Remnants of dark matter clumps

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What happened to the central cores of tidally destructed dark matter clumps in the Galactic halo? We calculate the probability of surviving of the remnants of dark matter clumps in the Galaxy by modelling the tidal destruction of the small-scale clumps. It is demonstrated that a substantial fraction of clump remnants may survive through the tidal destruction during the lifetime of the Galaxy if the radius of a core is rather small. The resulting mass spectrum of surviving clumps is extended down to the mass of the core of the cosmologically produced clumps with a minimal mass. Since the annihilation signal is dominated by the dense part of the core, destruction of the outer part of the clump affects the annihilation rate relatively weakly and the survived dense remnants of tidally destructed clumps provide a large contribution to the annihilation signal in the Galaxy. The uncertainties in the minimal clump mass resulting from the uncertainties in neutralino models are discussed.

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I. INTRODUCTION

According to current observations, about 30% of the mass of the Universe is in a form of cold dark matter (DM). The nature of DM particles is still unknown. The cold DM component is gravitationally unstable and is expected to form the gravitationally bounded clumpy structures from the scale of the superclusters of galaxies and down to very small clumps of DM. The large-scale DM structures are observed as the galactic halos and clusters of galaxies. They are also seen in numerical simulations. Theoretical studies of DM clumps are important for understanding the properties of DM particles because annihilation of DM particles in small dense clumps may result in a visible signal. The DM clumps in the Galaxy can produce the bright spots in the sky in the gamma or X-bands [1]. A local annihilation rate is proportional to the square of the DM particle number density. Thus, the annihilation signal from small clumps can dominate over diffuse components of DM in the halo.

The cosmological formation and evolution of small-scale DM clumps have been studied in numerous works [2–13]. The minimum mass of clumps (the cut-off of the mass spectrum), $M_{\min}$ is determined by the collisional and collisionless damping processes (see, e.g., [6] and references therein). Recent calculations [10] show that the cut-off mass is related to the friction between DM particles and cosmic plasma similar to the Silk damping. In the case of the Harrison-Zeldovich spectrum of primordial fluctuations with CMB normalization, the first small-scale DM clumps are formed at redshift $z \sim 60$ (for 2σ fluctuations) with a mean density 7×10^{-22} g cm^{-3}, virial radius 6×10^{-3} pc, and internal velocity dispersion 80 cm s^{-1}, respectively. Only a very small fraction of these clumps survive the early stage of tidal destruction during the hierarchical clustering [4]. Nevertheless, these survived clumps may provide the major contribution to the annihilation signal in the Galaxy [4,7,14–16]. At a high redshift, neutralinos, considered as DM particles, may cause the efficient heating of the diffuse gas [17] due to annihilation in the dense clumps.

One of the unresolved problems of DM clumps is a value of the central density or core radius. Numerical simulations give a nearly power density profile of DM clumps. Both the Navarro-Frenk-White and Moore profiles give formally a divergent density in the clump center. A theoretical modelling of the clump formation [18] predicts a power-law profile of the internal density of clumps

$$\rho_{\rm int}(r) = \frac{3-\beta}{3}\bar{\rho}\left(\frac{r}{R}\right)^{-\beta}, \tag{1}$$

where $\bar{\rho}$ and R are the mean internal density and a radius of clump, respectively, $\beta \simeq 1.8$–2 and $\rho_{\rm int}(r) = 0$ at $r > R$. A near isothermal power-law profile (1) with $\beta \simeq 2$ has been recently obtained in numerical simulations of a small-scale clump formation [19].

It must be noted that density profiles of small-scale DM clumps and large-scale DM haloes may be different. The Galactic halos are well approximated by the Navarro-Frenk-White profile outside of the central core where dynamical resolution of numerical simulations becomes insufficient. Different physical mechanisms are engaged for formation of a central core during the formation and evolution of clumps. A theoretical estimation of the relative core radius of a DM clump $x_c = R_c/R$ was obtained in [18] from energy criterion, $x_c \equiv R_c/R \simeq \delta_{\rm eq}^3$, where $\delta_{\rm eq}$ is a value of density fluctuation at the beginning of a matter-

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dominated stage. A similar estimate for DM clumps with the minimal mass $\sim 10^{-6} M_{\odot}$ originated from 2σ fluctuation peaks gives $\delta_{\rm eq} \simeq 0.013$ and $R_c/R \simeq 1.8 \times 10^{-5}$, respectively. In [4], the core radius $x_c \simeq 0.3\nu^{-2}$ has been obtained, where ν is a relative height of the fluctuation density peak in units of dispersion at the time of energy-matter equality (see also Sec. V). This value is a result of the influence of tidal forces on the motion of DM particles in the clump at the stage of formation. This estimate may be considered as an upper limit for the core radius or as the break scale in the density profile, e.g., a characteristic scale in the Navarro-Frenk-White profile. It could be that a real core radius, where the density ceases to grow, is determined by the relaxation of small-scale perturbations inside the forming clump [20]. Another mechanism for core formation arises in the "meta-cold dark matter model" due to the late decay of cold thermal relics into lighter nonrelativistic particles with low phase-space density [21,22].

Nowadays, numerical simulations have a rather low space resolution in the central region of clumps to determine the core radius. The only example with some indication to presence of a core with radius $x_c \simeq 10^{-2}$ is numerical simulation of small-scale clump formation [19]. The special numerical simulations with a sufficiently high space resolution to reveal the real core radius are very requested.

In this work, we consider the relative core radius $x_c = R_c/R$ of DM clumps as a free parameter in the range 0.001–0.1. We investigate the dependence of the probability of clump survival in the Galaxy on this parameter under the action of tidal forces from the Galactic disk and stars. As a preferred value, we consider $x_c \simeq 10^{-2}$ in spite of the numerical simulations [19]. The corresponding annihilation rate of DM particles is proportional to their squared number density, and thus is very sensitive to the value of the core radius.

In our earlier works [7,12], we used a simplified criterium for a tidal destruction of the clump. Namely, we postulated that the clump is destructed if a total tidal energy gain $\sum(\Delta E)_j$ after several disk crossings (or collisions with stars) becomes of order of initial binding energy of a clump $|E|$, i.e.,

$$\sum_j (\Delta E)_j \sim |E|, \tag{2}$$

where the summation goes over the successive disk crossings (or encounters with stars). This criterium is justified in the cosmological context of the DM clump formation because both the formation of the density profile of the clump and its tidal heating proceed during the same time of nonlinear evolution of density perturbation. For the Galaxy case, a more detailed consideration is needed to describe a tidal destruction of DM clumps by stars. An improved approach includes a gradual mass loss of systems [23–25], in particular, by small-scale DM clumps [5,26].

In this work, we will describe a gradual mass loss of small-scale DM clumps assuming that only the outer layers of clumps are involved and influenced by the tidal stripping. Additionally, we assume that the inner layers of a clump are not affected by tidal forces. In this approximation, we calculate a continuous diminishing of the clump mass and radius during the successive Galactic disk crossings and encounters with the stars. We accept now for criterium of clump destruction, the diminishing of the radius of tidally stripped clumps down to the core radius. An effective time of mass loss for the DM clump remains nearly the same as in our previous calculations [12]. However, the clump destruction time has now quite different physical meaning: it provides now a characteristic time scale for the diminishing of the clump mass and size instead of the total clump destruction. This means that small remnants of clumps may survive in the Galaxy. Respectively, these remnants would be an additional source of amplification of the DM annihilation signal in the Galaxy.

II. TIDAL DESTRUCTION OF CLUMPS BY DISK

The kinetic energy gain of a DM particle with respect to the center of a clump after one crossing of the Galactic disk is [27]

$$\delta E = \frac{4 g_m^2 (\Delta z)^2 m}{v_{z,c}^2} A(a), \tag{3}$$

where m is a constituent DM particle mass, Δz is a vertical distance (orthogonal to the disk plane) of a DM particle with respect to the center of the clump, $v_{z,c}$ is a vertical velocity of the clump with respect to the disk plane at the moment of disk crossing, and $A(a)$ is the adiabatic correction factor. A gravitational acceleration near the disk plane is

$$g_m(r) = 2\pi G \sigma_s(r), \tag{4}$$

where we use an exponential model for a surface density of disk

$$\sigma_s(r) = \frac{M_d}{2\pi r_0^2} e^{-r/r_0} \tag{5}$$

with $M_d = 8 \times 10^{10} M_{\odot}$, $r_0 = 4.5$ kpc.

The factor $A(a)$ in (3) describes the adiabatic protection from slow tidal effects [28]. This adiabatic correction, further on referred to as the Weinberg correction, is defined as an additional factor $A(a)$ to the values of energy gain in the momentum approximation. This factor satisfies the following asymptotic conditions: $A(a) = 1$ for $a \ll 1$ and $A(a) \ll 1$ for $a \gg 1$. In [23], the following fitting formula was proposed:

$$A(a) = (1 + a^2)^{-3/2}. \tag{6}$$

Here the adiabatic parameter $a = \omega\tau_d$, where ω is an orbital frequency of the DM particle in the clump, $\tau_d \simeq H_d/v_{z,c}$ is an effective duration of gravitational tidal shock produced by the disk with a half-thickness H_d. For tidal interactions of clumps with stars in the bulge and the halo the duration of the gravitational shock can be estimated as $\tau_s \sim l/v_{\rm rel}$, where l is an impact parameter and $v_{\rm rel}$ is a relative velocity of a clump with respect to a star.

As a representative example, we consider the isothermal internal density profile of a DM clump

$$\rho_{\rm int}(r) = \frac{1}{4\pi}\frac{v_{\rm rot}^2}{Gr^2} \tag{7}$$

with a cutoff at the virial radius R: $\rho(r) = 0$ at $r > R$. A corresponding mass profile of a clump is $M(r) = M_i(r/R)$, where M_i is an initial mass of a clump at the epoch of the Galaxy formation. With this mass distribution, a circular velocity inside a clump is independent of the radius, $v_{\rm rot} = (GM(r)/r)^{1/2} = (GM_i/R)^{1/2}$. A gravitational potential corresponding to the density profile (7) is $\phi(r) = v_{\rm rot}^2[\log(r/R) - 1]$. Let us define a dimensionless energy of the DM particle $\varepsilon = E/(mv_{\rm rot}^2)$ and gravitational potential $\psi(r) = \phi(r)/v_{\rm rot}^2 = \ln(r/R) - 1$. An internal density profile $\rho_{\rm int}(r)$ and the distribution function of DM particles in the clump $f_{\rm cl}(\varepsilon)$ are related by the integral relation [29]

$$\rho_{\rm int}(r) = 2^{5/2}\pi\int_{\psi(r)}^{0}\sqrt{\varepsilon - \psi(r)}f_{\rm cl}(\varepsilon)d\varepsilon. \tag{8}$$

The corresponding isothermal distribution function is

$$f_{\rm cl}(\varepsilon) \simeq \frac{v_{\rm rot}^2}{4\pi^{5/2}e^2GR^2}e^{-2\varepsilon}. \tag{9}$$

Note that this distribution function provides only an approximate representation of (7), far from the cutoff radius R. Nevertheless, this approximation is enough for our estimates of tidal destruction of the DM clumps.

By using the hypothesis of a tidal stripping of outer layers of a DM clump, we see that a tidal energy gain $\delta\varepsilon$ causes the stripping of particles with energies in the range $-\delta\varepsilon < \varepsilon < 0$. A corresponding variation of density at radius r is

$$\delta\rho(r) = 2^{5/2}\pi\int_{-\delta\varepsilon}^{0}\sqrt{\varepsilon - \psi(r)}f_{\rm cl}(\varepsilon)d\varepsilon. \tag{10}$$

In this equation, the tidal energy gain (3) by different DM particles is averaged over angles, so as $\langle(\Delta z)^2\rangle = r^2/3$. A resulting total mass loss by a DM clump during one crossing of the Galactic disk is

$$\delta M = -4\pi\int_0^R r^2\delta\rho(r)dr. \tag{11}$$

Let us specify the dimensionless quantities

$$Q_d = \frac{g_m^2}{2\pi v_{z,c}^2 G\bar{\rho}_i}, \qquad S_d = \frac{4\pi}{3}G\bar{\rho}_i\tau_d^2, \tag{12}$$

where $\bar{\rho}_i = 3M_i/(4\pi R^3)$ is an initial mean density of a clump. For most parts of the clumps $Q_d \ll 1$ with a typical value $Q_d \sim 0.03$. In the limiting case $Q_d \ll 1$ and in the absence of the adiabatic correction, $S_d = 0$, the integrals (10) can be calculated analytically. In a general case, the fitting formula for the mass loss of a clump during one passage through the Galactic disk is

$$\left(\frac{\delta M}{M}\right)_d \simeq -0.13Q_d\exp(-1.58S_d^{1/2}). \tag{13}$$

Now, we calculate the tidal mass loss by clumps using a realistic distribution of their orbits in the halo. The method of calculation is similar to the one used in [12], but instead of the rough energetic criterium for a tidal clump destruction (2) we will assume now a gradual decreasing of the clump mass and size.

Let us choose some particular clumps moving in the spherical halo with an orbital "inclination" angle γ between the normal vectors of the disk plane and orbit plane. The orbit angular velocity at a distance r from the Galactic center is $d\phi/dt = J/(mr^2)$, where J is an orbital angular momentum of a clump. A vertical velocity of a clump crossing the disk is

$$v_{z,c} = \frac{J}{mr_s}\sin\gamma, \tag{14}$$

where r_s is a radial distance of a crossing point from the Galaxy center. There are two crossing points (with different values of r_s) during an orbital period.

The standard Navarro-Frenk-White profile of the DM Galactic halo is

$$\rho_{\rm H}(r) = \frac{\rho_0}{(r/L)(1 + r/L)^2}, \tag{15}$$

where $L = 45$ kpc, $\rho_0 = 5\times 10^6 M_\odot$ kpc^{-3}. It is useful to introduce the dimensionless variables

$$x = \frac{r}{L}, \qquad \tilde{\rho}_{\rm H}(x) = \frac{\rho_{\rm H}(r)}{\rho_0}, \qquad y = \frac{J^2}{8\pi G\rho_0 L^4M^2}, \tag{16}$$

$$\varepsilon = \frac{E_{\rm orb}/M - \Phi_0}{4\pi G\rho_0L^2}, \qquad \psi = \frac{\Phi - \Phi_0}{4\pi G\rho_0L^2}, \tag{17}$$

where $\Phi_0 = -4\pi G\rho_0L^2$, $E_{\rm orb}$ is a total orbital energy of a clump. With these variables, the density profile of the halo (15) is written as

$$\tilde{\rho}_{\rm H}(x) = \frac{1}{x(1 + x)^2}. \tag{18}$$

A gravitational potential $\psi(x)$, corresponding to density profile (18) is

$$\psi(x) = 1 - \frac{\log(1+x)}{x}. \tag{19}$$

An equation for orbital turning points, $\dot{r}^2 = 0$, for DM clumps in the potential (19) is

$$1 - \frac{\log(1+x)}{x} + \frac{y}{x^2} = \varepsilon. \tag{20}$$

From (20), one can find numerically the minimum $x_{\min}$ and maximum $x_{\max}$ radial distance of a clump from the Galactic center as a function of orbital energy ε and the square of angular momentum y. Denoting $p = \cos\theta$, where θ is an angle between the radius-vector $\vec{r}$ and the orbital velocity $\vec{v}$, we have $y = (1-p^2)x^2[\varepsilon - \psi(x)]$. As we assumed above, the unit vectors $\vec{v}/v$ are distributed isotropically at each point x, and, therefore, p has a uniform distribution in the interval $[-1, 1]$.

The relation between the density profile $\tilde{\rho}_H(x)$ and the distribution function is given by the same Eq. (8) with an obvious substitution $f_{\rm cl} \Rightarrow F(\varepsilon)$, where the distribution function $F(\varepsilon)$ for a halo profile (18) can be fitted as [30]

$$F(\varepsilon) = F_1(1-\varepsilon)^{3/2}\varepsilon^{-5/2}\left[-\frac{\ln(1-\varepsilon)}{\varepsilon}\right]^q e^P. \tag{21}$$

Here $F_1 = 9.1968 \times 10^{-2}$, $q = -2.7419$, $P = \sum_i p_i \times (1-\varepsilon)^i$, $(p_1, p_2, p_3, p_4) = (0.3620, -0.5639, -0.0859, -0.4912)$. An interval of time for motion from $x_{\min}$ to $x_{\max}$ and back is

$$T_c(x, \varepsilon, p) = \frac{1}{\sqrt{2\pi G\rho_0}} \int_{x_{\min}}^{x_{\max}} \frac{ds}{\sqrt{\varepsilon - \psi(s) - y/s^2}}. \tag{22}$$

An angle of orbital precession during the time $T_c/2$ is

$$\tilde{\phi} = y^{1/2} \int_{x_{\min}}^{x_{\max}} \frac{ds}{s^2\sqrt{\varepsilon - \psi(s) - y/s^2}} - \pi < 0. \tag{23}$$

Therefore, an orbital period is longer than T_c and is given by

$$T_t = T_c(1 + \tilde{\phi}/\pi)^{-1}. \tag{24}$$

Choosing a time interval ΔT much longer than a clump orbital period T_t, but much shorter than the age of the Galaxy t_0, i.e., $T_t \ll \Delta T \ll t_0$, we define an average rate of mass loss by a selected clump under influence of tidal shocks in successive disk crossings

$$\frac{1}{M}\left(\frac{dM}{dt}\right)_d \simeq \frac{1}{\Delta T}\sum\left(\frac{\delta M}{M}\right)_d, \tag{25}$$

where $(\delta M/M)_d$ is given by (13) and the summation goes over all successive crossing points (odd and even) of a clump orbit with the Galactic disk during the time interval ΔT. According to (4) and (14) the g_m and $v_{z,c}$ both depend on the radius $x = r/L$. One simplification in the calculation of (25) follows from the fact that a velocity of orbit precession is constant. For this reason, the points of successive odd crossings are separated by the same angles $\tilde{\phi}$ from (23). The same is also true for successive even crossings. Using this simplification, we transform the summation in (25) to integration

$$\frac{1}{\Delta T}\sum\left(\frac{\delta M}{M}\right)_d \simeq \frac{2}{T_t|\tilde{\phi}|}\int_{x_{\min}}^{x_{\max}}\left(\frac{\delta M}{M}\right)_d \frac{d\phi}{dx}dx,$$

where

$$\frac{d\phi}{dx} = \frac{y^{1/2}}{x^2\sqrt{\varepsilon - \psi(x) - y/x^2}} \tag{26}$$

is an equation for the clump orbit in the halo. The method described will be used in Sec. IV for the final calculations.

III. TIDAL DESTRUCTION OF CLUMPS BY STARS

Now, we calculate the diminishing of a clump mass due to a tidal heating by stars in the Galaxy by using the same hypothesis of the preferable stripping of the outer clump layers. During a single close encounter of a DM clump with a star, the energy gain of a constituent DM particle in the clump with respect to the clump center is [7]

$$\delta E = \frac{2G^2 m_s^2 m \Delta z^2}{v_{\rm rel}^2 l^4}, \tag{27}$$

where m_* is a star mass, l is an impact parameter, $v_{\rm rel}$ is a relative star velocity with respect to a clump, $\Delta z = r\cos\psi$, r is a radial distance of a DM particle from the clump center, and ψ is an angle between the directions from the clump center to the DM particle and to the point of the closest approach of a star. Using the same method as in Sec. II, we calculate a relative mass loss by the clump $(\delta M/M)_s$ during a single encounter with a star and obtain the same fitting formula as (13) but with substituting the dimensionless parameters, $Q_d \Rightarrow Q_s$ and $S_d \Rightarrow S_s$, where

$$Q_s = \frac{Gm_*^2}{2\pi v_{\rm rel}^2 l^4 \bar{\rho}_i}, \qquad S_s = \frac{4\pi}{3}G\bar{\rho}_i\tau_s^2, \tag{28}$$

where $\tau_s \simeq l/v_{\rm rel}$.

A DM clump acquires the maximum energy gain during a single encounter with a star when the impact parameter is $l \sim R$. Using the relation

$$dt = \frac{1}{2\sqrt{2\pi G\rho_0}}\frac{dx}{\sqrt{\varepsilon - \psi(x) - y/x^2}}, \tag{29}$$

and integrating over all impact parameters $l > R$, we calculate an averaged rate of mass loss by a clump during successive encounters with stars

$$\frac{1}{M}\left(\frac{dM}{dt}\right)_s \simeq \frac{1}{2T_t\sqrt{2\pi G\rho_0}} \times \int_R^\infty 2\pi l dl \int_{x_{\min}}^{x_{\max}} \frac{dsn_*(s)v_{\rm rel}}{\sqrt{\varepsilon - \psi(s) - y/s^2}} \times \left(\frac{\delta M}{M}\right)_s, \tag{30}$$

where $n_*(r)$ is a radial number density distribution of stars in the bulge and halo. A DM clump moves through the medium with a varying value of n_* along the clump orbit. In contrast to the case of the disk crossing, the precession of the clump orbit during an orbital period does not influence the mass loss due to encounters with stars. Additionally, the mass loss due to encounters with stars is independent of the inclination of clump orbits in the case of a spherically symmetric distribution of stars in the bulge and halo.

Using the results of [31], we approximate the radial number density distribution of stars in the bulge in the radial range $r = 1$–3 kpc as

$$n_{b,*}(r) = (\rho_b/m_*)\exp[-(r/r_b)^{1.6}], \tag{31}$$

where $\rho_b = 8M_\odot/\mathrm{pc}^3$ and $r_b = 1$ kpc. A corresponding number density distribution of the halo stars at $r > 3$ kpc outside of the Galactic plane can be approximated as

$$n_{h,*}(r) = (\rho_h/m_*)(r_\odot/r)^3, \tag{32}$$

where $m_* = 0.4M_\odot$ and $r_\odot = 8.5$ kpc. According to [32], in the region between $r = 1$ and 40 kpc, a total mass of stars is $4\times 10^8 M_\odot$ with a star density profile $\propto r^{-3}$. These data correspond to $\rho_h = 1.4\times 10^{-5} M_\odot/\mathrm{pc}^3$ in (32). We neglect in our calculations the oblateness of the stellar halo [32].

IV. SURVIVING FRACTION OF CLUMPS

From Eq. (3) it is seen that the tidal forces influence mainly the outer part of the clump (where Δz is rather large). Further, we will use our basic assumption that only outer layers of a clump undergo the tidal stripping, while the inner parts of a clump are unaffected by tidal forces. Thus, we assume that a clump mass $M = M(t)$ and radius $R = R(t)$ are both diminishing in time due to the tidal stripping of outer layers, but its internal density profile remains the same as given by Eq. (7), e.g., for the isothermal density profile $M(t) \propto R(t)$ and $\bar\rho(t) \propto M(t)^{-2}$. Combining together the rates of mass loss (25) and (30) due to the tidal stripping of a clump by the disk and stars, respectively, we obtain the evolution equation for a clump mass

$$\frac{dM}{dt} = \left(\frac{dM}{dt}\right)_d + \left(\frac{dM}{dt}\right)_s. \tag{33}$$

In the following, we solve this equation numerically starting from the time of the Galaxy formation at $t_0 - t_G$ up to the present moment t_0. In numerical calculations, it is convenient to use the dimensionless variables: t/t_0 for time and M/M_i for a clump mass, where M_i is an initial clump mass. The adiabatic correction provides generally only a small effect. In the absence of adiabatic correction or, equivalently, at $S_d = S_s = 0$ the evolution Eq. (33) has a simple form

$$\frac{d\mu}{dt} = -\frac{\mu}{t_s} - \frac{\mu^3}{t_d}, \tag{34}$$

where $\mu = M(t)/M_i$ and parameters t_d and t_s are independent of μ. The solution of this equation

$$\mu^2(t_0) = \frac{2t_d}{(2t_d + t_s)\exp(2t_0/t_s) - t_s} \tag{35}$$

represents a good approximation to numerical solution of (33) with the adiabatic correction taken into account.

The most important astrophysical manifestation of DM clumps is a possible annihilation of constituent DM particles. The crucial point is a dominance of the central core of a clump in an annihilation signal if clumps have a steep enough density profile. Namely, annihilation of DM particles in a clump core will prevail in a total annihilation rate in a single clump with a power-law density profile (1) if $\beta > 3/2$ and $x_c = R_c/R \ll 1$. More specifically, the quantity $\dot N \propto \int_{r_0}^r 4\pi r'^2 dr' \rho_{\rm int}^2(r')$ practically does not depend on r, if $r \gg r_0$. As a result, the annihilation luminosity of a DM clump with approximately an isothermal density profile ($\beta \simeq 2$) will be nearly constant under influence of tidal stripping until a clump radius diminishes to its core radius. In other words, in the nowadays Galaxy the remnants of tidally stripped clumps with $x_c < \mu(t_0) \ll 1$, where $\mu(t) = M(t)/M_i$ and $t_0 \simeq 10^{10}$ yrs is the Galaxy age, obeys the evolution Eq. (33) and have the same annihilation luminosity as their progenitors with $\mu = 1$.

By using the evolution Eq. (33), we now calculate the probability P of the survival of the clump remnant during the lifetime of the Galaxy. Let us choose some arbitrary points in the halo with a radius-vector $\vec r$ and an angle α with a polar axis of the Galactic disk. Only the clump orbits with an inclination angle $\pi/2 - \alpha < \gamma < \pi/2$ pass through this point. A survival probability for clumps can be written now in the following form:

$$P(x,\alpha) = \frac{4\pi\sqrt{2}}{\tilde\rho(x)\sin\alpha}\int_0^1 dp \int_0^{\sin\alpha} d\cos\gamma \int_{\psi(x)}^1 d\varepsilon \times [\varepsilon - \psi(x)]^{1/2} F(\varepsilon)\Theta[\mu(t_0) - x_c]. \tag{36}$$

In this equation, $\tilde\rho(x)$ is a density profile of the halo from (18), $p = \cos\theta$, θ is an angle between the radius-vector $\vec r$ and the orbital velocity of clump, Θ is the Heviside function, $\psi(x)$ is the halo gravitational potential from (18), $F(\varepsilon)$ is a distribution function of clumps in the halo from (21), $\mu(t_0)$ depends on all variables of the integration, and $x_c = R_c/R$ is an initial value of the clump core. The function

$\mu(t_0)$ is calculated from the numerical solution of evolution Eq. (33). If $\mu(t_0) > x_c$, the clump remnant is survived through the tidal destruction by both the disk and stars. The annihilation rate in this remnant would be the same as in the initial clump. On the contrary, in the opposite case, when $\mu(t_G) < x_c$, the clump is totally destructed because (i) the core is not a dynamically separated system and composed of particles with extended orbits, and because (ii) a nearly homogeneous core is destructed easier than a similar object with the same mass but with a near isothermal density profile.

We consider the small-scale DM clumps in the initial mass interval $M_i = [10^{-6}M_\odot, 1M_\odot]$ originated from the 2σ (i.e., $\nu = 2$) peaks in the Harrison-Zeldovich perturbation spectrum. A reason is as follows. The DM clumps originated from initial density perturbations with $\nu < 1$ were almost completely destructed by tidal interactions during the early stage of hierarchical clustering as it can be seen from the distribution function of clumps (44) (see below). On the contrary, the most dense DM clumps with $\nu > 3$ are mostly survived in the stage of hierarchical clustering, but their number according to (44) exponentially falls with ν and is small. For this reason, we will use the following approximation: the DM clumps were originated on average from $\nu \simeq 2$ peaks, and for any given mass M we do not consider the distribution of clumps over their densities. In this approximation, the initial radius of the clump R_i depends only on the one parameter—the initial clump density, which also depends only on the initial clump mass M_i.

The crucial result of the numerical calculation of a survival probability (36) for clumps with $x_c \ll 0.05$ is that $P(x, \alpha) \sim 1$ everywhere. Even inside the bulge there are clumps which are flying through the bulge from external regions. This means that clump remnants are mostly survived through the tidal destruction in the Galaxy. A noticeable diminishing $P(x, \alpha) < 1$ near the center of the Galaxy becomes apparent for clumps with $x_c > 0.05$. It is understandable because with $x_c \to 1$, we return to the previous criterium of tidal destruction of clumps (2) and to a corresponding results for survival probability [7,12]. The survival probability $P(r, \alpha)$ numerically calculated from (36) for the cases $x_c \sim 0.05$ and 0.1 is shown in the Figs. 1 and 2. The dependence on α (an angle between a radius-vector $\vec{r}$ and a polar axis of the Galactic disk) is very weak as it was shown in [12]. For this reason, we present the results only for an intermediate value $\alpha = \pi/4$. The density of clumps is normalized to the density $7.3 \times 10^{-23}\ \mathrm{g\,cm^{-3}}$ valid for clumps with mass $M = 10^{-6}M_\odot$ originated from 2σ density peaks in the case of the power-law index of primordial spectrum of perturbations $n_p = 1$.

It is worth to note that a tidal radius of a clump in the bulge is [33]

$$r_t^3 = \frac{GM(r_t)}{\omega_p^2 - d^2\phi/dl^2}, \tag{37}$$

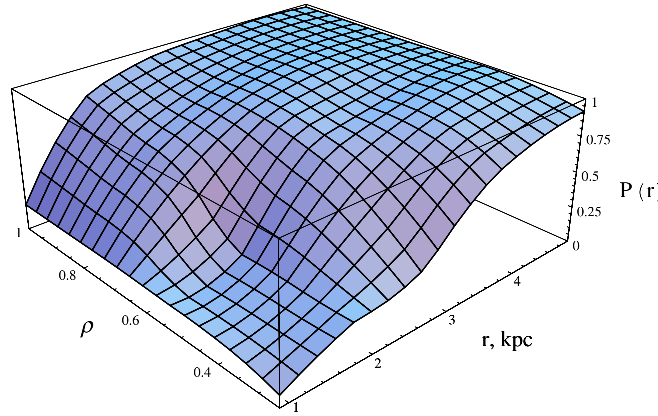

FIG. 1 (color online). The survival probability $P(r, \rho)$ plotted as a function of distance from the Galactic center r and a mean internal clump density ρ in the case $x_c = 0.1$. It gives the normalized fraction of DM clumps in the halo P calculated from (36), which survives the tidal destruction by the stellar disk and the halo stars.

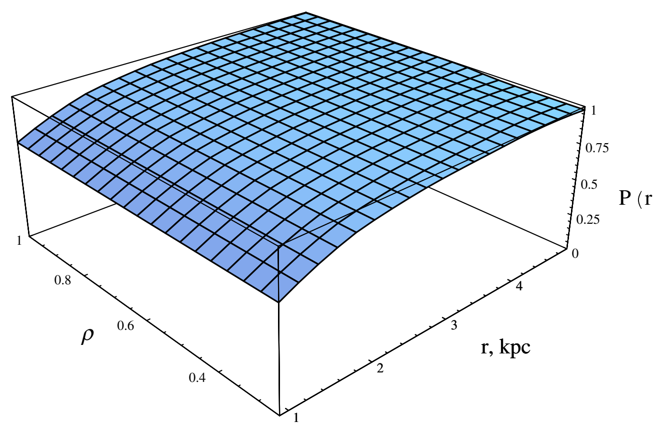

FIG. 2 (color online). The same as Fig. 1, but for the case $x_c = 0.05$.

where $\omega_p \simeq [GM_b(l)/l^3]^{1/2}$ is an angular velocity at the pericenter (we consider here a circular orbit for simplicity), $M_b(l)$ is a mass profile of the bulge, and $\phi(l)$ is a gravitational potential of the bulge. For the considered small-scale clumps $r_t \geq 0.2R_i$, and, therefore, the tidal radius is not a crucial factor for destruction of clumps.

V. COSMOLOGICAL DISTRIBUTION FUNCTION OF CLUMPS

In this section, we provide calculations of a mass function for the small-scale clumps in the Galactic halo by a more transparent method than in our previous works [4,7].

The first gravitationally bound objects in the Universe are the DM clumps of minimum mass $M_{\min}$. A numerical value of $M_{\min}$ depends strongly on the nature of the DM particle. Even in the case of a particular DM particle, e.g., neutralino, the calculated value of $M_{\min}$ can differ by many orders of magnitude for different sets of parameters in the mSUGRA model, see the Appendix. The clumps of larger scales are formed later. The larger scale clumps host the smaller ones and are hosted themselves by the next larger clumps. Major parts of small-scale clumps are destroyed by the tidal gravitational fields of their host clumps. At small-mass scales, the hierarchical clustering is a fast and complicated nonlinear process. The formation of new clumps and their capturing by the larger ones are nearly simultaneous processes because at small scales an effective index of the density perturbation power spectrum is very close to a critical value, $n \to -3$. The DM clumps are not totally virialized when they are captured by hosts. The adiabatic invariants cannot prevent the survival of cores at this stage because there is not enough time for the formation of the singular density profiles in clumps (an internal dynamical time of a clump is of the same order as its capture time by a host). We use a simplified model to take into account the most important features of hierarchical clustering.

In the model of spherical collapse (see for example [34]), a formation time for a clump with an internal density ρ is $t = (\kappa\rho_{\rm eq}/\rho)^{1/2} t_{\rm eq}$, where $\kappa = 18\pi^2$, $\rho_{\rm eq} = \rho_0(1+z_{\rm eq})^3$ is a cosmological density at the time of matter-radiation equality $t_{\rm eq}$, $1+z_{\rm eq} = 2.35\times 10^4\Omega_m h^2$, and $\rho_0 = 1.9\times 10^{-29}\Omega_m h^2$ g cm^{-3}. The index "eq" refers to quantities at the time of matter-radiation equality $t_{\rm eq}$. The DM clumps of mass M can be formed from density fluctuations of a different peak height $\nu = \delta_{\rm eq}/\sigma_{\rm eq}(M)$, where $\sigma_{\rm eq}(M)$ is a fluctuation dispersion on a mass scale M at the time $t_{\rm eq}$. A mean internal density of the clump ρ is fixed at the time of the clump formation and according to [34] is $\rho = \kappa\rho_{\rm eq}[\nu\sigma_{\rm eq}(M)/\delta_c]^3$, where $\delta_c = 3(12\pi)^{2/3}/20 \simeq 1.686$.

A tidal destruction of clumps is a complicated process and depends on many factors: the formation history of clumps, host density profile, the existence of another substructure inside the host, orbital parameters of individual clumps in the hosts, etc. Only in numerical simulations, all these factors can be taken into account properly. The first such simulation in the small-scale region was produced in [19]. We use a simplified analytical approach by parametrizing the energy gains in tidal interactions by the number of tidal shocks per dynamical time in the hosts. Using the model [35] for tidal heating, we determine the survival time T, i.e., time of tidal destruction, for a chosen small-scale clump due to the tidal heating inside of a host clump with larger mass. During the dynamical time $t_{\rm dyn} \simeq 0.5(G\rho_h)^{-1/2}$, where ρ_h is a mean internal density of the host, the chosen small-scale clump may belong to several successively destructed hosts. A clump trajectory in the host experiences successive turns accompanied by the "tidal shocks" [35]. Similar shocks come from interactions with other substructures, and in general due to any varying gravitational field. For the considered small-scale clump with a mass M and radius R, the corresponding internal energy increase after a single tidal shock is

$$\Delta E \simeq \frac{4\pi}{3}\gamma_1 G\rho_h M R^2, \tag{38}$$

where a numerical factor $\gamma_1 \sim 1$. Let us denote by γ_2 the number of tidal shocks per dynamical time $t_{\rm dyn}$. The corresponding rate of internal energy growth for a clump is $\dot{E} = \gamma_2\Delta E/t_{\rm dyn}$. A clump is destroyed in the host if its internal energy increase due to tidal shocks exceeds a total energy $|E| \simeq GM^2/2R$. As a result, for a typical time $T = T(\rho, \rho_h)$ of the tidal destruction of a small-scale clump with density ρ inside a more massive host with a density ρ_h we obtain

$$T^{-1}(\rho,\rho_h) = \frac{\dot{E}}{|E|} \simeq 4\gamma_1\gamma_2 G^{1/2}\rho_h^{3/2}\rho^{-1}. \tag{39}$$

It turns out that a resulting mass function of small-scale clumps (see Sec. VI) depends rather weakly on the value of the product $\gamma_1\gamma_2$.

During its lifetime, a small-scale clump can stay in many host clumps of larger mass. After tidal disruption of the first lightest host, a small-scale clump becomes a constituent part of a larger one, etc. The process of hierarchical transition of a small-scale clump from one host to another occurs almost continuously in time up to the final host formation, where the tidal interaction becomes inefficient. The probability of clump survival, determined as a fraction of the clumps with mass M surviving the tidal destruction in hierarchical clustering, is given by the exponential function e^{-J} with

$$J \simeq \sum_h \frac{\Delta t_h}{T(\rho,\rho_h)}. \tag{40}$$

Here, Δt_h is a difference of formation times t_h for two successive hosts, and summation goes over all clumps of intermediate mass scales, which successively host the considered small-scale clump of a mass M. Changing the summation by integration in (40) we obtain

$$J(\rho,\rho_f) = \int_{t_1}^{t_f} \frac{dt_h}{T(\rho,\rho_h)} \simeq \gamma\frac{\rho_1-\rho_f}{\rho} \simeq \gamma\frac{\rho_1}{\rho} \simeq \gamma\frac{t^2}{t_1^2}, \tag{41}$$

where

$$\gamma = 2\gamma_1\gamma_2\kappa^{1/2}G^{1/2}\rho_{\rm eq}^{1/2}t_{\rm eq} \simeq 14(\gamma_1\gamma_2/3), \tag{42}$$

and t, t_1, t_f, ρ, ρ_1, and ρ_f are, respectively, the formation times and internal densities of the considered clump and of its first and final hosts. One may see from Eq. (41) that the first host provides a major contribution to the tidal destruction of the considered small-scale clump, especially if the first host density ρ_1 is close to ρ, and consequently $e^{-J} \ll 1$. Therefore, Eq. (41) gives a qualitatively correct description of the tidal destruction, however in the more detailed approach one should take into account that the dependence of γ on another parameter is possible. As a reasonable estimate, we will use the ansatz given by Eq. (41) for further calculation of mass function.

Now we need to track the number of clumps M (originated from the density peak ν) which enter some larger host during time intervals Δt_1 around each t_1 beginning from the time t of the clump formation. A mass function of small-scale clumps (i.e., a differential mass fraction of DM in the form of clumps survived in hierarchical clustering) can be expressed as

$$\xi \frac{dM}{M} d\nu = dM d\nu \frac{e^{-\nu^2/2}}{\sqrt{2\pi}} \int_{t(\nu\sigma_{\rm eq})}^{t_0} dt_1 \times \left| \frac{\partial^2 F(M, t_1)}{\partial M \partial t_1} \right| e^{-J(t,t_1)}. \tag{43}$$

In this expression, t_0 is the Universe age and $F(M, t)$ is a mass fraction of unconfined clumps (i.e., clumps not belonging to more massive hosts) with a mass smaller than M at time t. According to [34], the mass fraction of unconfined clumps is $F(M, t) = \mathrm{erf}(\delta_c/[\sqrt{2}\sigma_{\rm eq}(M)D(t)])$, where $\mathrm{erf}(x)$ is the error function and $D(t)$ is the growth factor normalized by $D(t_{\rm eq}) = 1$. An upper limit of integration t_0 in Eq. (43) is not crucial and may be extrapolated to infinity because a main contribution to the tidal destruction of clumps is provided by the early formed hosts at the beginning of the hierarchical clustering.

Two processes are responsible for the time evolution of the fraction $\partial^2 F/(\partial M \partial t)$ for unconfined clumps in the mass interval dM: (i) the formation of new clumps and (ii) the capture of smaller clumps into the larger ones. Both of these processes are equally efficient at the time when $\partial^2 F/(\partial M \partial t) = 0$. To take into account the confined clumps (i.e., clumps in the hosts) we need only the second process (ii) for the fraction $\partial F(M, t)/\partial M$. Nevertheless, in Eq. (43), used the fraction $\partial F(M, t)/\partial M$, which depends on both processes. This is not accurate at a typical formation time of a clump with a mass M, when clump density is comparable with the density of hosts. Fortunately, for this time the exponent in Eq. (43) is very small, $e^{-J} \ll 1$, as it can be seen from (41) and (42). Respectively, an uncertain contribution from process (i) to the integral (43) is also very small. Meanwhile, only process (ii) dominates in the integration region where the exponent e^{-J} is not small. For this reason, Eq. (43) provides a suitable approximation for the mass fraction of clumps survived in the hierarchical clustering. The characteristic epoch t_* of the clumps M formation can be estimated from the equation $\sigma_{\rm eq}(M)D(t_*) \simeq \delta_c$. If one considers the times $t \gg t_*$, then the exponents $\exp\{-\delta_c^2/[2\sigma_{\rm eq}^2 D^2(t)]\}$ can be putted approximately to unity for simplification of integration in (43).

Finally, we transform the distribution function (43) to the following form:

$$\xi \frac{dM}{M} d\nu \simeq \frac{\nu d\nu}{\sqrt{2\pi}} e^{-\nu^2/2} f_1(\gamma) \frac{d \log\sigma_{\rm eq}(M)}{dM} dM, \tag{44}$$

where

$$f_1(\gamma) = \frac{2[\Gamma(1/3) - \Gamma(1/3, \gamma)]}{3\sqrt{2\pi}\gamma^{1/3}}, \tag{45}$$

$\Gamma(1/3)$ and $\Gamma(1/3, \gamma)$ are the Euler gamma function and incomplete gamma function, respectively. The function (45) is shown in Fig. 3. It is seen in this figure that $f_1(\gamma)$ varies rather slowly in the interesting interval of $14 < \gamma < 40$, and one may use $f_1(\gamma) \simeq 0.2$–0.3.

Physically, the first factor ν in (44) corresponds to a more effective survival of high-density clumps (i.e., with large values of ν) with respect to the low-density ones (with small values of ν). Integrating Eq. (44) over ν, we obtain

$$\xi_{\rm int} \frac{dM}{M} \simeq 0.02(n + 3) \frac{dM}{M}. \tag{46}$$

An effective power-law index n in Eq. (46) is determined as $n = -3(1 + 2\partial \log\sigma_{\rm eq}(M)/\partial \log M)$ and depends very weakly on M. Equation (46) implies that for suitable values of n, only a small fraction of clumps, about 0.1%–0.5%, survive the stage of hierarchical tidal destruction in each logarithmic mass interval $\Delta \log M \sim 1$. It must be stressed that a physical meaning of the survived clump distribution function $\xi dM/M$ is different from the similar one for the *unconfined* clumps given by the Press-Schechter mass function $\partial F/\partial M$.

The simple M^{-1} shape of the mass function (46) is in very good agreement with the corresponding numerical simulations [19], but our normalization factor is a few times smaller. One also can see a reasonable agreement between the extrapolation of our calculations and the corresponding numerical simulations of the large-scale clumps with $M \geq 10^6 M_\odot$ (for a comparison see [7]). The

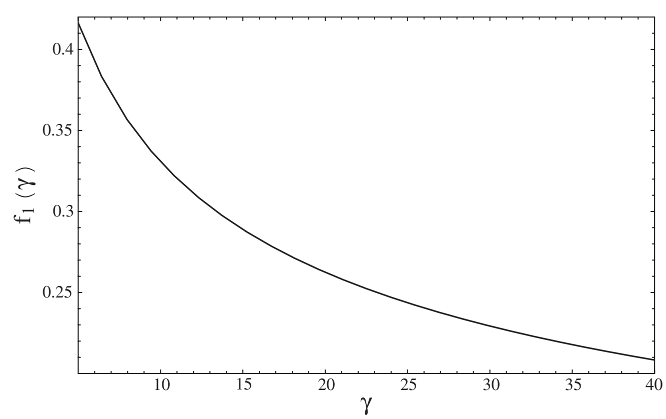

FIG. 3. The function $f_1(\gamma)$ from (45).

obtained mass function (46) is further transformed in the process of tidal destructions of clumps by stars in the Galaxy (see the previous sections).

VI. MODIFIED DISTRIBUTION FUNCTION OF CLUMPS

In this section, we calculate the modified mass function for the small-scale clumps in the Galaxy taking into account clump mass loss instead of the clump destruction considered in [4,12].

According to theoretical model [4] and numerical simulations [19], a differential number density of small-scale clumps in the comoving frame in the Universe is $n(M)dM \propto dM/M^2$. This distribution is shown in Fig. 4 by the solid line. The damping of small-scale perturbations with $M < M_{\rm min}$ provides an additional factor $\exp[-(M/M_{\rm min})^{2/3}]$ responsible for the fading of distribution at small M. The result of the numerical simulations [19] can be expressed in the form of a differential mass fraction of the DM clumps in the Galactic halo $f(M)dM \simeq \kappa(dM/M)$, where $\kappa \simeq 8.3 \times 10^{-3}$. The analytical estimation (46) gives approximately $\kappa \simeq 4 \times 10^{-3}$ for the mass interval $10^{-6}M_\odot < M < 1M_\odot$. The discrepancy by the factor $\simeq 2$ may be attributed to the approximate nature of our approach as well as to the well-known additional factor of 2 in the original Press-Schechter derivation of the mass function. In the later case, one must simply multiply Eq. (43) by a factor of 2. To clarify this discrepancy, the more sophisticated calculations are necessary.

As it was described earlier, we consider that DM clumps originated from 2σ density peaks. Therefore, in our approximation the density of clumps and their distribution depends only on one parameter M. In general, the distribution of DM clumps depends on the pair of parameters, e. g., mass and radius, mass and velocity dispersion, or mass and peak-high ν as in the distribution (44). Meanwhile, the authors of numerical simulations do not present a general distribution of clumps over two parameters. The general distribution of clumps can be in principle extracted from simulations and is very requested for further investigations of DM clumpiness.

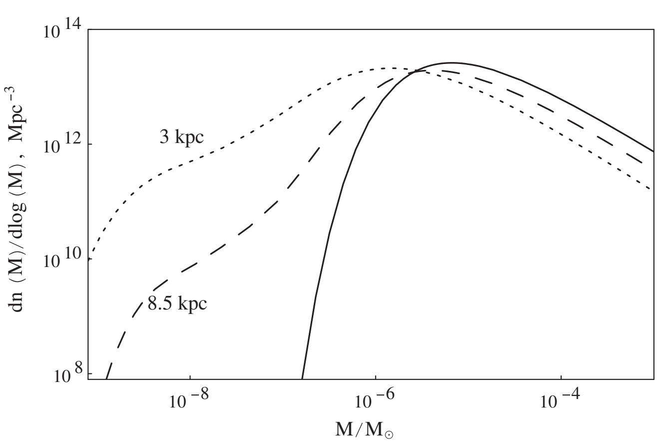

FIG. 4. Numerically calculated modified mass function of clump remnants for galactocentric distances 3 and 8.5 kpc. The solid curve shows the initial mass function.

By using the formalism of Sec. V, we derive the mass distribution of the clump remnants in dependence of the initial masses M_i of clumps. To do this, we calculate numerically the value of the mass μ of the clump remnant in dependence of the initial mass M_i for separate elements $\Delta p \Delta \gamma \Delta \varepsilon$ in the parameter space in (36). Then for fixed intervals $\Delta\mu$ of values of μ, we provide the summation of the weights of distribution function, which is given by Eq. (36) without symbols of integration and Θ function. By using the derived μ distribution, we transform the initial (cosmological) mass function of clumps to the final (nowadays) mass function in the halo at the present moment. This final mass function is shown in Fig. 4 for two distances from the Galactic center. We supposed in numerical calculations that a core radius is very small and all masses of remnants are admissible. With a finite core size, the final mass function has a cutoff near the core's mass of the clump with a minimal mass $M_{\rm min}$. The adiabatic correction leads to the accumulation of remnants of some mass corresponding to the violation of momentum approximation. One can see from Fig. 4 that clump remnants exist below the $M_{\rm min}$. Deep in the bulge (very near to the Galactic center) the clump remnants are more numerous because of intensive destructions of clumps in the dense stellar environment in comparison with the rarefied one in the halo. The main contribution to the low-mass tail of the mass function of remnants comes from the clumps with the near-disk orbits where the destructions are more efficient.

The another important point is an efficient destruction of clumps with orbits confined inside the stellar bulge. Nevertheless, a number of density of clumps inside the bulge is nonzero because a major part of the clumps have orbits extending far beyond the bulge. These "transit" clumps spend only a small part of their orbital time traversing the bulge and survive the tidal destruction.

VII. AMPLIFICATION OF THE ANNIHILATION SIGNAL

A local annihilation rate is proportional to the square of the DM particle number density. A number density of DM particles in a clump is much large than a corresponding number density of the diffuse (not clumped) component of DM. For this reason, an annihilation signal from even a small fraction of DM clumps can dominate over an annihilation signal from the diffuse component of DM in the halo. In this section, we calculate the amplification (or "boosting") of an annihilation signal due to the presence of the survived DM clump remnants in the Galactic halo. We consider here the Harrison-Zeldovich initial perturbation spectrum with power index $n_p = 1$ as a representative example. The value of n_p is not exactly fixed by the current observations of cosmic microwave background (CMB) anisotropy. In the case of $n_p < 1$, the DM clumps are less dense, and a corresponding amplification of the annihilation signal would be rather small [4].

The gamma-ray flux from the annihilation of the diffuse distribution (15) of DM in the halo is proportional to

$$I_{\rm H} = \int_0^{r_{\max}(\zeta)} \rho_H^2(\xi) dx, \qquad (47)$$

where the integration is over r along the line of sight, $\xi(\zeta, r) = (r^2 + r_\odot^2 - 2rr_\odot \cos\zeta)^{1/2}$ is the distance to the Galactic center, $r_{\max}(\zeta) = (R_{\rm H}^2 - r_\odot^2 \sin^2\zeta)^{1/2} + r_\odot \cos\zeta$ is a distance to the external halo border, ζ is an angle between the line of observation and the direction to the Galactic center, $R_{\rm H}$ is a virial radius of the Galactic halo, and $r_\odot = 8.5$ kpc is the distance between the sun and Galactic center. The corresponding signal from annihilations of DM in clumps is proportional to the quantity [4]

$$I_{\rm cl} = S \int_0^{r_{\max}(\zeta)} dx \int_{M_{\min}} f(M) dM \rho \rho_H(\xi) P(\xi, \rho), \qquad (48)$$

where $\rho(M)$ is the mean density of the clump. The function S depends on the clump density profile and core radius of the clump [4], and we use $S \simeq 14.5$ as a representative example. The observed amplification of the annihilation signal is defined as $\eta(\zeta) = (I_{\rm cl} + I_{\rm H})/I_{\rm H}$ and is shown in Fig. 6 for the case $x_c = 0.1$. It tends to unity at $\zeta \to 0$ because of the divergent form of the halo profile (15). The annihilation of diffuse DM prevails over signal from clumps at the Galactic center. The $\eta(\zeta)$ very slightly depends on x_c, and corresponding graphs for $x_c < 0.1$ are almost indistinguishable from the one in Fig. 5. This is because the observed signal is obtained by integration along the line of sight and the effect of the clump's destruction at the Galactic center is masked by the signal from another region of the halo.

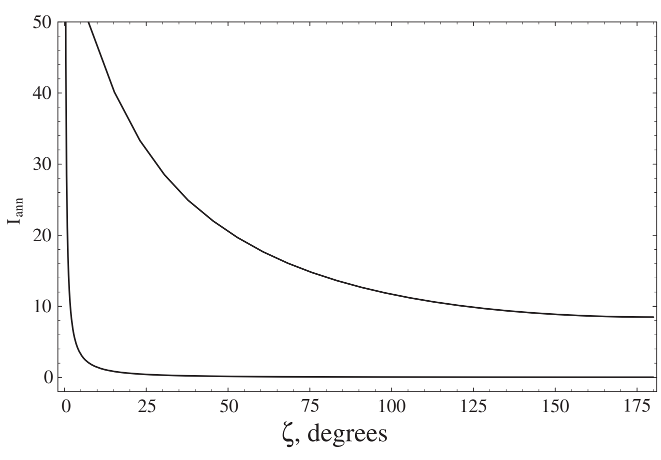

FIG. 5. The annihilation signal (48) (upper curve) as a function of the angle ζ between the line of observation and the direction to the Galactic center. For comparison the annihilation signal is also shown (by the bottom curve) from the Galactic halo without DM clumps (47). The values of both integrals (47) and (48) are multiplied by a factor of 10^{48}.

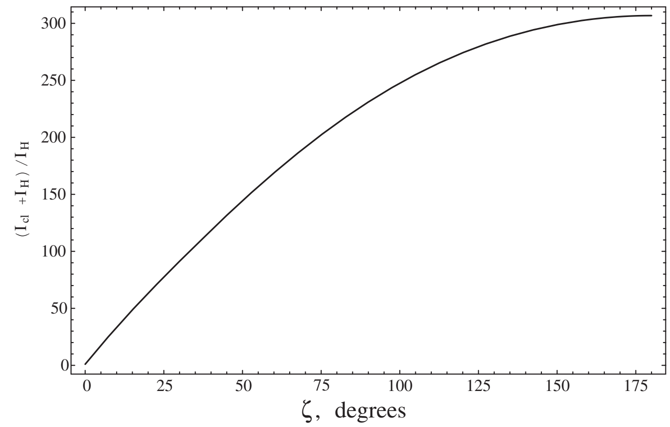

FIG. 6. The amplification of the annihilation signal $(I_{\rm cl} + I_{\rm H})/I_{\rm H}$ as a function of the angle between the line of observation and the direction to the Galactic center, where fluxes are given by (47) and (48).

This amplification of an annihilation signal is often called a "boost factor." A boost factor of the order of 10 is required for interpretation of the observed EGRET gamma-ray excess as a possible signature of DM neutralino annihilation [36].

VIII. CONCLUSION

In [5] it was found that almost all small-scale clumps in the Galaxy are destructed by tidal interactions with stars and transformed into "ministreams" of DM. The properties of these ministreams may be important for the direct detection of DM particles because DM particles in streams arrive anisotropically from several discrete directions. In this work, we demonstrate that the cores of clumps (or clump remnants) survive in general during the tidal destruction by stars in the Galaxy. Although their outer shells are stripped and produce the ministreams of DM, the central cores are protected by the adiabatic invariant and survived as the sources of annihilation signals. This conclusion depends crucially on the unknown sizes of the cores: the smaller cores are more protected because DM particles there have higher orbital frequencies and therefore the larger the adiabatic parameter.

Despite the small survival probability of clumps during early stage of hierarchical clustering, they provide the major contribution to the annihilation signal (in comparison with the unclumpy DM). The amplification (boost factor) can reach 10^2 or even 10^3 depending on the initial perturbation spectrum and minimum mass of clumps. This boost factor must be included in calculations of the annihilation signals. Some promising interpretations of observations and calculations of annihilation signal from the Galactic halo require this boost factor (see, e.g., [36]). The discussed dense remnants survive the tidal destruction and provide the enhancement of DM annihilation in the Galaxy. These remnants of DM clumps form the low-mass tail in the standard mass distribution of small-scale clumps extended much below $M_{\rm min}$ of the standard distribution. It does not mean of course the increasing of annihilation signal in comparison with the case without clump destruction. It only indicates that galactic clump destruction does not diminishe strongly the annihilation signal.

The principle simplifying assumption of this work is that only the outer layers of clumps are subjected by the tidal stripping. The main difficulties in considering the full problem with the mass loss from inner layers are in the complicated dynamical reconstruction of clumps just after tidal shocks. We believe that our approach provides a rather good result by the two reasons. First, the influence of tidal forces depends on the system size, and, therefore, the outer layers are greatly subjected to tidal forces. Second, the adiabatic protection is more efficient in the inner part of the clump because of higher orbital frequencies here. In reality, we expect some expansion of the clump and diminishing of its central density due to energy deposits from tidal forces. It would be a very interesting task to clarify this process in future works.

The numerical estimate of the boost factor for DM particle annihilation inside clumps is very model dependent. It depends on the nature of DM particles and on their interaction with ambient plasma. The important physical parameters, which affect the annihilation rate in clumps, are decoupling temperature T_d and minimal mass $M_{\rm min}$ in the clump mass distribution. The boost factor increases strongly for small $M_{\rm min}$. The minimal mass in standard calculations is determined by the escape of DM particles from a growing fluctuation due to, e.g., diffusion, free-streaming or Silk effect. Uncertainties in the calculated values of T_d and $M_{\rm min}$ are discussed in the Appendix. For the lightest neutralino as a DM particle, assuming it to be the pure bino B, one can see from Table III a huge difference in $M_{\rm min}$ caused by the variation of supersymmetry (SUSY) parameters m_χ and $\tilde{m}$. For these parameters, we use cosmologically allowed values from the benchmark scenarios of the work [37]. Moreover, inclusion of other neutralino compositions, e.g., mixed bino-Higgsino, the other allowed benchmark scenarios with coannihilation and focus-point regions, and some other modifications, may very considerably increase the allowed region of $M_{\rm min}$ values up to $(3\times10^{-12}$–$7\times10^{-4})M_\odot$ [38]. Inclusion of the other particle candidates further extends this region.

Another parameter variation which affects strongly the boost factor is the spectral index of density perturbation n_p (see [4]). We conclude thus that the annihilation boost (enhancement) factor even for neutralino has large uncertainties due to the difference in SUSY parameters and spectral index n_p. It can reach the factor 10^4 and even more, the largest values of boost factor can already be excluded by observations of indirect signal since mSUGRA parameters can be fixed for this largest value. On the contrary, a tidal destruction of clumps in the Galaxy affects the annihilation boost factor much less.

TABLE I. The values of decoupling temperature T_d and minimal clump mass $M_{\rm min}$ with $m_\chi = 100$ GeV and $\tilde{m} = 200$ GeV for different damping mechanisms: 1) free-streaming, 2) collision damping, 3) acoustic oscillations, 4) quasi-free-streaming with friction.

Reference:	[42] 1)	[4] 1)	[6] 2)	[41] 3)	[10] 4)
T_d, MeV	28	26	25	20	22.6
$M_{\rm min}/M_\odot$	2.5×10^{-7}	1.7×10^{-7}	1.5×10^{-6}	1.3×10^{-5}	8.4×10^{-6}

TABLE II. Selected benchmark scenarios from [37]. The masses of particles are given in GeV.

Scenario	χ	$\tilde{e}_L$	$\tilde{e}_R$	$\tilde{\nu}_e, \tilde{\nu}_\mu$	$\tilde{\nu}_\tau$
B′	95	188	117	167	167
E′	112	1543	1534	1539	1532
M′	794	1660	1312	1648	1492

TABLE III. Values of T_d and $M_{\rm min}$ for the Bertschinger [10] damping scenario and three benchmark scenarios [37] which close to scenarios B′, E′, and M′ shown in Table II.

m_χ	$\tilde{m}$	T_d	$M_{\rm min}$
100 GeV	200 GeV	22.6 MeV	$8.4\times10^{-6}M_\odot$
100 GeV	1500 GeV	196 MeV	$1.3\times10^{-8}M_\odot$
800 GeV	1600 GeV	305 MeV	$3.5\times10^{-9}M_\odot$

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APPENDIX: MINIMUM CLUMP MASS

1. Uncertainties in minimal clump mass and decoupling temperature

The low-mass cutoff of the clump mass spectrum accompanies the process of decoupling. It starts when DM particles coupled strongly with surrounding plasma in the growing density fluctuations. The smearing of the small-scale fluctuations is due to the collision damping occurring just before decoupling, in analogy with the Silk damping [39]. It occurs due to the diffusion of DM particles from a growing fluctuation, and only the small-scale fluctuations can be destroyed by this process. The corresponding diffusive cutoff $M_{\rm min}^{\rm diff}$ is very small. As coupling becomes weaker, the larger fluctuations are destroyed and $M_{\rm min}$ increases. One may expect that the largest value of $M_{\rm min}$ is related to a free-streaming regime. However, as recent calculations show [10], the largest $M_{\rm min}$ is related to some friction between DM particles and cosmic plasma similar to the Silk damping. The predicted minimal clump masses range from very low values, $M_{\rm min} \sim 10^{-12} M_{\odot}$ [40], produced by a diffusive escape of DM particles, up to $M_{\rm min} \sim 10^{-4} M_{\odot}$, caused by acoustics oscillations [41] and quasi-free-streaming with limited friction [10].

The calculations of minimal clump mass $M_{\rm min}$ and decoupling temperature T_d are determined by elastic scattering of DM particles off leptons $l = (\nu_L, e_L, e_R)$ in cosmic plasma. The uncertainties in cross section very strongly influence the resulting values of $M_{\rm min}$ and T_d. In all works cited above, the lightest neutralino (χ) in the form of pure bino ($\tilde{B}$) is assumed as a DM particle, and χl scattering occurs due to exchange by sleptons $\tilde{l} = (\tilde{\nu}_L, \tilde{e}_L, \tilde{e}_R)$.

The elastic cross sections for $l_L \chi$ and $l_R \chi$ scattering have been calculated in [4] as

$$\left(\frac{d\sigma}{d\Omega}\right)_{l_L\chi} = \frac{\alpha_{\rm em}^2}{8\cos^4\theta_{\rm W}} \frac{\omega^2(1+\cos\theta_{\rm cm})}{(\tilde{m}_L^2 - m_\chi^2)^2} \tag{A1}$$

and

$$\left(\frac{d\sigma}{d\Omega}\right)_{l_R\chi} = 16\left(\frac{d\sigma}{d\Omega}\right)_{l_L\chi}, \quad \text{if } \tilde{m}_L = \tilde{m}_R, \tag{A2}$$

where $\omega \gg m_l$ is a c.m. energy of l, $\theta_{\rm cm}$ is a scattering angle of l in a c.m. system, m_χ is a neutralino (bino) mass, $\tilde{m}_L$ and $\tilde{m}_R$ are, respectively, a mass of the left and right sfermions, and $\theta_{\rm W}$ is the Weinberg angle.

The values of T_d and $M_{\rm min}$ as cited in [4,6,41,42] differ very much from each other, but a very big contribution to this difference comes from the differences in the used values for m_χ and $\tilde{m}$. To see the difference, which must be attributed to the different damping mechanisms used in these works, we recalculated T_d and $M_{\rm min}$ with the same values of m_χ and $\tilde{m}$, for which we used 100 and 200 GeV, respectively. The results are presented in Table I. We did not include there the work [40] because a pure diffusive damping results in too low a value of $M_{\rm min}$. From Table I, one can see a reasonable agreement in values of T_d and $M_{\rm min}$ and a successive increasing of $M_{\rm min}$ from $2.5 \times 10^{-7} M_{\odot}$ for free-streaming to $\sim 110^{-5} M_{\odot}$ for oscillation damping and quasi-free-streaming with friction.

2. Uncertainties in SUSY parameters

We shall consider now the range of predictions for different values of SUSY parameters allowed in cosmology. For this aim, we shall use the SUSY benchmark scenarios from the work [37], which agrees with the Wilkinson microwave anisotropy probe (WMAP) and other cosmological data. These benchmark scenarios are obtained within the mSUGRA model with universal parameters at the grand unified theory scale: m_0 (the universal scalar soft breaking mass), $m_{1/2}$ (the universal gaugino soft breaking mass), A_0 (the universal cubic soft breaking terms), and $\tan\beta$ (the ratio of two Higgs v.e.v.'s). The LEP and $b \to s\gamma$ constraints are imposed. The resulting relic density of neutralinos from these scenarios is in agreement with the WMAP data or can be obtained with small changes of m_0 and $m_{1/2}$. In Table II, we display three benchmark scenarios from [37]. Scenario B′ gives the lower value $m_\chi \approx 100$ GeV and $\tilde{m}$ close to 200 GeV, which we discussed above. Scenario M′ gives the highest value $m_\chi \approx 800$ GeV and $\tilde{m} \approx 1600$ GeV. Respectively, scenario EE′ gives the intermediate value $m_\chi \approx 110$ GeV and $\tilde{m} \approx 1500$ GeV, similar to those we used in [4].

To illustrate the uncertainties in T_d and $M_{\rm min}$ due to uncertainties in m_χ and $\tilde{m}$ (in the simplifying assumption that $m_{\tilde{\nu}} = m_{\tilde{e}_L} = m_{\tilde{e}_R}$) we choose the calculations of Bertschinger [10] in the quasi-free-streaming scenario with friction, which seems to be at present the most detailed ones. We use the Bertschinger formulas

$$T_d = 7.65 C^{-1/4} g_*^{1/8} \left(\frac{m_\chi}{100\ \text{GeV}}\right)^{5/4} \text{MeV}, \tag{A3}$$

$$M_{\rm min} = 7.59 \times 10^{-3} C^{3/4} \left(\frac{m_\chi \sqrt{g_*}}{100\ \text{GeV}}\right)^{-15/4} M_{\odot}, \tag{A4}$$

with a dimensionless constant

$$C = 256 (G_F m_W^2)^2 \left(\frac{\tilde{m}^2}{m_\chi^2} - 1\right)^{-2} \sum_L (b_L^4 + c_L^4), \tag{A5}$$

where G_F is the Fermi coupling constant, b_L and c_L are left and right chiral vertices, and m_W, $\tilde{m}$, and m_χ are, respectively, the masses of the W boson $G_F m_W^2 = 0.0754$, the slepton, the neutralino, and the number of freedom at the decoupling epoch $g_* = 43/4$. (Our own calculations in [4] of C, which is related with the square of the matrix element for $l + \chi \to l + \chi$ scattering, differ from (A4) by a factor of 1.6.) As a result, we obtain for the benchmark scenarios which approximately coincide with model B′ (minimum m_χ and $\tilde{m}$), E′ (minimum m_χ and large $\tilde{m}$), and M′ (very large m_χ and $\tilde{m}$) the values of T_d and $M_{\rm min}$ listed in Table III. The predicted range of parameters for $M_{\rm min}$ from this Table, $(3.5 \times 10^{-9} - 8.4 \times 10^{-6}) M_\odot$ is not robust at all. It is obtained within mSUGRA assumptions about possible universality of SUSY parameters m_0, $m_{1/2}$, and A_0. Lifting the universality restriction, the mass of the neutralino can increase up to the TeV range scale (though $m_\chi > 200$ GeV needs a fine-tuning less than 1% in SUSY [43] or decreased down to a few GeV [44]).

In the numerical predictions above, we limited ourselves by rather restrictive assumptions on the mSUGRA model. The most important of them are assumptions that the neutralino is a pure bino state and a choice of a cosmologically allowed benchmark scenario. The detailed analysis made in [38] showed that the allowed parameters of mSUGRA result in much wider possibilities, e.g., neutralino as mixed bino-Higgsino and the other benchmark scenarios. These possibilities are considered under WMAP cosmological constraints and a condition of producing the corresponding DM density for each set of mSUGRA parameters. The considered modifications allow new channels of neutralino interactions with ordinary particles, e.g., the exchange by Z-boson, coannihilation and resonances in neutralino-fermion scattering. It results in a wide range (many orders of magnitude) of scattering cross sections, and, respectively, in a wide range of decoupling temperature from 5 MeV to 3 GeV. The corresponding range of $M_{\rm min}$ is given by $(3 \times 10^{-12}$–$7 \times 10^{-4}) M_\odot$. The authors consider also the Kaluza-Klein particle as a DM candidate.

The small-scale mass of $M_{\rm min}$ results in the large density of DM clumps, and thus in a much stronger annihilation signal from the Galactic halo. For typical values of the power index of the perturbation spectrum (from CMB observations) the small-scale mass of $M_{\rm min}$ results in the large density of DM clumps, and thus in a much stronger annihilation signal from the Galactic halo. However, the dependence of a mean clump density on the clump mass is rather weak due to the nearly flat form of the perturbation spectrum at small scales. The crucial factor for the amplification of the annihilation signal by clumps is the value of the perturbation power-law index n_p.

[1] C. Calcaneo-Roldan and B. Moore, Phys. Rev. D **62**, 123005 (2000).
[2] C. Schmid, D.J. Schwarz, and P. Widerin, Phys. Rev. D **59**, 043517 (1999).
[3] D.J. Schwarz and S. Hofmann, Nucl. Phys. B, Proc. Suppl. **87**, 93 (2000).
[4] V. Berezinsky, V. Dokuchaev, and Yu. Eroshenko, Phys. Rev. D **68**, 103003 (2003).
[5] H.S. Zhao, J. Taylor, J. Silk, and D. Hooper, arXiv :astro-ph/0502049v4.
[6] A.M. Green, S. Hofmann, and D.J. Schwarz, J. Cosmol. Astropart. Phys. 08 (2005) 003.
[7] V. Berezinsky, V. Dokuchaev, and Yu. Eroshenko, Phys. Rev. D **73**, 063504 (2006).
[8] J. Diemand, M. Kuhlen, and P. Madau, Astrophys. J. **649**, 1 (2006).
[9] A.M. Green and S.P. Goodwin, Mon. Not. R. Astron. Soc. **375**, 1111 (2007).
[10] E. Bertschinger, Phys. Rev. D **74**, 063509 (2006).
[11] G.W. Angus and H.S. Zhao, Mon. Not. R. Astron. Soc. **375**, 1146 (2007).
[12] V. Berezinsky, V. Dokuchaev, and Yu. Eroshenko, J. Cosmol. Astropart. Phys. 07 (2007) 011.
[13] C. Giocoli, L. Pieri, and G. Tormen, arXiv:0712.1476v1.
[14] M. Kamionkowsky and S.M. Koushiappas, arXiv:0801 .3269v1.
[15] L. Pieri, G. Bertone, and E. Branchini, arXiv:0706.2101.
[16] S. Ando and E. Komatsu, Phys. Rev. D **73**, 023521 (2006).
[17] Z. Myers and A. Nusser, arXiv:0710.0135v1.
[18] A.V. Gurevich and K.P. Zybin, Sov. Phys. JETP **67**, 1 (1988); **67**, 1957 (1988); Phys. Usp. **38**, 687 (1995).
[19] J. Diemand, B. Moore, and J. Stadel, Nature (London) **433**, 389 (2005).
[20] A.G. Doroshkevich, V.N. Lukash, and E.V. Mikheeva, arXiv:0712.1688v1.
[21] M. Kaplinghat, Phys. Rev. D **72**, 063510 (2005).
[22] L.E. Strigari, M. Kaplinghat, and J.S. Bullock, Phys. Rev. D **75**, 061303 (2007).
[23] O.Y. Gnedin and J.P. Ostriker, Astrophys. J. **513**, 626 (1999).
[24] J.E. Taylor and A. Babul, Astrophys. J. **559**, 716 (2001).
[25] J. Diemand, M. Kuhlen, and P. Madau, Astrophys. J. **667**, 859 (2007).
[26] T. Goerdt, O.Y. Gnedin, B. Moore, J. Diemand, and J. Stadel, Mon. Not. R. Astron. Soc. **375**, 191 (2007).
[27] J.P. Ostriker, L. Spitzer, Jr., and R.A. Chevalier, Astrophys. J. Lett. **176**, L51 (1972).
[28] M.D. Weinberg, Astron. J. **108**, 1403 (1994).
[29] A.S. Eddington, Mon. Not. R. Astron. Soc. **76**, 572 (1916).
[30] L.M. Widrow, Astrophys. J. Suppl. Ser. **131**, 39 (2000).
[31] R. Launhardt, R. Zylka, and P.G. Mezger, Astron. Astrophys. **384**, 112 (2002).

[32] E. F. Bell *et al.*, arXiv:0706.0004v1 [Astropys. J. (to be published)].
[33] I. King, Astron. J. **67**, 471 (1962).
[34] C. Lacey and S. Cole, Mon. Not. R. Astron. Soc. **262**, 627 (1993).
[35] O. Y. Gnedin, L. Hernquist, and J. P. Ostriker, Astrophys. J. **514**, 109 (1999).
[36] W. de Boer, C. Sander, V. Zhukov, A. V. Gladyshev, and D. I. Kazakov, Astron. Astrophys. **444**, 51 (2005).
[37] M. Battaglia, A. De Roeck, J. Ellis, F. Gianotti, K. A. Olive, and L. Pape, Eur. Phys. J. C **33**, 273 (2004).
[38] S. Profumo, K. Sigurdson, and M. Kamionkowski, Phys. Rev. Lett. **97**, 031301 (2006).
[39] J. Silk, Astrophys. J. **151**, 459 (1968).
[40] K. P. Zybin, M. I. Vysotsky, and A. V. Gurevich, Phys. Lett. **260A**, 262 (1999).
[41] A. Loeb and M. Zaldarriaga, Phys. Rev. D **71**, 103520 (2005).
[42] D. J. Schwarz, S. Hofmann, and H. Stocker, Phys. Rev. D **64**, 083507 (2001).
[43] V. Berezinsky, A. Bottino, J. Ellis, N. Fornengo, G. Mignola, and S. Scopel, Astropart. Phys. **5**, 333 (1996).
[44] A. Bottino, N. Fornengo, and S. Scopel, Phys. Rev. D **67**, 063519 (2003).