

Toward the AdS/CFT gravity dual for high energy collisions. II. The stress tensor on the boundary

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In this second paper of the series, we calculate the stress tensor of excited matter, created by debris of high energy collisions at the boundary. The falling open strings, connected to receding charges, produce a nonzero stress tensor which we found analytically from time-dependent linearized Einstein equations in the bulk. It corresponds to exploding nonequilibrium matter: we discuss its behavior in some detail, including its internal energy density in a comoving frame and the “freeze-out surfaces.” We then discuss what happens for the ensemble of strings.

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I. INTRODUCTION

This is the second paper of the series, and the first one [1] (to be referred to as [I] below) had a rather extensive introduction. Therefore we just briefly reiterate here the main goals of this study.

The holographic description of $\mathcal{N} = 4$ SYM theory in the strong coupling regime can be achieved via AdS/CFT correspondence, which relates it to string theory in $\text{AdS}_5 \times S^5$ space, in classical supergravity regime. A large number of applications use this tool to study properties of strongly coupled quark-gluon plasma: but most of them are done in a static setting, with fixed temperature via Witten’s AdS black hole metric.

High energy hadronic collisions in QCD are very difficult problems. They are time dependent and include non-equilibrium physics [only collisions of heavy ions can be approximated by hydrodynamics and locally equilibrated quark gluon plasma (QGP)]. On top of that, they involve different scales and different coupling regimes. Pure phenomenological approaches were developed long ago, such as e.g. Lund model [2], which are based on a picture of QCD strings stretched by departing partons. A more recent approach—known as the color glass picture—was proposed by McLerran and Venugopalan [3], who argued that since fluctuations of high energy collisions lead to large local color charges (in the transverse plane), they would lead to production of strong color fields. Those are treated by the classical Yang-Mills equation in a *weak coupling* regime.

Arguments suggested recently put forward a view that QCD has a certain “strong coupling window.” In particular, Brodsky and Teramond [4] have argued that the power scaling observed for a large number of exclusive processes is not due to perturbative QCD (as suggested originally in the 1970’s) but to a strong coupling regime in which the running is absent and the quasiconformal regime sets in. Polchinski and Strassler [5] have shown that, in spite of exponential string amplitudes, one does get power laws scaling for exclusive processes, due to convolution (integration over the z variable) with the power tails of hadronic wave functions. One of us proposed a scenario [6] for AdS/

QCD in which there are two domains, with weak and strong coupling. The gauge coupling rapidly rises at the “domain wall” associated with instantons. Pion diffractive dissociation is a process where a switch and weak coupling domains are observed: and cross section behavior is consistent with the Polchinski-Strassler expression and expected coupling change. Last but not least, such an approach looks now natural in comparison to what happens in heavy ion/finite T QCD, where we do know now that at comparable parton densities the system indeed is in a strong coupling regime.

Accepting the color glass picture as an asymptotic for very high parton density and large saturation scale $Q_s \rightarrow \infty$, one should ask what should happen in the case of saturation scale falling to intermediate momenta $Q_s \sim .3\text{--}1.5$ GeV associated with “strongly coupled window.” This is the issue we address in this work, using the AdS/CFT correspondence in its time-dependent version, as a tool to describe the evolution of the system.

The setting has been discussed in [I], where we extensively studied how exactly the “debris” produced in a collision—particles or open strings—are falling under gravity force into IR (the AdS throat). In this work we do the next technical step and calculate the backreaction of gravity by solving the linearized Einstein equations for metric perturbations, deducing the space-time dependence of the stress tensor $T_{\mu\nu}(x)$ of excited matter observed on the boundary.

The general setting is in fact rather similar to the Lund model: except that strings are departing from our world ($z = 0$ boundary) rather than breaking. Technically our work is a development along a line actively pursued by many authors. (Early work in a different scenario focused on the effective stress tensor on a brane [7]). In particular, it can be considered a continuation of our recent work [8] in which we calculated static (time-independent) stress tensor associated with Maldacena’s static dipole. It is also similar to recent AdS/CFT calculations of a hydrodynamical “conical flow” from quenching jet in QGP [9,10].

The process we describe resembles what happens in heavy ion collisions, but with very important distinction.

In the setting of this paper we treat debris as small perturbation, solving linearized Einstein equations in pure AdS₅ background. Therefore there is no black hole and/or temperature in this work, and our “miniexplosion” produces matter which is not equilibrated and the resulting stress tensor cannot be parametrized hydrodynamically.

To get all that one needs to proceed to nonlinear gravity and a nonlinear process of black hole formation: the problem which we hope to attack elsewhere.

II. SOLVING THE LINEARIZED EINSTEIN EQUATION

We want to solve the linearized Einstein equation in AdS₅ background in Poincare coordinates, with standard background metrics

$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2} + h_{\mu\nu}. \quad (1)$$

An axial gauge is chosen for metric perturbation, so $h_{z\mu} = 0$.

The linearized Einstein equation is well known and the tactics used in its solution are discussed in [8]: the present case is only different by appearance of time derivatives. We put it into the form

$$\frac{1}{2}\square h_{mn} - 2h_{mn} + \frac{z}{2}h_{mn,z} = s_{mn}, \quad (2)$$

where $\square = z^2(-\partial_t^2 + \partial_{\vec{x}}^2 + \partial_z^2)$, the indices are 0–3 and the right-hand side is the generalized source

$$s_{mn} = \delta S_{mn} - \int_0^z (\delta S_{zm,n} + \delta S_{zn,m}) dz + \frac{1}{2}h_{,m,n} + \frac{1}{2}\Gamma_{mn}^z h_{,z} \quad (3)$$

containing not only the stress tensor of the source

$$\delta S_{\mu\nu} = -\kappa^2 \left(T_{\mu\nu} - \frac{T}{3} g_{\mu\nu} \right) \quad (4)$$

but also the following combinations of perturbation metric which can be easily found from the equations:

$$h = \frac{1}{3} \int_0^z dz \cdot z \left[\delta S_{zz} + \delta S_{tt} - \Sigma_i \delta S_{x^i x^i} + 2 \int_0^z dz' (-\delta S_{zt,t} + \Sigma_i \delta S_{zx^i, x^i}) \right].$$

The source term for different objects is standard, obtained via variation of their action over the metric

$$S_{\text{NG}} = -\frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{-\det g_{\text{ind}}} \int d^5x \delta^{(5)}(x - X(\sigma)) \quad (5)$$

$$\begin{aligned} T^{\mu\nu} &= -\frac{2\delta S_{\text{NG}}}{\sqrt{-g}\delta g_{\mu\nu}} \\ &= \frac{1}{\sqrt{-g}2\pi\alpha'} \int d^2\sigma \delta^{(5)}(x - X(\sigma)) \partial_\alpha X^\mu \partial_\beta X^\nu g_{\text{ind}}^{\beta\alpha}. \end{aligned} \quad (6)$$

Here $g_{\mu\nu}$ and $g_{\text{ind},\alpha\beta}$ denote the AdS metric and the induced metric on the string world sheet, respectively.

In the next section we find an expression for Green’s function, which will provide h_{mn} for any given source s_{mn} . We will then extract an expression for the coefficient of the z^2 term in Taylor series of h_{mn} at the boundary, which by the rules of AdS/CFT correspondence gives us the boundary stress tensor.

III. THE GREEN’S FUNCTION FOR THE LINEARIZED GRAVITY IN AdS₅

The Green’s function we need satisfies the following equation:

$$\begin{aligned} \frac{z^2}{2} (\partial_z^2 - \partial_t^2 + \partial_{\vec{x}}^2) G(z, z') - 2G(z, z') + \frac{z}{2} \partial_z G(z, z') \\ = \delta(z - z') \delta(t - t') \delta^{(3)}(x - x') \end{aligned} \quad (7)$$

and the solution to (2) is then given by $h_{mn}(z) = \int G(z, z') s_{mn}(z') dz'$. Thus $G(z, z')$ should satisfy the same boundary condition as $h(z)$. Fourier transforming the 4-dim part of (7), we have the z -dependent equation

$$\begin{aligned} \frac{z^2}{2} (\partial_z^2 + \omega^2 - k^2) G(z, z') - 2G(z, z') + \frac{z}{2} \partial_z G(z, z') \\ = \delta(z - z'), \end{aligned} \quad (8)$$

where $G^k(z, z') = \int G(z, z') e^{-i\omega t + ik\vec{x}} dt d^3x$.

Equation (8) can be solved in terms of Bessel functions: For $|\omega| > k$, the solution is a linear combination of Bessel functions of the first and second kind. We impose the following boundary condition: at $z = 0$, $G(z, z') = 0$ ($h(z) = 0$); at $z \rightarrow \infty$, $G(z, z')$ ($h(z)$) contains outgoing the wave only, i.e. the wave propagates from the source to infinity [11]. The solution is composed of two homogeneous solutions:

$$G(z, z') = \begin{cases} AJ_2(\lambda z) & z < z' \\ B(J_2(\lambda z) - i\text{sgn}(\omega)Y_2(\lambda z)) & z > z' \end{cases} \quad (9)$$

with $\lambda = \sqrt{\omega^2 - k^2}$, A, B is fixed by matching the function itself and its first derivative at $z = z'$:

$$\begin{cases} A = \frac{\pi i \text{sgn}(\omega)}{z'} (J_2(\lambda z') - i\text{sgn}(\omega)Y_2(\lambda z')) \\ B = \frac{\pi i \text{sgn}(\omega)}{z'} J_2(\lambda z'). \end{cases} \quad (10)$$

For $|k| > \omega$, the solution can be built from the modified Bessel function. We choose the no exponential growth boundary condition at $z \rightarrow \infty$ [8]. The solution is given by

$$G(z, z') = \begin{cases} CI_2(\tilde{\lambda}z) & z < z' \\ DK_2(\tilde{\lambda}z) & z > z' \end{cases} \quad (11)$$

with $\tilde{\lambda} = \sqrt{k^2 - \omega^2}$

$$\begin{cases} C = \frac{-2K_2(\tilde{\lambda}z')}{z'} \\ D = \frac{-2I_2(\tilde{\lambda}z')}{z'} \end{cases} \quad (12)$$

It turns out the solution can be organized in a compact form using properties of Bessel function:

$$G(z, z') = \begin{cases} -\frac{2}{z'} I_2(i\lambda z_{<}) K_2(i\lambda z_{>}) & \omega > 0, \omega > k \\ -\frac{2}{z'} I_2(-i\lambda z_{<}) K_2(-i\lambda z_{>}) & \omega < 0, \omega > k \\ -\frac{2}{z'} I_2(\tilde{\lambda}z_{<}) K_2(\tilde{\lambda}z_{>}) & \omega < k, \end{cases} \quad (13)$$

where $z_{<} = \min(z, z')$, $z_{>} = \max(z, z')$.

Doing the inverse Fourier transform, $\frac{1}{(2\pi)^4} \times \int G^k(z, z') e^{i\omega t - i\vec{k}\vec{x}} d\omega d^3k$, we obtain a retarded propagator for the metric: (See Appendix A for the evaluation of the integral, a retarded scalar propagator was found in [12]. Other scalar and graviton propagators were also found in [13] with slightly different boundary conditions)

$$P_R = \frac{12iz}{(2\pi)^2} \left[\frac{1}{(t^2 - r^2 - z^2 + i\epsilon)^4} - \frac{1}{(t^2 - r^2 - z^2 - i\epsilon)^4} \right] \theta(t - r). \quad (14)$$

Several comments of the propagator are in order: (i) The theta function implies the propagator acts inside the light cone $t^2 - r^2 = z^2 > 0$ and moreover is retarded $t > r > 0$, which we indicate by the subscript R . It is also consistent with the outgoing boundary condition. Note that the propagator is also Lorentz invariant. (ii) The propagator relates the z^2 coefficient of metric perturbation Q_{mn} and the source s_{mn} in the following way [assuming $s_{mn}(z)$ does not contain z^0 and z^2 terms]: $Q_{mn}(t', x') = \int P_R(t' - t, x' - x, z) s_{mn}(z, t, x) dz dt d^3x$. The primed and unprimed coordi-

nates correspond to boundary and bulk, respectively. (iii) The propagator is dynamical. For static source, one can perform the t -integral to obtain a static propagator, which agrees with the one obtained in [8].

IV. THE STRESS TENSOR OF A FALLING OPEN STRING

We want to study the stress tensor by a falling string. A scaling string profile is obtained in [1] for a separating quark-antiquark pair, provided the separating velocity is not too large: $v < 0.6$. We briefly recall the scaling solution. The quark (antiquark) moves along the trajectory $x = \pm vt$. The string profile is given by

$$z = \frac{\tau}{f(y)} \\ y = f_0 \sqrt{\frac{f_0^2 - 1}{2f_0^2 - 1}} F\left(\sqrt{\frac{f^2 - f_0^2}{f^2 - 1}}, \frac{f_0}{\sqrt{2f_0^2 - 1}}\right) - \frac{1}{f_0} \\ \times \sqrt{\frac{(f_0^2 - 1)^3}{2f_0^2 - 1}} \Pi\left(\sqrt{\frac{f^2 - f_0^2}{f^2 - 1}}, \frac{1}{f_0}, \frac{f_0}{\sqrt{2f_0^2 - 1}}\right). \quad (15)$$

τ and y are proper time and space-time rapidity. The $y = Y$ limit of (15) relates the parameter f_0 and the quark rapidity $Y = \text{arctanh}(v)$. It is also very useful to write down the equations of motion (EOM) of $f(y)$:

$$f' = \frac{\sqrt{V(V - E^2)}}{E} \quad (16)$$

with $V = f^4 - f^2$, $f_0^4 - f_0^2 - E^2 = 0$.

We want the source term due to the scaling string. It is convenient to switch to a different parametrization:

$$z = z(t, x), \quad x_{\perp} = 0, \quad (17)$$

where x and x_{\perp} represent longitudinal and transverse coordinates, respectively. The above parametrization leads directly to

$$\delta S_{\mu\nu} = -\frac{1}{3} \frac{\kappa^2 z}{2\pi\alpha'} \delta(z - \bar{z}) \delta(x^2) \delta(x^3) \frac{1}{\sqrt{1 - z_t^2 + z_x^2}} \cdot \begin{pmatrix} 1 + 2z_t^2 + z_x^2 & 3z_t z_x & -3z_t & & & \\ 3z_t z_x & z_t^2 + 2z_x^2 - 1 & -3z_x & & & \\ -3z_t & -3z_x & 2 + z_t^2 - z_x^2 & & & \\ & & & 2(1 - z_t^2 + z_x^2) & & \\ & & & & 2(1 - z_t^2 + z_x^2) & \end{pmatrix}. \quad (18)$$

(In the matrices here and below we only show the nonzero entities: the adopted order of coordinate indices is t, z, x^1, x^2, x^3 .) With the help of string EOM, (15) of [14], h can be expressed in a very compact form: $h = -\frac{2}{3} \frac{\kappa^2}{2\pi\alpha'} \delta(x^2) \delta(x^3) \times \frac{z_t^2 - z_x^2 + 2}{\sqrt{1 - z_t^2 + z_x^2}} \frac{1}{2} (z^2 - \bar{z}^2) \theta(z - \bar{z})$. We also record the generalized source for later reference:

$$\begin{aligned}
s_{mn} = & \frac{1}{3} \frac{-\kappa^2}{2\pi\alpha'} \delta(x^2)\delta(x^3) \left[\frac{\bar{z}}{\sqrt{1-z_t^2+z_x^2}} \delta(z-\bar{z}) \begin{pmatrix} 1+2z_t^2+z_x^2 & 3z_t z_x & & & \\ & 3z_t z_x & z_t^2+2z_x^2-1 & & \\ & & & 2(1-z_t^2+z_x^2) & \\ & & & & 2(1-z_t^2+z_x^2) \end{pmatrix} \right. \\
& + \begin{pmatrix} 2\partial_t & \partial_x & \partial_{x_2} & \partial_{x_3} \\ \partial_x & & & \\ \partial_{x_2} & & & \\ \partial_{x_3} & & & \end{pmatrix} \frac{3\bar{z}z_t}{\sqrt{1-z_t^2+z_x^2}} \theta(z-\bar{z}) + \begin{pmatrix} \partial_t & 2\partial_x & \partial_{x_2} & \partial_{x_3} \\ \partial_{x_2} & & & \\ \partial_{x_3} & & & \end{pmatrix} \frac{3\bar{z}z_x}{\sqrt{1-z_t^2+z_x^2}} \theta(z-\bar{z}) \\
& \left. + \begin{pmatrix} \partial_t^2 & \partial_t\partial_x & \partial_t\partial_{x_2} & \partial_t\partial_{x_3} \\ \partial_t\partial_x & \partial_x^2 & \partial_x\partial_{x_2} & \partial_x\partial_{x_3} \\ \partial_t\partial_{x_2} & \partial_x\partial_{x_2} & \partial_{x_2}^2 & \partial_{x_2}\partial_{x_3} \\ \partial_t\partial_{x_3} & \partial_x\partial_{x_3} & \partial_{x_2}\partial_{x_3} & \partial_{x_3}^2 \end{pmatrix} \frac{z_t^2-z_x^2+2}{\sqrt{1-z_t^2+z_x^2}} \frac{1}{2} (z^2-\bar{z}^2) \theta(z-\bar{z}) + \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \frac{z_t^2-z_x^2+2}{\sqrt{1-z_t^2+z_x^2}} \theta(z-\bar{z}) \right].
\end{aligned} \tag{19}$$

With the source now at hand, we proceed to the calculation of stress tensor. We use similar substitutions as before: $\vec{\partial}_x = -\overleftarrow{\partial}_x = \overleftarrow{\partial}'_x$. Performing the derivative explicitly, we find the z -integral and x_\perp -integral can be done easily. We arrive at the following result:

$$\begin{aligned}
Q_{mn} = & \frac{1}{3} \frac{-\kappa^2}{2\pi\alpha'} \int dt dx \left[\frac{\bar{z}^2}{\sqrt{1-z_t^2+z_x^2}} P \begin{pmatrix} 1+2z_t^2+z_x^2 & 3z_t z_x & & & \\ & 3z_t z_x & z_t^2+2z_x^2-1 & & \\ & & & 2(1-z_t^2+z_x^2) & \\ & & & & 2(1-z_t^2+z_x^2) \end{pmatrix} \right. \\
& + \begin{pmatrix} 2(t'-t) & -(x'-x) & -x'_2 & -x'_3 \\ -(x'-x) & & & \\ -x'_2 & & & \\ -x'_3 & & & \end{pmatrix} \frac{3\bar{z}z_t}{\sqrt{1-z_t^2+z_x^2}} P + \begin{pmatrix} t'-t & -2(x'-x) & -x'_2 & -x'_3 \\ & -x'_2 & & \\ & -x'_3 & & \end{pmatrix} \\
& \left. \times \frac{3\bar{z}z_x}{\sqrt{1-z_t^2+z_x^2}} P + \begin{pmatrix} (t'-t)^2 & -(t'-t)(x'-x) & -(t'-t)x'_2 & -(t'-t)x'_3 \\ -(t'-t)(x'-x) & (x'-x)^2 & (x'-x)x'_2 & (x'-x)x'_3 \\ -(t'-t)x'_2 & (x'-x)x'_2 & x_2'^2 & x_2'x_3' \\ -(t'-t)x'_3 & (x'-x)x'_3 & x_2'x_3' & x_3'^2 \end{pmatrix} \frac{z_t^2-z_x^2+2}{\sqrt{1-z_t^2+z_x^2}} P \right]
\end{aligned} \tag{20}$$

with

$$\begin{aligned}
P = & \frac{12i}{(2\pi)^2} \left[\frac{1}{((t'-t)^2 - (x'-x)^2 - x_\perp'^2 - \bar{z}^2 + i\epsilon)^4} - \frac{1}{((t'-t)^2 - (x'-x)^2 - x_\perp'^2 - \bar{z}^2 - i\epsilon)^4} \right] \\
= & \frac{12i}{(2\pi)^2} \frac{\pm 1}{((t'-t)^2 - (x'-x)^2 - x_\perp'^2 - \bar{z}^2 \pm i\epsilon)^4} \theta(t'-t)
\end{aligned} \tag{21}$$

which is just the integrated propagator. The four matrices in the expression above we will refer to later as I, II, III, and IV, respectively.

Here we replaced the theta function of P_R by $\theta(t'-t)$. It is justified since the $\pm i\epsilon$ prescription encodes derivatives of the delta function, and the theta function picks up only one pole of the propagator.

In order to plug in the scaling solution for the string, we return to τ, y coordinates:

$$\begin{aligned}
z_t = & \frac{chy}{f} + \frac{shyf'}{f^2} & z_x = & -\frac{shy}{f} - \frac{chyf'}{f^2} \\
\int dt dx = & \int \tau d\tau dy.
\end{aligned}$$

The source together with the integration measure has one of the following simple τ -dependence: τ, τ^2, τ^3 . The propagator now is

$$\begin{aligned}
P &= \frac{12i}{(2\pi)^2} \\
&\times \frac{\pm\theta(\tau' - \tau)}{\left(\left(1 - \frac{1}{f^2}\right)\tau^2 + \tau'^2 - 2\tau\tau'ch(y - y') - x_{\perp}^2 \pm i\epsilon\right)^4} \\
&= \frac{12i}{(2\pi)^2} \frac{\pm 1}{\left(\left(1 - \frac{1}{f^2}\right)(\tau - \tau_+)(\tau - \tau_-) \pm i\epsilon\right)^4} \theta(\tau' - \tau)
\end{aligned} \tag{22}$$

with

$$\tau_{\pm} = \frac{\tau'ch(y' - y) \pm \sqrt{\tau'^2ch^2(y' - y) - (\tau'^2 - x_{\perp}^2)(1 - \frac{1}{f^2})}}{1 - \frac{1}{f^2}}. \tag{23}$$

$$\begin{aligned}
T_{mn} &= \frac{-\sqrt{\lambda}}{3\pi} \int_{-Y}^Y dy \left[\begin{pmatrix} 1 + 2z_t^2 + z_x^2 & 3z_t z_x & & & \\ 3z_t z_x & z_t^2 + 2z_x^2 - 1 & & & \\ & & 2(1 - z_t^2 + z_x^2) & & \\ & & & 2(1 - z_t^2 + z_x^2) & \\ & & & & \frac{1}{f^2\sqrt{1 - z_t^2 + z_x^2}} A \end{pmatrix} \right. \\
&+ \begin{pmatrix} 2t' & -x' & -x'_2 & -x'_3 \\ -x' & & & \\ -x'_2 & & & \\ -x'_3 & & & \end{pmatrix} \frac{3z_t}{f\sqrt{1 - z_t^2 + z_x^2}} B + \begin{pmatrix} -2chy & shy & 0 & 0 \\ shy & & & \\ 0 & & & \\ 0 & & & \end{pmatrix} \frac{3z_t}{f\sqrt{1 - z_t^2 + z_x^2}} A \\
&+ \begin{pmatrix} t' & & & \\ t' & -2x' & -x'_2 & -x'_3 \\ -x'_2 & & & \\ -x'_3 & & & \end{pmatrix} \frac{3z_x}{f\sqrt{1 - z_t^2 + z_x^2}} B + \begin{pmatrix} -chy & -chy & 0 & 0 \\ -chy & 2shy & 0 & 0 \\ 0 & & & \\ 0 & & & \end{pmatrix} \frac{3z_x}{f\sqrt{1 - z_t^2 + z_x^2}} A \\
&+ \begin{pmatrix} t'^2 & -t'x' & -t'x'_2 & -t'x'_3 \\ -t'x' & x'^2 & x'x'_2 & x'x'_3 \\ -t'x'_2 & x'_2x'_2 & x'_2x'_3 & \\ -t'x'_3 & x'_3x'_3 & x'_2x'_3 & x'_3x'_3 \end{pmatrix} \frac{z_t^2 - z_x^2 + 2}{\sqrt{1 - z_t^2 + z_x^2}} C + \begin{pmatrix} -2t'chy & t'shy + x'chy & x'_2chy & x'_3chy \\ t'shy + x'chy & -2x'shy & -x'_2shy & -x'_3shy \\ x'_2chy & -x'_2shy & & \\ x'_3chy & -x'_3shy & & \end{pmatrix} \\
&\times \left. \frac{z_t^2 - z_x^2 + 2}{\sqrt{1 - z_t^2 + z_x^2}} B + \begin{pmatrix} chy^2 & -chyshy \\ -chyshy & shy^2 \end{pmatrix} \frac{z_t^2 - z_x^2 + 2}{\sqrt{1 - z_t^2 + z_x^2}} A \right] \tag{24}
\end{aligned}$$

$$\begin{aligned}
A &= \frac{1}{\left(1 - \frac{1}{f^2}\right)^4} \frac{\tau_+^3 + 9\tau_+^2\tau_- + 9\tau_+\tau_-^2 + \tau_-^3}{(\tau_+ - \tau_-)^7} \\
B &= \frac{4}{\left(1 - \frac{1}{f^2}\right)^4} \frac{\tau_+^2 + 3\tau_+\tau_- + \tau_-^2}{(\tau_+ - \tau_-)^7} \\
C &= \frac{10}{\left(1 - \frac{1}{f^2}\right)^4} \frac{\tau_+ + \tau_-}{(\tau_+ - \tau_-)^7}.
\end{aligned} \tag{25}$$

A. A field near one charge

We first consider the stress tensor close to the quark. Note the quark moves with velocity v , the observer should

This propagator as a function of τ contains two fourth order poles, so the τ -integral is calculated by the residue theorem. Note that the theta function picks up only one pole at $\tau = \tau_-$. Since our τ -integral extends from zero to infinity, we must have a positive τ_- in order to have a nonvanishing result, which requires $\tau'^2 - x_{\perp}^2 = t'^2 - r'^2 > 0$, precisely the condition that the observer must stay inside the light cone. Since the quark and the antiquark are emerging from the space-time origin, the stress tensor is indeed expected to vanish outside the light cone.

Completing the τ integral and replacing $\frac{1}{3} \frac{-\kappa^2}{2\pi\alpha'}$ by $\frac{-\sqrt{\lambda}}{3\pi}$, we convert Q_{mn} to the boundary stress tensor T_{mn} (compare [8]). We thus get our final result:

also move in order to stay close to the quark. Thus it is convenient to switch to the rest coordinate of the moving quark, yet we still stay in the rest frame of the system. The rest coordinates of the quark, indicated by a tilde, relates the original coordinates in the following way:

$$\begin{aligned}
\tilde{t} &= chYt' - shYx' & \tilde{x} &= chYx' - shYt' \\
\tilde{x}_2 &= x'_2 & \tilde{x}_3 &= x'_3.
\end{aligned} \tag{26}$$

We expect to obtain the field by the quark only, provided we are sufficient close to the quark, which corresponds to the limit $\tilde{t} \gg \tilde{r}$, $\tilde{r} \equiv \sqrt{\tilde{x}^2 + \tilde{x}_2^2 + \tilde{x}_3^2} \rightarrow 0$. Comparing the string profile for the stretching dipole with that for a single

quark, we claim the leading order field near the quark receives contribution from the quark end of the string, which corresponds to the integration of y near $y = Y$ in (24). If we instead do the integral in f with $dy = \frac{df}{f'}$, we may only focus on large f integration.

Note $f' \sim \frac{f^4}{E} y - Y \sim \frac{1}{f^3}$. To the leading order, we can simply replace y by Y , which leads to the following relations:

$$\begin{aligned}\tau_+ - \tau_- &= \frac{2\sqrt{\tilde{r}^2 - (1 - \frac{1}{f^2})(\tilde{r}^2 - \tilde{r}^2)}}{1 - \frac{1}{f^2}} \\ \tau_+^3 + 9\tau_+^2\tau_- + 9\tau_+\tau_-^2 + \tau_-^3 &= \frac{8\tilde{r}^3}{(1 - \frac{1}{f^2})^3} + \frac{12(\tilde{r}^2 - \tilde{r}^2)\tilde{r}}{(1 - \frac{1}{f^2})^2} \\ \tau_+^2 + 3\tau_+\tau_- + \tau_-^2 &= \frac{4\tilde{r}^2}{(1 - \frac{1}{f^2})^2} + \frac{\tilde{r}^2 - \tilde{r}^2}{1 - \frac{1}{f^2}} \\ \tau_+ + \tau_- &= \frac{2\tilde{r}}{1 - \frac{1}{f^2}}.\end{aligned}\quad (27)$$

Performing the f integral near ∞ , and take the limit $\tilde{r} \gg \tilde{r}, \tilde{r} \rightarrow 0$, which is essentially a small \tilde{r} expansion, we find the leading order field given by source I and IV diverges as $O(\frac{1}{\tilde{r}^3})$, while sources II and III only yield subleading contribution $O(\frac{1}{\tilde{r}^2})$. We display the LO field near the quark as

$$\begin{aligned}P &= \frac{12i}{(2\pi)^2} \frac{\pm\theta(\tau' - \tau)}{((1 - \frac{1}{f^2})\tau^2 + \tau'^2 - 2\tau\tau'ch(y - y') - x_\perp'^2 \pm i\epsilon)^4} \\ &= \frac{12i}{(2\pi)^2} \left[\frac{\pm 1}{((1 - \frac{1}{f^2})\tau^2 + \tau'^2 - 2\tau\tau'ch(y' - Y) - x_\perp'^2 \pm i\epsilon)^4} + \frac{\pm 1}{((1 - \frac{1}{f^2})\tau^2 + \tau'^2 - 2\tau\tau'ch(y' - Y) - x_\perp'^2 \pm i\epsilon)^5} \right. \\ &\quad \left. \times (-8\tau\tau'sh(y' - Y)\Delta y) + \dots \right] \theta(\tau' - \tau) \\ &= \frac{12i}{(2\pi)^2} \left[\frac{\pm 1}{((1 - \frac{1}{f^2})\tau^2 + \tau'^2 - 2\tau\tau'ch(y' - Y) - x_\perp'^2 \pm i\epsilon)^4} + \frac{\pm 1}{((1 - \frac{1}{f^2})\tau^2 + \tau'^2 - 2\tau\tau'ch(y' - Y) - x_\perp'^2 \pm i\epsilon)^5} \right. \\ &\quad \left. \times \left(\frac{8E\tau\tilde{x}}{3f^3} \right) + \dots \right] \theta(\tau' - \tau) \\ &= \frac{12i}{(2\pi)^2} [P_4 + P_5 + \dots] \theta(\tau' - \tau)\end{aligned}\quad (29)$$

with

$$\begin{aligned}P_4 &= \frac{\pm 1}{((1 - \frac{1}{f^2})\tau^2 + \tau'^2 - 2\tau\tau'ch(y' - Y) - x_\perp'^2 \pm i\epsilon)^4} \\ P_5 &= \frac{\pm 1}{((1 - \frac{1}{f^2})\tau^2 + \tau'^2 - 2\tau\tau'ch(y' - Y) - x_\perp'^2 \pm i\epsilon)^5} \left(\frac{8E\tau\tilde{x}}{3f^3} \right).\end{aligned}$$

After a relatively lengthy calculation and comparison, we find that the NLO correction is composed of three pieces: the first one is source II and III convoluted with P_4 , the second corresponds to source IV and P_4 , and the third one comes from the convolution of source I and P_5 . Collecting all of them, we find the following result:

follows:

$$\begin{aligned}T_{mn} &= \frac{2\sqrt{\lambda}}{\pi^2} \left[\begin{pmatrix} 1 + 2\gamma^2\beta^2 & -2\gamma^2\beta & & \\ -2\gamma^2\beta & -1 + 2\gamma^2 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \frac{1}{24\tilde{r}^4} \right. \\ &\quad \left. - \begin{pmatrix} \gamma^2\beta^2\tilde{x}^2 & -\gamma^2\beta\tilde{x}^2 & -\gamma\beta\tilde{x}\tilde{x}_2 & -\gamma\beta\tilde{x}\tilde{x}_3 \\ -\gamma^2\beta\tilde{x}^2 & \gamma^2\tilde{x}^2 & \gamma\tilde{x}\tilde{x}_2 & \gamma\tilde{x}\tilde{x}_3 \\ -\gamma\beta\tilde{x}\tilde{x}_2 & \gamma\tilde{x}\tilde{x}_2 & \tilde{x}_2^2 & \tilde{x}_2\tilde{x}_3 \\ -\gamma\beta\tilde{x}\tilde{x}_3 & \gamma\tilde{x}\tilde{x}_3 & \tilde{x}_2\tilde{x}_3 & \tilde{x}_3^2 \end{pmatrix} \frac{1}{12\tilde{r}^6} \right]\end{aligned}\quad (28)$$

with $\gamma = chY$, $\gamma\beta = shY$.

This does not look very nice at first glance, actually it is just the stress tensor of a static quark boosted to a frame moving with velocity $-v$. It is clearly traceless and divergence-free. We confirm that the LO field near the quark contains a contribution from the quark only.

Next we would like to extend the result to next-to-leading order (NLO) to include the effect of the antiquark. Note there are two possible corrections relevant for NLO: correction to the source $\Delta y = y - Y = \frac{\Delta f}{f'} = \frac{E}{3f^3}$, thus $chy = chY + shY\Delta y$, $shy = shY + chY\Delta y$. The other is correction to the propagator:

$$T_{mn} = \frac{2\sqrt{\lambda E}}{\pi^2} \left[- \begin{pmatrix} 5 + 13\gamma^2\beta^2 & -13\gamma^2\beta & & & \\ -13\gamma^2\beta & -5 + 13\gamma^2 & & & \\ & & 8 & & \\ & & & 8 & \\ & & & & 8 \end{pmatrix} \frac{\tilde{x}}{144\tilde{r}^3\tilde{t}^2} + \begin{pmatrix} \gamma^2\beta^2\tilde{x}^2 & -\gamma^2\beta\tilde{x}^2 & -\gamma\beta\tilde{x}\tilde{x}_2 & -\gamma\beta\tilde{x}\tilde{x}_3 \\ -\gamma^2\beta\tilde{x}^2 & \gamma^2\tilde{x}^2 & \gamma\tilde{x}\tilde{x}_2 & \gamma\tilde{x}\tilde{x}_3 \\ -\gamma\beta\tilde{x}\tilde{x}_2 & \gamma\tilde{x}\tilde{x}_2 & \tilde{x}_2^2 & \tilde{x}_2\tilde{x}_3 \\ -\gamma\beta\tilde{x}\tilde{x}_3 & \gamma\tilde{x}\tilde{x}_3 & \tilde{x}_2\tilde{x}_3 & \tilde{x}_3^2 \end{pmatrix} \frac{\tilde{x}}{48\tilde{r}^5\tilde{t}^2} \right. \\ \left. + \begin{pmatrix} 2\gamma^2\beta^2\tilde{x} & -2\gamma^2\beta\tilde{x} & -\gamma\beta\tilde{x}_2 & -\gamma\beta\tilde{x}_3 \\ -2\gamma^2\beta\tilde{x} & 2\gamma^2\tilde{x} & \gamma\tilde{x}_2 & \gamma\tilde{x}_3 \\ -\gamma\beta\tilde{x}_2 & \gamma\tilde{x}_2 & & \\ -\gamma\beta\tilde{x}_3 & \gamma\tilde{x}_3 & & \end{pmatrix} \frac{1}{18\tilde{r}^3\tilde{t}^2} \right]. \quad (30)$$

We had found with satisfaction that this result is indeed traceless and divergence-free [to order $O(\frac{1}{\tilde{r}})$].

After this result is boosted to a frame moving with velocity $+v$, it reproduces the NLO field near the quark of a static dipole with the identification $\frac{E}{\tilde{r}^2} \rightarrow \frac{1}{z_m} \approx \frac{1.2^2}{L^2}$ (L is the quark-antiquark separation). It confirms that close to 1 charge it is not important to this accuracy what the other charge is doing. In fact in the quasistatic limit $v \rightarrow 0$, $\frac{E}{\tilde{r}^2} \approx \frac{1.2^2}{4v^2\tilde{r}^2} = \frac{1.2^2}{L^2}$, where \tilde{L} is the quark-antiquark separation at time \tilde{t} .

B. The slow-moving limit

The stretching string solution depends on only one parameter v , the ends velocity, which is bounded from above by the critical velocity. One interesting limit in which calculations can be pushed a step further is $v \rightarrow 0$, which correspond to slow motion. In practice, it corresponds to explanation of the stress tensor in inverse powers of f_0 .

We start with considering the large f_0 limit of (24). Define $\eta = \frac{f}{f_0} \eta \geq 1$ such that the range of η is independent from f_0 . The large f_0 expansion of y is a little complicated:

$$y = \frac{1}{2} \sqrt{f_0^4 - f_0^2} \left[\frac{2F\left(\frac{f^2 - f_0^2}{f^2 - 1}, \frac{f_0^2}{2f_0^2 - 1}\right)}{\sqrt{2f_0^2 - 1}} - \frac{2(f_0^2 - 1)\Pi\left(\frac{f^2 - f_0^2}{f^2 - 1}, \frac{1}{f_0^2}, \frac{f_0^2}{2f_0^2 - 1}\right)}{\sqrt{2f_0^2 - 1}f_0^2} \right] \\ = \frac{-F\left(\frac{\sqrt{\eta^2 - 1}}{\eta}, \frac{\sqrt{2}}{2}\right) + 2E\left(\frac{\sqrt{\eta^2 - 1}}{\eta}, \frac{\sqrt{2}}{2}\right)}{\sqrt{2}f_0} + O\left(\frac{1}{f_0^3}\right) \\ = \frac{G(\eta)}{f_0} + O\left(\frac{1}{f_0^3}\right), \quad (31)$$

$$\text{where } G(\eta) = \frac{-F\left(\frac{\sqrt{\eta^2 - 1}}{\eta}, \frac{\sqrt{2}}{2}\right) + 2E\left(\frac{\sqrt{\eta^2 - 1}}{\eta}, \frac{\sqrt{2}}{2}\right)}{\sqrt{2}}.$$

With the asymptotic expansion of y and f_0 , we are ready to proceed to the stress tensor. It seems at first glance the leading order is of $O(\frac{1}{f_0})$, given by source IV. Actually the prefactor, which is an integral of η vanishes. The order $O(\frac{1}{f_0^2})$ does not contribute either due to the symmetry $y \leftrightarrow -y$. Finally, we have to extend the calculation to order $O(\frac{1}{f_0^3})$. Expand all the relevant quantities in f_0 , and keep the order $O(\frac{1}{f_0^3})$ in the result of stress tensor. It is a quite lengthy but straightforward calculation. The result is displayed as follows (we have omitted the prime in boundary coordinates):

$$T_{tt} = \frac{2\sqrt{\lambda}}{f_0^3\pi^2} \frac{2t}{r^9} [(10a + 5e_1 + 5e_2)r^2t^2 + 45e_1x^2r^2 - 35e_1t^2x^2 - (9e_2 + 2f + 6a - 24c + 7e_1)r^4] \\ T_{tx} = \frac{2\sqrt{\lambda}}{f_0^3\pi^2} \frac{2x}{r^9} [(45e_1 + 15e_2 - 90d - 30c)r^2t^2 + 15e_1x^2r^2 - 105e_1t^2x^2 - (3e_2 + 7e_1 + 2f - 6c - 18d)r^4] \\ T_{tx_i} = \frac{2\sqrt{\lambda}}{f_0^3\pi^2} \frac{2x_i}{r^9} [(15e_2 + 15e_1 - 30c)r^2t^2 - 105e_1t^2x^2 + 15e_1x^2r^2] \\ T_{xx} = \frac{2\sqrt{\lambda}}{f_0^3\pi^2} \frac{2t}{r^{11}} [(20a - 30b + 10e_1 - 60d)r^4t^2 + (-12a + 18b - 6e_1 + 36d)r^6 + (420d - 175e_1 - 35e_2)r^2t^2x^2 \\ + (-180d + 15e_2 + 65e_1 + 10f)x^2r^4 + 315e_1x^4t^2 - 105e_1x^4r^2] \\ T_{xx_i} = \frac{2\sqrt{\lambda}}{f_0^3\pi^2} \frac{10txx_i}{r^{11}} [(42d - 21e_1 - 7e_2)t^2r^2 - 21e_1x^2r^2 + 63e_1t^2x^2] \\ T_{x_ix_j} = \frac{2\sqrt{\lambda}}{f_0^3\pi^2} \frac{8t}{r^7} (5at^2 - 3ar^2)\delta_{ij} - \frac{2\sqrt{\lambda}}{f_0^3\pi^2} \frac{10tx_ix_j}{r^{11}} [(7e_1 + 7e_2)r^2t^2 - (3e_2 + e_1 + 2f)r^4 + 21e_1x^2r^2 - 63e_1x^2t^2] \quad (32)$$

with

$$\begin{aligned}
 r &= \sqrt{x^2 + x_2^2 + x_3^2} & a &= \int_1^\infty \frac{1}{\eta^2 \sqrt{\eta^4 - 1}} d\eta = 0.5991 & b &= \int_1^\infty \frac{1}{\eta^6 \sqrt{\eta^4 - 1}} d\eta = 0.3594 \\
 c &= \int_1^\infty \left(\frac{1}{\eta} + G(\eta) \sqrt{\eta^4 - 1} \right) \frac{1}{\eta^5 \sqrt{\eta^4 - 1}} d\eta = 0.4493 & d &= \int_1^\infty \frac{G(\eta)}{\eta^5} d\eta = 0.0899 \\
 e_1 &= \int_1^\infty \frac{3 - \eta^4}{\eta^4 \sqrt{\eta^4 - 1}} G(\eta)^2 d\eta = -0.1797 & e_2 &= \int_1^\infty \frac{3 - \eta^4}{\eta^6 \sqrt{\eta^4 - 1}} d\eta = 0.4793 \\
 f &= \int_1^\infty \frac{-5\eta^4 + 6\eta^2 + 9}{2\sqrt{\eta^4 - 1}(\eta^2 + 1)\eta^6} d\eta = 0.7189.
 \end{aligned}
 \tag{33}$$

Several comments about the result are in order: (i) the result applies for the arbitrary point on the boundary, i.e. general t, x, x_2, x_3 , provided the point lies inside the light cone. The discontinuity of the stress tensor on the light cone is a consequence of the discontinuity in source at $t = 0$. ; (ii) Trace and divergence of the stress tensor vanish for any points away from the trajectory of the dipole ends, which is of course implicitly assumed in our calculation. (iii) If we consider the limit $t \gg r$, which amounts to keeping only the highest power of t . Recalling the quasistatic limit: $\frac{L^3}{f_0^3} \sim \left(\frac{vt}{0.6}\right)^3 = \left(\frac{L}{1.2}\right)^3$, where L is the dipole size at

time t . While for the case of static dipole: $z_m^3 = \left(\frac{L}{1.2}\right)^3$, we find the stretching dipole result agrees with static dipole in the double limits: $v \rightarrow 0, t \gg r$. The numerical factors match as well. (vi) The agreement of the quasistatic result and NLO near field with those of static dipole by the simple identification: $L = 2vt$ seems to suggest that the vacuum-quark (antiquark) interaction is instantaneous.

We plot the energy density T^{00} and the momentum density (energy flow) T^{0i} as a function of the spatial coordinates at three different times in Figs. 1 and 2, respectively. We observe that, although the shape of energy

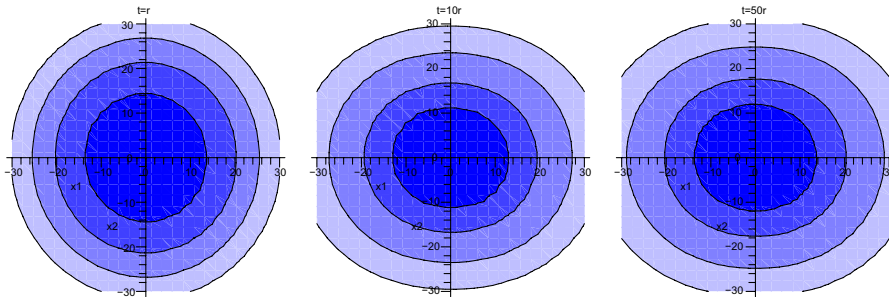


FIG. 1 (color online). The contours of energy density T^{00} , in units of $\frac{2\sqrt{\Lambda}}{f_0^3 \pi^2}$, in the $x_1 - x_2$ plane at different times. The three plots are made for $t = r, t = 10r$, and $t = 50r$ from top to bottom. Note the quark/antiquark is at $x_1 = \pm vt$. In the slow-moving limit, they are nearly at the origin. The magnitude of T^{00} is represented by the color, with darker color corresponding to greater magnitude. As time increases, the shape of the contours gets elongated along the x_1 axis

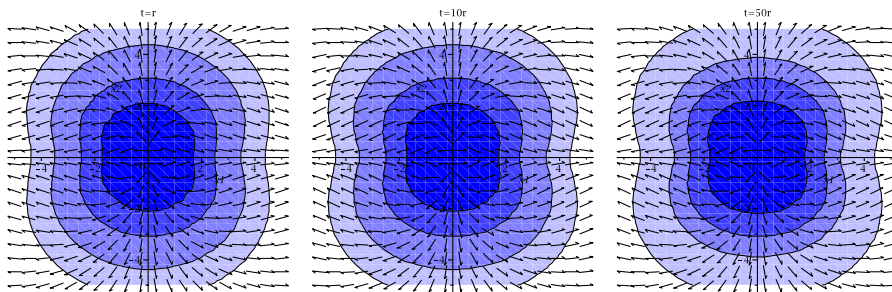


FIG. 2 (color online). The contours of momentum density T^{0i} , in units of $\frac{2\sqrt{\Lambda}}{f_0^3 \pi^2}$, in the $x_1 - x_2$ plane at different times. The three plots are made for $t = r, t = 10r$, and $t = 50r$ from top to bottom. Note the quark/antiquark is at $x_1 = \pm vt$. In the slow-moving limit, they are nearly at the origin. The magnitude is represented by color, with darker color corresponding to greater magnitude. The direction of the momentum density is indicated by normalized arrows

distribution becomes more elongated with time, reflecting changing shape of the string, the shape of the momentum flows seems to stay the same, with an interesting “eight” shape or forward-backward depletion. Small arrows display the direction of the energy flow. Although, overall, they show outgoing explosion away from the origin (the collision point), one can also see some “cumulative” flow with jets converging along the collision axes.

V. THE ENERGY DENSITY OF MATTER IN COMOVING FRAME AND FREEZE-OUT

It is illuminating to ask if the stress tensor we obtained can or cannot be described by some hydrodynamical flow. The latter is widely used in describing heavy ion collisions [15,16]. More precisely, the question is if our stress tensor (32) is that of a flowing ideal liquid

$$T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu + pg_{\mu\nu}, \quad (34)$$

where u_μ is the 4-velocity of the liquid and ϵ and p are the energy density and the pressure. (Tracelessness would of course demand that $\epsilon = 3p$.) It is not difficult to show that this is *not* the case: the structure of our answer is richer than this simple form.

Nevertheless, it is still possible to define a “comoving frame” of matter at any point, in which the (boosted) momentum density T'_{0i} vanishes. The (boosted) local value T'_{00} component is the energy density in such a comoving frame, which we denote by ϵ like for a liquid. We use the contour of ϵ to define the freeze-out surface.

However, the (boosted) local values of other spatial components are in general unrelated and can be viewed as “anisotropic pressure” of nonequilibrium matter. Needless to say, it remains unknown what combinations of fundamental fields of the $\mathcal{N} = 4$ theory—the gauge one, the fermion or the scalars—participate in this flow of produced matter: to learn that one should do “holography” for many more operators on the boundary.

Recall that the stress tensors in different frames are related by $T'_{\alpha\beta} = S^\mu_\alpha S^\nu_\beta T_{\mu\nu}$, where $S^\mu_\alpha = \frac{\partial x^\mu}{\partial x'^\alpha}$ are Lorentz transformation matrices. The primed quantities correspond to the new frame. Therefore the aim is to find such boost which kills all the $(0, i)$ components.

In practice, it is achieved numerically by the following recipe.

We pick up any point inside the light cone, calculate the eigenvalue, and associate eigenvectors of the corresponding stress tensor matrix. Out of the four eigenvalues, one is selected to be the local energy density based on its eigenvector (see Appendix B for a short explanation). Figure 3 is a surface plot of ϵ profile in spatial coordinate. The plot is

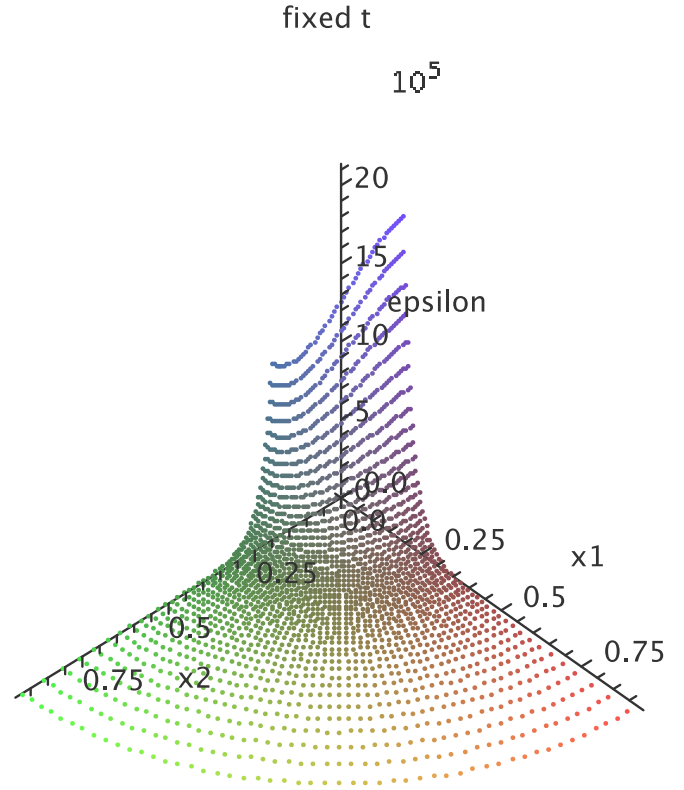


FIG. 3 (color online). The profile of ϵ , in units of $\frac{2\sqrt{\Lambda}}{f_0^2 \pi^2}$, at $t = 1$ with $r \approx 0.2-1$. The evolution of the shape of contour, i.e. the freeze-out surface is contained in this plot. The contours with large r (small ϵ) are nearly spherical while the contours with small r (large ϵ) are elongated along the x_1 axis

made for $t = 1$. It shows a nearly spherical shape for contour with large r and an elongated shape for contour with small r . By virtue of conformality of the setting, this translates to the following: At early time the local energy density contour is nearly spherical and at late time it gets elongated along x_1 axis.

VI. THE STRESS TENSOR OF MULTIPLE STRINGS

A simple extension of what is done above is to consider many colliding quark-antiquark pairs uniformly distributed in the transverse plane. Every pair with the same transverse coordinate is connected with a string. Let us assume quark and antiquark only interact pairwise, which in the dual picture means the strings do not interact with each other. As a result, the overall stress tensor induced by the multiple strings, in the linearized approximation simply amounts to integrating (32) over the transverse coordinates. Note in order to preserve causality, the integral is done for $0 < x_\perp^2 = r^2 - x^2 < t^2 - x^2$. We display the result as follows (we omit a factor of transverse string density, which does not alter space-time dependence of the stress tensor):

$$\begin{aligned}
T_{tt} &= \frac{8\sqrt{\lambda}}{f_0^3 \pi} \left[-4e_1 \frac{x^2}{t^4} + \left(\frac{4}{3}e_1 + \frac{2}{3}f + 2e_2 - 8c \right) \frac{1}{t^2} + \left(\frac{20}{3}e_1 - \frac{2}{3}f + 8c - 3e_2 - 2a \right) \frac{t}{x^3} + (e_2 + 2a - 4e_1) \frac{t^3}{x^5} \right] \\
T_{tx} &= \frac{8\sqrt{\lambda}}{f_0^3 \pi} \left[12e_1 \frac{x^3}{t^5} + \left(\frac{-20}{3}e_1 + 4c + 12d + \frac{2}{3}f - 2e_2 \right) \frac{x}{t^3} + \left(\frac{2}{3}e_1 + 2c - \frac{2}{3}f - e_2 + 6d \right) \frac{1}{x^2} \right. \\
&\quad \left. + (-6c - 6e_1 - 18d + 3e_2) \frac{t^2}{x^4} \right] \\
T_{tx_i} &= 0 \\
T_{xx} &= \frac{8\sqrt{\lambda}}{f_0^3 \pi} \left[-20e_1 \frac{x^4}{t^6} + (12e_1 + 2e_2 - 2f - 24d) \frac{x^2}{t^4} + (6b - 4a - 4e_1 + 3e_2 - 24d + 2f) \frac{t}{x^3} \right. \\
&\quad \left. + (12e_1 - 6b + 48d + 4a - 5e_2) \frac{t^3}{x^5} \right] \\
T_{xx_i} &= 0 \\
T_{x_i x_j} &= \frac{8\sqrt{\lambda}}{f_0^3 \pi} \left[10e_1 \frac{x^4}{t^6} + (f - e_2 - 14e_1) \frac{x^2}{t^4} + \left(e_2 - \frac{5}{3}f + \frac{8}{3}e_1 \right) \frac{1}{t^2} + \left(-4a + e_2 + \frac{2}{3}f - \frac{8}{3}e_1 \right) \frac{t}{x^3} \right. \\
&\quad \left. + (4a - e_2 + 4e_1) \frac{t^3}{x^5} \right]. \tag{35}
\end{aligned}$$

We plot the energy density profiles in Fig. 4.

VII. SUMMARY AND DISCUSSION

The main objective of this paper was to calculate a “hologram” of the falling open string, which has ends attached to heavy quarks moving with constant velocities $\pm v$. After an appropriate tool—Green function for time-

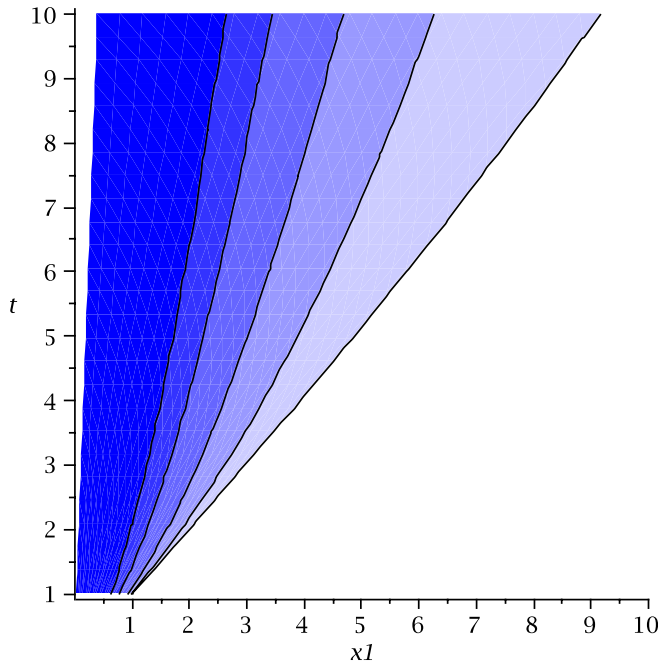


FIG. 4 (color online). The contours of energy density, in units of $\frac{8\sqrt{\lambda}}{f_0^3 \pi}$, as a function of t and x_1 . The magnitude is represented by color, with darker color corresponding to greater magnitude.

dependent linearized Einstein equation—was constructed, and stress tensor of the string calculated, a convolution of the two gave us the stress tensor of an “explosion” seen by an observer at the AdS boundary. Apart from analytical results in different limits, we have given pictures of the time evolution of the energy density and the Poynting vector in Figs. 1 and 2. In short, our main finding is that it looks like an explosion, with matter “fireball” expanding from the collision point, but a nonhydrodynamical explosion, in which fluid cannot be assigned temperature or entropy.

What can be a physical significance and applications of these results?

Literally, they describe energy/momentum flow following a collision of, for example, two heavy-quark mesons in a strongly coupled $\mathcal{N} = 4$ gauge theory). It would then be instructive to compare these results with those in a weakly coupled regime of the same theory, in which the appropriate calculation would be perturbative radiation of massless gluons and scalars. Those are well known to produce dipole radiation at small velocities and bremsstrahlung cones (or “jets”) in forward and backward directions. In QCD perturbative radiation is affected by confinement effects as well as the presence of light fermions: thus formation of QCD strings and their breaking by light quark pair production. All of it is well modeled by QCD “event generators,” one of which—the Lund model—we mentioned in the Introduction.

So, why would one be interested in a strongly coupled version of the “event generator”? One reason can be methodical: to better understand the difference between strongly coupled conformal regime and confining theories, as far as jet physics is concerned. It has been studied in

literature that hypothetical “hidden valleys theories” which may be found at LHC [17] are strongly coupled: so there is some nonzero (but tiny) chance that those can be used in real experiments one day.

However, as explained in the Introduction, those were not our motivations. We have done this calculation as a methodical step toward understanding heavy ion collisions, in the AdS/CFT setting. One obviously needs to study one falling string before considering many. Simply adding the effect of many strings, as we did above, is not yet sufficient to understand heavy ion collisions.

As a discussion item, we would like at the end of the paper to indicate where we will go from here. What we would like to understand in general is *how and under which conditions the equilibration and entropy production happen*, so that nonhydrodynamic explosion described above becomes hydrotype. In order to derive that, one has to abandon the “probe approximation” used above, and include the gravitational impact of falling matter (strings in our setting) in the metric. Only then one may see a transition from extremal black hole (AdS metric we use) to nonextremal black hole with matter mass added and a *nonzero horizon formed*. The horizon, when present, does provide both Hawking temperature and Bekenstein entropy. We expect to use in the next paper of this series a “two-membrane paradigm,” in which collision debris are represented by one (falling) membrane, and the (rising and then falling because of stretching) horizon membrane by

the other. (These two membranes can be associated with the so-called top-down and down-up equilibration scenarios, proposed in literature in various model settings.) When and where most of the entropy is produced is the major issue to be addressed. How that is reflected in the hologram observed in the gauge theory could then be calculated in the linearized approximation, as above.

ACKNOWLEDGMENTS

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APPENDIX A: INVERSE FOURIER TRANSFORM OF GREEN’S FUNCTION

Let us recall the expression of Green’s function in momentum space:

$$G(z, z') = \begin{cases} -\frac{2}{z} I_2(i\lambda z_{<}) K_2(i\lambda z_{>}) & \omega > 0, \omega > k \textcircled{1} \\ -\frac{2}{z} I_2(-i\lambda z_{<}) K_2(-i\lambda z_{>}) & \omega < 0, \omega > k \textcircled{2} \\ -\frac{2}{z} I_2(\tilde{\lambda} z_{<}) K_2(\tilde{\lambda} z_{>}) & \omega < k \textcircled{3} \end{cases} \quad (\text{A1})$$

We will use $\textcircled{1}$, $\textcircled{2}$, $\textcircled{3}$ to refer to the three cases as indicated above. In order to do the inverse Fourier transform: $\frac{1}{(2\pi)^4} \int G(z, z') e^{i\omega t - i\vec{k}\vec{x}} d\omega d^3k$, we make the change of variable for each case:

$$\begin{aligned} \textcircled{1}. \quad \omega = \sqrt{k^2 + \lambda^2} \frac{1}{(2\pi)^4} \int G(z, z') e^{i\omega t - i\vec{k}\vec{x}} d\omega d^3k &= \frac{1}{(2\pi)^3} \int G(z, z') e^{i\sqrt{k^2 + \lambda^2} t} \frac{2 \sin(kr)}{kr} k^2 dk \frac{\lambda d\lambda}{\sqrt{k^2 + \lambda^2}} \\ \textcircled{2}. \quad \omega = -\sqrt{k^2 + \lambda^2} \frac{1}{(2\pi)^4} \int G(z, z') e^{i\omega t - i\vec{k}\vec{x}} d\omega d^3k &= \frac{1}{(2\pi)^3} \int G(z, z') e^{-i\sqrt{k^2 + \lambda^2} t} \frac{2 \sin(kr)}{kr} k^2 dk \frac{\lambda d\lambda}{\sqrt{k^2 + \lambda^2}} \\ \textcircled{3}. \quad \omega = \pm\sqrt{k^2 - \tilde{\lambda}^2} \frac{1}{(2\pi)^4} \int G(z, z') e^{i\omega t - i\vec{k}\vec{x}} d\omega d^3k &= \frac{1}{(2\pi)^3} \int G(z, z') 2 \cos\sqrt{k^2 - \tilde{\lambda}^2} t \frac{2 \sin(kr)}{kr} k^2 dk \frac{\lambda d\lambda}{\sqrt{k^2 - \tilde{\lambda}^2}}. \end{aligned} \quad (\text{A2})$$

We use the following formulas to evaluate the k -integrals:

$$\begin{aligned} 2 \int_0^\infty \frac{\cos(b\sqrt{x^2 + a^2})}{\sqrt{x^2 + a^2}} \sin(\xi x) x dx &= \pi Y'_0(a\sqrt{b^2 - \xi^2}) \frac{-a\xi}{\sqrt{b^2 - \xi^2}} \theta(b - \xi) - 2K'_0(a\sqrt{\xi^2 - b^2}) \frac{a\xi}{\sqrt{\xi^2 - b^2}} \theta(\xi - b) 2 \\ &\times \int_0^\infty \frac{\cos(b\sqrt{x^2 - a^2})}{\sqrt{x^2 - a^2}} \sin(\xi x) x dx \\ &= -2K'_0(a\sqrt{b^2 - \xi^2}) \frac{-a\xi}{\sqrt{b^2 - \xi^2}} \theta(b - \xi) + \pi Y'_0(a\sqrt{\xi^2 - b^2}) \frac{a\xi}{\sqrt{\xi^2 - b^2}} \theta(\xi - b) 2 \\ &\times \int_0^\infty \frac{\sin(b\sqrt{x^2 + a^2})}{\sqrt{x^2 + a^2}} \sin(\xi x) x dx \\ &= -\pi J'_0(a\sqrt{b^2 - \xi^2}) \frac{-a\xi}{\sqrt{b^2 - \xi^2}} \theta(b - \xi) + \pi \delta(b - \xi) \end{aligned} \quad (\text{A3})$$

with $a, b, \xi > 0$.

These formulas are obtained by differentiating with respect ξ the cosine-transform of $\frac{\cos(b\sqrt{x^2+a^2})}{\sqrt{x^2+a^2}}$, $\frac{\cos(b\sqrt{x^2-a^2})}{\sqrt{x^2-a^2}}$, and $\frac{\sin(b\sqrt{x^2+a^2})}{\sqrt{x^2+a^2}}$.

Let us focus on the case $t > 0$ at the moment. With the help of (A2), (A3) becomes

$$\begin{aligned} \textcircled{1}. & \int -\frac{2}{z'} I_2(i\lambda z_{<}) K_2(i\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[\pi Y'_0(\lambda\sqrt{t^2-r^2}) \frac{-\lambda}{\sqrt{t^2-r^2}} \theta(t-r) - 2K'_0(\lambda\sqrt{r^2-t^2}) \frac{\lambda}{\sqrt{r^2-t^2}} \theta(r-t) \right. \\ & \left. - i\pi J'_0(\lambda\sqrt{t^2-r^2}) \frac{-\lambda}{\sqrt{t^2-r^2}} \theta(t-r) + i\pi \frac{\delta(t-r)}{r} \right] \\ \textcircled{2}. & \int -2\frac{2}{z'} I_2(-i\lambda z_{<}) K_2(-i\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[\pi Y'_0(\lambda\sqrt{t^2-r^2}) \frac{-\lambda}{\sqrt{t^2-r^2}} \theta(t-r) - 2K'_0(\lambda\sqrt{r^2-t^2}) \frac{\lambda}{\sqrt{r^2-t^2}} \theta(r-t) \right. \\ & \left. + i\pi J'_0(\lambda\sqrt{t^2-r^2}) \frac{-\lambda}{\sqrt{t^2-r^2}} \theta(t-r) - i\pi \frac{\delta(t-r)}{r} \right] \\ \textcircled{3}. & 2 \int -\frac{2}{z'} I_2(\lambda z_{<}) K_2(\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[-2K'_0(\lambda\sqrt{t^2-r^2}) \frac{-\lambda}{\sqrt{t^2-r^2}} \theta(t-r) + \pi Y'_0(\lambda\sqrt{r^2-t^2}) \frac{\lambda}{\sqrt{r^2-t^2}} \theta(r-t) \right]. \end{aligned} \quad (\text{A4})$$

Suppose we replace $z_{>}$ by $z_{>} - i\epsilon$, convergence of the integral enables us to rotate the contour of $\textcircled{3}$ and $\textcircled{1}$.

$$\begin{aligned} & \int -\frac{2}{z'} I_2(\lambda z_{<}) K_2(\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[-2K'_0(\lambda\sqrt{t^2-r^2}) \frac{-\lambda}{\sqrt{t^2-r^2}} \theta(t-r) \right] \\ & = \int -\frac{2}{z'} I_2(i\lambda z_{<}) K_2(i\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[(-\pi Y'_0(\lambda\sqrt{t^2-r^2}) - i\pi J'_0(\lambda\sqrt{t^2-r^2})) \frac{-\lambda}{\sqrt{t^2-r^2}} \theta(t-r) \right] \\ & \quad \times \int -\frac{2}{z'} I_2(i\lambda z_{<}) K_2(i\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[-2K'_0(\lambda\sqrt{r^2-t^2}) \frac{\lambda}{\sqrt{r^2-t^2}} \theta(r-t) \right] \\ & = \int -\frac{2}{z'} I_2(\lambda z_{<}) K_2(\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[(-\pi Y'_0(\lambda\sqrt{r^2-t^2}) + i\pi J'_0(\lambda\sqrt{r^2-t^2})) \frac{\lambda}{\sqrt{r^2-t^2}} \theta(r-t) \right]. \end{aligned} \quad (\text{A5})$$

Similarly, suppose $z_{>} \rightarrow z_{>} + i\epsilon$, we can rotate the contour of $\textcircled{3}$ and $\textcircled{2}$.

$$\begin{aligned} & \int -\frac{2}{z'} I_2(\lambda z_{<}) K_2(\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[-2K'_0(\lambda\sqrt{t^2-r^2}) \frac{-\lambda}{\sqrt{t^2-r^2}} \theta(t-r) \right] \\ & = \int -\frac{2}{z'} I_2(-i\lambda z_{<}) K_2(-i\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[(-\pi Y'_0(\lambda\sqrt{t^2-r^2}) + i\pi J'_0(\lambda\sqrt{t^2-r^2})) \frac{-\lambda}{\sqrt{t^2-r^2}} \theta(t-r) \right] \\ & \quad \times \int -\frac{2}{z'} I_2(-i\lambda z_{<}) K_2(-i\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[-2K'_0(\lambda\sqrt{r^2-t^2}) \frac{\lambda}{\sqrt{r^2-t^2}} \theta(r-t) \right] \\ & = \int -\frac{2}{z'} I_2(\lambda z_{<}) K_2(\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[(-\pi Y'_0(\lambda\sqrt{r^2-t^2}) - i\pi J'_0(\lambda\sqrt{r^2-t^2})) \frac{\lambda}{\sqrt{r^2-t^2}} \theta(r-t) \right]. \end{aligned} \quad (\text{A6})$$

Summing up piece $\textcircled{1}$, $\textcircled{2}$, and $\textcircled{3}$, we find various terms cancel against each other. We are left with

$$\begin{aligned} z_{>} \rightarrow z_{>} - i\epsilon: & \int -\frac{2}{z'} I_2(i\lambda z_{<}) K_2(i\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[-2\pi i J'_0(\lambda\sqrt{t^2-r^2}) \frac{-\lambda}{\sqrt{t^2-r^2}} \theta(t-r) + \pi i \frac{\delta(t-r)}{r} \right] \\ & + \int -\frac{2}{z'} I_2(\lambda z_{<}) K_2(\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[\pi i J'_0(\lambda\sqrt{r^2-t^2}) \frac{\lambda}{\sqrt{r^2-t^2}} \theta(r-t) \right] \\ z_{>} \rightarrow z_{>} + i\epsilon: & \int -\frac{2}{z'} I_2(-i\lambda z_{<}) K_2(-i\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[2\pi i J'_0(\lambda\sqrt{t^2-r^2}) \frac{-\lambda}{\sqrt{t^2-r^2}} \theta(t-r) - \pi i \frac{\delta(t-r)}{r} \right] \\ & + \int -\frac{2}{z'} I_2(\lambda z_{<}) K_2(\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} \left[-\pi i J'_0(\lambda\sqrt{r^2-t^2}) \frac{\lambda}{\sqrt{r^2-t^2}} \theta(r-t) \right]. \end{aligned} \quad (\text{A7})$$

If we are only interested in the coefficient of the $z_{<}^2$ term [18], we may make the following substitution: $I_2(\pm i\lambda z_{<}) \rightarrow -\frac{1}{8}\lambda^2$, $I_2(\lambda z_{<}) \rightarrow \frac{1}{8}\lambda^2$, $z_{>} \rightarrow z'$.

Further evaluation of the integral involves the two formulas:

$$\int_0^\infty x^{\mu+\nu+1} J_\mu(ax) K_\nu(bx) dx = 2^{\mu+\nu} a^\mu b^\nu \frac{\Gamma(\mu + \nu + 1)}{(a^2 + b^2)^{\mu+\nu+1}} \quad (\text{A8})$$

$$\int_0^\infty x^\mu K_\nu(ax) dx = 2^{\mu-1} a^{-\mu-1} \Gamma\left(\frac{1 + \mu + \nu}{2}\right) \Gamma\left(\frac{1 + \mu - \nu}{2}\right).$$

It is easy to find $\int \lambda^3 d\lambda K_2(\pm i\lambda z_{>})$ and $\int \lambda^4 d\lambda K_2(\lambda z_{>}) J_1(\lambda\sqrt{r^2 - t^2}) \theta(r - t)$ contain no singularity in $z_{>}$, thus the contributions from $z_{>} \pm i\epsilon$ cancel each other. We end up with

$$\begin{aligned} & \int -\frac{2}{z'} \frac{-\lambda^2}{8} K_2(i\lambda z_{>}) \frac{\lambda d\lambda}{(2\pi)^3} (-2\pi i) J_1(\lambda\sqrt{t^2 - r^2}) \frac{\lambda}{\sqrt{t^2 - r^2}} \theta(t - r)|_{z_{>} \rightarrow z_{>} - i\epsilon} + \int -\frac{2}{z'} \frac{-\lambda^2}{8} K_2(-i\lambda z_{>}) \\ & \times \frac{\lambda d\lambda}{(2\pi)^3} 2\pi i J_1(\lambda\sqrt{t^2 - r^2}) \frac{\lambda}{\sqrt{t^2 - r^2}} \theta(t - r)|_{z_{>} \rightarrow z_{>} + i\epsilon} \\ & = \frac{12iz'}{(2\pi)^2} \left[\frac{1}{(t^2 - r^2 - z'^2 + i\epsilon)^4} - \frac{1}{(t^2 - r^2 - z'^2 - i\epsilon)^4} \right] \theta(t - r). \end{aligned} \quad (\text{A9})$$

For $t < 0$, a similar procedure leads to a vanishing result. Therefore the $z_{<}^2$ term of the Green's function in coordinate space, which is exactly the propagator we are looking for, can be expressed as [19]

$$P_R = \frac{12iz'}{(2\pi)^2} \left[\frac{1}{(t^2 - r^2 - z'^2 + i\epsilon)^4} - \frac{1}{(t^2 - r^2 - z'^2 - i\epsilon)^4} \right] \theta(t - r). \quad (\text{A10})$$

APPENDIX B: EXTRACT ENERGY DENSITY IN COMOVING FRAME

The aim is to kill the $(0, i)$ components (momentum density) by local Lorentz transformation: $T' = STS^T$, where

$$T' = T'_{\alpha\beta}, \quad T = T_{\mu\nu}, \quad S_{\alpha\mu} = \frac{\partial x^\mu}{\partial x^\alpha}. \quad (\text{B1})$$

The local Lorentz transformation matrix is a product of matrices, which are either rotations, e.g. ① or boosts, e.g. ②. This is a consequence of the fact: $SgS^T = g$, where $g = \text{diag}(-1, 1, 1, 1)$ is the Minkowski metric

$$\begin{aligned} & \textcircled{1} \begin{pmatrix} 1 & & & \\ & \cos(\theta) & \sin(\theta) & \\ & -\sin(\theta) & \cos(\theta) & \\ & & & 1 \end{pmatrix} \\ & \textcircled{2} \begin{pmatrix} chy & shy & & \\ shy & chy & & \\ & & 1 & \\ & & & 1 \end{pmatrix}. \end{aligned} \quad (\text{B2})$$

A nice property of g is $g = g^T = g^{-1}$; therefore gT' and Tg are related by similarity transformation:

$$gT' = gSTg^T S^T = gS(Tg)(gS)^{-1}. \quad (\text{B3})$$

Since gT' does not alter the zero entries of T' , now the problem becomes to kill the $(0, i)$ components of Tg by similarity transformation. The original matrix Tg can be viewed as an operator L acting on a set of basis, while the similarity transformation is just a change of basis:

$$\begin{aligned} (Tg)_{mn} &= (e_n, Le_m) & (gT')_{mn} &= (e'_n, Le'_m) \\ e'_n &= (gS)_{nm} e_m. \end{aligned} \quad (\text{B4})$$

We want to find a basis e'_0 such that $Le'_0 = \lambda e'_0$ with $\lambda = (gT')_{00}$. Denote $e'_0 = x_m e_m$, it is easy to show $x_m (Tg)_{mn} = \lambda x_n$. This is exactly an eigenvalue problem for matrix $(Tg)^T$. The restriction in transformation matrix is translated to $x_0^2 - x_1^2 - x_2^2 - x_3^2 > 0$. It turns out this condition is just enough to determine a unique eigenvalue out of four possible eigenvalues. The energy density is given by $\epsilon = -\lambda$.

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