ, $\nu = 0, 1, 2, 3$. So it in fact couples to the four-dimensional part of the energy-momentum tensor of the D8-brane Yang-Mills action. Therefore the coupling should be of the form

$$\int d^4x G^{\mu\nu} \eta^{\rho\sigma}\, \text{Tr}(F_{\mu\rho} F_{\nu\sigma}),$$

$$\int d^4x G^{\mu\nu}\, \text{Tr}(\partial_\mu \pi \partial_\nu \pi), \qquad \int d^4x G^{\mu\nu}\, \text{Tr}(\rho_\mu \rho_\nu),$$

$$\quad (2.14)$$

at the quadratic order in π and ρ_μ. Here $F_{\mu\nu}$ is the field strength of the ρ meson at its linear order, $F_{\mu\nu} \equiv \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$.

Also the second lightest 0^{++} glueball is simple, since the supergravity fluctuation is involved with the dilaton that has a very simple coupling to the D8-brane Yang-Mills fields. We expect that this kind of simplicity in the glueball couplings may give some constraint on the decay products and decay widths, in particular, the spin dependence of the decay product, and may serve as a smoking gun for identifying the higher glueball states in the meson spectroscopy.[10] In the next subsection, we explicitly compute the interaction Lagrangian of the lightest scalar glueball.

C. Interaction of the lightest scalar glueball

In this subsection we derive the interaction Lagrangian of glueballs with light $q\bar{q}$ mesons (the pions and the ρ mesons). First, we need to fix the normalization of eigenfunctions in higher dimensions, in the dual side. Then, we substitute all the fluctuations into the D8-brane action and perform integration over extra dimensions to obtain the interaction Lagrangian. The interaction Lagrangian includes a possible mixing between the glueball states and the $q\bar{q}$ mesons; however, we show that there is no mixing for the lightest glueball.

1. Normalization of the fluctuation fields

Sakai and Sugimoto [17,18] have determined the normalization of the eigenfunctions $\psi_1(z)$ and $\phi_0(z)$ for the ρ meson and the pion, respectively:

$$\frac{2}{3} R_{SS}^{9/2} U_{KK}^{1/2} \mathcal{T}_{D8} V_4 g_s^{-1} (2\pi\alpha')^2 \int dZ K^{-1/3}(\psi_1)^2 = 1,$$

$$\quad (2.15)$$

$$\frac{3}{2} R_{SS}^{3/2} U_{KK}^{7/2} \mathcal{T}_{D8} V_4 g_s^{-1} (2\pi\alpha')^2 \int dZ K(\phi_0)^2 = 1. \quad (2.16)$$

With these, substituting (2.11) into the D8-brane action (2.9) (the metric is fixed with its background value), we obtain canonically normalized kinetic terms for the ρ meson and the pion,

$$S_0 = -\text{Tr}\int d^4x \left\{ \frac{1}{2}(\partial_\mu \pi)^2 + \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2}\lambda_1 M_{KK}^2 \rho_\mu^2 \right\}.$$

$$\quad (2.17)$$

The field strength F is that of the ρ meson, $F_{\mu\nu} \equiv \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$.

On the other hand, the normalization of the fluctuation eigenfunction $H(Z)$ for the glueball has not been carried out in the past. The normalization of $H(Z)$ in (2.4) should

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[10]In fact, we will find in the next subsection that there is no mixing between the lightest 0^{++} glueball (or the lightest 2^{++}) with $q\bar{q}$ mesons, at the leading $1/\sqrt{N_c}$ order.

be fixed in such a way that substitution of the expressions (2.4) into the supergravity action (2.2) results in, after the integration of the extra dimensions, a canonical kinetic term for the glueball field $G(x)$,

$$S = \text{const} - \int d^4x \Big(\frac{1}{2} (\partial_\mu G)^2 + \frac{1}{2} M^2 G^2 \Big) + \mathcal{O}(G^3).$$
(2.18)

We numerically solve the differential equation (2.7) for $H(Z)$, to get[11]

$$\int d^7x \sqrt{-\det G_{MN}} \Big(R(G) + \frac{30}{L^2} \Big) = -0.0574 \frac{R^4}{L^3} (H(Z=0))^2$$
$$\times \int d^4x d\tau dx^4 [\partial_\mu G \partial^\mu G$$
$$+ M^2 G^2].$$
(2.19)

So, using the expressions for the M theory gravity coupling κ_{11}, and also the relations between the supergravity parameters and the QCD parameters [17,18]

$$R_{\text{SS}}^3 = \frac{1}{2} \frac{g_{\text{YM}}^2 N_c l_s^2}{M_{\text{KK}}}, \qquad U_{\text{KK}} = \frac{2}{9} g_{\text{YM}}^2 N_c M_{\text{KK}} l_s^2,$$
$$g_s = \frac{1}{2\pi} \frac{g_{\text{YM}}^2}{M_{\text{KK}} l_s},$$
(2.20)

we obtain the normalization

$$(H(Z=0))^{-1} = 0.009\,78 g_{\text{YM}} N_c^{3/2} M_{\text{KK}}.$$
(2.21)

2. Glueball interaction Lagrangian

Once the normalization of the eigenfunctions $H(Z)$, $\psi_1(Z)$, and $\phi_0(Z)$ are determined, substituting all the fluctuations (+ background) into the D8-brane action (2.9) gives us the glueball-$q\bar{q}$ meson couplings. We concentrate on interactions linear in the glueball field G, since we are interested in the glueball decays.

First, because the D8-brane action is written in terms of the type IIA string metric and the dilaton field, we need a dimensional reduction from the 11-dimensional fields to the 10-dimensional fields. We find

$$g_{rr} = \frac{L}{rf} \Big(1 + \frac{L^2}{2r^2} h_{44} + \frac{r^2 f}{L^2} h_{rr} \Big),$$
$$g_{\mu\nu} = \frac{r^3}{L^3} \Big(\Big(1 + \frac{L^2}{2r^2} h_{44} \Big) \eta_{\mu\nu} + \frac{L^2}{r^2} h_{\mu\nu} \Big),$$
$$g_{r\mu} = \frac{r}{L} h_{r\mu},$$
(2.22)
$$g_{\tau\tau} = \frac{r^3}{L^3} f \Big(1 + \frac{L^2}{2r^2} h_{44} + \frac{L^2}{r^2 f} h_{\tau\tau} \Big),$$
$$e^{4\Phi/3} = \frac{r^2}{L^2} + h_{44}.$$

Substituting these and all the expressions for the fluctuations (2.4) and (2.11) into the D8-brane action (2.9), we obtain the interaction action S_{int} before the integration over the extra dimension Z:

$$S_{\text{int}} = \frac{-\mathcal{T}_{\text{D8}}(2\pi\alpha')^2 V_4}{4g_s} \text{Tr} \int d^4x dZ \Big[3R_{\text{SS}}^{3/2} U_{\text{KK}}^{5/2} K \Big\{ \frac{1}{2} ((\partial_\mu \pi)^2 \phi_0^2 + \rho_\mu^2 \psi_1^2) \tilde{H} \Big(1 - \frac{\Box}{M^2} \Big) G + (\partial_\mu \pi \partial_\nu \pi \phi_0^2$$
$$+ \rho_\mu \rho_\nu \psi_1^2) \tilde{H} \frac{\partial^\mu \partial^\nu}{M^2} G \Big\} + \frac{2}{3} K^{-1/3} R_{\text{SS}}^{9/2} U_{\text{KK}}^{-1/2} \Big\{ -\frac{1}{2} F_{\mu\nu}^2 \psi_1^2 \tilde{H} \Big(1 + \frac{\Box}{M^2} \Big) G + 2 F_{\mu\rho} F_\nu{}^\rho \psi_1^2 \tilde{H} \frac{\partial^\mu \partial^\nu}{M^2} G \Big\}$$
$$- \frac{180 K}{(5K-2)^2} R_{\text{SS}}^{3/2} U_{\text{KK}}^{3/2} Z \psi_1 (\partial_Z \psi_1) H \rho_\nu F_\mu{}^\nu \frac{\partial^\mu}{M} G \Big].$$
(2.23)

We have defined $\tilde{H}(Z) \equiv ((1/4) + 3/(5K-2)) H(Z)$. In this action (2.23) we have kept only terms quadratic in π and ρ_μ, for simplicity. Note that the first line in the interaction action (2.23) vanishes for an on-shell glueball, $(\Box - M^2) G = 0$.

Terms of higher order in π and ρ in the list (2.13) can also be computed in the same manner. (Other couplings do not appear at this order in the large 't Hooft coupling expansion.) Among these additional couplings, only the $G\pi\pi\rho$ coupling shown below will be relevant for the later

computations of the decay width:

$$S_{\text{int}}' = \frac{6i \mathcal{T}_{\text{D8}}(2\pi\alpha')^2 V_4}{4g_s} \int d^4x dZ$$
$$\times \Big[R_{\text{SS}}^{3/2} U_{\text{KK}}^{5/2} K \text{Tr}(\partial_\mu \pi [\rho_\nu, \pi]) \phi_0^2 \psi_1 \tilde{H} \frac{\partial^\mu \partial^\nu}{M^2} G \Big].$$
(2.24)

In addition to this, there is another term for $G\pi\pi\rho$ which vanishes for the on-shell glueball.

Finally, performing the Z integration, we obtain the following interaction Lagrangian (in this expression, again we have kept only terms quadratic in π and ρ_μ):

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[11]Note that there are no higher derivative terms on the right-hand side of this expression. This is due to a useful gauge choice for the gravity fluctuations (2.4), introduced in [10].

$$S_{\text{int}} = -\text{Tr} \int d^4x \Big\{ c_1 \frac{1}{4} (\partial_\mu \pi)^2 \Big(1 - \frac{\square}{M^2}\Big) G$$

$$+ c_2 \frac{1}{4} M_{KK}^2 \rho_\mu^2 \Big(1 - \frac{\square}{M^2}\Big) G + c_1 \frac{1}{2} (\partial_\mu \pi \partial_\nu \pi) \frac{\partial^\mu \partial^\nu}{M^2} G$$

$$+ c_2 \frac{1}{2} M_{KK}^2 \rho_\mu \rho_\nu \frac{\partial^\mu \partial^\nu}{M^2} G - c_3 \frac{1}{8} F_{\mu\nu}^2 \Big(1 + \frac{\square}{M^2}\Big) G$$

$$+ c_3 \frac{1}{2} F_{\mu\rho} F_\nu{}^\rho \frac{\partial^\mu \partial^\nu}{M^2} G - c_4 \frac{3}{2} \rho_\nu F_\mu{}^\nu \frac{\partial^\mu}{M^2} G \Big\}. \quad (2.25)$$

Here G is the lightest scalar glueball field with the mass M, $J^{PC} = 0^{++}$, and $F_{\mu\nu} \equiv \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$ is the field strength of the rho meson ρ_μ, and the coefficients $c_1 \sim c_4$ are defined as follows:

$$c_1 \equiv \int dZ \frac{1}{K\pi} \tilde{H}, \quad (2.26)$$

$$c_2 \equiv \frac{2}{3} R_{\text{SS}}^{9/2} U_{\text{KK}}^{1/2} \mathcal{T}_{\text{D8}} V_4 (2\pi\alpha'^2) g_s^{-1} \int dZ K (\partial_Z \psi_1)^2 \tilde{H}, \quad (2.27)$$

$$c_3 \equiv \frac{2}{3} R_{\text{SS}}^{9/2} U_{\text{KK}}^{1/2} \mathcal{T}_{\text{D8}} V_4 (2\pi\alpha'^2) g_s^{-1} \int dZ K^{-1/3} (\psi_1)^2 \tilde{H}, \quad (2.28)$$

$$c_4 \equiv \frac{2}{3} R_{\text{SS}}^{9/2} U_{\text{KK}}^{1/2} \mathcal{T}_{\text{D8}} V_4 (2\pi\alpha'^2) g_s^{-1} M_{KK}^2$$

$$\times \int dZ \frac{20 KZ}{(5K-2)^2} \psi_1 (\partial_Z \psi_1) H. \quad (2.29)$$

These are evaluated numerically,

$$c_1 = \frac{44.3}{g_{\text{YM}} N_c^{3/2} M_{\text{KK}}}, \quad c_2 = \frac{5.03}{g_{\text{YM}} N_c^{3/2} M_{\text{KK}}},$$

$$c_3 = \frac{49.3}{g_{\text{YM}} N_c^{3/2} M_{\text{KK}}}, \quad c_4 = \frac{-0.0732 M_{\text{KK}}}{g_{\text{YM}} N_c^{3/2}}. \quad (2.30)$$

The $G\pi\pi\rho$ coupling (2.24) is integrated to give

$$S'_{\text{int}} = i c_5 \int d^4x \, \text{Tr}(\partial_\mu \pi [\rho_\nu, \pi]) \frac{\partial^\mu \partial^\nu}{M^2} G, \quad \text{with}$$

$$c_5 \equiv \int dZ \frac{1}{K\pi} \psi_1 \tilde{H} = \frac{1.43 \times 10^3}{g_{\text{YM}}^2 N_c^{5/2} M_{\text{KK}}}. \quad (2.31)$$

These are the basic ingredients for computing the decay of the lightest glueball in Sec. III.

3. Mixing of glueball with $q\bar{q}$ mesons

For the identification of the glueball state in the data of the real hadronic spectra, mixing with other states possessing the same quantum number (for the scalar glueball of our interest, it is 0^{++}) is quite essential. Generically, the mixing is expected to appear, because no symmetry can prohibit it. In holographic QCD, mixing can be computed explicitly.[12] Here we show that the lightest scalar glueball has no mixing, in the leading order interaction Lagrangian of our concern. This means that the mixing is largely suppressed, so the decay of the lightest scalar glueball is dominated by a direct decay (not through the mixing of the mass matrix). Note that for generic glueball excitations, this is not the case, as we will see below.

First, let us give a generic argument on the mixing of a generic glueball and $q\bar{q}$ mesons, in the holographic QCD. We can show that the mixing is suppressed by $1/\sqrt{N_c}$. The mixing, a linear coupling between a glueball and a $q\bar{q}$ meson, originates in a linear coupling between supergravity fields and Yang-Mills/scalar fields in the D8-brane action (2.9). The order of the mixing can be identified after canonically normalizing the glueball field G and a meson field X. We already know that the normalization of the glueball field G is given by (2.21), while that of the X meson can be characterized by that of the ρ meson, (2.15). Using the relations (2.20), the prefactor in (2.15) can be computed as $g_{\text{YM}}^2 N_c^2 / 108 \pi^3$. The D8-brane action has the same prefactor (since the prefactor in (2.15) is basically for canonically normalizing the ρ meson kinetic term in the D8-brane action), so the mixing term can be written as

$$S_{\text{mix}} \sim \frac{g_{\text{YM}}^2 N_c^2}{108 \pi^3} \int d^4x X G \int dZ \psi_1^{(X)} H$$

$$\sim \frac{(g_{\text{YM}}^2 N_c^2)}{(g_{\text{YM}}^2 N_c^2)^{1/2} (g_{\text{YM}} N_c^{3/2})} \int d^4x X G$$

$$\sim \frac{1}{\sqrt{N_c}} \int d^4x X G. \quad (2.32)$$

This means that the mixing is of order $1/\sqrt{N_c}$.[13]

Although it is suppressed in the large N_c limit, this has a significant effect on the decay process. The direct meson decay process comes from meson interactions which are of order $g_{\text{YM}}^{-1} N_c^{-1}$. So, combining this with the mixing, the total decay amplitude through the mixing is $\sim g_{\text{YM}}^{-1} N_c^{-3/2}$. On the other hand, the couplings computed in (2.30) mean that the direct decay amplitude of the glueball is of order $g_{\text{YM}}^{-1} N_c^{-3/2}$, which is the same as the mixing decay amplitude. Therefore in generic glueball decay, direct decay process is comparable to the decay through the mixing term.

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[12]See [20] for glueball mixing to η' meson in a different holographic model of QCD based on flavor D6-branes [16].

[13]This suppression can be understood more easily. The supergravity fields are normalized with the $1/g_s^2$ factor in front of the supergravity action, while the D8-brane gauge fields are normalized with the $1/g_s$ factor in the tension of the D8-brane $\mathcal{T}_{\text{D8}}$. Therefore, if one canonically normalizes the kinetic terms of the fluctuation fields, the above-mentioned mixing coupling receives a $\sqrt{g_s}$ factor, and in view of the AdS/CFT correspondence this factor is just a $1/\sqrt{N_c}$ correction.

Our interest here is primarily on the lightest glueball, and let us show that there is no mixing for this lightest glueball. First, note that the ρ meson and the pion appear quadratically in the D8-brane action. This already shows that for the glueballs originating in the dilaton and the graviton fluctuations have no mixing with the ρ meson and the pion. (For glueballs coming from Neveu-Schwarz (NS)-NS B-field or Ramond-Ramond (RR) gauge fields may have mixings.) So the lightest glueball can mix only with other types of mesons which are not in the higher-dimensional Yang-Mills field; that is the transverse scalar field y on the D8-brane. This y is not written explicitly in (2.9) but included in the induced metric and the dilaton. The KK decomposition of the field y produces scalar mesons with quantum number 0^{++}. This y is again an $N_f \times N_f$ matrix. When $N_f = 3$, among $N_f^2 = 9$ matrix elements, we have two elements with isospin zero, which mix with the glueball. These two $q\bar{q}$ meson states can be identified with f_0 mesons (other than the glueball candidate $f_0(1500)$); near $f_0(1500)$, there are $f_0(1370)$ and $f_0(1710)$, which may be identified with these two $q\bar{q}$ mesons coming from y.

Possible mixings among these three f_0 mesons have been studied phenomenologically (see for example [4,21]). However, we show below that holographic QCD predicts there is no mixing at the leading order.[14] From the induced metric in the D8-brane action, we are interested in terms linear in the field y, which would lead to possible mixing. These are

$$g_{\nu y}|_{y=0}\partial_\mu y(z, x^\mu), \qquad g_{\nu y}|_{y=0}\partial_z y(z, x^\mu),$$
$$g_{zy}|_{y=0}\partial_\mu y(z, x^\mu), \qquad g_{zy}|_{y=0}\partial_z y(z, x^\mu), \qquad (2.33)$$
$$y[\partial_y g_{\tau\tau, rr, \mu r, \mu\nu}]_{y=0}, \qquad y[\partial_y \phi]_{y=0},$$

where the last term is of course from the dilaton. Here the bulk coordinates y and $z = ZU_{KK}$ are [17]

$$y = \left(U_{KK}\sqrt{\frac{r^6}{R^6} - 1}\right)\cos\theta,$$

$$z = \left(U_{KK}\sqrt{\frac{r^6}{R^6} - 1}\right)\sin\theta, \qquad \theta \equiv \frac{3}{2}\frac{U_{KK}^{1/2}}{R_{SS}^{3/2}}\tau. \qquad (2.34)$$

It is easy to show that in fact the metrics and the dilaton appearing in (2.33) which include the glueball fluctuations (2.4) disappear at $y = 0$ where the D8-brane is located, after transforming the metric by using the (r, μ, τ) space-time coordinates. So we conclude that there is no mixing of the lightest glueball with mesons, at the leading order in $1/\sqrt{N_c}$.[15] Precisely the same argument shows that the

lightest 2^{++} glueball also does not participate in mixing at this order.

III. DECAY OF THE LIGHTEST SCALAR GLUEBALL

Starting from the interaction Lagrangian (2.25) and (2.31), we can directly study the decay products and their decay widths. In this section, we first enumerate the kinematically allowed decay processes by analyzing the masses of particles involved. This provides a list of decay products for the lightest glueball. We then compute the decay widths by using the interaction Lagrangians (2.25) and (2.31) including explicit numerical coefficients (2.30). Finally, we compare the widths with experimental data for the glueball candidate $f_0(1500)$. We find that the prediction of holographic QCD qualitatively reproduces the total width as well as the branching ratios of the $f_0(1500)$.

A. Decay products

Let us study the kinematical constraints on the decay of the lightest glueball in the holographic QCD. The lightest glueball mass M is given [12] by $M = \sqrt{7.31/9}M_{KK}$, while the ρ meson mass is $m_\rho = \sqrt{\lambda_1}M_{KK} = \sqrt{0.669}M_{KK}$. So we have a relation

$$m_\rho < M < 2m_\rho \qquad (3.1)$$

in the holographic QCD. This means, our lightest glueball cannot decay to two on-shell ρ mesons.[16]

In (2.13), we listed all the coupling terms appearing in the interaction Lagrangian with a single glueball field G. From those terms one can construct Feynman diagrams for the decay processes. We will work with two flavors, for definiteness. The list (2.13) can be grouped into two categories, as follows:
  (i) $G \operatorname{Tr}(\pi^2)$, $G \operatorname{Tr}(\pi, [\pi, \rho])$, $G \operatorname{Tr}(\rho^2)$, $G \eta'\eta'$
  (ii) $G \operatorname{Tr}([\pi, \rho]^2)$, $G \operatorname{Tr}(\rho[\rho, \rho])$, $G \operatorname{Tr}([\rho, \rho]^2)$, $G\omega\omega$
      We have written explicitly and separately the trace part of the $q\bar{q}$ mesons: η' for the pions and ω for the ρ mesons. In addition to these, there are couplings coming from the Chern-Simons term,
  (iii) $G \operatorname{Tr}(\pi\rho\rho)$, $G\eta' \operatorname{Tr}(\rho\rho)$, $G\omega \operatorname{Tr}(\pi\rho)$, $G\eta'\omega\omega$ for which the spacetime indices are contracted by the epsilon tensor. The category (i) is important for the decay processes, while the categories (ii) and (iii) are almost irrelevant kinematically, since with the couplings (ii) and (iii) the final decay product includes more than five pions (or four pions and one η'). To understand this, note that the ρ meson can decay to two pions, and ω decays to three pions.[17] When the

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[14]The components with isospin = 1 are identified as $a_0(1450)$ meson [17].

[15]Here we only consider the $U(N_f)$ singlet of the transverse scalars. We can show that the other non-Abelian scalars do not mix with the lightest glueball as in the case of the ρ mesons.

[16]For the most probable candidate of the glueball $f_0(1500)$, $M = 1507$ MeV, so this mass relation is satisfied in the experimental data ($m_\rho = 776$ MeV).

[17]Couplings to photons are described in the last section.
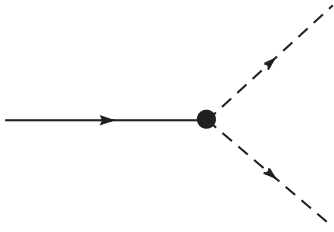
FIG. 1.   A glueball G decaying to two pions π.

number of the pions are large, typical momentum for the final pion state is small. Pion couplings are accompanied by derivatives, then the amplitude is expected to be suppressed. By this reason, we restrict our analysis to the cases where the final decay product is induced by the couplings (i).

Since the mass of the glueball is not larger than twice the ρ meson mass, the final decay products should have less than two ρ mesons. All possible decay chains obtained by these couplings are categorized by the decay products:

(a) $G \to \pi\pi$ (Fig. 1)
(b) $G \to \rho\pi\pi$, $G \to \rho\rho \to \rho\pi\pi$ (Fig. 2)
(c) $G \to \rho\pi\pi \to \pi\pi\pi\pi$, $G \to \rho\rho \to \pi\pi\pi\pi$ (Fig. 3)
(d) $G \to \eta'\eta'$ (Fig. 1)

If we think of G as the $f_0(1500)$, then this list is consistent with what is known in the particle data book [1]. The branching ratio given in [1] is 35% for (a), 49% for $G \to$
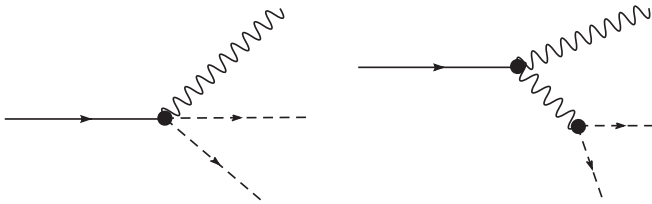
FIG. 2.   A glueball G decaying to two pions π and a single ρ. There are two graphs, the decay with a single vertex (left) and the decay with two vertices (right). In both graphs, solid lines are glueballs, dashed lines are π^1 and π^3, and wavy lines are ρ^2.
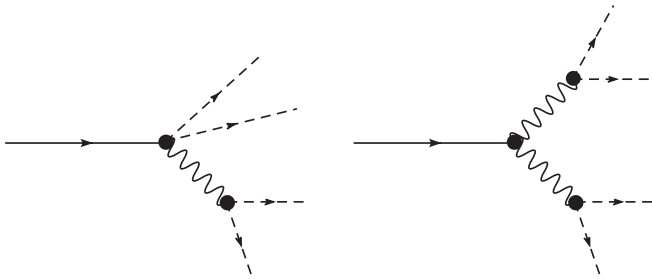
FIG. 3.   A glueball G decaying to four pions π. There are two graphs, the decay with two vertices (left) and the decay with three vertices (right). Pions (dashed lines) in both the graphs are labeled from the top to the bottom as π^a, π^c, π^d, and π^f. ρ mesons (wavy lines) are (ρ^e (left), and (ρ^b (top) and ρ^e (bottom)) (right).

4π (corresponding to (b) + (c)[18]), and 7% for (d).[19] So we can reproduce the main decay channels of the $f_0(1500)$. In the next subsection, we compute the decay widths for each of these decay branches.

B. Decay widths

Let us evaluate the decay widths for these groups. Groups (a) and (d) are two-body decays so that the decay widths can be computed analytically. For the remaining groups, (b) and (c), integrations over final momenta are complicated that we computed the decay widths numerically.[20]

1. $G \to \pi\pi$, $G \to \eta\eta$

Two-body decays are simple to analyze, for which we have for the decay width,

$$\Gamma = \frac{|\mathbf{p}|}{8\pi M^2} |\mathcal{M}|^2, \qquad (3.4)$$

where $\mathcal{M}$ is the amplitude of the graph responsible for the decay, and $\mathbf{p}$ is the final momentum of one of the identical particles in the decay product.

In the rest frame of the glueball, the first line in the interaction Lagrangian (2.25) vanishes. For the 2π decay, the relevant coupling in that frame is

$$\tfrac{1}{2} c_1 \partial_0 \pi^a \partial_0 \pi^a G. \qquad (3.5)$$

For definiteness we consider a specific adjoint index for the pion π^a ($a = 1, 2, 3$). We have two pions as a final state, $\pi^{(1)}$ and $\pi^{(2)}$, then

$$\mathcal{M} = \tfrac{1}{2} c_1 i p_0^{\pi^{(1)}} i p_0^{\pi^{(2)}} \times 2, \qquad (3.6)$$

where the last factor 2 is for the symmetry of exchanging

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[18]In [1], (b) is not explicitly written, but we interpret that (b) is included in the $G \to 4\pi$ decay in [1] because the on-shell ρ meson in (b) would decay to 2π.

[19]We are working in the case of two flavors, so we do not distinguish η and η', and ignore K.

[20]We worked in the supergravity convention, but to compute the decay widths it is convenient to go to the metric convention $(+1, -1, -1, -1)$. The new action $S = S_0 + S_{\text{int}}$ quadratic in the $q\bar{q}$ meson fields is

$$S_0 = \text{Tr} \int d^4x \left\{ \frac{1}{2} (\partial_\mu \pi)(\partial^\mu \pi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \lambda_1 M_{KK}^2 \rho_\mu \rho^\mu \right\}, \qquad (3.2)$$

$$\begin{aligned} S_{\text{int}} = \text{Tr} \int d^4x \Bigg\{ & c_1 \frac{1}{4} (\partial_\mu \pi)(\partial^\mu \pi)\left(1 + \frac{\square}{M^2}\right) G \\ & + c_2 \frac{1}{4} M_{KK}^2 \rho_\mu \rho^\mu \left(1 + \frac{\square}{M^2}\right) G - c_1 \frac{1}{2} (\partial_\mu \pi \partial_\nu \pi) \frac{\partial^\mu \partial^\nu}{M^2} G \\ & - c_2 \frac{1}{2} M_{KK}^2 \rho_\mu \rho_\nu \frac{\partial^\mu \partial^\nu}{M^2} G + c_3 \frac{1}{8} F_{\mu\nu} F^{\mu\nu}\left(1 - \frac{\square}{M^2}\right) G \\ & + c_3 \frac{1}{2} F_{\mu\rho} F^{\nu\rho} \frac{\partial^\mu \partial_\nu}{M^2} G + c_4 \frac{3}{2} \rho_\nu F^{\mu\nu} \frac{\partial_\mu}{M^2} G \Bigg\}. \qquad (3.3) \end{aligned}$$

the two final identical particles. The kinematics shows that $p_0^{\pi^{(1)}} = p_0^{\pi^{(2)}} = |\mathbf{p}| = M/2$ because the pions are massless, so we obtain

$$\mathcal{M} = -c_1 M^2/4. \qquad (3.7)$$

The decay width summed over $a = 1, 2, 3$ is

$$\Gamma_{G \to \pi\pi} = \frac{|c_1|^2 M^3}{256\pi} \times 3 \times \frac{1}{2}. \qquad (3.8)$$

The last factor $1/2$ is necessary because the final state has two identical particles.

For evaluating the numerical value of the decay width, we need $\kappa \equiv g_{\mathrm{YM}}^2 N_c^2/108\pi^3 = 7.45 \times 10^{-3}$ which was used in [17] to fit the pion decay constant, and $N_c = 3$. Using these as inputs, we finally obtain the decay width divided by the glueball mass

$$\begin{aligned}
\frac{\Gamma_{G \to \pi\pi}}{M} &= \frac{3}{512\pi}\left(\frac{44.3}{g_{\mathrm{YM}} N_c^{3/2} M_{\mathrm{KK}}}\right)^2 \left(\sqrt{\frac{7.31}{9}} M_{\mathrm{KK}}\right)^2 \\
&= 0.040.
\end{aligned} \qquad (3.9)$$

This is to be compared with the experimental data in [1],

$$\frac{\Gamma_{G \to \pi\pi}^{(\mathrm{ex})}}{M} = \frac{109}{1507} \times 34.9\% = 0.0252, \qquad (3.10)$$

with which we find a qualitatively good agreement.

Another two-body decay channel is for $G \to \eta\eta$. The η' mass evaluated in the holographic QCD in [17][21] is found to be too large, $2m_{\eta'} > M$, so in the holographic QCD this decay channel cannot be described. However, if we adopt, as a trial, the η' mass as a free parameter in this holographic QCD, then we obtain the decay width

$$\frac{\Gamma_{G \to \eta\eta}}{M} = \frac{\Gamma_{G \to \pi\pi}}{M} \times \frac{1}{3} \times \sqrt{1 - \frac{4m_\eta^2}{M^2}}. \qquad (3.11)$$

The factor $1/3$ is to suppress the effect of the three kinds of the pions, and the last factor is necessary to replace $|\mathbf{p}|$ of the pion with that of the η meson. If we substitute the real observed ratio $m_\eta/M_{f_0(1500)} = 547.5/1507$, we obtain

$$\frac{\Gamma_{G \to \eta\eta}}{M} = 0.0090. \qquad (3.12)$$

We compare this with the experimental data,

$$\frac{\Gamma_{G \to \eta\eta}^{(\mathrm{ex})} + \Gamma_{G \to \eta\eta'}^{(\mathrm{ex})}}{M} = \frac{109}{1507} \times 7.0\% = 0.005\,06, \qquad (3.13)$$

again this is qualitatively in agreement with our result.

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[21]The expression is $m_{\eta'} = \frac{1}{3\sqrt{6}\pi}\sqrt{\frac{N_f}{N_c}}(g_{\mathrm{YM}}^2 N_c)M_{\mathrm{KK}} \sim 17.4 M_{\mathrm{KK}}$. This value, which is a subleading term in the large N_c expansion, is too large, if compared with the mass of the real η'. We expect that the mass formula above, derived in supergravity approximation, would receive large corrections of N_f. Note that the ρ mesons and other massive mesons have masses independent of N_f, in the leading approximation of [17]. Taking into account the supergravity backreaction of the flavor D8-branes (equivalently, the dependence in N_f) is quite difficult technically.

2. $G \to \rho\pi\pi$

First we describe the decay $G \to \rho\pi\pi$ which uses the single vertex (2.31), see Fig. 2 (left). The $SU(2)$ generators are normalized as $\sigma^a/\sqrt{2}$, so the interaction (2.31) is written explicitly as

$$-\sqrt{2}c_5 \epsilon_{abc}\partial_\mu \pi^a \rho_\nu^b \pi^c \frac{\partial^\mu \partial^\nu}{M^2} G = \sqrt{2}c_5 \epsilon_{abc}\partial_0 \pi^a \rho_0^b \pi^c G, \qquad (3.14)$$

where we have used a relation in the rest frame of the glueball G,

$$\frac{\partial_\mu \partial_\nu}{M^2} G = \frac{\delta_0^\mu \delta_0^\nu (ip_G^0)^2}{M^2} G = -\delta_0^\mu \delta_0^\nu G. \qquad (3.15)$$

Labeling the decay products as $\pi^1(\mathbf{p}_{(1)}) + \rho_\nu^2(\mathbf{p}_{(\rho)}) + \pi^3(\mathbf{p}_{(2)})$, the amplitude is

$$\mathcal{M} = \sqrt{2}c_5(ip_{(1)0} - ip_{(2)0})\epsilon_0, \qquad (3.16)$$

where $\epsilon_0 = |\mathbf{p}_{(\rho)}|/m_\rho$ is the zeroth component of the ρ meson polarization vector,

$$\epsilon^\mu = \left(\frac{|\mathbf{p}_{(\rho)}|}{m_\rho}, \frac{-p_{(\rho)0}}{|\mathbf{p}_{(\rho)}|m_\rho}\mathbf{p}_{(\rho)}\right). \qquad (3.17)$$

Other polarization vectors have vanishing ϵ_0.

Next, we evaluate the amplitude for the process $G \to \rho\rho \to \rho\pi\pi$, see Fig. 2 (right). Let us list the Feynman rules: The $\pi^a \rho_\nu^b \pi^c G$ vertex is the same as before, $\sqrt{2}c_5 ip_0^{(\pi^a)}\delta_0^\nu \epsilon_{abc}$, and the $\pi^d \rho_\mu^e \pi^f$ vertex was obtained in [17] as

$$\sqrt{2}c_6 ip_\mu^{(\pi^d)}\epsilon_{def}, \quad \text{where } c_6 \equiv \int_{-\infty}^{\infty} dZ \frac{1}{\pi K}\psi_1 = \frac{24.0}{N_c g_{\mathrm{YM}}}. \qquad (3.18)$$

The $\rho_\mu^a \rho_\nu^a G$ vertex (no sum over a) is

$$\begin{aligned}
&\frac{1}{2}c_2 M_{\mathrm{KK}}^2 \delta_0^\mu \delta_0^\nu - \frac{1}{2}c_3(p_{(\nu)}^\sigma p_{(\mu)\sigma}\eta^{\mu\nu} - p_{(\mu)}^\nu p_{(\nu)}^\mu) \\
&+ \frac{1}{2}c_3(p_{(\mu)0}p_{(\nu)0}\eta^{\mu\nu} - p_{(\mu)}^\nu p_{(\nu)0}\delta_0^\mu - p_{(\nu)}^\mu p_{(\mu)0}\delta_0^\nu \\
&+ p_{(\mu)}^\sigma p_{(\nu)\sigma}\delta_0^\nu \delta_0^\mu) - \frac{3}{4}c_4 \frac{1}{M}(p_{(\nu)0}\eta^{\mu\nu} - p_{(\nu)}^\mu \delta_0^\nu \\
&+ p_{(\mu)0}\eta^{\mu\nu} - p_{(\mu)}^\nu \delta_0^\mu),
\end{aligned} \qquad (3.19)$$

and the ρ meson propagator is

$$\frac{1}{p_{(\rho)}^2 - m_\rho^2 + im_\rho\Gamma_\rho}\left(\delta_\nu^\mu - \frac{p_{(\rho)}^\mu p_{(\rho)\nu}}{m_\rho^2}\right)\delta^{be}. \qquad (3.20)$$

Here we need the ρ meson decay width, $\Gamma_\rho/m_\rho = c_6^2/24\pi$, which can be evaluated as $\Gamma_\rho/m_\rho = 0.307$ in the holographic QCD [17,18]. This is close to the experimental value $\Gamma_\rho/m_\rho = 149.4/775.5 = 0.1927$ in [1]. As for the polarization vectors, in addition to (3.17), we have two more vectors. One is

$$\epsilon^\mu = \left(0, \frac{p_2^{(\rho)}}{\sqrt{(p_1^{(\rho)})^2 + (p_2^{(\rho)})^2}}, \frac{-p_1^{(\rho)}}{\sqrt{(p_1^{(\rho)})^2 + (p_2^{(\rho)})^2}}, 0\right) \qquad (3.21)$$

and the other gives the same decay width with that of the polarization (3.21). Using all of these Feynman rules, we can compute the decay amplitude for the process $G \to \rho\rho \to \rho\pi\pi$.

The total expression for the decay width is lengthy, and we here provide only the numerical results after substituting the necessary inputs used also in the evaluation of the $\Gamma_{G\to\pi\pi}$. We obtain

$$\frac{\Gamma_{G\to\rho\pi\pi}}{M} = 3 \times (1.1 \times 10^{-8} + 2 \times (2.1 \times 10^{-7}))$$
$$\sim 1.3 \times 10^{-6}. \qquad (3.22)$$

The factor 3 is for the sum over possible flavor indices. (The decay width we computed is for the decay $G \to \pi^1\rho^2\pi^3$, and there are two other possible combinations for the indices.) Terms in the parentheses are for different polarizations of the final ρ meson.

Our result (3.22) is very small. The smallness mainly comes from the fact that the integration region of the momentum is so small because m_ρ is very close to M in our holographic computation. In reality, the mass of $f_0(1500)$ is much larger than the mass of the ρ meson. So, as a trial, in our computation of the decay width, let us modify[22] the input glueball mass M such that M/m_ρ coincides with the experimental value in [1]. Then, we obtain

$$\frac{\Gamma_{G\to\rho\pi\pi}}{M} = 0.096. \qquad (3.23)$$

Since the decay product $\rho\pi\pi$ seems to be included in $G \to 4\pi$ in [1], we compare our result with the experimental value after adding the decay width of $G \to 4\pi$ which we compute next.

3. $G \to 4\pi$

The computation of this amplitude is done in the same manner, and we do not write it explicitly, except for some important points. First, it is easy to find out that the decay product is only in the combination $G \to 2\pi^i 2\pi^j (i \neq j)$, so one can specifically choose $i = 1$, $j = 2$ for the computation. (This determines the index for the ρ meson as ρ^3.) Since the amplitude is proportional to

$$\delta^{be}\epsilon_{abc}\epsilon_{def} = \delta_{ad}\delta_{cf} - \delta_{af}\delta_{cd}, \qquad (3.24)$$

all possible ways to assign the adjoint index for each of the final pions are

$\pi^1(p_{(1)})$	a	d	a	d	a	f	a	f	c	f	c	f	c	d	c	d
$\pi^1(p_{(2)})$	d	a	d	a	f	a	f	a	f	c	f	c	d	c	d	c
$\pi^1(p_{(3)})$	c	c	f	f	c	c	d	d	a	a	d	d	a	a	f	f
$\pi^1(p_{(4)})$	f	f	c	c	d	d	c	c	d	d	a	a	f	f	a	a
sign	+	+	+	+	−	−	−	−	+	+	+	+	−	−	−	−

We have to sum the amplitude with these substitutions, with the sign indicated in the table.

The main difficulty in the computation resides in the evaluation of the integration of the final momenta, $\int d^3\mathbf{p}_{(1)} d^3\mathbf{p}_{(2)} d^3\mathbf{p}_{(3)} d^3\mathbf{p}_{(4)}$. In the integrand of the decay width, there is a four-dimensional delta function coming from the total energy-momentum conservation. The integration $\int d^3\mathbf{p}_{(4)}$ trivially eliminates three delta functions of the momentum conservation. Furthermore, using spatial rotation symmetry, we can orient one of the remaining momenta as $\vec{\mathbf{p}}_{(3)} = (p, 0, 0)$, then the remaining single delta function for the energy conservation can be eliminated by the integration $\int d^3\mathbf{p}_{(3)} = 4\pi \int p^2 dp$. Specifically, the delta function is expressed as

$$\delta\left(p + \sqrt{p^2 + 2p(p_{(1)x} + p_{(2)x}) + |\mathbf{p}_{(1)} + \mathbf{p}_{(2)}|^2} + |\mathbf{p}_{(1)}| + |\mathbf{p}_{(2)}| - M \right)$$
$$= \frac{2(M - |\mathbf{p}_{(1)}| - |\mathbf{p}_{(2)}|)(p_{(1)x} + p_{(2)x}) + |\mathbf{p}_{(1)} + \mathbf{p}_{(2)}|^2 + (M - |\mathbf{p}_{(1)}| - |\mathbf{p}_{(1)}|)^2}{2(M - |\mathbf{p}_{(1)}| - |\mathbf{p}_{(2)}| + p_{(1)x} + p_{(2)x})^2}$$
$$\times \delta\left(p - \frac{(M - |\mathbf{p}_{(1)}| - |\mathbf{p}_{(2)}|)^2 - |\mathbf{p}_{(1)} + \mathbf{p}_{(2)}|^2}{2(M - |\mathbf{p}_{(1)}| - |\mathbf{p}_{(2)}| + p_{(1)x} + p_{(2)x})} \right). \qquad (3.25)$$

So the integration over p results in a constraint for the remaining momenta $\mathbf{p}_{(1)}$ and $\mathbf{p}_{(2)}$. This constraint corresponds to a restriction on the integration region, $|\mathbf{p}_{(1)}| + |\mathbf{p}_{(2)}| + |\mathbf{p}_{(1)} + \mathbf{p}_{(2)}| \leq M$.

Numerical integration for the remaining momenta gives the decay width,[23]

$$\frac{\Gamma_{G\to4\pi}}{M} \sim 2.2 \times 10^{-5}. \qquad (3.26)$$

If we adjust the glueball mass to the experimental value

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[22]There is a possibility that this hand-waving argument may be justified by slightly modifying the probe D8-brane configurations. (For example, the location of the D8-brane can be shifted off the tip of the bulk geometry.) It may change the mass scale of the ρ mesons compared to the mass scale of the glueballs, while the structure of the glueball-meson couplings is maintained. In this paper, we have not calculated anything on possible modifications of the probe D8-brane configuration. Here we simply shifted only the glueball mass as a first trial.

[23]This value includes a factor 3 accounting for different combinations of the species of the final decay product.

(while fixing the ρ meson mass by $m_\rho = \sqrt{\lambda_1} M_{KK}$) as was done before, we obtain

$$\frac{\Gamma_{G\to 4\pi}}{M} \sim 0.0087. \tag{3.27}$$

Let us compare our results (3.23) and (3.27) with the experimental values. Adding these two, we obtain

$$\frac{\Gamma_{G\to 4\pi} + \Gamma_{G\to\rho\pi\pi}}{M} \sim 0.105, \tag{3.28}$$

while the experimental data [1] shows

$$\frac{\Gamma_{G\to 4\pi}^{(ex)}}{M} = \frac{109}{1507} \times 49.5\% = 0.0358. \tag{3.29}$$

One can see that the order of this decay width is reproduced in the holographic QCD.[24]

IV. CONCLUDING REMARKS

We have presented here the first attempt in computing decays of glueballs to $q\bar{q}$ mesons using holographic QCD. We have adopted a string-theoretic setup, (which is of the so-called "top-down" type), the Sakai-Sugimoto model. The glueball sector lives in supergravity fluctuations in the Witten's background of non-Bogomol'nyi-Prasad-Sommerfeld (BPS) black 4-branes, and the mesons live on the probe D8-branes. The coupling between the two sectors is encoded in the D8-brane action, and KK decomposition and integration over extra dimensions gives the desired couplings in four spacetime dimensions.

Explicit couplings between the lightest glueball and the $q\bar{q}$ mesons are given, and the associated decay products/widths are calculated. We find that our results are consistent with the experimental data of the decay for the $f_0(1500)$ which is thought to be the best candidate of a glueball in the hadronic spectrum.

The most important merit of the holographic QCD is that one can go beyond the chiral perturbation theory; one can compute coefficients which cannot be fixed solely by the chiral symmetry. At low energy the chiral perturbation works well, but at the energy scale of the glueball mass the derivative expansion in the chiral perturbation becomes unreliable. Furthermore, glueballs are flavor-blind, so it is quite difficult to constrain possible interactions from the chiral symmetry. The holographic description obtained in the holographic QCD is, in principle, equivalent to QCD, though in the large N_c and large 't Hooft coupling limit.[25]

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[24]If we include the mass of the pions in some way, the decay width (3.28) to the four pions is expected to become significantly smaller and close to the experimental data.

[25]Precisely speaking, the limit to QCD includes $M_{KK} \to \infty$ (there should be a double-scaling limit with a simultaneous scaling of the 't Hooft coupling), after incorporating an infinite number of $1/N_c$ corrections. The background receives large stringy corrections and becomes essentially a purely stringy background.

We therefore expect that the holographic approach should provide interesting information on strong coupling physics of QCD. In fact, we have discussed generic features of the glueball interactions predicted in holographic QCD (see Sec. II B). For example, we have argued that, among the decay products of glueballs, $4\pi_0$ should be suppressed.

One of the reasons why the $f_0(1500)$ is expected to be a glueball state is that the $f_0(1500)$ does not decay to 2γ. In the holographic QCD, we can compute relevant photon coupling in the same manner, and find that $G\gamma\gamma$ coupling is vanishing at the leading order (see [17,18] for the way to introduce the electromagnetic field as an external background of the massless fields on the D8-branes). Since we have shown that there is no mixing with $q\bar{q}$ mesons at the leading order, our result of the suppressed photon coupling reproduces the experimental data.

In this paper, we have explicitly computed for the decay of the lightest glueball, which is of the most phenomenological interest. There are also many other interesting directions, e.g., generalizing our results using various approaches in holographic QCD. Here are some examples:

(i) Multi-glueball couplings.—Self-couplings of the glueballs can be computed in the supergravity sector. Emission of mesons from a propagating glueball can be described by the D8-brane action similarly. For highly spinning glueballs whose holographic dual are closed fundamental strings in the confining supergravity background, their decay into two glueballs was briefly described in [22].

(ii) Universally narrow width of glueballs.—If one can show in the holographic QCD that the total decay width of any glueball state is narrow, that would provide support for this widely held belief. In this paper we have shown the narrowness only for the lightest glueball. Explicit calculation of the widths is possible for other glueball excitations, as they are available in [12]. The 2^{++} glueball coupling has been described in (2.14) for example.

(iii) Glueballs with other J^{PC} quantum numbers.—Glueball states originating in the RR fields in the supergravity may possess interesting structure in the meson couplings. The 0^{-+} glueball is described by a RR one-form C_τ whose fluctuation is completely decoupled from the others, and it appears in the Chern-Simons coupling in the D8-brane action. The 1^{+-} glueballs reside in the NS-NS two-form field, and it should have a large mixing with the meson fields. This is a consequence of the gauge invariance in the supergravity, requiring the gauge-invariant combination $B_{NSNS} + F$ in the D8-brane action.

(iv) Thermal/dense QCD.—One can modify the supergravity background, or introduce a background for the D8-brane fields, to describe the finite tempera-

ture/baryon density,[26] that surely will modify the couplings which we have computed. Glueball couplings should be sensitive to the deconfinement temperature, and, near the transition temperature, they should become singular in some sense. This is of phenomenological interest in view of the onset of the LHC.

(v) Computation of the glueball couplings in other models of holographic QCD.—For a single-flavor case, flavor D6-branes enable one to introduce easily the quark mass [16], which might shed some light on how our results may be modified by the pion masses. To apply our strategy to the so-called bottom-up

phenomenological approach in holographic QCD may reveal how universal the glueball couplings obtained in our paper are. All of these would be interesting to investigate.

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[26]There are many recent works on introducing finite temperature or finite baryon density (or chemical potential) in holographic models of QCD. Specifically, for the Sakai-Sugimoto model, see [23].

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