Long-distance contributions to the $\eta_b \to J/\psi J/\psi$ decay

Pietro Santorelli

Dipartimento di Scienze Fisiche, Università di Napoli "Federico II", Italy and Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, Italy

(Received 26 March 2007; revised manuscript received 25 January 2008; published 17 April 2008)

It was argued long ago that η_b could be observed through the $\eta_b \to J/\psi(\to \mu^+\mu^-)J/\psi(\to \mu^+\mu^-)$ decay chain. Recent calculations indicate that the width of η_b into two J/ψ is almost 3 orders of magnitude smaller than the one into the $D\bar{D}^*$. We study the effects of final-state interactions due to the $D\bar{D}^*$ intermediate state on the $J/\psi J/\psi$ final state. We find that the inclusion of this contribution may enhance the short-distance branching ratio up to about 2 orders of magnitude.

DOI: 10.1103/PhysRevD.77.074012 PACS numbers: 13.25.Hw, 13.25.Gv

Six years ago the authors of Ref. [1] encouraged by the large measured width of $\eta_c \to \phi\phi$ suggested to observe η_b through the $\eta_b \to J/\psi J/\psi$ decay process. By using the measured branching ratio of $\eta_c \to \phi\phi$ and scaling laws with heavy quark masses the authors of Ref. [1] obtained

$$\begin{aligned} \mathcal{B}r[\eta_b \to J/\psi J/\psi] &= 7 \times 10^{-4\pm1}, \\ \mathcal{B}r[\eta_b \to (J/\psi J/\psi) \to 4\mu] &= 2.5 \times 10^{-6\pm1}. \end{aligned} \quad (1)$$

Following this suggestion the CDF Collaboration has searched for the $\eta_b \to J/\psi J/\psi \to 4\mu$ events in the full run I data sample [2]. In the search window, where a background of 1.8 events is expected, a set of seven events are seen. This result seems to confirm the predictions in Eq. (1). Recently, Maltoni and Polosa [3] criticized the scaling procedure adopted in Ref. [1] whose validity should reside only in the domain of perturbative QCD. The nonperturbative effects, which are dominant in $\eta_c \to \phi\phi$ as a consequence of its large branching fraction, cannot be rescaled by the same factor of the perturbative ones. In [3], to obtain an upper limit on $\mathcal{B}r[\eta_b \to J/\psi J/\psi]$, the authors evaluated the inclusive decay rate of η_b into 4-charm states:

$$\mathcal{B}r[\eta_b \to c\bar{c}c\bar{c}] = 1.8^{+2.3}_{-0.8} \times 10^{-5}, \quad (2)$$

which is even smaller than the lower limit on $\mathcal{B}r[\eta_b \to J/\psi J/\psi]$ estimated in Ref. [1].

Very recently Jia [4] has performed an explicit calculation of the same exclusive $\eta_b \to J/\psi J/\psi$ decay process in the framework of the color-singlet model

$$\mathcal{B}r[\eta_b \to J/\psi J/\psi] \sim (0.5 \div 6.6) \times 10^{-8}, \quad (3)$$

which is 3 orders of magnitude smaller than the inclusive result in [3]. The result in Eq. (3) indicates that the cluster reported by CDF [2] is extremely unlikely to be associated with η_b. Moreover, the potential of discovering η_b through this decay mode is hopeless even in Tevatron run II. Another interesting decay channel to observe η_b, $\eta_b \to D^{(*)}\bar{D}^*$, has been proposed in [3] where the range $10^{-3} < \mathcal{B}r[\eta_b \to D\bar{D}^*] < 10^{-2}$ was predicted. Finally, in Ref. [4] by doing reasonable physical considerations the author obtained

$$\mathcal{B}r[\eta_b \to D\bar{D}^*] \sim 10^{-5}, \qquad \mathcal{B}r[\eta_b \to D^*\bar{D}^*] \sim 10^{-8}, \quad (4)$$

which are at odds with the ones obtained in [3].

In this paper we start from the following assumptions:

(a) the short-distance branching ratio of $\eta_b \to J/\psi J/\psi$ is too small to look at this channel to detect η_b ($\sim 10^{-8}$ [4]);

(b) the branching ratio $\mathcal{B}r[\eta_b \to D\bar{D}^*]$ is either of the order of 10^{-5} [4] or it is in the range $10^{-3} \div 10^{-2}$ [3,5];

(c) the $\mathcal{B}r[\eta_b \to D^*\bar{D}^*]$ is negligible in comparison with $\mathcal{B}r[\eta_b \to D\bar{D}^*]$.

We also will consider the effect of $D\bar{D}^* \to J/\psi J/\psi$ rescattering (cf. Fig. 1) which should dominate the long-distance contribution to the decay under analysis. The dominance of the $D\bar{D}^*$ intermediate state is a consequence of the large coupling of $D^{(*)}\bar{D}^*$ to J/ψ as a result of quark models and QCD sum rules calculations (see later). In this respect, in our analysis we do not take into account contributions coming from others intermediate states with large branching ratios [4] because they

(i) do not couple to the $J/\psi J/\psi$ (as in the case of KK^*);

(ii) have small couplings to the η_b (as in the case of $D^*\bar{D}^*$).

The main result in this paper is the estimation of the contributions coming from the triangle graph in Fig. 1. The absorbitive part of the diagram is given by

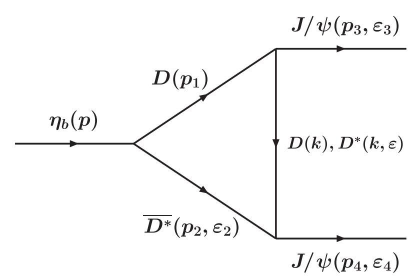

FIG. 1. Long-distance t-channel rescattering contributions to $\eta_b \to J/\psi J/\psi$.

$$\begin{aligned}\mathrm{Abs\,(Fig.~1)} &= \frac{1}{16\pi m_{\eta_b}\sqrt{m_{\eta_b}^2-4m_{J/\psi}^2}}\int_{t_m}^{t_M} dt\mathcal{A}(\eta_b\to D\bar{D}^*)\mathcal{A}(D\bar{D}^*\to J/\psi J/\psi)\\ &= \frac{\imath g_{\eta_b DD^*}\varepsilon_{\alpha\beta\gamma\delta}p_3^\alpha p_4^\beta\epsilon_3^{*\gamma}\epsilon_4^{*\delta}}{16\pi m_{\eta_b}\sqrt{m_{\eta_b}^2-4m_{J/\psi}^2}}\int_{t_m}^{t_M}\frac{dt}{t-m_D^2}\frac{g_{JDD^*}}{m_{J/\psi}(m_{\eta_b}^2-4m_{J/\psi}^2)}F(t)^2\Big\{2g_{JDD}[(m_D^2-m_{J/\psi}^2)^2+(m_{\eta_b}^2-2m_D^2\\ &\quad -2m_{J/\psi}^2)t+t^2]-\frac{g_{JD^*D^*}}{m_D^2}[(m_D^2-m_{J/\psi}^2)^2(2m_D^2+m_{\eta_b}^2)-2m_D^2(2(m_D^2+m_{J/\psi}^2)-m_{\eta_b}^2)t+(2m_D^2+m_{\eta_b}^2)t^2]\Big\}\\ &\equiv\left(\frac{A_{LD}}{m_{\eta_b}}g_{\eta_b DD^*}\right)\imath\varepsilon_{\alpha\beta\gamma\delta}p_3^\alpha p_4^\beta\epsilon_3^{*\gamma}\epsilon_4^{*\delta},\end{aligned} \quad (5)$$

where the two contributions coming from the D and D^* in the t channel are explicitly written, although we neglect D and D^* mass difference in order to have a simple expression. However, in the numerical calculations we use the physical masses of the involved charmed mesons. The integration domain is given by $[t_m, t_M] \approx [-60, -0.6]\ \mathrm{GeV}^2$. The numerical values of the on-shell strong couplings g_{JDD}, g_{JDD^*}, and $g_{JD^*D^*}$ [6] are taken from QCD sum rules [7], from the constituent quark meson model [8], and from relativistic quark model [9] findings which are compatible with each other. We used $(g_{JDD}, g_{JDD^*}, g_{JD^*D^*}) = (6, 12, 6)$. To take into account the off-shellness of the exchanged $D^{(*)}$ mesons in Fig. 1, we have introduced the t dependance of these couplings [cf. Eq. (5)] by means of the function

$$F(t) = \frac{\Lambda^2 - m_{D^{(*)}}^2}{\Lambda^2 - t}, \quad (6)$$

which satisfies QCD counting rules. Λ should be not far from the mass of the exchanged particle. However, a first-principles calculation of Λ does not exist. Thus, following the authors of [10] we write $\Lambda = m_R + \alpha\Lambda_{\mathrm{QCD}}$, where m_R is the mass of the exchanged particle (D or D^*), $\Lambda_{\mathrm{QCD}} = 220$ MeV and $\alpha \in [0.8, 2.2]$ [10]; with these values, the allowed range for Λ is given by $2.1 < \Lambda < 2.5$ GeV.

Regarding the dispersive contribution, an estimate of it can be obtained by a dispersion relation from the absorbitive part. It should be observed that this procedure suffers from the uncertainty related to possible subtractions. However, here we just want to estimate the order of magnitude of the contribution. In this respect the real part of the long-distance contribution is given by

$$\mathrm{Dis\ (Fig.\ 1)} = \frac{1}{\pi}\mathcal{P}\int_{s_0}^{\infty}\frac{\mathrm{Abs}(s)}{s - m_{\eta_b}^2}ds, \quad (7)$$

where the Abs(s) is the expression in Eq. (5) in which the substitution $m_{\eta_b}^2 \to s$ was done, and $s_0 = (m_D + m_{D^*})^2$. $\mathcal{P}$ indicates the principal value. Note that in this calculation we neglect the off-shellness of the $\eta_b D\bar{D}^*$ coupling because the wide range of values quoted for $g_{\eta_b DD^*}/g_{\eta_b JJ}$ should take into account also this effect.

Using the definition in Eqs. (5) and (7), the full amplitude for the $\eta_b \to J/\psi J/\psi$ process can be written as

$$\begin{aligned}\mathcal{A}_f(\eta_b(p)\to J/\psi(p_3,\varepsilon_3)J/\psi(p_4,\varepsilon_4)) &= \imath\frac{g_{\eta_b JJ}}{m_{\eta_b}}\varepsilon_{\alpha\beta\gamma\delta}p_3^\alpha p_4^\beta\epsilon_3^{*\gamma}\epsilon_4^{*\delta}\\ &\quad\times\left[1+3\frac{g_{\eta_b DD^*}}{g_{\eta_b JJ}}(\imath A_{LD}+D_{LD})\right],\end{aligned} \quad (8)$$

where A_{LD} and D_{LD} stand for absorbitive and dispersive contributions, respectively. The factor 3 is due to the three different charge assignments to the $D\bar{D}^*$ intermediate state. In Eq. (5) we have introduced the (on-shell) effective couplings $g_{\eta_b DD^*}$ and $g_{\eta_b JJ}$ defined by

$$\mathcal{A}\,(\eta_b(p)\to D(p_1)\bar{D}^*(p_2,\varepsilon_2)) = 2g_{\eta_b DD^*}(\varepsilon_2^*\cdot p), \quad (9)$$

$$\begin{aligned}\mathcal{A}\,(\eta_b(p)\to J/\psi(p_3,\varepsilon_3)J/\psi(p_4,\varepsilon_4)) &= \frac{\imath g_{\eta_b JJ}}{m_{\eta_b}}\varepsilon_{\alpha\beta\gamma\delta}p_3^\alpha p_4^\beta\varepsilon_3^{*\gamma}\varepsilon_4^{*\delta}.\end{aligned} \quad (10)$$

The ratio in Eq. (8) is obtained in terms of the existing theoretical estimate of the $\mathcal{B}r[\eta_b \to D\bar{D}^*]/\mathcal{B}r[\eta_b \to J/\psi J/\psi] = (0.3/3.6) \times 10^{+3} \times (1 \text{ or } 10^{+2} \div 10^{+3})$, i. e. $g_{\eta_b DD^*}/g_{\eta_b JJ} \approx 1.1$ or $11 \div 35$. In Fig. 2 the ratio $r = 3A_{LD}g_{\eta_b DD^*}/g_{\eta_b JJ}$ is plotted as a function of α for the allowed value and the range of couplings ratio. Moreover, the dashed line is for $g_{\eta_b DD^*}/g_{\eta_b JJ} \approx 26$ which corresponds to the central value in the allowed range for $\eta_b \to D\bar{D}^*$ estimated in Ref. [3]. Looking at the figure we see that the long-distance absorbitive contribution coming from the graphs in Fig. 1 is at the most about 10 times larger than the short-distance amplitude.

The numerical evaluation of the dispersive contribution, Eq. (7), gives numbers of the same order of magnitude of the absorbitive one. In Fig. 3 the ratio $R = 3D_{LD}g_{\eta_b DD^*}/g_{\eta_b JJ}$ is plotted as a function of α for the same cases of Fig. 2.

Looking at Figs. 2 and 3 we are able to identify two possible scenarios. In the first scenario, the coupling of η_b to $D\bar{D}^*$ is very small (in agreement with the prediction in [4]) and so the effects of final-state interactions result to be

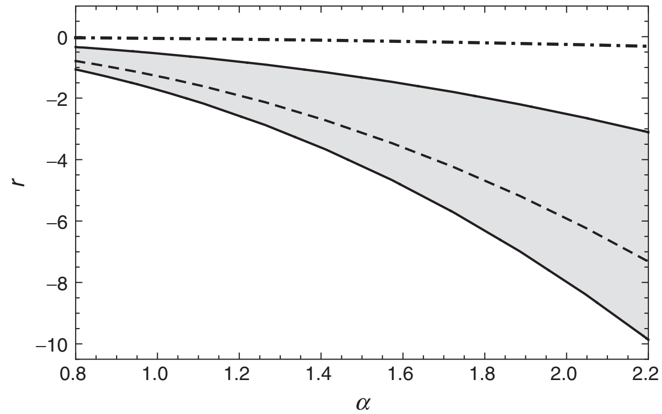

FIG. 2. The ratio r (see text for definition) is plotted vs α for $g_{\eta_b DD^*}/g_{\eta_b JJ} \approx 1$ (dashed-dotted line) and $g_{\eta_b DD^*}/g_{\eta_b JJ} \approx \{11, 35\}$ (solid lines). The dashed line corresponds to $g_{\eta_b DD^*}/g_{\eta_b JJ} \approx 26$.

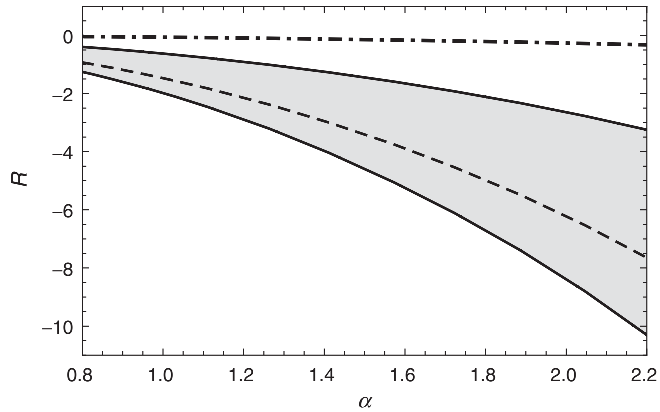

FIG. 3. The ratio R (see text for definition) is plotted vs α for the same cases of Fig. 2.

negligible independently of α. In the second scenario, in agreement with the predictions in [3], the effects of final-state interactions could be large as a consequence of the large $\mathcal{B}r[\eta_b \to D\bar{D}^*]$. Moreover, in this scenario, the long-distance contribution depends strongly on the value of α (cf. gray bands in Figs. 2 and 3).

Starting from the estimate of the short-distance part in Eq. (3) we are able to give the allowed range for the full branching ratio

$$\mathcal{B}r[\eta_b \to J/\psi J/\psi] = 0.5 \times 10^{-8} \div 1.2 \times 10^{-5}, \quad (11)$$

where the lower bound corresponds to the one in Eq. (3) while the upper bound is obtained using the upper value in Eq. (3) and for $\alpha = 2.2$, $g_{\eta_b DD^*}/g_{\eta_b JJ} = 35$. Note that the upper bound almost saturates the inclusive branching ratio resulting from the calculation in [3] [cf. Eq. (2)]. The wide range for $\mathcal{B}r[\eta_b \to J/\psi J/\psi]$ in Eq. (11) depends on the large uncertainty on the $\mathcal{B}r[\eta_b \to D\bar{D}^*]$ and on the dependence on the α parameter of the loop contribution. The choice between the two scenarios can be done only by the experimental measurement of the $\mathcal{B}r[\eta_b \to D\bar{D}^*]$ which can be measured at Tevatron. The dependence of our results on the α parameter or, more generally, the off-shellness of the couplings entering the calculation can be studied in the framework of a model. Obviously, once the experimental data on the $\mathcal{B}r[\eta_b \to J/\psi J/\psi]$ will be available, the couplings and their off-shellness can be obtained by using data and the results of this paper. QCD sum rules findings [7] on the $g_{JD^{(*)}D^{(*)}}$ and their off-shellness allow us to evaluate, for the second scenario, the long-distance contribution in a specific approach. For the absorbitive term r we have the range $2 \leq r \leq 6$ and $R \approx -2$. Thus in the framework of the second scenario plus QCD sum rules, we get the results $\mathcal{B}r[\eta_b \to J/\psi J/\psi] = 2.5 \times 10^{-8} \div 2.4 \times 10^{-6}$.

As far as the number of events in Tevatron run I data (100 pb^{-1}) is concerned, one should take into account the $\mathcal{B}r[J/\psi \to \mu^+\mu^-] \approx 6\%$ [11] and the total cross section for η_b production at Tevatron energy, $\sigma_{\rm tot}(\eta_b) = 2.5\mu b$ [3], obtaining between 0.004 and 11 produced η_b to the allowed range for $\mathcal{B}r[\eta_b \to J/\psi J/\psi]$. However, if we take into account the acceptance (± 0.6) and efficiency for detecting muons (10%), the previous range becomes 0 to 0.1 events. This is at odds with the experimental data from the CDF Collaboration on the run I data set [2]. However, preliminary results from CDF Collaboration run II data with 1.1 fb^{-1} [12] seem to be compatible with the predicted range in Eq. (11). In Ref. [12], in fact, CDF observed three candidates while expecting 3.6 background events in the search window from 9.0 to 9.5 GeV. For the run II data set, we estimate that there are $0.04 \div 120$ produced events which become $0 \div 1$ event by taking into account acceptance and efficiency to detect muons [12,13].

In conclusion, we have shown that, if the branching ratio of η_b into $D\bar{D}^*$ is large ($10^{-3} \div 10^{-2}$), the effect of final-state interactions, i.e. the rescattering $D\bar{D}^* \to J/\psi J/\psi$, may increase the short-distance $\eta_b \to J/\psi J/\psi$ branching ratio [cf. Eq. (3)] by a factor of about 200.

This result first of all calls for a direct calculation or measurement of the $\eta_b \to D\bar{D}^*$ decay process and, in any case, it supports the experimental search of η_b by looking at its decay into $J/\psi J/\psi$, which has very clean signature.

I would like to thank G. Nardulli for useful discussions.

[1] E. Braaten, S. Fleming, and A. K. Leibovich, Phys. Rev. D **63**, 094006 (2001).
[2] J. Tseng (CDF collaboration), Report No. FERMILAB-CONF-02-348-E.
[3] F. Maltoni and A. D. Polosa, Phys. Rev. D **70**, 054014 (2004).
[4] Y. Jia, arXiv:hep-ph/0611130.
[5] $\mathcal{B}r[\eta_b \to D\bar{D}^*]$ denotes the sum over the branching ratios of the three different charge assignments to the $D\bar{D}^*$ final state. In the following we assume, as in [3], they occur with the same probability of $1/3$.
[6] We use dimensionless strong coupling constants in all cases. In particular, we use the ratio $g_{JDD^*}/m_{J/\psi}$ instead of the dimensional $g_{JDD^*}(\mathrm{GeV}^{-1})$ usually found in literature.
[7] R. D. Matheus, F. S. Navarra, M. Nielsen, and R. Rodrigues da Silva, Int. J. Mod. Phys. E **14**, 555 (2005); M. E. Bracco, M. Chiapparini, F. S. Navarra, and M. Nielsen, Phys. Lett. B **605**, 326 (2005).
[8] A. Deandrea, G. Nardulli, and A. D. Polosa, Phys. Rev. D **68**, 034002 (2003).
[9] M. A. Ivanov and P. Santorelli, Phys. Lett. B **456**, 248 (1999); M. A. Ivanov, J. G. Korner, and P. Santorelli, Phys. Rev. D **70**, 014005 (2004).
[10] H. Y. Cheng, C. K. Chua, and A. Soni, Phys. Rev. D **71**, 014030 (2005).
[11] W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).
[12] M. Paulini (CDF II Collaboration and D0 Collaboration), *Heavy Quarks and Leptons, Munich, Germany, 2006*, http://hql06.physik.tu-muenchen.de/talks/20061019_1114_paulini_Paulini_HQL06.pdf.
[13] I thank S. D'auria for having submitted to my attention the preliminary results in Ref. [12].