# Flavor changing neutral Higgs bosons in a supersymmetric extension based on a $Q_6$ family symmetry

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A supersymmetric extension of the standard model based on the discrete $Q_6$ family symmetry is considered, and we investigate flavor changing neutral current (FCNC) processes, especially those mediated by heavy flavor changing neutral Higgs bosons. Because of the family symmetry the number of the independent Yukawa couplings is smaller than that of the observed quantities such as the Cabibbo-Kobayashi-Maskawa matrix and the quark masses, so that the FCNCs can be parametrized only by the mixing angles and masses of the Higgs fields. We focus our attention on the mass differences of the neutral $K$, $D$, and $B$ mesons. All the constraints including that from the ratio $\Delta M_{B_s}/\Delta M_{B_d}$ can be satisfied, if the heavy Higgs bosons are heavier than $\sim$1.5 TeV. If the constraint from $\Delta M_K$ is slightly relaxed, the heavy Higgs bosons can be as light as $\sim$0.4 TeV, which is within the accessible range of LHC.

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## I. INTRODUCTION

In recent studies on flavor symmetries[1] it has become clear that a flavor symmetry can be realized at low energies. As long as this possibility is not excluded, theoretical as well as experimental searches for a low energy flavor symmetry should be continued. An important prediction of any viable low energy flavor symmetry, which is broken only spontaneously or at most softly, is the existence of multiple $SU(2)_L$ doublet Higgs fields, as one could read off from a sort of no-go theorem of [4]. This implies that there should exist several neutral Higgs fields that have flavor changing couplings to the fermions at the tree level. Therefore, an observation of a nonstandard flavor changing neutral current (FCNC) process, at LHC for instance, is not necessarily an indication of supersymmetry [5,6].

In Ref. [7] a supersymmetric flavor model based on a dicyclic dihedral group $Q_6$ has been suggested.[2] The main motivation there was to derive a modified Fritzsch mass matrix for the quarks from a flavor symmetry. With an assumption that $CP$ is spontaneously broken, the model can fix six quark masses and four Cabibbo-Kobayashi-Maskawa (CKM) parameters in term of nine parameters of the model. It has been later realized in Refs. [21,22] that through an appropriate change of the lepton assignment, the leptonic sector can be brought into the same form as that of the model of [23,24]. Then there are only seven parameters in the leptonic sector of the model to fix six lepton masses and six Maki-Nakagawa-Sakata (MNS) parameters. The discrete flavor group $Q_6$ is the smallest non-Abelian group with which the above situation can be achieved.

However, it turned out that one has to introduce a certain set of $SU(2)_L \times U(1)_Y$ singlet fields and also additional Abelian global symmetries to make the model viable. Nothing is wrong with this situation, but in this paper we would like to stress the minimal content of the Higgs fields and at the same time a "one + two" structure for each family; one $Q_6$ singlet and one $Q_6$ doublet for each family including the $SU(2)_L$ doublet Higgs fields. In Sec. II we will shed light upon the relation between the nonrenormalization theorem and flavor symmetry, and will show that different flavor symmetries can be consistently introduced into a softly broken supersymmetric gauge theory. We will systematically investigate this possibility in a general framework. With this observation we will find in Sec. III that the one + two structure of family in a minimal $Q_6$ extension of the supersymmetric standard model (MSSM) can be consistently realized.

In Sec. IV we will consider the Higgs sector. Because of the one + two structure the Higgs sector is much simpler than that of [7,21,22], and therefore the sector can be investigated with much less assumptions. We will explicitly show that it is possible to fine-tune the soft-supersymmetry-breaking (SSB) parameters so as to make the heavy Higgs bosons much heavier (several TeV) than $M_Z$ and at the same time to obtain a desired size of spontaneous $CP$ violation to reproduce the Kobayashi-Maskawa $CP$-violating phase.

In Sec. V we will first calculate the unitary matrices that diagonalize the fermion mass matrices, which are needed to write down the Yukawa couplings in terms of mass eigenstates. We only briefly mention FCNCs and $CP$ violations in the SSB sector and in the lepton sector, because detailed investigations on these subjects have been recently carried out in Ref. [22] and in Ref. [25], respectively. Instead we investigate FCNC processes mediated by neu-

[1]For recent reviews see, for instance, [1–3].

[2]$Q_6$ is one of $Q_{2N}$ with $N = 2, 3, \ldots$, which are the "covering groups" of the dihedral groups $D_N$ [8,9]. In recent years there are a number of interesting flavor models based on $Q_{2N}$ and $D_N$. For instance, $D_4$ has been used as a flavor symmetry in Refs. [10–14], while $D_5$, $D_6$, $D_7$, and $Q_4$ have been considered in Refs. [15–18], respectively. See also Refs. [19,20].

 

tral heavy Higgs fields. We concentrate on the constraints coming from the mass differences in the neutral meson systems, $\Delta M_K$, $\Delta M_{B_s}$, $\Delta M_{B_d}$, and $\Delta M_D$, in a similar spirit as Refs. [26–32] and references therein. We express the relevant flavor changing neutral Yukawa couplings in terms of the mass eigenstates, where except the phases the size of the Yukawa couplings are basically fixed. Allowed ranges in which the constraints are satisfied are shown in different figures. We find that the heavy Higgs bosons should be heavier than ~1.5 TeV, although it is possible to fine-tune the parameters such that the constraints can be satisfied for lighter mass values.

Section VI is devoted for conclusion.

## II. NONRENORMALIZATION THEOREM AND FLAVOR SYMMETRY

A flavor symmetry can control the structure of the independent parameters of a theory. In supersymmetric theories, moreover, the nonrenormalization theorem allows to suppress certain couplings and also to relate them with each other, without facing contradictions with renormalization. What is therefore the (technical) role of a flavor symmetry in supersymmetric theories? We recall that the D-terms are renormalized and the wave function renormalization can mix matter superfields $\Phi_i$'s in general. Therefore, starting with diagonal kinetic terms $\Phi_i^*\Phi_i$ is not always consistent with renormalization. If a nondiagonal (infinite) kinetic term is induced, a corresponding nondiagonal counterterm should be added. Then after the diagonalization the originally assumed structure of the couplings in the superpotential will receive large quantum corrections. In other words, we have in spite of the nonrenormalization theorem more parameters in the superpotential, when written in terms of the bare fields, than originally assumed. The undesired mixing among $\Phi_i$'s and large quantum corrections can be avoided if an appropriate flavor symmetry is present.

We will see below that the nonrenormalization theorem and the renormalization properties of the soft-supersymmetry-breaking (SSB) terms allow us to introduce in a consistent manner different flavor symmetries for different sectors of a softly broken supersymmetric theory to control the independent parameters of the theory.

To be more specific, we consider an $N = 1$ supersymmetric gauge theory whose superpotential is given by

$$W(\Phi) = W_Y(\Phi) + W_\mu(\Phi), \tag{1}$$

with

$$W_Y(\Phi) = \tfrac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k \quad \text{and} \quad W_\mu(\Phi) = \tfrac{1}{2}\mu^{ij}\Phi_i\Phi_j. \tag{2}$$

The SSB Lagrangian can be written as

$$\begin{aligned} L(\Phi, W) = &-\Big(\int d^2\theta\,\eta\Big(\frac{1}{6}h^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}b^{ij}\Phi_i\Phi_j \\ &+ \frac{1}{2}M_g W_A^\alpha W_{A\alpha}\Big) + \text{H.c.}\Big) \\ &- \int d^4\theta\,\tilde{\eta}\eta\bar{\Phi}^j(m^2)^i_j(e^{2gV})^k_i\Phi_k, \end{aligned} \tag{3}$$

where $\eta = \theta^2$, $\tilde{\eta} = \tilde{\theta}^2$ are the external spurion superfields and $M_g$ is the gaugino mass. The $\beta$ functions of the $Y$, $\mu$, $h$, and $m^2$ are given by Refs. [33–41]

$$\beta_Y^{ijk} = \gamma^i{}_l Y^{ljk} + \gamma^j{}_l Y^{ilk} + \gamma^k{}_l Y^{ijl}, \tag{4}$$

$$\beta_\mu^{ij} = \gamma^i{}_l\mu^{lj} + \gamma^j{}_l\mu^{il}, \tag{5}$$

$$\begin{aligned} \beta_h^{ijk} = &\gamma^i{}_l h^{ljk} + \gamma^j{}_l h^{ilk} + \gamma^k{}_l h^{ijl} - 2\gamma^i_{1l}Y^{ljk} \\ &- 2\gamma^j_{1l}Y^{ilk} - 2\gamma^k_{1l}Y^{ijl}, \end{aligned} \tag{6}$$

$$\beta_b^{ij} = \gamma^i{}_l b^{lj} + \gamma^j{}_l b^{il} - 2\gamma^i_{1l}\mu^{lj} - 2\gamma^j_{1l}\mu^{il}, \tag{7}$$

$$(\beta_{m^2})^i{}_j = \Big[\Delta + X\frac{\partial}{\partial g}\Big]\gamma^i{}_j, \tag{8}$$

$$\mathcal{O} = \Big(M_g g^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial Y^{lmn}}\Big), \tag{9}$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2|M_g|^2 g^2\frac{\partial}{\partial g^2} + \tilde{Y}_{lmn}\frac{\partial}{\partial Y_{lmn}} + \tilde{Y}^{lmn}\frac{\partial}{\partial Y^{lmn}}, \tag{10}$$

where $(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j$, $Y_{lmn} = (Y^{lmn})^*$, and

$$\tilde{Y}^{ijk} = (m^2)^i{}_l Y^{ljk} + (m^2)^j{}_l Y^{ilk} + (m^2)^k{}_l Y^{ijl}, \tag{11}$$

$$X = \frac{-|M_g|^2 C(G) + \sum_l m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2}. \tag{12}$$

Here $X$ of (12) is the expression in the renormalization scheme of Novikov *et al.* [42], $T(R_l)$ is the Dynkin index of $R_l$, and $C_2(G)$ is the quadratic Casimir of the adjoint representation of the gauge group $G$. From Eqs. (4)–(12) we now derive the hierarchical structure of the renormalization properties of the theory, which is basically the Symanzik theorem applied to softly broken supersymmetric gauge theories:

(1) The (infinite) renormalization of the supersymmetric parameters $Y^{ijk}$, $\mu^{ij}$ is not influenced by the SSB terms, in accord with the definition of the SSB terms.

(2) The (infinite) renormalization of the trilinear couplings $h^{ijk}$ does not depend on $\mu^{ij}$. It is also independent on $(m^2)^i_j$ and $b^{ij}$.

(3) The (infinite) renormalization of the soft scalar masses $(m^2)^i_j$ does not depend on $b^{ij}$ and $\mu^{ij}$, as one can see from Eqs. (8)–(12).

(4) The (infinite) renormalization of $b^{ij}$ does not depend on $(m^2)^i_j$ and $h^{ijk}$, which is the consequence of (7).

Because of these renormalization properties we can consistently introduce different symmetries for different sectors.

To begin with we assume the existence of a flavor symmetry in the Yukawa sector which protects the mixing (of the wave function renormalization) among the matter superfields $\Phi_i$'s.[3] This implies that the anomalous dimensions $\gamma^i_j$ are diagonal, i.e.,

$$\gamma^i{}_j = \delta^i{}_j \gamma_j. \tag{13}$$

Then Eqs. (4)–(8) become

$$\beta_Y^{ijk} = Y^{ijk}(\gamma_i + \gamma_j + \gamma_k), \qquad \beta_\mu^{ij} = \mu^{ij}(\gamma_i + \gamma_j), \tag{14}$$

$$\begin{aligned} \beta_h^{ijk} &= (h^{ijk} - 2Y^{ijk}\mathcal{O})(\gamma_i + \gamma_j + \gamma_k), \\ \beta_b^{ij} &= (b^{ij} - 2\mu^{ij}\mathcal{O})(\gamma_i + \gamma_j), \end{aligned} \tag{15}$$

$$(\beta_{m^2})_l = \left[\Delta + X\frac{\partial}{\partial g}\right]\gamma_l, \tag{16}$$

with $\tilde{Y}^{ijk} = Y^{ijk}(m_i^2 + m_j^2 + m_k^2)$. From these equations we observe:

(a) The $\mu$ sector can have a flavor symmetry which is different from the flavor symmetry of the Yukawa sector if both symmetries are compatible with respect to renormalization of $\mu_{ij}$.

(b) It is consistent to introduce into the trilinear couplings the same flavor symmetry as that of the Yukawa couplings, even if it is violated in other sectors.

(c) The flavor symmetry which protects the mixing among $\Phi_i$'s ensures that $(m^2)^i_j$ is diagonal. If the Yukawa couplings and trilinear couplings have the flavor symmetry, the soft scalar mass terms, too, can have the flavor symmetry, even if the $\mu$ and $b$ terms do not respect the flavor symmetry.

(d) The $b$ terms associated with the $\mu$ terms should always exist (see (16)). But the $b$ sector has no influence on the infinite renormalization of the parameters in other sectors. So the violation of a symmetry in the $b$ sector is absolutely soft.

In the next section we reconsider the supersymmetric flavor model of [7,21,22] along the line of thought about a flavor symmetry in this section.

## III. THE MODEL

The supersymmetric flavor model of [7,21,22] is based on a dicyclic dihedral group $Q_6$. If $CP$ is spontaneously broken, the nine parameters of the model express six quark masses and four CKM parameters. In the leptonic sector there are only seven parameters to fix six lepton masses and six MNS parameters. As we announced in the introduction we would like to stress the one + two structure for each family; a $Q_6$ singlet and a $Q_6$ doublet for each family including the $SU(2)_L$ doublet Higgs fields.

TABLE I. The $Q_6 \times R$ assignment of the chiral matter supermultiplets, where $R$ is the $R$ parity. The group theory notation is given in Ref. [7].

| | $Q$ | $Q_3$ | $U^c, D^c$ | $U_3^c, D_3^c$ | $L$ | $L_3$ | $E^c, N^c$ | $E_3^c$ | $N_3^c$ | $H^u, H^d$ | $H_3^u, H_3^d$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $Q_6$ | $\mathbf{2}_1$ | $\mathbf{1}_{+,2}$ | $\mathbf{2}_2$ | $\mathbf{1}_{-,1}$ | $\mathbf{2}_2$ | $\mathbf{1}_{+,0}$ | $\mathbf{2}_2$ | $\mathbf{1}_{+,0}$ | $\mathbf{1}_{-,3}$ | $\mathbf{2}_2$ | $\mathbf{1}_{-,1}$ |
| $R$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $+$ | $+$ |

### A. The Yukawa sector

As in the original model of [7,21,22] we assume that the flavor symmetry of the Yukawa sector is based on $Q_6$. In Table I we write the $Q_6$ assignment of the quark, lepton, and Higgs chiral supermultiplets,[4] where $Q$, $Q_3$, $L$, $L_3$ and $H^u$, $H_3^u$, $H^d$, $H_3^d$ stand for $SU(2)_L$ doublet supermultiplets for the quarks, leptons, and Higgs bosons, respectively. Similarly, $SU(2)_L$ singlet supermultiplets for quarks, charged leptons, and neutrinos are denoted by $U^c$, $U_3^c$, $D^c$, $D_3^c$, $E^c$, $E_3^c$ and $N^c$, $N_3^c$. From Table I we see that the one + two structure of family is realized, and because of this structure the $Q_6$ flavor symmetry can ensure that no nondiagonal kinetic term can be induced. So (13) is satisfied.

We then write down the most general, renormalizable, $Q_6 \times R$ invariant superpotential $W$ ($R$ is the $R$ parity.):

$$W_Y = W_Q + W_L, \tag{17}$$

where

$$W_Q = \sum_{I,i,j,k=1,2,3} (Y_{ij}^{uI} Q_i U_j^c H_I^u + Y_{ij}^{dI} Q_i D_j^c H_I^d), \tag{18}$$

$$W_L = \sum_{I,i,j,k=1,2,3} (Y_{ij}^{eI} L_i E_j^c H_I^d + Y_{ij}^{\nu I} L_i N_j^c H_I^u). \tag{19}$$

The Yukawa matrices $Y$'s are given by

$$\begin{aligned} \mathbf{Y}^{u1(d1)} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & Y_b^{u(d)} \\ 0 & Y_{b'}^{u(d)} & 0 \end{pmatrix}, \\ \mathbf{Y}^{u2(d2)} &= \begin{pmatrix} 0 & 0 & Y_b^{u(d)} \\ 0 & 0 & 0 \\ -Y_{b'}^{u(d)} & 0 & 0 \end{pmatrix}, \\ \mathbf{Y}^{u3(d3)} &= \begin{pmatrix} 0 & Y_c^{u(d)} & 0 \\ Y_c^{u(d)} & 0 & 0 \\ 0 & 0 & Y_a^{u(d)} \end{pmatrix}, \end{aligned} \tag{20}$$

[3] We also assume that the flavor symmetry is not gauged.

[4] The same model exists for $Q_{2N}$ if $N$ is odd and a multiple of 3.

$$\mathbf{Y}^{e1} = \begin{pmatrix} -Y_c^e & 0 & Y_b^e \\ 0 & Y_c^e & 0 \\ Y_{b'}^e & 0 & 0 \end{pmatrix},$$
$$\mathbf{Y}^{e2} = \begin{pmatrix} 0 & Y_c^e & 0 \\ Y_c^e & 0 & Y_b^e \\ 0 & Y_{b'}^e & 0 \end{pmatrix}, \tag{21}$$
$$\mathbf{Y}^{e3} = 0,$$

$$\mathbf{Y}^{\nu 1} = \begin{pmatrix} -Y_c^\nu & 0 & 0 \\ 0 & Y_c^\nu & 0 \\ Y_{b'}^\nu & 0 & 0 \end{pmatrix},$$
$$\mathbf{Y}^{\nu 2} = \begin{pmatrix} 0 & Y_c^\nu & 0 \\ Y_c^\nu & & 0 \\ 0 & Y_{b'}^\nu & 0 \end{pmatrix}, \tag{22}$$
$$\mathbf{Y}^{\nu 3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Y_a^\nu \end{pmatrix}.$$

All the parameters appearing above are real, because we assume that $CP$ is spontaneously broken. We will shortly come back to this issue.

### B. The $\mu$ sector

The most general $Q_6 \times R$ invariant renormalizable $\mu$ part of the superpotential is

$$W_\mu^{(Q_6)} = \mu H_I^u H_I^d + \frac{m}{2} N_I^c N_I^c. \tag{23}$$

Note that no mass terms for $H_3^{u,d}$ and $N_3^c$ are allowed by $Q_6$ and that the superpotential $W_\mu^{(Q_6)}$ has an accidental $O(2)$ symmetry. For phenomenological reasons we, however, need mass terms for $H_3^{u,d}$ and $N_3^c$. Therefore, we assume that the flavor symmetry of the $\mu$ sector is $O(2)$ and that $H_3^{u,d}$ and $N_3^c$ are singlets of $O(2)$, and add

$$W_\mu^{(\not{Q}_6)} = \mu_3 H_3^u H_3^d + \frac{m_3}{2} N_3^c N_3^c \tag{24}$$

to (23). Then the total $\mu$ part of the superpotential is $W_\mu = W_\mu^{(Q_6)} + W_\mu^{(\not{Q}_6)}$. The $O(2) \times R$ symmetry of $W_\mu$ is compatible with $Q_6 \times R$ of the Yukawa sector, because $Q_6$ ensures

$$\gamma_{H_1^u} = \gamma_{H_2^u} \quad \text{and} \quad \gamma_{H_1^d} = \gamma_{H_2^d}. \tag{25}$$

### C. Soft-supersymmetry-breaking sector

#### *1. The trilinear couplings and soft scalar mass terms*

We require that the trilinear couplings and soft scalar mass terms have the same flavor symmetry as that of the Yukawa sector, that is, $Q_6 \times R$. Therefore, the trilinear couplings and soft scalar mass matrices have the following form:

$$\mathbf{h}_{ij}^k = A_{ij} \mathbf{Y}_{ij}^k, \qquad k = u1, u2, \ldots, \nu_3, \tag{26}$$

where $\mathbf{Y}_{ij}^k$ are given in [20–22], and

$$\mathbf{m}^2 \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & f \end{pmatrix} \tag{27}$$

for all the bosonic scalar partners. This is very crucial to suppress FCNCs in the SSB sector as we will see later on.

#### *2. The b terms*

The $b$ sector should contain at least terms which correspond to the $\mu$ terms $W_\mu = W_\mu^{(Q_6)} + W_\mu^{(\not{Q}_6)}$, where $W_\mu^{(Q_6)}$ and $W_\mu^{(\not{Q}_6)}$ are given in (23) and (24), respectively, i.e.

$$\mathcal{L}_b^{(O_2)} = b\hat{H}_I^u \hat{H}_I^d + b_{33} \hat{H}_3^u \hat{H}_3^d + b_N \hat{N}_I^c \hat{N}_I^c + b_{N_3} \hat{N}_3^c \hat{N}_3^c + \text{H.c.} \tag{28}$$

(The hatted fields are bosonic components.) Because of the $O(2)$ symmetry in the $\mu$ and $b$ sectors and the $Q_6$ symmetry in the soft scalar mass terms, the Higgs scalar potential also respects the $O(2)$ symmetry, so that there is a Nambu-Goldstone boson corresponding to this symmetry because in the $O(2)$ symmetry the gauge symmetry is spontaneously broken, together with $SU(2)_L \times U(1)_Y$. Moreover, we face the domain wall problem when the discrete flavor symmetries are spontaneously broken. To overcome these problems we add terms which explicitly break $O_2$ down to $Z_2$:

$$\begin{aligned}\mathcal{L}_b^{(O_2/)} = {} & b_{++} \hat{H}_+^u \hat{H}_+^d + b_{--} \hat{H}_-^u \hat{H}_-^d + b_{+3} \hat{H}_+^u \hat{H}_3^d \\ & + b_{3+} \hat{H}_3^u \hat{H}_+^d + b_{N_+} \hat{N}_+^c \hat{N}_+^c + b_{N_-} \hat{N}_-^c \hat{N}_-^c \\ & + \hat{N}_3^c \hat{N}_+^c + \text{H.c.},\end{aligned} \tag{29}$$

where

$$H_\pm^{u,d} = \tfrac{1}{\sqrt{2}}(H_1^{u,d} \pm H_2^{u,d}), \qquad N_\pm^c = \tfrac{1}{\sqrt{2}}(N_1^c \pm N_2^c). \tag{30}$$

($H_+^{u,d}$, $H_3^{u,d}$, $N_+^c$, and $N_3^c$ are $Z_2$ even, while $H_-^{u,d}$ and $N_-^c$ are $Z_2$ odd.) This $Z_2$ is indeed broken by the Yukawa and trilinear couplings, but is compatible with $Q_6$, i.e., $\gamma_{H_1^{u,d}} = \gamma_{H_2^{u,d}}$.

We allow the $b$ parameters to be complex, because $CP$ cannot be broken if all the $b$ parameters are real as we will find in the next subsection. So $CP$ is explicitly, but only softly broken in this sector. In Table II we give the symmetry of each sector.

## IV. THE HIGGS SECTOR

### A. The Higgs potential

Given the $O(2) \times R$ invariant superpotential $W_\mu$ in the $\mu$ sector (23) and (24) along with the $Q_6 \times R$ invariant soft scalar masses (27) and the $Z_2 \times R$ invariant $b$ terms (28) and (29), we can now write down the scalar potential. For simplicity we assume that only the neutral scalar compo-

TABLE II. The symmetry of the different sectors. **Y**, **h**, and **m** stand for the Yukawa, trilinear, and soft scalar mass sector, respectively. $Q_6$ ensures that all the anomalous dimensions $\gamma$'s are diagonal, and that the two components of a $Q_6$ doublet have the same anomalous dimension. Therefore, $Q_6$ in the Yukawa and trilinear sectors and $O_2$ in the $\mu$ sector are compatible with each other. $O_2$ in the soft scalar mass sector is accidental. $Z_2$ is a subgroup of $O_2$, which implies the compatibility of $O_2$ and $Z_2$. $CP$ is explicitly broken only by the $b$ terms, which is (super) soft because the propagation of its violation to the other sectors is calculable and small. So, all the symmetries are compatible with each other.

| | **Y**, **h** | **m** | $\mu$ sector | $b$ terms |
|---|---|---|---|---|
| $Q_6$ | ○ | ○ | × | × |
| $O_2$ | × | ○ | ○ | × |
| $Z_2$ | × | ○ | ○ | ○ |
| $CP$ | ○ | ○ | ○ | × |
| R | ○ | ○ | ○ | ○ |

nents (denoted by a superscript 0) of the Higgs supermultiplets acquire vacuum expectation values (VEVs):

$$\begin{aligned} V &= m^2_{H^u_+}(|\hat{H}^{0u}_+|^2 + |\hat{H}^{0u}_-|^2) + m^2_{H^d_+}(|\hat{H}^{0d}_+|^2 + |\hat{H}^{0d}_-|^2) \\ &\quad + m^2_{H^u_3}|\hat{H}^{0u}_3|^2 + m^2_{H^d_3}|\hat{H}^{0d}_3|^2 + \tfrac{1}{8}(\tfrac{3}{5}g_1^2 + g_2^2) \\ &\quad \times (|\hat{H}^{0u}_+|^2 + |\hat{H}^{0u}_-|^2 + |\hat{H}^{0u}_3|^2 - |\hat{H}^{0d}_+|^2 - |\hat{H}^{0d}_-|^2 \\ &\quad - |\hat{H}^{0d}_3|^2)^2 + [b'_{++}\hat{H}^{0u}_+\hat{H}^{0d}_+ + b'_{--}\hat{H}^{0u}_-\hat{H}^{0d}_- \\ &\quad + b_{+3}\hat{H}^{0u}_+\hat{H}^{0d}_3 + b_{3+}\hat{H}^{0u}_3\hat{H}^{0d}_+ + b_{33}\hat{H}^{0u}_3\hat{H}^{0d}_3 + \text{H.c.}], \end{aligned} \tag{31}$$

where $b'_{++(--)} = b + b_{++(--)}$, $g_{1,2}$ are the gauge coupling constants for the $U(1)_Y$ and $SU(2)_L$ gauge groups, and $H_\pm$'s are defined in (30). Note that the scalar potential (31) has the same $Z_2$ symmetry as that of the $b$ sector. ($H_+$'s and $H_3$'s are $Z_2$ even, and $H_-$'s are $Z_2$ odd.) Therefore,

$$\begin{aligned} \langle \hat{H}^{0u,d}_- \rangle = 0, \qquad \langle \hat{H}^{0u,d}_+ \rangle = \frac{v^{u,d}_+}{\sqrt{2}}\exp i\theta^{u,d}_+, \\ \langle \hat{H}^{0u,d}_3 \rangle = \frac{v^{u,d}_3}{\sqrt{2}}\exp i\theta^{u,d}_3 \end{aligned} \tag{32}$$

can become a local minimum, where we assume that $v^{u,d}_+$ and $v^{u,d}_3$ are real. We recall that the $Z_2$ is an accidental symmetry expect for the $b$ sector.[5] Therefore, the VEV structure (32) is stable against (infinite) renormalization.

We investigate whether the potential energy at the VEV (32) can become negative so that $SU(2)_L \times U(1)_Y$ is spontaneously broken. To this end we consider the quadratic part of the scalar potential

$$V^{(2)} = \mathcal{H}^I \mathcal{M}_{IJ} \mathcal{H}^J, \tag{33}$$

where

$$\mathcal{M} = \begin{pmatrix} m^2_{H^u_+} & 0 & \Re(b'_{++}) & -\Im(b'_{++}) & 0 & 0 & \Re(b_{+3}) & -\Im(b_{+3}) \\ 0 & m^2_{H^u_+} & -\Im(b'_{++}) & -\Re(b'_{++}) & 0 & 0 & -\Im(b_{+3}) & -\Re(b_{+3}) \\ \Re(b'_{++}) & -\Im(b'_{++}) & m^2_{H^d_+} & 0 & \Re(b_{3+}) & -\Im(b_{3+}) & 0 & 0 \\ -\Im(b'_{++}) & -\Re(b'_{++}) & 0 & m^2_{H^d_+} & -\Im(b_{3+}) & -\Re(b_{3+}) & 0 & 0 \\ 0 & 0 & \Re(b_{3+}) & -\Im(b_{3+}) & m^2_{H^u_3} & 0 & \Re(b_{33}) & -\Im(b_{33}) \\ 0 & 0 & -\Im(b'_{3+}) & -\Re(b_{3+}) & 0 & m^2_{H^u_3} & -\Im(b_{33}) & -\Re(b_{33}) \\ \Re(b_{+3}) & -\Im(b_{+3}) & 0 & 0 & \Re(b_{33}) & -\Im(b_{33}) & m^2_{H^d_3} & 0 \\ -\Im(b_{+3}) & -\Re(b_{+3}) & 0 & 0 & -\Im(b_{33}) & -\Re(b_{33}) & 0 & m^2_{H^d_3} \end{pmatrix}, \tag{34}$$

and

$$\begin{aligned} \mathcal{H} = (&\Re(\hat{H}^{0u}_+), \Im(\hat{H}^{0u}_+), \Re(\hat{H}^{0d}_+), \Im(\hat{H}^{0d}_+), \Re(\hat{H}^{0u}_3), \\ &\Im(\hat{H}^{0u}_3), \Re(\hat{H}^{0d}_3), \Im(\hat{H}^{0d}_3)). \end{aligned} \tag{35}$$

We find that all the eigenvalues of $\mathcal{M}$ are doubly generate, and that two orthogonal eigenvectors of the same eigenvalue can be always written in the form

$$\begin{aligned} \vec{u}_A &= (u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8) \quad \text{and} \\ \vec{u}_B &= (u_2, -u_1, -u_4, u_3, u_6, -u_5, -u_8, u_7). \end{aligned} \tag{36}$$

This is due to the $U(1)_Y$ gauge invariance: All the directions defined by a linear combination of $\vec{u}_A$ and $\vec{u}_B$ are physically equivalent. If all the imaginary parts of $b$'s vanish, then we find $u_2 = u_4 = u_6 = u_8 = 0$, which means that $CP$ cannot be spontaneously broken, because

[5] It is accidental in the part of (31) coming from the D-terms (the second line). The $Q_6$ invariant soft scalar mass terms respect automatically this $Z_2$, although it is not contained in $Q_6$. This $Z_2$ is a part of the $O(2)$ symmetry of the $\mu$ sector, which is only softly broken down to the $Z_2$ in the $b$ sector.

the imaginary parts $\Im(\mathcal{H}_I)$ along the direction defined by $(u_1, 0, u_3, 0, u_5, 0, u_7, 0)$ stay at zero. So at least one of the $b$'s should be complex so that $CP$ is spontaneously broken.[6] The product of the four independent eigenvalues is $\det\mathcal{M}$. Therefore, if $\det\mathcal{M}$ is negative, one or three independent eigenvalues are negative. If $\det\mathcal{M}$ is positive, there may be zero, two, or four negative eigenvalues. In this case one should compute the eigenvalues explicitly. A local minimum lies along the direction of a negative eigenvalue. Further, the potential (31) along the D-term flat direction should not be unbounded below. This condition requires

$$\begin{aligned} &m^2_{H^u_+} + m^2_{H^d_+} - 2|b'_{++}| > 0, \\ &m^2_{H^u_+} + m^2_{H^d_+} - 2|b'_{--}| > 0, \\ &m^2_{H^u_+} + m^2_{H^d_3} - 2|b_{+3}| > 0, \\ &m^2_{H^u_3} + m^2_{H^d_+} - 2|b_{3+}| > 0, \\ &m^2_{H^u_3} + m^2_{H^d_3} - 2|b_{33}| > 0. \end{aligned} \tag{37}$$

We have to make the flavor changing neutral Higgs bosons sufficiently heavy to suppress FCNCs. (This will be discussed in Sec. V.) So we need a certain fine-tuning among the SSB parameters, because the size of the VEVs is bounded from above. To achieve this situation, we have to so fine-tune the parameters that one negative eigenvalue at the origin of the potential becomes very small.[7] Then the potential energy falls only slowly when moving from the origin, and the quartic terms in the potential (31) coming from the D-terms start to dominate, so that the energy scale of the VEVs at the bottom of the potential can be much smaller then the energy scale of the SSB parameters. Here is such an example:

$$\begin{aligned} &\Im(b_{++})/\Re(b'_{++}) = 0.747, \qquad \Re(b_{33})/\Re(b'_{++}) = 0.852, \\ &\Im(b_{33})/\Re(b'_{++}) = 1.399, \qquad \Re(b_{+3})/\Re(b'_{++}) = 0.667, \\ &\Im(b_{+3})/\Re(b'_{++}) = 0.31, \qquad \Re(b_{3+})/\Re(b'_{++}) = 1.3, \\ &\Im(b_{3+})/\Re(b'_{++}) = 0.42, \qquad m^2_{H^u_+}/\Re(b'_{++}) = 3.13, \\ &m^2_{H^d_+}/\Re(b'_{++}) = 2.69, \qquad m^2_{H^u_3}/\Re(b'_{++}) = 1.39, \\ &m^2_{H^d_3}/\Re(b'_{++}) = 5.93. \end{aligned} \tag{38}$$

The four independent eigenvalues are $-5.4 \times 10^{-5}$, 2.27, 4.16, 6.70 in the unit of $b'_{++}$, and two eigenvectors for the smallest eigenvalue correspond to

$$\begin{aligned} &u_1 = -0.1070, \quad u_2 = 0.2232, \quad u_3 = 0.4091, \\ &u_4 = 0.3081, \quad u_5 = -0.4216, \quad u_6 = 0.6636, \\ &u_7 = 0.2408, \quad u_8 = 0.0154, \end{aligned} \tag{39}$$

where $u$'s are defined in (36). Along the direction defined by (39) the potential energy falls very slowly when moving from the origin. So the $SU(2)_L \times U(1)_Y$ invariant point is a saddle point, and we find that the size of $\sqrt{b'_{++}}$ may be estimated as

$$\begin{aligned} \sqrt{b'_{++}} &\simeq \left(\frac{0.13(g_2^2 + 3g_1^2/5)/8}{5.4 \times 10^{-5}}\right)^{1/2} \times (246\ \text{GeV}) \\ &\simeq 3.2\ \text{TeV}. \end{aligned} \tag{40}$$

$CP$ is also spontaneously broken, because it is not possible to obtain a vector of the form $(\bullet, 0, \bullet, 0, \bullet, 0, \bullet, 0)$ through a linear combination of $\vec{u}_A$ and $\vec{u}_B$ for (39). Therefore, the angle $\theta_q$ that enters in the calculation of the CKM (given in (67)) is nonzero for (39). We find

$$\begin{aligned} \theta_q &= \theta^u_+ - \theta^d_+ - \theta^u_3 + \theta^d_3 \\ &= \arctan(u_2/u_1) - \arctan(u_4/u_3) \\ &\quad - \arctan(u_6/u_5) + \arctan(u_8/u_7) \\ &\simeq -0.701, \end{aligned} \tag{41}$$

which is the size of $\theta_q$ we need to produce the correct CKM parameters as we will see in Sec. V.

### B. The heavy neutral Higgs fields

Now redefine the Higgs fields as follows: First we define the tilde fields

$$\begin{aligned} \tilde{H}^{0u,0d}_+ &= \hat{H}^{0u,0d}_+ \exp -i\theta^{u,d}_+, \\ \tilde{H}^{0u,0d}_3 &= \hat{H}^{0u,0d}_3 \exp -i\theta^{u,d}_3, \end{aligned} \tag{42}$$

and then

$$\begin{aligned} \phi^u_L &= \cos\gamma^u \tilde{H}^{0u}_3 + \sin\gamma^u \tilde{H}^{0u}_+, \\ \phi^u_H &= -\sin\gamma^u \tilde{H}^{0u}_3 + \cos\gamma^u \tilde{H}^{0u}_+, \end{aligned} \tag{43}$$

where

$$\cos\gamma^u = \frac{v^u_3}{\sqrt{(v^u_3)^2 + (v^u_+)^2}}, \qquad \sin\gamma^u = \frac{v^u_+}{\sqrt{(v^u_3)^2 + (v^u_+)^2}}, \tag{44}$$

and similarly for the down sector. As we see from (44), only $\phi^u_L$ and $\phi^d_L$ have a nonvanishing VEV, which we denote by

$$\langle \phi^{u,d}_L \rangle = \frac{\sqrt{(v^{u,d}_3)^2 + (v^{u,d}_+)^2}}{\sqrt{2}} = \frac{v_{u,d}}{\sqrt{2}}. \tag{45}$$

[6] Spontaneous $CP$ violation in supersymmetric models and two Higgs doublet models have been discussed in Refs. [43–49], Ref. [50], and references therein.

[7] By one eigenvalue we mean one of four eigenvalues. All the eigenvalues are doubly degenerate.

The neutral light and heavy scalars of the MSSM are given by

$$\frac{1}{\sqrt{2}}(v + h) = \mathrm{Re}(\phi_L^{d*})\cos\beta + \mathrm{Re}(\phi_L^u)\sin\beta, \tag{46}$$

$$\frac{1}{\sqrt{2}}(H + iA) = -(\phi_L^{d*})\sin\beta + (\phi_L^u)\cos\beta, \tag{47}$$

where as in the MSSM

$$v = \sqrt{v_u^2 + v_d^2}, \qquad \tan\beta = \frac{v_u}{v_d}. \tag{48}$$

As in the case of the MSSM, the couplings of $\phi_L^{u,d}$ are flavor-diagonal, and so we do not have to consider them below when discussing FCNCs. Therefore, only the heavy fields $\hat{H}_-^{0u,0d} = \phi_-^{u,d}$ and $\phi_H^{u,d}$ can have flavor changing couplings. Their mass matrix can be written as

$$\begin{pmatrix} m^2_{\phi_I^u} & 0 & 0 & b_I^* \\ 0 & m^2_{\phi_I^u} & b_I & 0 \\ 0 & b_I^* & m^2_{\phi_I^d} & 0 \\ b_I & 0 & 0 & m^2_{\phi_I^d} \end{pmatrix} \tag{49}$$

in the $(\phi_I^u, \phi_I^{u*}, \phi_I^d, , \phi_I^{d*})$ basis, where $I = -, H$,

$$\begin{aligned} m^2_{\phi_-^{u,d}} &= m^2_{H_-^{u,d}}, \qquad b_- = b'_{--}, \\ m^2_{\phi_H^{u,d}} &= m^2_{H_+^{u,d}}\cos^2\gamma^{u,d} + m^2_{H_3^{u,d}}\sin^2\gamma^{u,d}, \\ b_H &= b'_{++}e^{-i(\theta_+^u+\theta_+^d)}\cos\gamma^u\cos\gamma^d \\ &\quad - b_{+3}\cos\gamma^u\sin\gamma^d e^{-i(\theta_+^u+\theta_3^d)} \\ &\quad - b_{3+}\sin\gamma^u\cos\gamma^d e^{-i(\theta_3^u+\theta_+^d)} \\ &\quad + b_{33}\sin\gamma^u\sin\gamma^d e^{-i(\theta_3^u+\theta_3^d)}, \end{aligned} \tag{50}$$

and the mass parameters on the right-hand side (rhs) are given in (31) and $\gamma^{u,d}$ are defined in (44). The inverse of the matrix (49) is given by

$$\frac{1}{(M_{I1})^2(M_{I2})^2}\begin{pmatrix} m^2_{\phi_I^d} & 0 & 0 & -b_I^* \\ 0 & m^2_{\phi_I^d} & -b_I & 0 \\ 0 & -b_I^* & m^2_{\phi_I^u} & 0 \\ -b_I & 0 & 0 & m^2_{\phi_I^u} \end{pmatrix}, \qquad (I = -, H), \tag{51}$$

where $M_{1,2}$ are approximate pole masses and given by

$$(M_{I1(2)})^2 = \frac{1}{2}(m^2_{\phi_I^u} + m^2_{\phi_I^d})\left(1 + (-)\times\left[\frac{4|b_I|^2 + (m^2_{\phi_I^u} - m^2_{\phi_I^d})^2}{(m^2_{\phi_I^u} + m^2_{\phi_I^d})^2}\right]^{1/2}\right), \tag{52}$$

and we find

$$(M_{I1})^2(M_{I2})^2 = -|b_I|^2 + m^2_{\phi_I^u}m^2_{\phi_I^d}. \tag{53}$$

(51) is the inverse propagator at the zero momentum. We will be using it later on. For the parameter values in the example (39) we find

$$\begin{aligned} &\tan\gamma^u = 0.315, \quad \tan\gamma^d = 2.122, \\ &\tan\beta = -1.456, \quad M_{H1} = 2.31\sqrt{b'_{++}} \simeq 7.3\,\mathrm{TeV}, \\ &M_{H2} = 1.72\sqrt{b'_{++}} \simeq 5.5\,\mathrm{TeV}, \end{aligned} \tag{54}$$

where we have used (40). So, what we have numerically shown in A and B in this section is that it is possible to fine-tune the SSB parameters so as to make the heavy Higgs bosons much heavier than $M_Z$ (see (54)) and at the same time to obtain a desired size of spontaneous $CP$ violation (see (41)).

## V. FCNCS

### A. The physical quarks and leptons

From the Yukawa interactions (18) and (19) along with the form of the VEVs (32) we obtain the fermion mass matrices.

#### *1. Quark sector*

The quark mass matrices are given by

$$\mathbf{m}^u = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2}Y_c^u v_3^u e^{-i\theta_3^u} & Y_b^u v_+^u e^{-i\theta_+^u} \\ \sqrt{2}Y_c^u v_3^u e^{-i\theta_3^u} & 0 & Y_b^u v_+^u e^{-i\theta_+^u} \\ -Y_{b'}^u v_+^u e^{-i\theta_+^u} & Y_{b'}^u v_+^u e^{-i\theta_+^u} & \sqrt{2}Y_a^u v_3^u e^{-i\theta_3^u} \end{pmatrix}, \tag{55}$$

$$\mathbf{m}^d = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2}Y_c^d v_3^d e^{-i\theta_3^d} & Y_b^d v_+^d e^{-i\theta_+^d} \\ \sqrt{2}Y_c^d v_3^d e^{-i\theta_3^d} & 0 & Y_b^d v_+^d e^{-i\theta_+^d} \\ -Y_{b'}^d v_+^d e^{-i\theta_+^d} & Y_{b'}^d v_+^d e^{-i\theta_+^d} & \sqrt{2}Y_a^d v_3^d e^{-i\theta_3^d} \end{pmatrix}. \tag{56}$$

Then using the phase matrices defined below

$$R_L = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}, \quad R_R = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}, \tag{57}$$

$$P_L^u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \exp(i2\Delta\theta^u) & 0 \\ 0 & 0 & \exp(i\Delta\theta^u) \end{pmatrix}, \tag{58}$$

$$P_R^u = \begin{pmatrix} \exp(i2\Delta\theta^u) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \exp(i\Delta\theta^u) \end{pmatrix}\exp(i\theta_3^u), \tag{59}$$

$$\Delta\theta^u = \theta_3^u - \theta_+^u, \tag{60}$$

and similarly for the down sector, we can bring $\mathbf{m}^u$ into a real form

$$\hat{\mathbf{m}}^u = P_L^{u\dagger} R_L^T \mathbf{m}^u R_R P_R^u = m_t \begin{pmatrix} 0 & q_u/y_u & 0 \\ -q_u/y_u & 0 & b_u \\ 0 & b_u' & y_u^2 \end{pmatrix}. \tag{61}$$

The mass matrix $\hat{\mathbf{m}}^u$ can then be diagonalized as

$$O_L^{uT} \hat{\mathbf{m}}^u O_R^u = \begin{pmatrix} m_u & 0 & \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \tag{62}$$

and similarly for $\mathbf{m}^d$, where $O_{L,R}^{u,d}$ are orthogonal matrices. So the mass eigenstates $u_{iL}' = (u_L', c_L', t_L')$, etc. can be obtained from

$$u_L = U_L^u u_L', \qquad u_R = U_R^u u_R', d_L = U_L^d d_L', \qquad d_R = U_R^d d_R', \tag{63}$$

where

$$U_{uL(R)} = R_{L(R)} P_{L(R)}^u O_{L(R)}^u. \tag{64}$$

Therefore, the CKM matrix $V_{\rm CKM}$ is given by

$$V_{\rm CKM} = O_L^{uT} P_L^{u\dagger} R_L^T R_L P_L^d O_L^d = O_L^{uT} P_q O_L^d, \tag{65}$$

where

$$P_q = P_L^{u\dagger} P_L^d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \exp(i2\Delta\theta_q) & 0 \\ 0 & 0 & \exp(i\Delta\theta_q) \end{pmatrix}. \tag{66}$$

For the set of the parameters

$$\begin{aligned} &\theta_q = \theta_3^d - \theta_+^d - \theta_3^u + \theta_+^u = -0.7, \\ &q_u = 0.000\,179\,9, \qquad b_u = 0.059\,79, \\ &b_u' = 0.070\,54, \qquad y_u = 0.997\,86, \\ &q_d = 0.003\,784, \qquad b_d = 0.032\,68, \\ &b_d' = 0.4620, \qquad y_d = -0.9415, \end{aligned} \tag{67}$$

we obtain

$$\begin{aligned} &m_u/m_t = 0.766 \times 10^{-5}, \qquad m_c/m_t = 4.23 \times 10^{-3}, \\ &m_d/m_b = 0.895 \times 10^{-3}, \qquad m_s/m_b = 1.60 \times 10^{-2}, \\ &|V_{\rm CKM}| = \begin{pmatrix} 0.9740 & 0.2266 & 0.003\,62 \\ 0.2265 & 0.9731 & 0.0417 \\ 0.008\,49 & 0.0410 & 0.9991 \end{pmatrix}, \\ &|V_{td}/V_{ts}| = 0.207, \end{aligned} \tag{68}$$

$$\sin 2\beta(\phi_1) = 0.690, \qquad \gamma(\phi_3) = 63.4°. \tag{69}$$

The experimental values to be compared are [51] (see also [52]):

$$|V_{\rm CKM}^{\rm exp}| = \begin{pmatrix} 0.973\,83\,{}^{+0.000\,24}_{-0.000\,23} & 0.2272\,{}^{+0.0010}_{-0.0010} & 0.003\,96\,{}^{+0.000\,09}_{-0.000\,09} \\ 0.2271\,{}^{+0.0010}_{-0.0010} & 0.972\,96\,{}^{+0.000\,24}_{-0.000\,24} & 0.04221\,{}^{+0.000\,10}_{-0.000\,80} \\ 0.008\,14\,{}^{+0.000\,32}_{-0.000\,64} & 0.041\,61\,{}^{+0.000\,12}_{-0.000\,78} & 0.999\,100\,{}^{+0.000\,034}_{-0.000\,004} \end{pmatrix},$$
$$\sin 2\beta(\phi_1) = 0.687 \pm 0.032,, \qquad |V_{td}/V_{ts}| = 0.208\,{}^{+0.008}_{-0.006}. \tag{70}$$

The quark masses at $M_Z$ are given by [53]

$$\begin{aligned} m_u/m_d &= 0.541 \pm 0.086(0.51), \\ m_s/m_d &= 18.9 \pm 1.6(17.9), \\ m_c &= 0.73 \pm 0.17(0.74)\ \text{GeV}, \\ m_s &= 0.058 \pm 0.015(0.046)\ \text{GeV}, \\ m_t &= 175 \pm 6\ \text{GeV}, \\ m_b &= 2.91 \pm 0.07\ \text{GeV}, \end{aligned} \tag{71}$$

where the values in the parentheses are the theoretical values obtained from (68) for $m_t = 174$ GeV and $m_b = 2.9$ GeV. So, we see that the model can well reproduce the experimentally measured parameters.

The orthogonal matrices (62) are found to be

$$O_{uL} \simeq \begin{pmatrix} 0.9991 & -0.042\,52 & 1.269 \times 10^{-5} \\ 0.04244 & 0.9973 & 0.059\,64 \\ -2.548 \times 10^{-3} & -0.059\,58 & 0.9982 \end{pmatrix}, \tag{72}$$

$$O_{uR} \simeq \begin{pmatrix} -0.9991 & -0.04255 & -1.075 \times 10^{-5} \\ 0.04244 & -0.9966 & 0.07042 \\ -3.007 \times 10^{-3} & 0.07035 & 0.9975 \end{pmatrix}, \tag{73}$$

$$O_{dL} \simeq \begin{pmatrix} 0.9764 & 0.2160 & -1.856\times 10^{-3} \\ -0.2159 & 0.9760 & 0.028\,99 \\ 8.074\times 10^{-3} & -0.027\,90 & 0.9996 \end{pmatrix}, \tag{74}$$

$$O_{dR} \simeq \begin{pmatrix} -0.9695 & 0.2452 & 1.165\times 10^{-4} \\ -0.2174 & -0.8599 & 0.4618 \\ 0.1133 & 0.4477 & 0.8870 \end{pmatrix}. \tag{75}$$

### *2. Lepton sector*

The charged lepton mass matrix becomes

$$\mathbf{m}_e = \begin{pmatrix} -m_2 & m_2 & m_5 \\ m_2 & m_2 & m_5 \\ m_4 & m_4 & 0 \end{pmatrix} \exp(-i\theta_+^d), \tag{76}$$

where

$$m_2 = \tfrac{1}{2}Y_c^e v_+^d, \qquad m_4 = \tfrac{1}{2}Y_{b'}^e v_+^d, \qquad m_5 = \tfrac{1}{2}Y_b^e v_+^d. \tag{77}$$

The phase $\exp(-i\theta_+^d)$ can be rotated away, and all the mass parameters appearing in (76) are real. Diagonalization of the mass matrices is straightforward.

We would like to mention that the model has many predictions in this sector, because there are only four parameters to describe three light neutrino masses, three angles, and three $CP$-violating phases of $V_{\rm MNS}$. Since the details of the predictions are presented in Refs. [23,24,54], we do not repeat them here again.[8] Furthermore, the FCNC processes in the lepton sector have been very recently analyzed in detail in Ref. [25], concluding that the model predictions of tree-level FCNC processes are at least 5 orders of magnitude smaller than the experimental upper bounds (The mass of the heavy neutral Higgs fields are assumed to be 120 GeV.) For instance, the branching fraction for $\mu \to e\gamma$ is 7 orders of magnitude smaller than the expected experimental sensitivity [25]. Therefore, we shall not consider FCNCs in the leptonic sector in the following discussions.

## B. *CP* violations and FCNCs in the SSB sector

If three generations of a family have the one + two structure, then the soft scalar mass matrices for the sfermions have a diagonal form (27):

$$\tilde{\mathbf{m}}^2{}_{aLL(RR)} = m_{\tilde{a}}^2 \begin{pmatrix} a_{L(R)}^a & 0 & 0 \\ 0 & a_{L(R)}^a & 0 \\ 0 & 0 & b_{L(R)}^a \end{pmatrix} (a = u, d, e), \tag{78}$$

where $m_{\tilde{a}}$ denotes the average of the squark and slepton masses, respectively, and $(a_{L(R)}, b_{L(R)})$ are dimensionless free real parameters of $O(1)$. Because of the $Q_6$ flavor symmetry in the trilinear interactions, all the soft left-right mass matrices assume the form

$$(\tilde{\mathbf{m}}^2{}_{aLR})_{ij} = A_{ij}^a (\mathbf{m}^a)_{ij} \qquad (a = u, d, e), \tag{79}$$

where $A_{ij}^a$ are free parameters of dimension one (see (26)). They are also real, because we impose $CP$ invariance in the trilinear couplings.

The quantities [56,57]

$$\Delta_{LL(RR)}^a = U_{aL}^\dagger \tilde{\mathbf{m}}^2{}_{aLL(RR)} U_{aL(R)} \quad \text{and} \quad \Delta_{LR}^a = U_{aL}^\dagger \tilde{\mathbf{m}}^2{}_{aLR} U_{aR} \tag{80}$$

in the super CKM basis are used widely to parametrize FCNCs and $CP$ violations coming from the SSB sector, where the unitary matrices $U$'s are given in [58–61].

### *1. CP violations*

The imaginary parts of $\Delta$'s (80) contribute to $CP$-violating processes in the SSB sector. Recall that the soft scalar mass matrices $\mathbf{m}_{aLL,RR}^2$ are real, because they are diagonal, and that the phases of $\mathbf{m}_{aLR}^2$ come from the complex VEVs (32), because $CP$ is only spontaneously broken in this sector. The unitary matrices $U$'s are complex, and so $\Delta$'s can be complex, too. Note that the unitary matrices have the form $U = RPO$, where only $P$'s (given in (58)) are complex. Since $P$'s are diagonal, they commute with $\mathbf{m}_{aLL,RR}^2$, so that $\Delta_{LL,RR}^a$ have no imaginary part. Further $\mathbf{m^2}_{aLR}$ has the same phase structure as the corresponding fermion mass matrix $\mathbf{m}^a$, which can be made real according to [57,62–65]. Therefore, $\Delta_{LR}^a$, too, are real. Consequently, there is no $CP$ violation originating from the SSB sector. The stringent constraints on $\Delta$'s (80) coming from the electric dipole moments (EDMs) [62,63,65] are automatically satisfied in this way of phase alignment.[9]

### *2. FCNC*

In Refs. [26–31,56,57,62–65], experimental bounds on the dimensionless quantities

$$\delta_{LL,RR,LR}^a = \Delta_{LL,RR,LR}^a / m_{\tilde{a}}^2 \qquad (a = u, d), \tag{81}$$

are given. The theoretical values of $\delta$'s for the present model have been calculated in Ref. [22] as a function of the average sfermion masses and fine-tuning parameters. The

[8] See also [55] for the predictions of the model on $R$-parity violating processes. The leptonic sector of the present model is basically the same as the model of [23,24], except for the spontaneous breaking of $CP$, which reduces one more independent phase in the leptonic sector.

[9] This does not mean that there is no $CP$ violation in the SSB sector. Because of the existence of the multiple Higgs fields, there are one-loop diagrams contributing to the EDMs, even if all the SSB parameters are real. The diagrams typically contain the $b$ terms, and we find that in the case of the present model $b_- \ll m_{H_-^{u,d}}^2$, $b_H < m_{\phi_H^{u,d}}^2$ (given in (50)) should be satisfied to satisfy the experimental constraints.

results may be summarized as follows. For the slepton sector where the average slepton mass $m_{\tilde{e}}$ is assumed to be 500 GeV, the theoretical values of $(\delta^{\ell}_{ij})_{LL,RR,LR}$, except for $(\delta^{\ell}_{12})_{LL}$, are several orders of magnitude smaller than the current experimental bounds, while $(\delta^{\ell}_{12})_{LL}$ is of the same order as that of the experimental bound which comes from $\mu \to e\gamma$. In the squark sector, we find:

*Up quark sector:*

$$\begin{aligned}(\delta^u_{12})_{LL} &= (\delta^u_{21})_{LL} \simeq -1.5\times 10^{-4}\Delta a^q_L,\\ (\delta^u_{12})_{RR} &= (\delta^u_{21})_{RR} \simeq -2.1\times 10^{-4}\Delta a^u_R,\\ (\delta^u_{12})_{LR} &\simeq -(\delta^u_{21})_{LR}\\ &\simeq 6.2\times 10^{-5}(-\tilde{A}^u_a + \tilde{A}^u_b + \tilde{A}^u_{b'} - \tilde{A}^u_c)\\ &\quad\times\left(\frac{500\ \text{GeV}}{m_{\tilde{q}}}\right),\end{aligned} \tag{82}$$

*Down quark sector:*

$$\begin{aligned}(\delta^d_{12})_{LL} &= (\delta^d_{21})_{LL} \simeq 2.2\times 10^{-4}\Delta a^q_L,\\ (\delta^d_{13})_{LL} &= (\delta^d_{31})_{LL} \simeq -8.1\times 10^{-3}\Delta a^q_L,\\ (\delta^d_{23})_{LL} &= (\delta^d_{32})_{LL} \simeq 2.8\times 10^{-2}\Delta a^q_L,\\ (\delta^d_{12})_{RR} &= (\delta^d_{21})_{RR} \simeq -5.1\times 10^{-2}\Delta a^d_R,\\ (\delta^d_{13})_{RR} &= (\delta^d_{31})_{RR} \simeq -0.1\Delta a^d_R,\\ (\delta^d_{23})_{RR} &= (\delta^d_{32})_{RR} \simeq -0.4\Delta a^d_R,\end{aligned} \tag{83}$$

where

$$\Delta a^q_L = a^q_L - b^q_L, \qquad \Delta a^a_R = a^a_R - b^a_R, \qquad \tilde{A}^a_i = \frac{A^a_i}{m_{\tilde{q}}}(a = u, d). \tag{84}$$

These parameters, $a_{L,R}$ and $\tilde{A}_i$, are free dimensionless parameters, so that they are $O(1)$ if we do not fine-tune them. The most stringent constraint in the up-sector comes from $\Delta M_D$ [30,31]:

$$\Delta M_D = >|(\delta^u_{12})_{LL}|, |(\delta^u_{12})_{RR}| \lesssim 6\times 10^{-2}, \quad |(\delta^u_{12})_{LR}|, |(\delta^u_{21})_{LR}| \lesssim 10^{-2} \tag{85}$$

for $m_{\tilde{q}} = 0.5$ TeV. As we can see from (82) this constraint can be satisfied without a fine-tuning. As for the down-sector we have to satisfy the constraints coming from $\Delta M_K$, $\Delta M_{B_s}$, and $\Delta M_{B_d}$ [26,29]:

$$\Delta M_K = >|(\delta^d_{12})_{LL}|, \quad |(\delta^d_{12})_{RR}|, \quad |(\delta^d_{12})_{LR}|, \quad |(\delta^d_{21})_{LR}| \lesssim 10^{-3}, \tag{86}$$

$$\Delta M_{B_d} = >|(\delta^d_{13})_{LL}|, \quad |(\delta^d_{13})_{RR}|, \quad |(\delta^d_{13})_{LR}|, \quad |(\delta^d_{31})_{LR}| \lesssim 10^{-2}, \tag{87}$$

$$\Delta M_{B_s} = >|(\delta^d_{23})_{LL}|, \quad |(\delta^d_{23})_{RR}|, \quad |(\delta^d_{23})_{LR}|, \quad |(\delta^d_{32})_{LR}| \lesssim 10^{-1}. \tag{88}$$

Comparing these constraints with (83) we see that $\Delta a^d_R$ should be fine-tuned at the level of few percent.[10] In the next subsections we assume that $\Delta a^d_R$ is so small that only the heavy flavor changing neutral Higgs fields contribute to the mass differences of the neutral mesons.

### C. Flavor changing neutral Higgs couplings

In Sec. IV we found that only the Higgs fields $\phi^{u,d}_{H,-}$ have flavor changing neutral couplings to the fermions, and that they have a definite form of mixing (see (49)). These are consequences of the $Z_2$ symmetry which is a part of the $O(2)$ flavor symmetry in the $\mu$ sector (as discussed in Sec. III B). In the basis of the fermion mass eigenstates these Higgs couplings have the following form:

$$\begin{aligned}\mathcal{L}_{\text{FCNC}} = &-[Y^{uH}_{ij}\phi^u_H + Y^{u-}_{ij}\phi^u_-]^*\bar{u}'_{iL}u'_{jR}\\ &-[Y^{dH}_{ij}\phi^d_H + Y^{d-}_{ij}\phi^d_-]^*\bar{d}'_{iL}d'_{jR}\\ &-[Y^{eH}_{ij}\phi^d_H + Y^{e-}_{ij}\phi^d_-]^*\bar{e}'_{iL}e'_{jR} + \text{H.c.},\end{aligned} \tag{89}$$

where the Higgs fields are defined in (43), and

$$\begin{aligned}\mathbf{Y}^{uH} &= U^{u\dagger}_L\left[\frac{1}{\sqrt{2}}\cos\gamma^u e^{-i\theta^u_+}(\mathbf{Y}^{u1} + \mathbf{Y}^{u2}) - \sin\gamma^u e^{-i\theta^u_3}\mathbf{Y}^{u3}\right]U^u_R\\ &= O^{u\dagger}_L\left[\frac{1}{\sqrt{2}}\cos\gamma^u(\mathbf{Y}^{u1} + \mathbf{Y}^{u2}) - \sin\gamma^u\mathbf{Y}^{u3}\right]O^u_R,\end{aligned} \tag{90}$$

$$\begin{aligned}\mathbf{Y}^{dH} &= U^{d\dagger}_L\left[\frac{1}{\sqrt{2}}\cos\gamma^d e^{-i\theta^d_+}(\mathbf{Y}^{d1} + \mathbf{Y}^{d2}) - \sin\gamma^d e^{-i\theta^d_3}\mathbf{Y}^{d3}\right]U^d_R\\ &= O^{d\dagger}_L\left[\frac{1}{\sqrt{2}}\cos\gamma^d(\mathbf{Y}^{d1} + \mathbf{Y}^{d2}) - \sin\gamma^d\mathbf{Y}^{d3}\right]O^d_R,\end{aligned} \tag{91}$$

$$\mathbf{Y}^{I-} = U^{I\dagger}_L\left[\frac{1}{\sqrt{2}}(\mathbf{Y}^{I1} - \mathbf{Y}^{I2})\right]U^I_R(I = u, d). \tag{92}$$

The Yukawa matrices $\mathbf{Y}^{u1}$, etc. are given in (20), and the unitary matrices are given in [57–65].

The present model is consistent with the experimental observations in a certain region in the parameter space of the Yukawa couplings. An example of the choice of the nine parameters is given in (67), where we emphasize that

[10] We find that, as in the case of $(\delta^u_{12})_{LR}$ of (82), the left-right insertions $|(\delta^d_{12,21,13,31,23,32})_{LR}|$ are much smaller than these constraints.

this set of the nine parameters describe 10 physical independent quantities of the SM; six quark masses and four CKM parameters. Therefore, the consistent region in the space of the Yukawa couplings is very restricted, and we will be using only this set of the parameter values in the following discussion. Accordingly, for the values given in (67) we find the actual size of the Yukawa couplings:

$$Y_a^u = \frac{\sqrt{2}m_t y_u^2}{v_u \cos\gamma^u} \simeq \frac{0.9957}{\sin\beta\cos\gamma^u}, \quad Y_b^u = \frac{\sqrt{2}m_t b_u}{v_u \sin\gamma^u} \simeq \frac{0.059\,79}{\sin\beta\sin\gamma^u}, \tag{93}$$

$$Y_{b'}^u = \frac{\sqrt{2}m_t b'_u}{v_u \sin\gamma^u} \simeq \frac{0.070\,54}{\sin\beta\sin\gamma^u}, \quad Y_c^u = \frac{\sqrt{2}m_t q_u}{y_u v_u \cos\gamma^u} \simeq \frac{1.802\times 10^{-4}}{\sin\beta\cos\gamma^u}, \tag{94}$$

$$Y_a^d = \frac{\sqrt{2}m_b y_d^2}{v_d \cos\gamma^d} \simeq \frac{0.014\,78}{\cos\beta\cos\gamma^d}, \quad Y_b^d = \frac{\sqrt{2}m_b b_d}{v_d \sin\gamma^d} \simeq \frac{5.449\times 10^{-4}}{\cos\beta\sin\gamma^d}, \tag{95}$$

$$Y_{b'}^d = \frac{\sqrt{2}m_b b'_d}{v_d \sin\gamma^d} \simeq \frac{7.702\times 10^{-3}}{\cos\beta\sin\gamma^d}, \quad Y_c^d = \frac{\sqrt{2}m_b q_d}{y_d v_d \cos\gamma^d} \simeq \frac{-6.701\times 10^{-5}}{\cos\beta\cos\gamma^d}, \tag{96}$$

where $\gamma$'s and $\beta$ are given in (44) and (48), respectively, and we have used: $m_t = 174$ GeV, $m_b = 2.9$ GeV, and $v = \sqrt{v_u^2 + v_d^2} = 246$ GeV. These parameters are defined in the $\overline{\text{MS}}$ scheme and evaluated at the scale $M_Z$. With these numerical values we then obtain:

$$\mathbf{Y}^{uH} \simeq \frac{1}{\tan\gamma^u \sin\beta} \times \begin{pmatrix} -2.65\times 10^{-4} & 3.22\times 10^{-3} & 0.0439 \\ -3.22\times 10^{-3} & 5.68\times 10^{-3} & 0.0400 \\ 0.0519 & -0.0473 & 6.02\times 10^{-3} \end{pmatrix} - \frac{\tan\gamma^u}{\sin\beta} \times \begin{pmatrix} 7.63\times 10^{-6} & -3.58\times 10^{-4} & -2.52\times 10^{-3} \\ -1.54\times 10^{-6} & -4.17\times 10^{-3} & -0.0592 \\ -2.99\times 10^{-3} & 0.0699 & 0.991 \end{pmatrix}, \tag{97}$$

$$\mathbf{Y}^{u-} \simeq \frac{\exp i(2\theta_3^u - \theta_+^u)}{\sin\gamma^u \sin\beta} \times \begin{pmatrix} 0 & -4.21\times 10^{-3} & -0.0596 \\ -4.21\times 10^{-3} & 0 & 2.54\times 10^{-3} \\ 0.0704 & 3.00\times 10^{-3} & 0 \end{pmatrix}, \tag{98}$$

$$\mathbf{Y}^{dH} \simeq \frac{1}{\tan\gamma^d \cos\beta} \times \begin{pmatrix} 6.63\times 10^{-5} & 8.26\times 10^{-5} & 2.80\times 10^{-4} \\ -6.224\times 10^{-5} & 3.74\times 10^{-4} & 3.37\times 10^{-4} \\ 4.10\times 10^{-3} & -6.01\times 10^{-3} & 2.52\times 10^{-3} \end{pmatrix} - \frac{\tan\gamma^d}{\cos\beta} \times \begin{pmatrix} 1.37\times 10^{-5} & 1.13\times 10^{-4} & 7.56\times 10^{-5} \\ 1.98\times 10^{-5} & -1.88\times 10^{-4} & -3.72\times 10^{-4} \\ 1.67\times 10^{-3} & 6.61\times 10^{-3} & 0.0131 \end{pmatrix}, \tag{99}$$

$$\mathbf{Y}^{d-} \simeq \frac{\exp i(2\theta_3^d - \theta_+^d)}{\sin\gamma^d \cos\beta} \times \begin{pmatrix} 0 & -2.53\times 10^{-4} & -4.72\times 10^{-4} \\ -2.22\times 10^{-4} & 0 & -1.04\times 10^{-4} \\ 7.46\times 10^{-3} & -1.89\times 10^{-3} & 0 \end{pmatrix}. \tag{100}$$

The phases appearing in the matrices are given in (55) and (56). As we can see from these Yukawa matrices the size of the entries is fixed once the ratios of the VEVs ( $\sin\beta$, $\sin\gamma^u$, etc.) are fixed. For the down-type Yukawa matrices (99) and (100), for instance, all the entries (except the (3, 3) entry) are at most $O(10^{-3})$. All these facts originate from the flavor symmetries of the model. Needless to say that in multi-Higgs models without a flavor symmetry this situation is completely different.

### D. FCNC

The most severe FCNC constraints on the theory come from the mass differences in the neutral meson systems; $\Delta M_D$, $\Delta M_K$, $\Delta M_{B_s}$, and $\Delta M_{B_d}$.[11] The Yukawa interaction terms that contribute to them can be found from (89):

[11] The contribution to $\epsilon'/\epsilon$ is negligibly small, at most $O([10^{-7}/\alpha_S^2][\tilde{m}_q^2/M^2])$, where $\sim 10^{-7}$ originates from the Yukawa couplings relevant to this quantity, and $\tilde{m}_q$ and $M$ stand for the generic average squark and charged Higgs masses. See [66] and references therein for the constraint from the oblique corrections due to multiple $SU(2)_L$ doublet Higgs fields.

$$\begin{aligned}\mathcal{L}_{\Delta M_B} = &-[Y^{uH}_{uc}\phi^u_H + Y^{u-}_{uc}\phi^u_-]^*(\bar{u}_L c_R - [Y^{uH}_{cu}\phi^u_H \\ &+ Y^{u-}_{cu}\phi^u_-]\bar{u}_R c_L - [Y^{dH}_{sd}\phi^d_H + Y^{d-}_{sd}\phi^d_-]^*\bar{s}_L d_R \\ &- [Y^{dH}_{ds}\phi^d_H + Y^{d-}_{ds}\phi^d_-]\bar{s}_R b_L - [Y^{dH}_{bd}\phi^d_H \\ &+ Y^{d-}_{bd}\phi^d_-]^*\bar{B}_L d_R - [Y^{dH}_{db}\phi^d_H + Y^{d-}_{db}\phi^d_-]\bar{B}_R d_L \\ &- [Y^{dH}_{bs}\phi^d_H + Y^{d-}_{bs}\phi^d_-]^*\bar{B}_L s_R \\ &- [Y^{dH}_{sb}\phi^d_H + Y^{d-}_{sb}\phi^d_-]\bar{b}_R s_L, \end{aligned} \tag{101}$$

where the values of the Yukawa couplings can be read off from (97)–(100). In (101) we have dropped the prime on the fields, which was indicating the mass eigenstate. As we can see from (51), no $\phi - \phi$ and $\phi^* - \phi^*$ type propagators contribute to the mass differences. So, only the $\phi - \phi^*$ type propagators can contribute, implying the phases in the Yukawa couplings (101) cancel in the tree-level diagrams contributing to the mass differences.

The independent parameters entering into $\Delta M_D$ are

$$\sin\beta, \sin\gamma^u, (M^u_H)^2 = \frac{(M_{H1}M_{H2})^2}{m^2_{\phi^d_H}}, \quad (M^u_-)^2 = \frac{(M_{-1}M_{-2})^2}{m^2_{\phi^d_-}}, \tag{102}$$

where they are given, respectively, in (48), (44), (50), and (52). Similarly,

$$\cos\beta, \sin\gamma^d, (M^d_H)^2 = \frac{(M_{H1}M_{H2})^2}{m^2_{\phi^u_H}}, \quad (M^d_-)^2 = \frac{(M_{-1}M_{-2})^2}{m^2_{\phi^u_-}} \tag{103}$$

enter into $\Delta M_K$, $\Delta M_{B_s}$, and $\Delta M_{B_d}$. With these remarks in mind, we proceed.

*D1: Constraint from* $\Delta M_D$

As we can see from Fig. 1, only the $\bar{u}_R c_L \bar{u}_L c_R$ type operator contributes to $\Delta M_D$ at the tree level. The mass difference $\Delta M_D$ can then be obtained from

$$\Delta M_D = 2|(M^{\rm SM}_D)_{12} + (M^{\rm EXTRA}_D)_{12}|, \tag{104}$$

where $(M^{\rm SM}_D)_{12}$ is the SM contribution, and

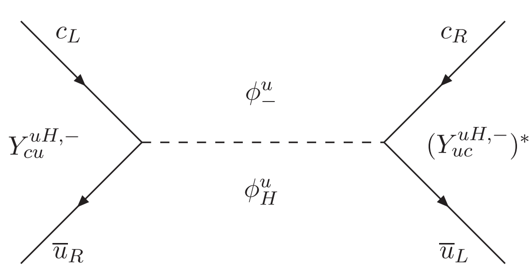


FIG. 1. The tree diagram contributing to $(M^{\rm EXTRA}_D)_{12}$. Tree diagrams contributing to $M_K$ and $M_{B_{d,s}}$ are similar to this diagram. Leading QCD corrections [76] will be included, except for $\Delta M_K$.

$$(M^{\rm EXTRA}_D)_{12} = 2C_D(\mu)\langle\bar{D}^0|\bar{u}^\alpha_R c^\alpha_L \bar{u}^\beta_L c^\beta_R|D^0\rangle(\mu), \tag{105}$$

$$C_D(\mu) = \eta(\mu)\left[\frac{Y^{uH}_{cu}(Y^{uH}_{uc})^*}{(M^u_H)^2} + \frac{Y^{u-}_{cu}(Y^{u-}_{uc})^*}{(M^u_-)^2}\right] \tag{106}$$

with the QCD correction $\eta(\mu)$. The operator $\bar{u}_R c_L \bar{u}_L c_R$ can mix with $\bar{u}_L\gamma^\mu c_L \bar{u}_R \gamma_\mu c_R$ even at the leading order in QCD in principle [67]. However, if $\bar{u}_L\gamma^\mu c_L \bar{u}_R \gamma_\mu c_R$ is absent at $\mu =$ some energy, it will not be induced, at least in the leading order in QCD. Note that the values of the Yukawa matrices (97)–(100) are defined at $\mu = M_Z$, so that there are corrections if $\mu \neq M_Z$. We here take into account only QCD corrections because they are most dominant. The leading-order QCD correction $\eta$ takes the form [67]

$$\eta(\mu_c = 2.8\ {\rm GeV}) = \left[\frac{\alpha_s(m_b)}{\alpha_s(\mu_c)}\right]^{-24/25}\left[\frac{\alpha_s(m_t)}{\alpha_s(m_b)}\right]^{-24/23} \times \left[\frac{\alpha_s(M)}{\alpha_s(m_t)}\right]^{-8/7}\left[\frac{\alpha_s(M_Z)}{\alpha_s(M)}\right]^{-8/7} \tag{107}$$

$$\simeq 2.3, \tag{108}$$

where we have used the two-loop running of $\alpha_s(\mu)$ with $\alpha_s(M_Z) = 0.119$, and the last factor is the QCD correction to the Yukawa matrices. So, the $M$ (which is supposed to be of the order of the heavy Higgs masses) dependence cancels nicely. The matrix element in the vacuum saturation approximation is given by [26]

$$\langle\bar{D}^0|\bar{u}^\alpha_R c^\alpha_L \bar{u}^\beta_L c^\beta_R|D^0\rangle(\mu_c = 2.8\ {\rm GeV}) = \frac{1}{4}f^2_D B'_D M_D\left(\frac{M_D}{m_c}\right)^2 \simeq 3.1\times 10^{-2}\ {\rm GeV}^3, \tag{109}$$

where we have used the central values of the parameters[12] given in Table III. ($m_c$(2.8 GeV) $= 1.0$ GeV which corresponds to $m_c(m_c) = 1.3$ GeV.)

Clearly, the larger $(M^u_H)^2$ and $(M^u_-)^2$ are, the smaller are the extra contributions. Here we are interested in the minimal values of $(M^d_H)^2$ and $(M^d_-)^2$, which are consistent with the observations. We find that the Wilson coefficient $C_D$ becomes

$$C_D(\mu_c) = \frac{\eta(\mu_c)}{\sin^2\beta}\left[\frac{1\ {\rm TeV}}{M^u_H}\right]^2 \times 10^{-11} \times \left(\frac{1.772}{r^2_u \sin^2\gamma^u} - \frac{1.037}{\tan^2\gamma^u} - 0.115 + 5.5\times 10^{-5}\tan^2\gamma^u\right)\ {\rm GeV}^{-2}, \tag{110}$$

where

[12]Since we take here a conservative standpoint that the extra contribution can be as large as the experimental value, we ignore the details of uncertainties.

TABLE III. Parameter values used in the text (see also Ref. [52]). $f_D$ is taken from [68], and we use $B'_D$ and $x_D$ of [30,69], respectively. $M_D$, $\tau_D$, $f_K$, $M_K$, $\Delta M_K^{\rm exp}$, $M_{B_s}$, $M_{B_d}$, $\Delta M_{B_d}^{\rm exp}$ are from [51]. $f_{B_s}$ (I) and $f_{B_s}\sqrt{B_s}$ (I) are the conservative sets of [58], and $f_{B_s}\sqrt{B_s}$ (II) is found in [70], while $f_{B_s}$ (II) and $\xi$ are taken from [71], and $f_{B_d}$ is obtained from $f_{B_s}/\xi$. (See [72] for a more conservative estimate of $\xi$, and references therein.) $B'_s$ and $B'_d$ are found in [73]. $\Delta M_{B_s}^{\rm exp}$ is from [74]. $m_u(2\ {\rm GeV})$ and $m_d(2\ {\rm GeV})$ are from [51], while the mass values of the other quarks are taken from [58], in which the relevant references are given.

| Input | | Input | |
|---|---|---|---|
| $f_D$ | $(222.6 \pm 16.7 {}^{+2.8}_{-3.4}) \times 10^{-3}$ GeV | $B'_D(2.8\ {\rm GeV})$ | $1.08 \pm 0.03$ |
| $M_D$ | $1.8645 \pm 0.0004$ GeV | $\tau_D$ | $(410.1 \pm 1.5) \times 10^{-3}$ ps |
| $x_D$ | $(5.3 - 11.7) \times 10^{-3}$ | $f_K$ | $(159.8 \pm 1.4 \pm 0.44) \times 10^{-3}$ GeV |
| $f_{B_s}$ | I: $0.240 \pm 0.040$ II: $0.245 \pm 0.013$ GeV | $B'_s(m_b)$ | $1.16 \pm 0.02 {}^{+0.05}_{-0.07}$ |
| $f_{B_s}\sqrt{B_s}$ | I: $0.221 \pm 0.046$ II: $0.227 \pm 0.017$ GeV | $\xi$ | $1.24 \pm 0.04$ |
| $f_{B_d}$ | $0.198 \pm 0.017$ GeV | $B'_d(m_b)$ | $1.15 \pm 0.03 {}^{+0.05}_{-0.07}$ |
| $M_K$ | $0.497\,648 \pm 0.000\,022$ GeV | $\Delta M_K^{\rm exp}$ | $(0.5292 \pm 0.0009) \times 10^{-2} {\rm ps}^{-1}$ |
| $M_{B_s}$ | $5.3661 \pm 0.0006$ GeV | $\Delta M_{B_s}^{\rm exp}$ | $17.77 \pm 0.10 \pm 0.07\ {\rm ps}^{-1}$ |
| $M_{B_d}$ | $5.279\,50 \pm 0.000\,33$ GeV | $\Delta M_{B_d}^{\rm exp}$ | $0.507 \pm 0.005\ {\rm ps}^{-1}$ |
| $m_u(2\ {\rm GeV})$ | $(3 \pm 1) \times 10^{-3}$ GeV | $m_c(m_c)$ | $1.30 \pm 0.05$ GeV |
| $m_d(2\ {\rm GeV})$ | $(6.0 \pm 1.5) \times 10^{-3}$ GeV | $m_s(2\ {\rm GeV})$ | $0.10 \pm 0.02$ GeV |
| $m_d(m_b)$ | $(5.1 \pm 1.3) \times 10^{-3}$ GeV | $m_s(m_b)$ | $0.085 \pm 0.017$ GeV |
| $m_t(m_t)$ | $163.8 \pm 2.0$ GeV | $m_b(m_b)$ | $4.22 \pm 0.08$ GeV |

$$r_u = \frac{M_-^u}{M_H^u} = \left(\frac{M_{-1}M_{-2}}{M_{H1}M_{H2}}\right)\left(\frac{m_{\phi_H^d}}{m_{\phi_-^d}}\right), \tag{111}$$

and the mass parameters are defined in (102). If each term in (110) should satisfy the constraint,

$$|\Delta M_D^{\rm EXTRA}| = 2|(M_D^{\rm EXTRA})_{12}| < \Delta M_D^{\rm exp} = x_D/\tau_D \simeq 1.4 \times 10^{-14}\ {\rm GeV}, \tag{112}$$

one finds that $\sin\beta M_H^u \gtrsim 17$ TeV and $\sin\beta M_-^u \gtrsim 22$ TeV should be satisfied. We, however, observe that the terms in (110) can cancel each other, so that no lower bounds on $M_H^u$

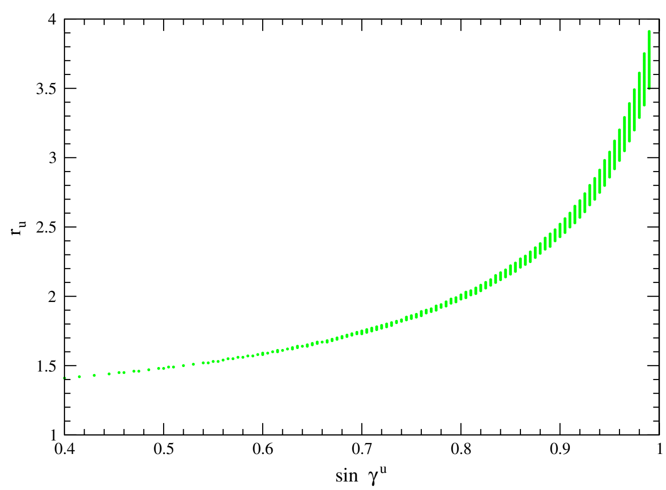


FIG. 2 (color online). The region in the $\sin\gamma^u - r_u$ plane, in which the constraint (113) coming from $\Delta M_D$ is satisfied for $\sin\beta M_H^u = 2$ TeV, where $r_u$, $\sin\gamma^u$ and $M_H^u$ are defined in (111), (44), and (102), respectively.

and $M_-^u$ can be obtained. In Fig. 2 we show the region in the $\sin\gamma^u - r_u$ plane for $\sin\beta M_H^u = 2$ TeV, in which $|\Delta M_D^{\rm EXTRA}|$ is smaller than the smallest $\Delta M_D^{\rm exp}$, i.e.,

$$|\Delta M_D^{\rm EXTRA}| < 8 \times 10^{-15}\ {\rm GeV}. \tag{113}$$

We see from Fig. 2 that to satisfy the constraint (113), we have to fine-tune $r_u$ and $\sin\gamma^u$ even for $\sin\beta M_H^u = 2$ TeV.

The neutral Higgs bosons in question can induce processes such as $D^0 \to e^+e^-$ and $D^0 \to \mu^+\mu^-$ which are strongly suppressed. The experimental upper bounds of the branching ratios are smaller than $O(10^{-6})$. From a rough estimate we find that $M_-^u$, , $M_H^u > M_Z$ is more than sufficient to suppress these processes. So, in principle, $M_-^u$, $M_H^u$ could be light, although one needs an extreme fine-tuning between $r_u$ and $\sin\gamma^u$.

*D2:* Constraint from $\Delta M_K$

As in the case of $\Delta M_D$, the interaction Lagrangian generates only one type of the $\Delta S = 2$ operator at the tree level. So, the relevant matrix element is

$$\langle \bar{K}^0|\bar{s}_R^\alpha d_L^\alpha \bar{s}_L^\beta d_R^\beta|K^0\rangle = \frac{1}{4} f_{B_K}^2 B'_K M_K \left(\frac{M_K}{m_s + m_d}\right)^2 \simeq 0.28\ {\rm GeV}^3, \tag{114}$$

where we have used the central values of the parameters given in Table III. (As in the case of $\Delta M_D$ we ignore the details of uncertainties involved in $\Delta M_D$.) As far as we understand, there is no reliable calculation of $B'_K$ for the present case (114),[13] and so we assume that $B'_K = 1$.

[13]See [59] for a lattice calculation of $B'_K$ of the present case, and also comments of [60].

Correspondingly, we do not take into account QCD corrections for the present case.

The tree-level coefficient is given by

$$C_K = \left[\frac{Y^{dH}_{ds}(Y^{dH}_{sd})^*}{(M^d_H)^2} + \frac{Y^{d-}_{ds}(Y^{d-}_{sd})^*}{(M^d_-)^2}\right] = \frac{1}{\cos^2\beta}\left[\frac{1\ \text{TeV}}{M^d_H}\right]^2 \times 10^{-14} \times \left(\frac{5.617}{r_d^2\sin^2\gamma^d} - \frac{0.514}{\tan^2\gamma^d} - 0.539 + 0.224\tan^2\gamma^d\right)\ \text{GeV}^{-2}, \quad (115)$$

where

$$r_d = \frac{M^d_-}{M^d_H} = \left(\frac{M_{-1}M_{-2}}{M_{H1}M_{H2}}\right)\left(\frac{m_{\phi^d_H}}{m_{\phi^d_-}}\right). \quad (116)$$

In Fig. 3 we show the region in the $r_d - \sin\gamma^d$ plane in which

$$\Delta M_K = 2 \times 0.28 \times C_K \text{GeV} < \Delta M_K^{\text{exp}} \simeq 3.49 \times 10^{-15}\ \text{GeV} \quad (117)$$

is satisfied.

*D3:* Constraint from $\Delta M_{B_s}$, $\Delta M_{B_d}$

As in the previous cases, the mass differences can be obtained from

$$\Delta M_{B_{s,d}} = 2|\langle\bar{B}^0|(M^{\text{SM}}_{s,d})_{12} + (M^{\text{EXTRA}}_{s,d})_{12}|B^0\rangle|. \quad (118)$$

The SM contributions to $\Delta M_{B_s}$, $\Delta M_{B_d}$ are well controlled up to the numerical uncertainty in the decay constants. Here following [58], which is based on the NLO-QCD calculations in Refs. [61,75], we consider two sets of the uncertainties for the $B$ system, I and II, as one can see in Table III. Since the uncertainties in the decay constants are much larger than those of other quantities, we assume that

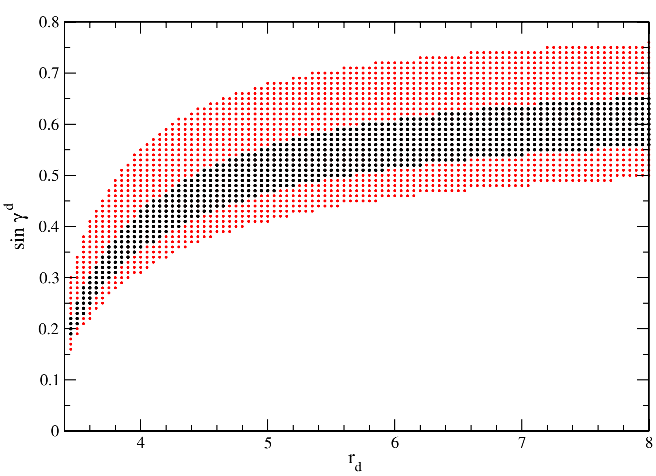


FIG. 3 (color online). The region in the $r_d - \sin\gamma^d$ plane for $\cos\beta M^d_H = 0.5$ TeV (red (dark gray)) and 0.3 TeV (black), in which $|\Delta M_K^{\text{EXTRA}}| < \Delta M_K^{\text{exp}}$ is satisfied. $r_d$ and $\sin\gamma^d$ are defined in (116) and (44), respectively.

$$f_{B_d}\sqrt{B_s} = \begin{cases} 0.221 \pm 0.046 & \text{for the parameter set I} \\ 0.227 \pm 0.017 & \text{for the parameter set II} \end{cases}, \quad (119)$$

$$f_{B_d}\sqrt{B_d} = \begin{cases} 0.180 \pm 0.043 & \text{for the parameter set I} \\ 0.184 \pm 0.020 & \text{for the parameter set II} \end{cases} \quad (120)$$

are the only uncertainties for the SM model contributions $M^{\text{SM}}_{s,d}$, where $f_{B_d}\sqrt{B_d}$ is obtained from $\xi = f_{B_s}\sqrt{B_s}/f_{B_d}\sqrt{B_d}$. To simplify the situation further, we assume that this is also true for the extra contributions $M^{\text{EXTRA}}_{s,d}$.

To calculate $(M^{\text{SM}}_{s,d})_{12}$ we use the parameter values (68) which are predicted in the present model:

$$|V_{\text{CKM}}|_{us} = 0.2266, \quad |V_{\text{CKM}}|_{ub} = 0.003\,62, \quad |V_{\text{CKM}}|_{cb} = 0.0417, \quad \phi_3(\gamma) = 1.107. \quad (121)$$

Then we follow the calculation of [58] and obtain:

$$2(M^{\text{SM}}_{B_s})_{12} = 2|(\bar{M}^{\text{SM}}_s)_{12}|(1 \pm \delta_s)\exp i\phi_s = \begin{cases} 20.1(1 \pm 0.40)\exp(-i0.0035) \\ 20.6(1 \pm 0.16)\exp(-i0.0035) \end{cases} \text{ps}^{-1} \quad \text{for} \begin{cases} \text{I} \\ \text{II} \end{cases}, \quad (122)$$

$$2(M^{\text{SM}}_{B_s})_{12} = 2|(\bar{M}^{\text{SM}}_d)_{12}|(1 \pm \delta_d)\exp i\phi_d = \begin{cases} 0.56(1 \pm 0.45)\exp(i0.77) \\ 0.56(1 \pm 0.21)\exp(i0.77) \end{cases} \text{ps}^{-1} \quad \text{for} \begin{cases} \text{I} \\ \text{II} \end{cases}, \quad (123)$$

where $(\bar{M}^{\text{SM}}_{s,d})_{12}$ are the SM contributions which are obtained with the central values of $f_{B_s}\sqrt{B_s}$, $\xi$, $M_{B_{s,d}}$ and the quark masses given[14] in III and $\alpha_s(M_Z) = 0.119$, and $\delta_s$ and $\delta_d$ correspond to the uncertainties in $f_{B_s}\sqrt{B_s}$ and $f_{B_d}\sqrt{B_d}$ given in (120), respectively. As we can see from Table III, the SM values are slightly larger than the experimental values.

As for the extra contributions, only the matrix elements

$$\langle\overline{B_s}^0|\bar{b}^\alpha_R s^\alpha_L \bar{b}^\beta_L s^\beta_R|B^0_s\rangle = \frac{1}{4}f^2_{B_s}B'_s M_{B_s}\left(\frac{M_{B_s}}{m_b + m_s}\right)^2 \simeq \begin{cases} 0.29(\text{I}) \\ 0.30(\text{II}) \end{cases} \text{GeV}^3 \quad (124)$$

and

[14]The model does not predict the absolute scale for the quark masses. If we use the mass ratio given in (68), we obtain a slightly smaller value for $m_b(m_b)$ (while we obtain the same value for $m_c(m_c)$). This difference has only a negligible effect on the SM contributions.

$$\langle \overline{B_d}^0 | \bar{b}_R^\alpha d_L^\alpha \bar{b}_L^\beta d_R^\beta | B_d^0 \rangle = \frac{1}{4} f_{B_d}^2 B_d' M_{B_d} \left( \frac{M_{B_d}}{m_b + m_d} \right)^2 \simeq 0.18(\mathrm{I, II})\ \mathrm{GeV}^3 \quad (125)$$

are relevant for $\Delta M_{B_s}$, $\Delta M_{B_d}$, where the tree-level diagrams similar to Fig. 1 contribute to these mass differences, and we have used the central values of the parameters in Table III. The leading-order Wilson coefficients are

$$C_{B_s} = \eta_B(m_b) \frac{1}{\cos^2\beta} \left[ \frac{1\ \mathrm{TeV}}{M_H^d} \right]^2 \times 10^{-12} \times \left( \frac{0.197}{r_d^2 \sin^2\gamma^d} - \frac{2.025}{\tan^2\gamma^d} - 4.463 - 2.459 \tan^2\gamma^d \right) \mathrm{GeV}^{-2}, \quad (126)$$

$$C_{B_d} = \eta_B(m_b) \frac{1}{\cos^2\beta} \left[ \frac{1\ \mathrm{TeV}}{M_H^d} \right]^2 \times 10^{-12} \times \left( -\frac{3.521}{r_d^2 \sin^2\gamma^d} + \frac{1.148}{\tan^2\gamma^d} - 0.780 + 0.127 \tan^2\gamma^d \right) \mathrm{GeV}^{-2}, \quad (127)$$

where

$$\eta_B(m_b = 4.22\ \mathrm{GeV}) = \left[ \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right]^{-24/23} \left[ \frac{\alpha_s(M)}{\alpha_s(m_t)} \right]^{-8/7} \times \left[ \frac{\alpha_s(M_Z)}{\alpha_s(M)} \right]^{-8/7} \simeq 2.0. \quad (128)$$

Then we require that

$$\Delta M_{B_s} = \Delta M_{B_s}^{\mathrm{exp}} = 2 |(\bar{M}_{B_s}^{\mathrm{SM}})_{12} + 0.58(0.60) \times C_{B_s}| (1 \pm \delta_s) = 17.77\ \mathrm{ps}^{-1} = 1.17 \times 10^{-11}\ \mathrm{GeV}, \quad (129)$$

$$\Delta M_{B_d} = \Delta M_{B_d}^{\mathrm{exp}} = 2 |(\bar{M}_{B_d}^{\mathrm{SM}})_{12} + 0.36 \times C_{B_d}| (1 \pm \delta_d) = 0.507\ \mathrm{ps}^{-1} = 3.34 \times 10^{-13}\ \mathrm{GeV}. \quad (130)$$

Note that according to our assumption the uncertainties factorize as $(1 \pm \delta_{s,d})[(\bar{M}_{s,d}^{\mathrm{SM}})_{12} + (M_{s,d}^{\mathrm{EXTRA}})_{12}]$, where $\bar{M}_{s,d}$ are the central values and $\delta_{s(d)}$ are given in (122) and (123). In Figs. 4 and 5 we show the allowed region in the $r_d - \sin\gamma^d$ plane for the parameter sets I and II, respectively, in which (129) and (130) are satisfied. We find that (129) and (130) can be simultaneously satisfied even for small $\cos\beta M_H^d \gtrsim 0.50$ TeV. The allowed region shrinks as $M_H^d$ increases. At $\cos\beta M_H^d = 1$ TeV the allowed region is very small. But a wide allowed region exists for $\cos\beta M_H^d = 2$ TeV. The reason that the allowed region first decreases and then increases as $M_H^d$ increases starting from $\simeq 0.50$ TeV is the following. The constraint (129) and (130) can be written as

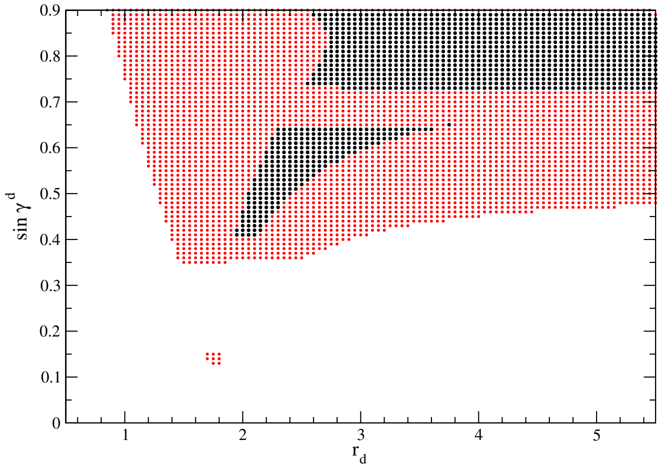


FIG. 4 (color online). The allowed region for the parameter set I with $\cos\beta M_H^d = 0.50$ (black) and 1.5 (red (dark gray)) TeV in which the constraints (129) and (130) are simultaneously satisfied. $r_d$ and $\sin\gamma^d$ are defined in (116) and (44), respectively. Two sets of values I and II are given in Table III.

$$[\Delta M_{B_{s,d}}^{\mathrm{exp}}]^2 (1 + \delta_{s,d})^{-2} \leq \Delta_{s,d} + 4[(\bar{M}_{s,d}^{\mathrm{SM}})_{12}]^2 \leq [\Delta M_{B_{s,d}}^{\mathrm{exp}}]^2 (1 - \delta_{s,d})^{-2}, \quad (131)$$

where

$$\Delta_{s,d} = 4[(M_{s,d}^{\mathrm{EXTRA}})_{12}]^2 + 8 \cos\phi_{s,d} (\bar{M}_{s,d}^{\mathrm{SM}})_{12} (M_{s,d}^{\mathrm{EXTRA}})_{12}, \quad (132)$$

and $(\bar{M}_{s,d}^{\mathrm{SM}})_{12}$ and $\phi_{s,d}$ are given in (122) and (123). For a large $M_H^d$ the second term of $\Delta_{s,d}$ is dominant. However, for a small $M_H^d$, two terms can become of the same order, and since $(M_{B_s}^{\mathrm{EXTRA}})_{12}$ and $(M_{B_d}^{\mathrm{EXTRA}})_{12}$ can simultaneously become negative, these two terms can cancel each other, so that both constraints (131) for $\Delta M_{B_s}$ and $\Delta M_{B_d}$ can be

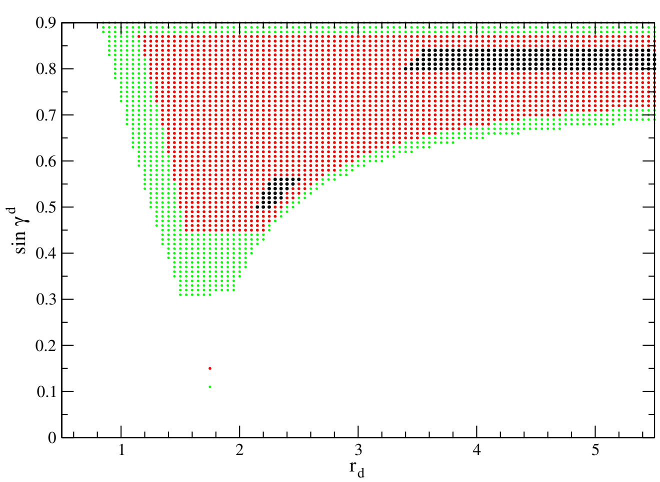


FIG. 5 (color online). The same as Fig. 4 for the parameter set II with $\cos\beta M_H^d = 0.50$ (black), 1.5 (red (dark gray)), and 2 (green (gray)) TeV.

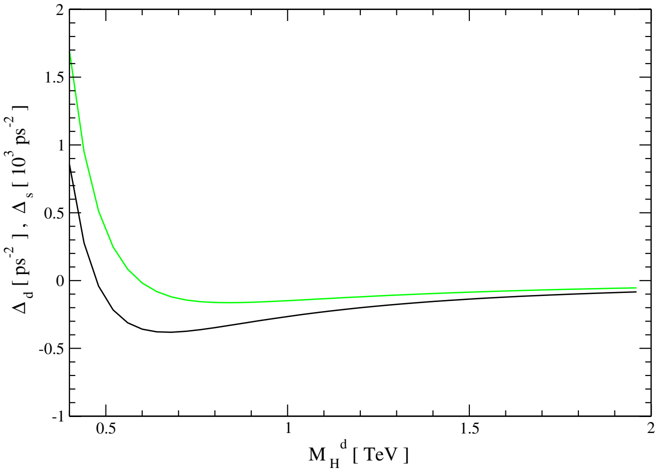


FIG. 6 (color online). $\Delta_s$ (green ( gray)) and $\Delta_d$ (black) for the parameter set I with $r_d = 3$ and $\sin\gamma^d = 0.8$, where they are defined in (132). This graph explains why the allowed region in the $r_d - \sin\gamma^d$ plane first shrinks and then extends as $M_H^d$ increases. For the parameter set II we obtain a similar result.

simultaneously satisfied. In Fig. 6 we show $\Delta_s$ (green) and $\Delta_d$ (black) for the parameter set I as a function of $M_H^d$ for $r_d = 3$ and $\sin\gamma^d = 0.8$, where we vary $M_H^d$ from 0.4 TeV to 2 TeV ( $\cos\beta = 1$). We see from the figure that $\Delta_s$ and $\Delta_d$ decrease as $M_H^d$ increases for $M_H^d \lesssim 0.6$ TeV. Note that the constraint from $\Delta M_{B_d}$ (130) is stronger than that from $\Delta M_{B_s}$ (129). In this region the constraint from $\Delta_K$ is not satisfied. But if we relax the constraint (because nonperturbative contributions to $\Delta_K$ suffer from large uncertainties) to $\Delta M_K^{\mathrm{EXTRA}} < 2\Delta M_K^{\mathrm{exp}}$, then it is satisfied.

Next we consider the region in which all the three constraints (117), (129) and (130) are satisfied. We find that the small $M_H^d$ region in Figs. 4 and 5 disappears, and

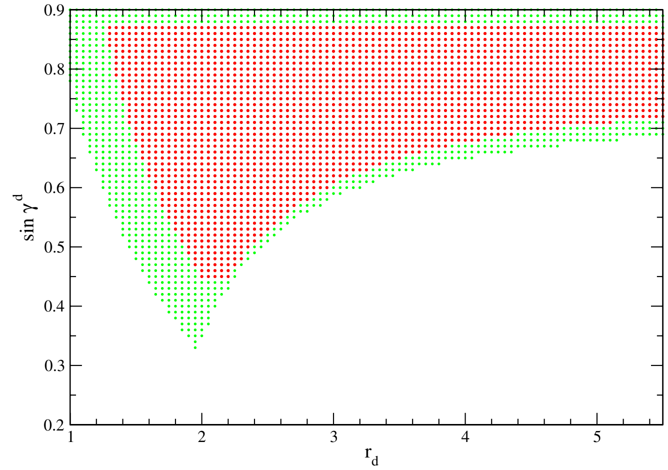


FIG. 8 (color online). The same as Fig. 7 for the parameter set II with $\cos\beta M_H^d = 1.5$ (red (dark gray)) and 2 (green (gray)) TeV.

that $M_H^d \gtrsim 1.0$ (I) and 1.3 (II) TeV have to be satisfied. In Figs. 7 and 8 we show the allowed region in which all the constraints (117), (129), and (130) are satisfied for $\cos\beta M_H^d = 1.5$ TeV (red) $\cos\beta M_H^d = 2$ TeV (green).

*D4:* Constraint from $\Delta M_{B_s}/\Delta M_{B_d}$

This ratio is important to determine experimentally $|V_{td}/V_{ts}|$. This is true only if there is no other contribution than the SM ones. In the presence of the extra neutral Higgs bosons, the situation changes. Here we ask ourselves how heavy the extra neutral Higgs bosons should be, or where the allowed region in the $r_d - \sin\gamma^d$ plane for a given $\cos\beta M_H^d$ is, such that the determination of $|V_{td}/V_{ts}|$ from the ratio $\Delta M_{B_s}/\Delta M_{B_d}$ is not influenced.

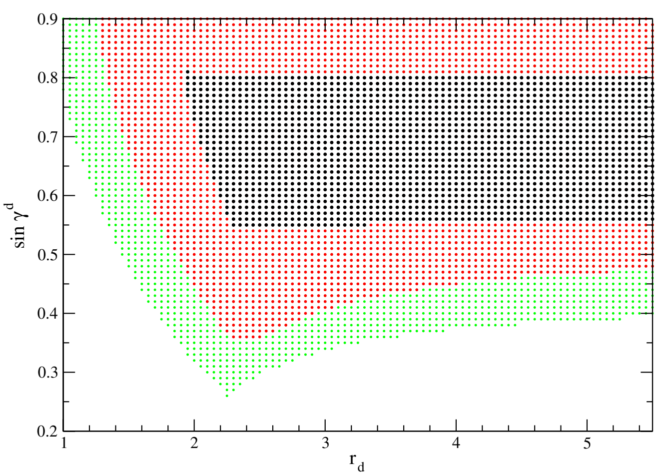


FIG. 7 (color online). The region in which the constraints (117), (129), and (130), coming from $\Delta M_K$, $\Delta M_{B_{s,d}}$ are satisfied for the parameter set I with $\cos\beta M_H^d = 1.1$ (black), $\cos\beta M_H^d = 1.5$ (red (dark gray)), and 2 (green (gray)) TeV. $r_d$ and $\sin\gamma^d$ are defined in (116) and (44), respectively.

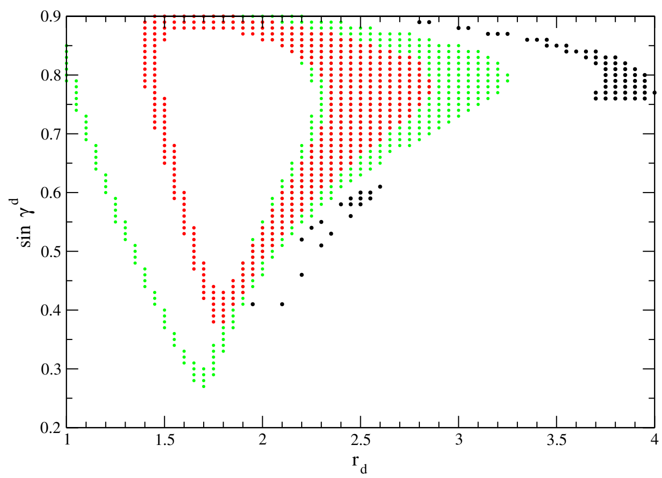


FIG. 9 (color online). The allowed region for the parameter set I with $\cos\beta M_H^d = 0.50$ (black), 1.5 (red (dark gray)), and 2 (green (gray)) TeV, in which the constraints (129), (130), and (133) coming from $\Delta M_{B_{s,d}}$ and $\Delta M_{B_s}/\Delta M_{B_d}$ are simultaneously satisfied.

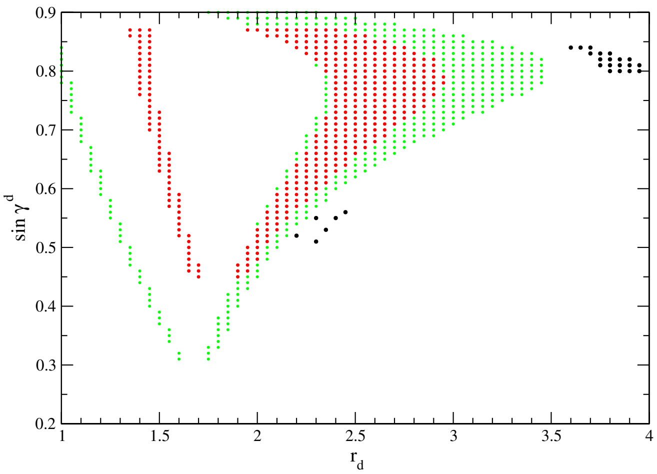


FIG. 10 (color online). The same as Fig. 9 for the parameter set II. Two set of values I and II are given in Table III.

The largest theoretical uncertainty in the mass ratio is contained in $\xi = f_{B_s}\sqrt{B_s}/f_{B_d}\sqrt{B_d} = 1.24 \pm 0.04$ (see Table III), that is, 3.2% uncertainty, which is lager than the experimental ones. Accordingly, we require that the theoretical value of $\Delta M_{B_s}/\Delta M_{B_d}$ should be equal to the experimental central value 35.05 within an error of 5% (the mass ratio is proportional to $\xi^2$), i.e.

$$\Delta M_{B_s}/\Delta M_{B_d} = 35.05(1 \pm 0.05). \tag{133}$$

We require that (129), (130), and (133) are simultaneously satisfied. The allowed region is shown in Figs. 9 and 10 for $\cos\beta M_H^d = 0.50$ (black), 1.5 (red), and 2 (green) TeV. We see that the small $M_H^d$ region of Figs. 4 and 5 is still there. We also find that $\cos\beta M_H^d \gtrsim 1.1(1.3)$ for the parameter set I (II) TeV or 0.39 TeV $\lesssim \cos\beta M_H^d \lesssim 0.65$ (0.45 TeV $\lesssim \cos\beta M_H^d \lesssim 0.6$) TeV for the parameter set I (II) if $\Delta M_{B_s}/\Delta M_{B_d}$ is equal to the experimental central value 35.05 within an error of 1%.

## VI. CONCLUSION

We have considered a supersymmetric extension of the SM based on the discrete $Q_6$ family symmetry, which has been recently proposed in Refs. [7,21,22]. We have stressed the one + two structure for each family; one $Q_6$ singlet and one $Q_6$ doublet for each family including the $SU(2)_L$ doublet Higgs fields. We have found that it is possible to realize the one + two structure in a renormalizable way, so that the Higgs sector becomes minimal and much simpler than that of the original model of [7,21,22]. In this way the Higgs sector can be investigated with much less assumptions. It is explicitly shown that the SSB parameters can be fine-tuned so as to make the heavy Higgs bosons much heavier than $M_Z$ and at the same time to obtain a desired size of spontaneous $CP$ violation to reproduce the Kobayashi-Maskawa $CP$-violating phase.

We have investigated the FCNC processes, especially those mediated by heavy neutral Higgs bosons. Because of the $Q_6$ family symmetry, the number of the independent Yukawa couplings is smaller than that of the observed quantities such as the CKM matrix and the quark masses. Therefore, the FCNCs can be parametrized only by the mixing angles and masses of the Higgs fields: There are two angles and four mass parameters that enter into the FCNCs for a given $\tan\beta$; a set of three parameters for $\Delta M_D$ and another set of three parameters for $\Delta M_K$ and $\Delta M_{B_{d,s}}$. We have expressed the mass differences of the neutral mesons $\Delta M_K$, $\Delta M_D$, and $\Delta M_{B_{d,s}}$ in terms of these parameters.

Since the SM contributions to $\Delta M_{B_s}$ and $\Delta M_{B_d}$ are well controlled, we haven taken them into account to obtain the constraints from $\Delta M_{B_s}$ and $\Delta M_{B_d}$. That is, we have assumed that the extra contributions are allowed only in a small window in which the SM values differ from the experimental values. Allowed ranges in which the constraints are satisfied are shown in various figures, where $\Delta M_K$, $\Delta M_{B_s}$, and $\Delta M_{B_d}$ take values in the common parameter space. We have also investigated the ratio $\Delta M_{B_s}/\Delta M_{B_d}$ in the region, in which all the constraints from $\Delta M_{B_s}$ and $\Delta M_{B_d}$ are simultaneously satisfied, and found that in a wide subregion the ratio differs from the experimental central value only by less than 5%. If we require that all the constraints from $\Delta M_K$, $\Delta M_{B_s}$, and $\Delta M_{B_d}$ including the ratio $\Delta M_{B_s}/\Delta M_{B_d}$ are satisfied, we have found that the heavy Higgs bosons should be heavier than ~1.5 TeV. If we relax the constraint from $\Delta M_K$ to $\Delta M_K^{\rm EXTRA} < 2\Delta M_K^{\rm exp}$ (because of the reason that nonperturbative contributions suffer from large uncertainties), the heavy Higgs bosons can be as light as ~0.4 TeV, which is within the accessible range of LHC [5].

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