Charmed baryon strong decays in a chiral quark model

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(Received 3 December 2007; published 14 April 2008)

Charmed baryon strong decays are studied in a chiral quark model. The data for the decays of $\Lambda_c^+(2593)$, $\Lambda_c^+(2625)$, $\Sigma_c^{++,+,0}$, and $\Sigma_c^{+,0}(2520)$ are accounted for successfully, which allows one to fix the pseudoscalar-meson-quark couplings in an effective chiral Lagrangian. Extending this framework to analyze the strong decays of the newly observed charmed baryons, we classify that $\Lambda_c(2880)$ and $\Lambda_c(2940)$ as D-wave states in the $N=2$ shell; $\Lambda_c(2880)$ could be $|\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$ and $\Lambda_c(2940)$ could be $|\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$. Our calculation also suggests that $\Lambda_c(2765)$ is very likely a ρ-mode P-wave excited state in the $N=1$ shell, and favors a $|\Lambda_c{}^4P_{\rho}\frac{1}{2}^-\rangle$ configuration. The $\Sigma_c(2800)$ favors being a $|\Sigma_c{}^2P_{\lambda}\frac{1}{2}^-\rangle$ state. But its being $|\Sigma_c^{++4}P_{\lambda}\frac{5}{2}^-\rangle$ cannot be ruled out.

DOI: 10.1103/PhysRevD.77.074008 PACS numbers: 13.30.−a, 14.20.Lq, 12.39.Jh

I. INTRODUCTION

In the past years, some new charmed baryons, such as $\Lambda_c(2880)$, $\Lambda_c(2940)$, $\Lambda_c(2765)$, and $\Sigma_c^{++,+,0}(2800)$, were observed by Belle, *BABAR*, and CLEO [1–4]. This initiated great interests in the heavy flavor baryon spectrum in both experiment and theory. At present, the experimental information is still limited and nearly nothing is known for their spin-parity quantum numbers (some review of the charmed baryons can be found in [5–8]). How to understand the properties of these new charmed baryons, e.g. their structures, and interactions with known particles in their production and decay, and how to establish the charmed baryon spectroscopy have been hot topics in both experiment and theory.

By studying the transitions of their different decay modes, one expects to extract information about their structures and the underlying dynamics. Several classes of models have been developed to deal with the strong decays of baryons. One is the hadrodynamic model, in which all hadrons are treated as pointlike objects. The heavy hadron chiral perturbation theory (HHChPT) approach [5,9] belongs to this class. The second class of models treats the baryons as a three-quark system, while the meson behaves as a pointlike particle emitted from active quarks when the initial baryon decays. Typical models of this class were reviewed in Refs. [10,11]. The third class of models is the pair creation model, in which both the baryon and meson have internal structures. The decay of a hadron is recognized by the creation of a quark-antiquark pair from vacuum, which combines with the initial quarks to form a meson and a baryon in the final state. Typical ways of treating the pair creations include the $^3P_0$ model [6,12], the string-breaking model [13,14], and the flux-tube breaking models [15–18]. Detailed review of these phenomenologies can be found in Ref. [19]. A large number of recent papers have contributed to the determination of the quantum numbers of these newly observed states [5,6,9,20–29].

In this work, we will analyze the strong decays of charmed baryons in the nonrelativistic chiral quark model, which belongs to the second class of models and had been well developed and widely used for the processes of meson photoproductions [30–37]. An extension of this approach to describe the process of πN scattering also turns out to be successful and inspiring [38]. In this framework, the charmed baryon spatial wave functions are described by harmonic oscillators. An effective chiral Lagrangian is then introduced to account for the quark-meson coupling. Since the quark-meson coupling is invariant under the chiral transformation, some of the low-energy properties of QCD are retained [33,35,38]. This approach is similar to that used in [10,11]; the only difference is that two constants in the decay amplitudes in Refs. [10,11] are replaced by two energy-dependent factors deduced from the chiral Lagrangian in our model.

In this work, we will first study the strong decays of the well-determined charmed baryons, $\Lambda_c^+(2593)$, $\Lambda_c^+(2625)$, $\Sigma_c^{++,+,0}$, and $\Sigma_c^{++,+,0}(2520)$. Using the measurement of $\Sigma_c^{++}(2520)$ as an input, we then determine the only free parameter δ in our model, with which we calculate the strong decays of $\Lambda_c^+(2593)$, $\Lambda_c^+(2625)$, $\Sigma_c^{++,+,0}$, and $\Sigma_c^{+,0}(2520)$ as a prediction. By comparing with the data we can extract information about these states, in particular, about these structures and quantum numbers [39].

Finally, we analyze the strong decays of the new observed charmed baryons $\Lambda_c(2880)$, $\Lambda_c(2940)$, $\Lambda_c(2765)$, and $\Sigma_c^{++,+,0}(2800)$. We predict that both $\Lambda_c(2880)$ and $\Lambda_c(2940)$ are D-wave states in the $N=2$ shell. $\Lambda_c(2880)$ could be a $|\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$ state and $\Lambda_c(2940)$ could be a $|\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$ state. We suggest that $\Lambda_c(2765)$ is most likely a ρ-mode P-wave excitation charmed baryon in the $N=1$ shell. The most possible state is $|\Lambda_c{}^4P_{\rho}\frac{1}{2}^-\rangle$. The calculations indicate that $\Sigma_c(2800)$ favors a $|\Sigma_c{}^2P_{\lambda}\frac{1}{2}^-\rangle$ state over the other ones for its broad width in experiment.

The paper is organized as follows. In the subsequent section, the charmed baryons in the quark model are outlined. Then, the nonrelativistic quark-meson couplings are given in Sec. III. The decay amplitudes are deduced in Sec. IV. We present our calculations and discussions in Sec. V. Finally, a summary is given in Sec. VI.

II. CHARMED BARYONS IN THE QUARK MODEL

A. Oscillator states

A state in a udc basis contains two light quarks (1 and 2) with equal mass m, and a heavy charmed quark (3) with mass m'. The basis states are generated by the Hamiltonian [40]

$$\mathcal{H} = \frac{1}{2m}(\mathbf{p}_1^2 + \mathbf{p}_2^2) + \frac{1}{2m'}\mathbf{p}_3^2 + \frac{1}{2}K\sum_{i<j}(\mathbf{r}_i - \mathbf{r}_j)^2. \quad (1)$$

In the above nonrelativistic expansions, vectors $\mathbf{r}_j$ and $\mathbf{p}_j$ are the coordinate and momentum for the jth quark in the baryon rest frame. The quarks are confined in an oscillator potential in which the potential parameter K independent of the flavor quantum number. One defines the Jacobi coordinates to eliminate the c.m. variables:

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2), \quad (2)$$

$$\vec{\lambda} = \frac{1}{\sqrt{6}}(\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3), \quad (3)$$

$$\mathbf{R}_{\text{c.m.}} = \frac{m(\mathbf{r}_1 + \mathbf{r}_2) + m'\mathbf{r}_3}{2m + m'}. \quad (4)$$

With the above relations (2)–(4), the oscillator Hamiltonian (1) is reduced to

$$\mathcal{H} = \frac{P_{\text{c.m.}}^2}{2M} + \frac{1}{2m_\rho}\mathbf{p}_\rho^2 + \frac{1}{2m_\lambda}\mathbf{p}_\lambda^2 + \frac{3}{2}K(\rho^2 + \lambda^2), \quad (5)$$

where

$$\mathbf{p}_\rho = m_\rho\dot{\vec{\rho}}, \qquad \mathbf{p}_\lambda = m_\lambda\dot{\vec{\lambda}}, \qquad \mathbf{P}_{\text{c.m.}} = M\dot{\mathbf{R}}_{\text{c.m.}}, \quad (6)$$

with

$$M = 2m + m', \qquad m_\rho = m, \qquad m_\lambda = \frac{3mm'}{2m + m'}. \quad (7)$$

With Eqs. (2)–(4) and (6) the coordinate $\mathbf{r}_j$ can be translated into functions of the Jacobi coordinates λ and ρ:

$$\mathbf{r}_1 = \mathbf{R}_{\text{c.m.}} + \frac{1}{\sqrt{6}}\frac{3m'}{2m + m'}\vec{\lambda} + \frac{1}{\sqrt{2}}\vec{\rho}, \quad (8)$$

$$\mathbf{r}_2 = \mathbf{R}_{\text{c.m.}} + \frac{1}{\sqrt{6}}\frac{3m'}{2m + m'}\vec{\lambda} - \frac{1}{\sqrt{2}}\vec{\rho}, \quad (9)$$

$$\mathbf{r}_3 = \mathbf{R}_{\text{c.m.}} - \sqrt{\frac{2}{3}}\frac{3m}{2m + m'}\vec{\lambda}, \quad (10)$$

and the momentum $\mathbf{p}_j$ is given by

$$\mathbf{p}_1 = \frac{m}{M}\mathbf{P}_{\text{c.m.}} + \frac{1}{\sqrt{6}}\mathbf{p}_\lambda + \frac{1}{\sqrt{2}}\mathbf{p}_\rho, \quad (11)$$

$$\mathbf{p}_2 = \frac{m}{M}\mathbf{P}_{\text{c.m.}} + \frac{1}{\sqrt{6}}\mathbf{p}_\lambda - \frac{1}{\sqrt{2}}\mathbf{p}_\rho, \quad (12)$$

$$\mathbf{p}_3 = \frac{m'}{M}\mathbf{P}_{\text{c.m.}} - \sqrt{\frac{2}{3}}\mathbf{p}_\lambda. \quad (13)$$

The spatial wave function is a product of the ρ-oscillator and the λ-oscillator states. With the standard notation, the principal quantum numbers of the ρ oscillator and λ oscillator are $N_\rho = (2n_\rho + l_\rho)$ and $N_\lambda = (2n_\lambda + l_\lambda)$, and the energy of a state is given by

$$E_N = (N_\rho + \tfrac{3}{2})\omega_\rho + (N_\lambda + \tfrac{3}{2})\omega_\lambda. \quad (14)$$

The total principal quantum number (i.e. shell number) N is defined as

$$N = N_\rho + N_\lambda, \quad (15)$$

and the frequencies of the ρ mode and λ mode are

$$\omega_\rho = (3K/m_\rho)^{1/2}, \qquad \omega_\lambda = (3K/m_\lambda)^{1/2}. \quad (16)$$

In the quark model two useful oscillator parameters, i.e. the potential strengths, are defined by

$$\alpha_\rho = (m_\rho\omega_\rho)^{1/2}, \qquad \alpha_\lambda = (m_\lambda\omega_\lambda)^{1/2}. \quad (17)$$

Combining Eqs. (7) and (16) with (17), we obtain the relation between these two parameters:

$$\alpha_\lambda^2 = \sqrt{\frac{3m'}{2m + m'}}\alpha_\rho^2. \quad (18)$$

Then, the wave function of an oscillator is given by

$$\psi^{n_\sigma}_{l_\sigma m}(\sigma) = R_{n_\sigma l_\sigma}(\sigma)Y_{l_\sigma m}(\sigma), \quad (19)$$

where $\sigma = \rho, \lambda$. The total orbital angular-momentum $\mathbf{L}$ of a state is obtained by coupling $\boldsymbol{l}_\rho$ to $\boldsymbol{l}_\lambda$:

$$\mathbf{L} = \boldsymbol{l}_\rho + \boldsymbol{l}_\lambda. \quad (20)$$

The total spatial wave function can then be constructed. All the functions with principal quantum number $N \leq 2$ are listed in Table I.

B. Flavor and spin wave functions

For the udc basis states which violate $SU(4)$ symmetry, as done in Ref. [11] we introduce

TABLE I. The spatial wave functions with principal quantum number $N \leq 2$, denoted by ${}^{N}\Psi^{\sigma}_{LL_z}$, where $\sigma = s, \lambda\, \rho, A, \rho\rho, \lambda\lambda$ stand for different excitation modes in the quark model.

N	**L**	L	l_ρ	l_λ	n_ρ	n_λ	L_z	Wave function
0	0	0	0	0	0	0	0	${}^{0}\Psi^{S}_{00} = \psi_{00}(\rho)\psi_{00}(\lambda)$
1	1	1	0	1	0	0	m	${}^{1}\Psi^{\lambda}_{1m} = \psi_{00}(\rho)\psi_{1m}(\lambda)$
1	1	1	1	0	0	0	m	${}^{1}\Psi^{\rho}_{1m} = \psi_{1m}(\rho)\psi_{00}(\lambda)$
2	2	0	1	1	0	0	0	${}^{2}\Psi^{A}_{00} = \sqrt{\frac{1}{3}}R_{01}(\rho)R_{01}(\lambda)\{Y_{11}(\lambda)Y_{1-1}(\rho) - Y_{10}(\lambda)Y_{10}(\rho) + Y_{1-1}(\lambda)Y_{11}(\rho)\}$
2	2	1	1	1	0	0	1	${}^{2}\Psi^{A}_{11} = \sqrt{\frac{1}{2}}R_{01}(\rho)R_{01}(\lambda)\{Y_{11}(\lambda)Y_{10}(\rho) - Y_{10}(\lambda)Y_{11}(\rho)\}$
2	2	1	1	1	0	0	0	${}^{2}\Psi^{A}_{10} = \sqrt{\frac{1}{2}}R_{01}(\rho)R_{01}(\lambda)\{Y_{11}(\lambda)Y_{1-1}(\rho) - Y_{1-1}(\lambda)Y_{11}(\rho)\}$
2	2	1	1	1	0	0	-1	${}^{2}\Psi^{A}_{1-1} = \sqrt{\frac{1}{2}}R_{01}(\rho)R_{01}(\lambda)\{Y_{10}(\lambda)Y_{1-1}(\rho) - Y_{1-1}(\lambda)Y_{10}(\rho)\}$
2	2	2	1	1	0	0	2	${}^{2}\Psi^{A}_{22} = R_{01}(\rho)R_{01}(\lambda)Y_{11}(\lambda)Y_{11}(\rho)$
2	2	2	1	1	0	0	1	${}^{2}\Psi^{A}_{21} = \sqrt{\frac{1}{2}}R_{01}(\rho)R_{01}(\lambda)\{Y_{11}(\lambda)Y_{10}(\rho) + Y_{10}(\lambda)Y_{11}(\rho)\}$
2	2	2	1	1	0	0	0	${}^{2}\Psi^{A}_{20} = \sqrt{\frac{1}{6}}R_{01}(\rho)R_{01}(\lambda)\{Y_{11}(\lambda)Y_{1-1}(\rho) + 2Y_{10}(\lambda)Y_{10}(\rho) + Y_{1-1}(\lambda)Y_{11}(\rho)\}$
2	2	2	1	1	0	0	-1	${}^{2}\Psi^{A}_{2-1} = \sqrt{\frac{1}{2}}R_{01}(\rho)R_{01}(\lambda)\{Y_{10}(\lambda)Y_{1-1}(\rho) + Y_{1-1}(\lambda)Y_{10}(\rho)\}$
2	2	2	1	1	0	0	-2	${}^{2}\Psi^{A}_{2-2} = R_{01}(\rho)R_{01}(\lambda)Y_{1-1}(\lambda)Y_{1-1}(\rho)$
2	2	2	2	0	0	0	m	${}^{2}\Psi^{\rho\rho}_{2m} = \psi^{0}_{2m}(\rho)\psi^{0}_{00}(\lambda)$
2	2	2	0	2	0	0	m	${}^{2}\Psi^{\lambda\lambda}_{2m} = \psi^{0}_{00}(\rho)\psi^{0}_{2m}(\lambda)$
2	0	0	0	0	1	0	0	${}^{2}\Psi^{\rho\rho}_{00} = \psi^{1}_{00}(\rho)\psi^{0}_{00}(\lambda)$
2	0	0	0	0	0	1	0	${}^{2}\Psi^{\lambda\lambda}_{00} = \psi^{0}_{00}(\rho)\psi^{1}_{00}(\lambda)$

$$\phi_{\Lambda_c} = \frac{1}{\sqrt{2}}(ud - du)c, \tag{21}$$

and

$$\phi_{\Sigma_c} = \begin{cases} ddc & \text{for } \Sigma_c^0, \\ \frac{1}{\sqrt{2}}(ud + du)c & \text{for } \Sigma_c^+, \\ uuc & \text{for } \Sigma_c^{++}, \end{cases} \tag{22}$$

for the Λ_c- and Σ_c-type flavor wave functions, respectively.

For the spin wave functions, the usual ones are adopted [10,11]:

$$\begin{aligned} \chi^s_{3/2} &= \uparrow\uparrow\uparrow, \qquad \chi^s_{-3/2} = \downarrow\downarrow\downarrow, \\ \chi^s_{1/2} &= \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow), \\ \chi^s_{-1/2} &= \frac{1}{\sqrt{3}}(\uparrow\downarrow\downarrow + \downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow), \end{aligned} \tag{23}$$

for the spin-3/2 states;

$$\chi^\rho_{1/2} = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \qquad \chi^\rho_{-1/2} = \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow), \tag{24}$$

for the spin-1/2 states, in which the first two quark spins are antisymmetric; and

$$\begin{aligned} \chi^\lambda_{1/2} &= -\frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow), \\ \chi^\lambda_{-1/2} &= -\frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow), \end{aligned} \tag{25}$$

for the spin-1/2 states, in which the first two quark spins are symmetric.

C. The total wave functions

The spin-flavor and spatial wave functions of baryons must be symmetric since the color wave function is antisymmetric. The flavor wave functions of the Λ_c-type charmed baryons, ϕ_{Λ_c}, are antisymmetric under the interchange of the u and d quarks; thus, their spin-space wave functions must be symmetric. In contrast, the spin-spatial wave functions of Σ_c-type charmed baryons are required to be antisymmetric due to their symmetric flavor wave functions under the interchange of the u and d quarks. The wave functions of the Λ_c-type and Σ_c-type charmed baryons are listed in Tables II and III, respectively.

III. THE QUARK-MESON COUPLINGS

In the chiral quark model, the low-energy quark-meson interactions are described by an effective Lagrangian [33,35],

$$\mathcal{L} = \bar{\psi}[\gamma_\mu(i\partial^\mu + V^\mu + \gamma_5 A^\mu) - m]\psi + \cdots, \tag{26}$$

where V^μ and A^μ correspond to vector and axial currents, respectively. They are given by

$$\begin{aligned} V^\mu &= \frac{1}{2}(\xi\partial^\mu\xi^\dagger + \xi^\dagger\partial^\mu\xi), \\ A^\mu &= \frac{1}{2i}(\xi\partial^\mu\xi^\dagger - \xi^\dagger\partial^\mu\xi), \end{aligned} \tag{27}$$

TABLE II. The total wave functions of Λ_c-type charmed baryons, denoted by $|\Lambda_c{}^{2S+1}L_\sigma J^P\rangle$ as used in Ref. [11]. The Clebsch-Gordan series for the spin and angular-momentum addition $|\Lambda_c{}^{2S+1}L_\sigma J^P\rangle = \sum_{L_z+S_z=J_z}\langle LL_z, SS_z|JJ_z\rangle^N\Psi^\sigma_{LL_z}\chi_{S_z}\phi_{\Lambda_c}$ has been omitted.

State	N	**J**	L	S	J^P	Wave function
$\|\Lambda_c{}^2S\frac{1}{2}^+\rangle$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}^+$	${}^0\Psi^S_{00}\chi^\rho_{S_z}\phi_{\Lambda_c}$
$\|\Lambda_c{}^2P_\lambda\frac{1}{2}^-\rangle$	1	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}^-$	${}^1\Psi^\lambda_{1L_z}\chi^\rho_{S_z}\phi_{\Lambda_c}$
$\|\Lambda_c{}^2P_\lambda\frac{3}{2}^-\rangle$	1	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^-$	
$\|\Lambda_c{}^2P_\rho\frac{1}{2}^-\rangle$	1	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}^-$	${}^1\Psi^\rho_{1L_z}\chi^\lambda_{S_z}\phi_{\Lambda_c}$
$\|\Lambda_c{}^2P_\rho\frac{3}{2}^-\rangle$	1	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^-$	
$\|\Lambda_c{}^4P_\rho\frac{1}{2}^-\rangle$	1	$\frac{5}{2}$	1	$\frac{3}{2}$	$\frac{1}{2}^-$	
$\|\Lambda_c{}^4P_\rho\frac{3}{2}^-\rangle$	1	$\frac{5}{2}$	1	$\frac{3}{2}$	$\frac{3}{2}^-$	${}^1\Psi^\rho_{1L_z}\chi^S_{S_z}\phi_{\Lambda_c}$
$\|\Lambda_c{}^4P_\rho\frac{5}{2}^-\rangle$	1	$\frac{5}{2}$	1	$\frac{3}{2}$	$\frac{5}{2}^-$	
$\|\Lambda_c{}^2S_A\frac{1}{2}^+\rangle$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}^+$	${}^2\Psi^A_{00}\chi^\lambda_{S_z}\phi_{\Lambda_c}$
$\|\Lambda_c{}^4S_A\frac{3}{2}^+\rangle$	2	$\frac{3}{2}$	0	$\frac{3}{2}$	$\frac{3}{2}^+$	${}^2\Psi^A_{00}\chi^s_{S_z}\phi_{\Lambda_c}$
$\|\Lambda_c{}^2P_A\frac{1}{2}^-\rangle$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}^-$	${}^2\Psi^A_{1L_z}\chi^\lambda_{S_z}\phi_{\Lambda_c}$
$\|\Lambda_c{}^2P_A\frac{3}{2}^-\rangle$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^-$	
$\|\Lambda_c{}^4P_A\frac{1}{2}^-\rangle$	2	$\frac{5}{2}$	1	$\frac{3}{2}$	$\frac{1}{2}^-$	
$\|\Lambda_c{}^4P_A\frac{3}{2}^-\rangle$	2	$\frac{5}{2}$	1	$\frac{3}{2}$	$\frac{3}{2}^-$	${}^2\Psi^A_{1L_z}\chi^S_{S_z}\phi_{\Lambda_c}$
$\|\Lambda_c{}^4P_A\frac{5}{2}^-\rangle$	2	$\frac{5}{2}$	1	$\frac{3}{2}$	$\frac{5}{2}^-$	
$\|\Lambda_c{}^2D_A\frac{3}{2}^+\rangle$	2	$\frac{5}{2}$	2	$\frac{1}{2}$	$\frac{3}{2}^+$	
$\|\Lambda_c{}^2D_A\frac{5}{2}^+\rangle$	2	$\frac{5}{2}$	2	$\frac{1}{2}$	$\frac{5}{2}^+$	${}^2\Psi^A_{2L_z}\chi^\lambda_{S_z}\phi_{\Lambda_c}$
$\|\Lambda_c{}^4D_A\frac{1}{2}^+\rangle$	2	$\frac{7}{2}$	2	$\frac{3}{2}$	$\frac{1}{2}^+$	
$\|\Lambda_c{}^4D_A\frac{3}{2}^+\rangle$	2	$\frac{7}{2}$	2	$\frac{3}{2}$	$\frac{3}{2}^+$	
$\|\Lambda_c{}^4D_A\frac{5}{2}^+\rangle$	2	$\frac{7}{2}$	2	$\frac{3}{2}$	$\frac{5}{2}^+$	${}^2\Psi^A_{2L_z}\chi^S_{S_z}\phi_{\Lambda_c}$
$\|\Lambda_c{}^4D_A\frac{7}{2}^+\rangle$	2	$\frac{7}{2}$	2	$\frac{3}{2}$	$\frac{7}{2}^+$	
$\|\Lambda_c{}^2D_{\rho\rho}\frac{3}{2}^+\rangle$	2	$\frac{5}{2}$	2	$\frac{1}{2}$	$\frac{3}{2}^+$	
$\|\Lambda_c{}^2D_{\rho\rho}\frac{5}{2}^+\rangle$	2	$\frac{5}{2}$	2	$\frac{1}{2}$	$\frac{5}{2}^+$	${}^2\Psi^{\rho\rho}_{2L_z}\chi^\rho_{S_z}\phi_{\Lambda_c}$
$\|\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$	2	$\frac{5}{2}$	2	$\frac{1}{2}$	$\frac{3}{2}^+$	
$\|\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$	2	$\frac{5}{2}$	2	$\frac{1}{2}$	$\frac{5}{2}^+$	${}^2\Psi^{\lambda\lambda}_{2L_z}\chi^\rho_{S_z}\phi_{\Lambda_c}$
$\|\Lambda_c{}^2S_{\rho\rho}\frac{1}{2}^+\rangle$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}^+$	${}^2\Psi^{\rho\rho}_{00}\chi^\rho_{S_z}\phi_{\Lambda_c}$
$\|\Lambda_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}^+$	${}^2\Psi^{\lambda\lambda}_{00}\chi^\rho_{S_z}\phi_{\Lambda_c}$

with $\xi = \exp(i\phi_m/f_m)$, where f_m is the meson decay constant. In the flavor $SU(3)$ sector, the pseudoscalar-meson octet ϕ_m can be expressed as

$$\phi_m = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}, \quad (28)$$

and the quark field ψ is given by

$$\psi = \begin{pmatrix} \psi(u) \\ \psi(d) \\ \psi(s) \end{pmatrix}. \quad (29)$$

TABLE III. The total wave functions of Σ_c-type charmed baryons, denoted by $|\Sigma_c{}^{2S+1}L_\sigma J^P\rangle$. The Clebsch-Gordan series for the spin and angular-momentum addition $|\Sigma_c{}^{2S+1}L_\sigma J^P\rangle = \sum_{L_z+S_z=J_z}\langle LL_z, SS_z|JJ_z\rangle^N\Psi^\sigma_{LL_z}\chi_{S_z}\phi_{\Sigma_c}$ has been omitted.

State	N	**J**	L	S	J^P	Wave function
$\|\Sigma_c{}^2S\frac{1}{2}^+\rangle$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}^+$	${}^0\Psi^S_{00}\chi^\lambda_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^4S\frac{3}{2}^+\rangle$	0	$\frac{3}{2}$	0	$\frac{3}{2}$	$\frac{3}{2}^+$	${}^0\Psi^S_{00}\chi^s_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^2P_\lambda\frac{1}{2}^-\rangle$	1	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}^-$	${}^1\Psi^\lambda_{1L_z}\chi^\lambda_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^2P_\lambda\frac{3}{2}^-\rangle$	1	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^-$	
$\|\Sigma_c{}^4P_\lambda\frac{1}{2}^-\rangle$	1	$\frac{5}{2}$	1	$\frac{3}{2}$	$\frac{1}{2}^-$	
$\|\Sigma_c{}^4P_\lambda\frac{3}{2}^-\rangle$	1	$\frac{5}{2}$	1	$\frac{3}{2}$	$\frac{3}{2}^-$	${}^1\Psi^\lambda_{1L_z}\chi^s_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^4P_\lambda\frac{5}{2}^-\rangle$	1	$\frac{5}{2}$	1	$\frac{3}{2}$	$\frac{5}{2}^-$	
$\|\Sigma_c{}^2P_\rho\frac{1}{2}^-\rangle$	1	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}^-$	${}^1\Psi^\rho_{1L_z}\chi^\rho_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^2P_\rho\frac{3}{2}^-\rangle$	1	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^-$	
$\|\Sigma_c{}^2S_A\frac{1}{2}^+\rangle$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}^+$	${}^2\Psi^A_{00}\chi^\rho_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^2P_A\frac{1}{2}^-\rangle$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}^-$	${}^2\Psi^A_{1L_z}\chi^\rho_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^2P_A\frac{3}{2}^-\rangle$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}^-$	
$\|\Sigma_c{}^2D_A\frac{3}{2}^+\rangle$	2	$\frac{5}{2}$	2	$\frac{1}{2}$	$\frac{3}{2}^+$	
$\|\Sigma_c{}^2D_A\frac{5}{2}^+\rangle$	2	$\frac{5}{2}$	2	$\frac{1}{2}$	$\frac{5}{2}^+$	${}^2\Psi^A_{2L_z}\chi^\rho_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^2D_{\rho\rho}\frac{3}{2}^+\rangle$	2	$\frac{5}{2}$	2	$\frac{1}{2}$	$\frac{3}{2}^+$	
$\|\Sigma_c{}^2D_{\rho\rho}\frac{5}{2}^+\rangle$	2	$\frac{5}{2}$	2	$\frac{1}{2}$	$\frac{5}{2}^+$	${}^2\Psi^{\rho\rho}_{2L_z}\chi^\lambda_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^4D_{\rho\rho}\frac{1}{2}^+\rangle$	2	$\frac{7}{2}$	2	$\frac{3}{2}$	$\frac{1}{2}^+$	
$\|\Sigma_c{}^4D_{\rho\rho}\frac{3}{2}^+\rangle$	2	$\frac{7}{2}$	2	$\frac{3}{2}$	$\frac{3}{2}^+$	${}^2\Psi^{\rho\rho}_{2L_z}\chi^s_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^4D_{\rho\rho}\frac{5}{2}^+\rangle$	2	$\frac{7}{2}$	2	$\frac{3}{2}$	$\frac{5}{2}^+$	
$\|\Sigma_c{}^4D_{\rho\rho}\frac{7}{2}^+\rangle$	2	$\frac{7}{2}$	2	$\frac{3}{2}$	$\frac{7}{2}^+$	
$\|\Sigma_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$	2	$\frac{5}{2}$	2	$\frac{1}{2}$	$\frac{3}{2}^+$	
$\|\Sigma_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$	2	$\frac{5}{2}$	2	$\frac{1}{2}$	$\frac{5}{2}^+$	${}^2\Psi^{\lambda\lambda}_{2L_z}\chi^\lambda_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^4D_{\lambda\lambda}\frac{1}{2}^+\rangle$	2	$\frac{7}{2}$	2	$\frac{3}{2}$	$\frac{1}{2}^+$	
$\|\Sigma_c{}^4D_{\lambda\lambda}\frac{3}{2}^+\rangle$	2	$\frac{7}{2}$	2	$\frac{3}{2}$	$\frac{3}{2}^+$	${}^2\Psi^{\lambda\lambda}_{2L_z}\chi^s_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^4D_{\lambda\lambda}\frac{5}{2}^+\rangle$	2	$\frac{7}{2}$	2	$\frac{3}{2}$	$\frac{5}{2}^+$	
$\|\Sigma_c{}^4D_{\lambda\lambda}\frac{7}{2}^+\rangle$	2	$\frac{7}{2}$	2	$\frac{3}{2}$	$\frac{7}{2}^+$	
$\|\Sigma_c{}^2S_{\rho\rho}\frac{1}{2}^+\rangle$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}^+$	${}^2\Psi^{\rho\rho}_{00}\chi^\lambda_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^4S_{\rho\rho}\frac{3}{2}^+\rangle$	2	$\frac{3}{2}$	0	$\frac{3}{2}$	$\frac{3}{2}^+$	${}^2\Psi^{\rho\rho}_{00}\chi^s_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}^+$	${}^2\Psi^{\lambda\lambda}_{00}\chi^\lambda_{S_z}\phi_{\Sigma_c}$
$\|\Sigma_c{}^4S_{\lambda\lambda}\frac{3}{2}^+\rangle$	2	$\frac{3}{2}$	0	$\frac{3}{2}$	$\frac{3}{2}^+$	${}^2\Psi^{\lambda\lambda}_{00}\chi^s_{S_z}\phi_{\Sigma_c}$

The tree-level quark-meson pseudovector coupling is thus given by

$$H_m = \sum_j \frac{1}{f_m}\bar{\psi}_j\gamma^j_\mu\gamma^j_5\psi_j\partial^\mu\phi_m, \quad (30)$$

where ψ_j represents the jth quark field in a baryon. This effective quark-meson pseudovector coupling can be used for D mesons as well, if we extend the $SU(3)$ case to the $SU(4)$ case.

In the quark model, the nonrelativistic form of Eq. (30) is written as [33,35,38]

$$H_m^{\rm nr} = \sum_j \left\{ \frac{\omega_m}{E_f + M_f} \boldsymbol{\sigma}_j \cdot \mathbf{P}_f + \frac{\omega_m}{E_i + M_i} \boldsymbol{\sigma}_j \cdot \mathbf{P}_i - \boldsymbol{\sigma}_j \cdot \mathbf{q} + \frac{\omega_m}{2\mu_q} \boldsymbol{\sigma}_j \cdot \mathbf{p}'_j \right\} I_j \varphi_m, \tag{31}$$

where $\boldsymbol{\sigma}_j$ and μ_q correspond to the Pauli spin vector and the reduced mass of the jth quark in the initial and final baryons, respectively. For emitting a meson, we have $\varphi_m = e^{-i\mathbf{q}\cdot\mathbf{r}_j}$, and for absorbing a meson we have $\varphi_m = e^{i\mathbf{q}\cdot\mathbf{r}_j}$. In the above nonrelativistic expansions, $\mathbf{p}'_j = \mathbf{p}_j - (m_j/M)\mathbf{P}_{\rm c.m.}$ is the internal momentum for the jth quark in the baryon rest frame. ω_m and $\mathbf{q}$ are the energy and three-vector momentum of the meson, respectively. The isospin operator I_j in Eq. (31) is expressed as

$$I_j = \begin{cases} a_j^\dagger(u) a_j(c) & \text{for } D^0, \\ a_j^\dagger(u) a_j(d) & \text{for } \pi^-, \\ a_j^\dagger(d) a_j(u) & \text{for } \pi^+, \\ \frac{1}{\sqrt{2}}[a_j^\dagger(u) a_j(u) - a_j^\dagger(d) a_j(d)] & \text{for } \pi^0, \end{cases} \tag{32}$$

where $a_j^\dagger(u, d, c)$ and $a_j(u, d, c)$ are the creation and annihilation operators for the u, d, and c quarks.

IV. DECAY OF THE CHARMED BARYON IN THE QUARK MODEL

In the calculations, we select the initial-baryon-rest system for the decay precesses. The energies and momenta of the initial charmed baryons are denoted by $(E_i, \mathbf{P}_i)$, while those of the final state mesons and baryons are denoted by $(\omega_f, \mathbf{q})$ and $(E_f, \mathbf{P}_f)$. Note that $\mathbf{P}_i = 0$ ($E_i = M_i$) and $\mathbf{P}_f = -\mathbf{q}$.

A. $\mathcal{B}_c \to \mathcal{B}'_c \pi(\mathbf{q})$

Because the π meson only couples to light quark 1 or 2 in a udc basis state, the strong decay amplitudes for the process $\mathcal{B}_c \to \mathcal{B}'_c \pi(\mathbf{q})$ can be written as

$$\mathcal{M}[\mathcal{B}_c \to \mathcal{B}'_c \pi(\mathbf{q})] = 2\langle \mathcal{B}'_c | \{ G\boldsymbol{\sigma}_1 \cdot \mathbf{q} + h \boldsymbol{\sigma}_1 \cdot \mathbf{p}'_1 \} \times I_1 e^{-i\mathbf{q}\cdot\mathbf{r}_1} | \mathcal{B}_c \rangle, \tag{33}$$

with

$$G \equiv -\frac{\omega_\pi}{E_f + M_f} - 1, \qquad h \equiv \frac{\omega_\pi}{m}, \tag{34}$$

where $\mathcal{B}_c$ and $\mathcal{B}'_c$ stand for the initial and final charmed baryon wave functions, which are listed in Tables II and III. Similar expressions were also derived in Refs. [10,11]. By selecting $\mathbf{q} = q\hat{z}$, namely, the meson moves along the z axial, we can simplify the amplitude to

$$\begin{aligned} \mathcal{M}[\mathcal{B}_c \to \mathcal{B}'_c \pi(\mathbf{q})] &= 2\left\{ Gq - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{3}} q_\lambda + q_\rho \right) h \right\} \langle \mathcal{B}'_c | \sigma_{1z} \phi I_1 | \mathcal{B}_c \rangle \\ &\quad - i\sqrt{\frac{2}{3}} h \langle \mathcal{B}'_c | (\boldsymbol{\sigma}_1 \cdot \vec{\nabla}_\lambda - \alpha_\lambda^2 \boldsymbol{\sigma}_1 \cdot \vec{\lambda}) \phi I_1 | \mathcal{B}_c \rangle \\ &\quad - i\sqrt{2} h \langle \mathcal{B}'_c | (\boldsymbol{\sigma}_1 \cdot \vec{\nabla}_\rho - \alpha_\rho^2 \boldsymbol{\sigma}_1 \cdot \vec{\rho}) \phi I_1 | \mathcal{B}_c \rangle, \end{aligned} \tag{35}$$

where $\vec{\nabla}_\lambda$ and $\vec{\nabla}_\rho$ are the derivative operators on the spatial wave function of the final baryon except the factor $\exp[(-\alpha_\lambda^2 \lambda^2 - \alpha_\rho^2 \rho^2)/2]$ which has been worked out, and

$$q_\lambda = \frac{1}{\sqrt{6}} \frac{3m'}{2m + m'} q, \qquad q_\rho = \frac{1}{\sqrt{2}} q, \tag{36}$$

and

$$\phi = \exp(-iq_\lambda \lambda_z) \exp(-iq_\rho \rho_z). \tag{37}$$

In Eq. (35), the first term comes from the c.m. motion of the system, while the last two terms attribute to the λ- and ρ-mode orbital excitations of the charmed baryons, respectively.

For example, we calculate the decay process $|\Lambda_c\, {}^2P_\lambda \frac{1}{2}^-\rangle \to \Sigma_c \pi$. The initial and final charmed baryon wave functions are given by (see Table II)

$$|\mathcal{B}_c\rangle = \left[\sqrt{\tfrac{1}{3}} \Psi_{11}^\lambda \chi_{-1/2}^\rho + \sqrt{\tfrac{2}{3}} \Psi_{10}^\lambda \chi_{1/2}^\rho \right] \phi_{\Lambda_c}, \tag{38}$$

$$|\mathcal{B}'_c\rangle = \Psi_{00}^S \chi_{1/2}^\lambda \phi_{\Sigma_c}. \tag{39}$$

Substituting into Eq. (35), we obtain the decay amplitude

$$\mathcal{M} = i g_1 g_I \left\{ \sqrt{\frac{2}{3}} \left[Gq - \frac{h}{2\sqrt{2}} \left(\frac{1}{\sqrt{3}} q_\lambda + q_\rho \right) \right] \frac{q_\lambda}{\alpha_\lambda} + h\alpha_\lambda \right\} F(q_\lambda, q_\rho), \tag{40}$$

where the spin and isospin factors are

$$g_1 = \langle \chi_{1/2}^\lambda | \sigma_{1z} | \chi_{1/2}^\rho \rangle \tag{41}$$

and

$$g_I = \langle \phi_{\Sigma_c} | \sigma_{1z} | \phi_{\Lambda_c} \rangle. \tag{42}$$

The spatial integral gives

$$F(q_\lambda, q_\rho) = \exp\left(-\frac{q_\lambda^2}{4\alpha_\lambda^2} - \frac{q_\rho^2}{4\alpha_\rho^2} \right), \tag{43}$$

which plays the role of form factor.

The corresponding spin factors are listed in Table IV. Some of the decay amplitudes for $|\Lambda_c\, {}^{2S+1}L_\sigma J^P\rangle \to \Sigma_c \pi$, $|\Lambda_c\, {}^{2S+1}L_\sigma J^P\rangle \to \Sigma_c(2520)\pi$, $|\Sigma_c\, {}^{2S+1}L_\sigma J^P\rangle \to \Lambda_c \pi$,

TABLE IV. The spin factors used in this work.

$g_1=\langle\chi^\lambda_{1/2}\vert\sigma_{1z}\vert\chi^\rho_{1/2}\rangle=-\frac{1}{\sqrt{3}}$	$g^D_1=\langle\chi^\rho_{1/2}\vert\sigma_{3z}\vert\chi^\lambda_{1/2}\rangle=0$
$g_2=\langle\chi^\lambda_{1/2}\vert\sigma^+_1\vert\chi^\rho_{-1/2}\rangle=-\frac{1}{\sqrt{3}}$	$g^D_2=\langle\chi^\rho_{1/2}\vert\sigma^+_3\vert\chi^\lambda_{-1/2}\rangle=0$
$g_7=\langle\chi^\lambda_{1/2}\vert\sigma_{1z}\vert\chi^s_{1/2}\rangle=\frac{\sqrt{2}}{3}$	$g^D_3=\langle\chi^\rho_{1/2}\vert\sigma_{3z}\vert\chi^s_{1/2}\rangle=0$
$g_3=\langle\chi^\lambda_{1/2}\vert\sigma^-_1\vert\chi^s_{3/2}\rangle=-\frac{1}{\sqrt{6}}$	$g^D_4=\langle\chi^\rho_{1/2}\vert\sigma^+_3\vert\chi^s_{-1/2}\rangle=0$
$g_4=\langle\chi^\lambda_{1/2}\vert\sigma^+_1\vert\chi^s_{-1/2}\rangle=\frac{\sqrt{2}}{6}$	$g^D_5=\langle\chi^\rho_{1/2}\vert\sigma^-_3\vert\chi^s_{3/2}\rangle=0$
$g_5=\langle\chi^\lambda_{1/2}\vert\sigma_{1z}\vert\chi^\lambda_{1/2}\rangle=\frac{2}{3}$	$g^D_6=\langle\chi^\rho_{1/2}\vert\sigma_{3z}\vert\chi^\rho_{1/2}\rangle=1$
$g_6=\langle\chi^\lambda_{1/2}\vert\sigma^+_1\vert\chi^\lambda_{-1/2}\rangle=-\frac{2}{3}$	$g^D_7=\langle\chi^\rho_{1/2}\vert\sigma^+_3\vert\chi^\rho_{-1/2}\rangle=1$
$g^\Sigma_1=\langle\chi^\rho_{1/2}\vert\sigma_{1z}\vert\chi^\lambda_{1/2}\rangle=-\frac{1}{\sqrt{3}}$	$g^*_1=\langle\chi^s_{3/2}\vert\sigma^+_1\vert\chi^\lambda_{1/2}\rangle=-\frac{1}{\sqrt{6}}$
$g^\Sigma_2=\langle\chi^\rho_{1/2}\vert\sigma^+_1\vert\chi^\lambda_{-1/2}\rangle=\frac{1}{\sqrt{3}}$	$g^*_2=\langle\chi^s_{1/2}\vert\sigma_{1z}\vert\chi^\lambda_{1/2}\rangle=\frac{\sqrt{2}}{3}$
$g^\Sigma_3=\langle\chi^\rho_{1/2}\vert\sigma_{1z}\vert\chi^s_{1/2}\rangle=\frac{\sqrt{6}}{3}$	$g^*_3=\langle\chi^s_{1/2}\vert\sigma^+_1\vert\chi^\lambda_{-1/2}\rangle=\frac{\sqrt{2}}{6}$
$g^\Sigma_4=\langle\chi^\rho_{1/2}\vert\sigma^+_1\vert\chi^s_{-1/2}\rangle=\frac{1}{\sqrt{6}}$	$g^*_4=\langle\chi^s_{3/2}\vert\sigma_{1z}\vert\chi^s_{3/2}\rangle=1$
$g^\Sigma_5=\langle\chi^\rho_{1/2}\vert\sigma^-_1\vert\chi^s_{3/2}\rangle=-\frac{1}{\sqrt{2}}$	$g^*_5=\langle\chi^s_{3/2}\vert\sigma^+_1\vert\chi^s_{1/2}\rangle=\frac{1}{\sqrt{3}}$
$g^\Sigma_6=\langle\chi^\rho_{1/2}\vert\sigma_{1z}\vert\chi^\rho_{1/2}\rangle=0$	$g^*_6=\langle\chi^s_{1/2}\vert\sigma^-_1\vert\chi^s_{3/2}\rangle=\frac{1}{\sqrt{3}}$
$g^\Sigma_7=\langle\chi^\rho_{1/2}\vert\sigma^+_1\vert\chi^\rho_{-1/2}\rangle=0$	$g^*_7=\langle\chi^s_{1/2}\vert\sigma_{1z}\vert\chi^s_{1/2}\rangle=\frac{1}{3}$
$g^*_8=\langle\chi^s_{1/2}\vert\sigma^+_1\vert\chi^s_{-1/2}\rangle=\frac{2}{3}$	$g^*_9=\langle\chi^s_{1/2}\vert\sigma_{1z}\vert\chi^\rho_{1/2}\rangle=\frac{\sqrt{6}}{3}$
$g^*_{10}=\langle\chi^s_{3/2}\vert\sigma^+_1\vert\chi^\rho_{1/2}\rangle=-\frac{1}{\sqrt{2}}$	$g^*_{11}=\langle\chi^s_{1/2}\vert\sigma^+_1\vert\chi^\rho_{-1/2}\rangle=-\frac{1}{\sqrt{6}}$

$|\Sigma_c{}^{2S+1}L_\sigma J^P\rangle\to\Sigma_c\pi$, and $|\Sigma_c{}^{2S+1}L_\sigma J^P\rangle\to\Sigma_c(2520)\pi$ are listed in Tables V, VI, VII, VIII, and IX, respectively.

B. $\mathcal{B}_c\to D(\mathbf{q})p$

For a charmed baryon decaying into Dp, since the D meson only couples to charm quark 3 in a udc basis state, the strong decay amplitudes for the process $\mathcal{B}_c\to D(\mathbf{q})p$ can be written as

$$\mathcal{M}[\mathcal{B}_c\to D(\mathbf{q})p]=\langle p|\{G\boldsymbol{\sigma}_3\cdot\mathbf{q}+h\boldsymbol{\sigma}_3\cdot\mathbf{p}'_3\}\times I_3e^{-i\mathbf{q}\cdot\mathbf{r}_3}|\mathcal{B}_c\rangle, \tag{44}$$

where the wave function of a proton in the quark model is expressed as

$$|p\rangle=\frac{1}{\sqrt{2}}(\Phi_\rho\chi^\rho+\Phi_\lambda\chi^\lambda)^0\Psi^S_{00}, \tag{45}$$

with

TABLE V. The decay amplitudes for all the states $|\Lambda_c{}^{2S+1}L_\sigma J^P\rangle$ up to the $N=2$ shell in the $\Sigma_c\pi$ channel (a factor 2 is omitted). g_I is an isospin factor which is defined by $g_I=\langle\phi_\Sigma|I_1|\phi_\Lambda\rangle$. F, as the decay form factor, is defined in Eq. (43).

Initial state	Amplitude
$\vert\Lambda_c{}^2S\frac{1}{2}^+\rangle$	Forbidden by the kinematics
$\vert\Lambda_c{}^2P_\lambda\frac{1}{2}^-\rangle$	$i\frac{\sqrt{6}}{6}g_1g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F+i\frac{1}{2}g_1g_Ih\alpha_\lambda F$
$\vert\Lambda_c{}^2P_\lambda\frac{3}{2}^-\rangle$	$-i\frac{\sqrt{3}}{3}g_1g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F$
$\vert\Lambda_c{}^2P_\rho\frac{1}{2}^-\rangle$	$i\frac{\sqrt{6}}{6}g_5g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\rho}{\alpha_\rho}F+i\frac{\sqrt{3}}{6}(g_5+2g_6)g_Ih\alpha_\rho F$
$\vert\Lambda_c{}^2P_\rho\frac{3}{2}^-\rangle$	$-i\frac{\sqrt{3}}{3}g_5g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\rho}{\alpha_\rho}F-i\frac{\sqrt{6}}{6}(g_5-g_6)g_Ih\alpha_\rho F$
$\vert\Lambda_c{}^4P_\rho\frac{1}{2}^-\rangle$	$-i\frac{\sqrt{6}}{6}g_7g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\rho}{\alpha_\rho}F-i\frac{\sqrt{3}}{6}(\sqrt{3}g_3-g_4-g_7)g_Ih\alpha_\rho F$
$\vert\Lambda_c{}^4P_\rho\frac{3}{2}^-\rangle$	$-i\frac{\sqrt{30}}{30}g_7g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\rho}{\alpha_\rho}F-i\frac{\sqrt{15}}{30}(g_7+2\sqrt{3}g_3+4g_4)g_Ih\alpha_\rho F$
$\vert\Lambda_c{}^4P_\rho\frac{5}{2}^-\rangle$	$-i\frac{\sqrt{30}}{10}g_7g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\rho}{\alpha_\rho}F-i\frac{\sqrt{15}}{10}(g_7+\frac{1}{\sqrt{3}}g_3-g_4)g_Ih\alpha_\rho F$
$\vert\Lambda_c{}^2S_A\frac{1}{2}^+\rangle$	$\frac{\sqrt{3}}{6}g_5g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F+\frac{\sqrt{2}}{12}g_5g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4S_A\frac{3}{2}^+\rangle$	$\frac{\sqrt{3}}{6}g_7g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F+\frac{\sqrt{2}}{12}g_7g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^2P_A\frac{1}{2}^-\rangle$	$\frac{1}{6}g_6g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}-\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^2P_A\frac{3}{2}^-\rangle$	$\frac{\sqrt{2}}{12}g_6g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}-\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4P_A\frac{1}{2}^-\rangle$	$\frac{1}{12}(\sqrt{3}g_3-g_4)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}-\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4P_A\frac{3}{2}^-\rangle$	$\frac{\sqrt{5}}{30}(\sqrt{3}g_3-2g_4)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}-\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4P_A\frac{5}{2}^-\rangle$	Forbidden
$\vert\Lambda_c{}^2D_A\frac{3}{2}^+\rangle$	$\frac{\sqrt{15}}{15}g_5g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F+\frac{\sqrt{10}}{60}(2g_5+3g_6)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^2D_A\frac{5}{2}^+\rangle$	$-\frac{\sqrt{10}}{10}g_5g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F-\frac{\sqrt{15}}{30}(g_5-g_6)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4D_A\frac{1}{2}^+\rangle$	$-\frac{\sqrt{30}}{30}g_7g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F-\frac{\sqrt{5}}{60}(3g_4-\sqrt{3}g_3+2g_7)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4D_A\frac{3}{2}^+\rangle$	$\frac{\sqrt{30}}{30}g_7g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F-\frac{\sqrt{5}}{30}(\sqrt{3}g_3-g_7)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4D_A\frac{5}{2}^+\rangle$	$\frac{\sqrt{70}}{70}g_7g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F+\frac{\sqrt{105}}{420}(3\sqrt{3}g_3+5g_4+2g_7)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4D_A\frac{7}{2}^+\rangle$	$-\frac{\sqrt{105}}{35}g_7g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F-\frac{\sqrt{210}}{210}(g_3-\sqrt{3}g_4+\sqrt{3}g_7)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^2D_{\rho\rho}\frac{3}{2}^+\rangle$	$\frac{\sqrt{30}}{30}g_1g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F+\frac{\sqrt{15}}{6}g_1g_Ihq_\rho F$
$\vert\Lambda_c{}^2D_{\rho\rho}\frac{5}{2}^+\rangle$	$-\frac{\sqrt{5}}{10}g_1g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F$
$\vert\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$	$\frac{\sqrt{30}}{30}g_1g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F+\frac{\sqrt{5}}{6}g_1g_Ihq_\lambda F$
$\vert\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$	$-\frac{\sqrt{5}}{10}g_1g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F$
$\vert\Lambda_c{}^2S_{\rho\rho}\frac{1}{2}^+\rangle$	$\frac{\sqrt{6}}{12}g_1g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F+\frac{\sqrt{3}}{6}g_1g_Ihq_\rho F$
$\vert\Lambda_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$	$\frac{\sqrt{6}}{12}g_1g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F+\frac{1}{6}g_1g_Ihq_\lambda F$

TABLE VI. The decay amplitudes for all the states $\vert\Lambda_c{}^{2S+1}L_\sigma J^P\rangle$ up to the $N=2$ shell in the $\Sigma_c(2520)\pi$ channel (a factor 2 is omitted). F, as the decay form factor, is defined in Eq. (43).

Initial state	J_z	Amplitude
$\vert\Lambda_c{}^2S_{A}\frac{1}{2}^+\rangle$	$\frac{1}{2}$	$\frac{\sqrt{3}}{6}g_2^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F+\frac{\sqrt{2}}{12}g_2^*g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4S_{A}\frac{3}{2}^+\rangle$	$\frac{1}{2}$	$\frac{\sqrt{3}}{6}g_7^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F+\frac{\sqrt{2}}{12}g_7^*g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
	$\frac{3}{2}$	$\frac{\sqrt{3}}{6}g_4^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F+\frac{\sqrt{2}}{12}g_4^*g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^2P_{A}\frac{1}{2}^-\rangle$	$\frac{1}{2}$	$\frac{1}{6}g_3^*g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}-\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^2P_{A}\frac{3}{2}^-\rangle$	$\frac{1}{2}$	$\frac{\sqrt{2}}{12}g_3^*g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}-\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
	$\frac{3}{2}$	$\frac{\sqrt{6}}{12}g_1^*g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}-\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4P_{A}\frac{1}{2}^-\rangle$	$\frac{1}{2}$	$\frac{1}{12}(\sqrt{3}g_6^*-g_8^*)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}-\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4P_{A}\frac{3}{2}^-\rangle$	$\frac{1}{2}$	$\frac{\sqrt{5}}{30}(\sqrt{3}g_6^*-2g_8^*)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}-\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
	$\frac{3}{2}$	$-\frac{\sqrt{15}}{30}g_5^*g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}-\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4P_{A}\frac{5}{2}^-\rangle$	$\frac{1}{2}$	$\frac{\sqrt{15}}{60}(\sqrt{3}g_8^*+g_6^*)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}-\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
	$\frac{3}{2}$	$\frac{\sqrt{10}}{20}g_5^*g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}-\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^2D_{A}\frac{3}{2}^+\rangle$	$\frac{1}{2}$	$\frac{\sqrt{15}}{15}g_2^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F+\frac{\sqrt{10}}{60}(2g_2^*+3g_1^*)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
	$\frac{3}{2}$	$\frac{\sqrt{30}}{60}g_1^*g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^2D_{A}\frac{5}{2}^+\rangle$	$\frac{1}{2}$	$-\frac{\sqrt{10}}{10}g_2^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F-\frac{\sqrt{15}}{30}(g_2^*-g_1^*)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
	$\frac{3}{2}$	$\frac{\sqrt{30}}{30}g_1^*g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4D_{A}\frac{1}{2}^+\rangle$	$\frac{1}{2}$	$-\frac{\sqrt{30}}{30}g_7^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F-\frac{\sqrt{5}}{60}(3g_8^*-\sqrt{3}g_6^*+2g_7^*)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c^4D_{A}\frac{3}{2}^+\rangle$	$\frac{1}{2}$	$-\frac{\sqrt{30}}{30}g_7^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F-\frac{\sqrt{5}}{30}(\sqrt{3}g_6^*-g_7^*)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
	$\frac{3}{2}$	$-\frac{\sqrt{30}}{30}g_4^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F-\frac{\sqrt{5}}{30}(\sqrt{3}g_5^*+g_4^*)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4D_{A}\frac{5}{2}^+\rangle$	$\frac{1}{2}$	$\frac{\sqrt{70}}{70}g_7^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F+\frac{\sqrt{105}}{420}(3\sqrt{3}g_6^*+5g_8^*+2g_7^*)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
	$\frac{3}{2}$	$\frac{\sqrt{105}}{35}g_4^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F+\frac{\sqrt{210}}{420}(2\sqrt{3}g_4^*+g_5^*)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^4D_{A}\frac{7}{2}^+\rangle$	$\frac{1}{2}$	$-\frac{\sqrt{105}}{35}g_7^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F-\frac{\sqrt{210}}{210}(g_6^*-\sqrt{3}g_8^*+\sqrt{3}g_7^*)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
	$\frac{3}{2}$	$-\frac{\sqrt{21}}{21}g_4^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}\frac{q_\rho}{\alpha_\rho}F-\frac{\sqrt{14}}{42}(g_4^*-\sqrt{3}g_5^*)g_Ih(\alpha_\lambda\frac{q_\rho}{\alpha_\rho}+\sqrt{3}\alpha_\rho\frac{q_\lambda}{\alpha_\lambda})F$
$\vert\Lambda_c{}^2D_{\rho\rho}\frac{3}{2}^+\rangle$	$\frac{1}{2}$	$\frac{\sqrt{30}}{30}g_9^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F+\frac{\sqrt{15}}{30}(2g_9^*+3g_{11}^*)g_Ihq_\rho F$
	$\frac{3}{2}$	$-\frac{\sqrt{5}}{10}g_{10}^*g_Ihq_\rho F$
$\vert\Lambda_c{}^2D_{\rho\rho}\frac{5}{2}^+\rangle$	$\frac{1}{2}$	$-\frac{\sqrt{5}}{10}g_9^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F-\frac{\sqrt{10}}{10}(g_9^*-g_{11}^*)g_Ihq_\rho F$
	$\frac{3}{2}$	$\frac{\sqrt{5}}{5}g_{10}^*g_Ihq_\rho F$
$\vert\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$	$\frac{1}{2}$	$\frac{\sqrt{30}}{30}g_9^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F+\frac{\sqrt{5}}{30}(2g_9^*+3g_{11}^*)g_Ihq_\lambda F$
	$\frac{3}{2}$	$-\frac{\sqrt{15}}{30}g_{10}^*g_Ihq_\lambda F$
$\vert\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$	$\frac{1}{2}$	$-\frac{\sqrt{5}}{10}g_9^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F-\frac{\sqrt{30}}{30}(g_9^*-g_{11}^*)g_Ihq_\lambda F$
	$\frac{3}{2}$	$\frac{\sqrt{15}}{15}g_{10}^*g_Ihq_\lambda F$
$\vert\Lambda_c{}^2S_{\rho\rho}\frac{1}{2}^+\rangle$	$\frac{1}{2}$	$\frac{\sqrt{6}}{12}g_9^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F+\frac{\sqrt{3}}{6}g_9^*g_Ihq_\rho F$
$\vert\Lambda_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$	$\frac{1}{2}$	$\frac{\sqrt{6}}{12}g_9^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F+\frac{1}{6}g_9^*g_Ihq_\lambda F$

$$\Phi_\rho=\frac{1}{\sqrt{2}}(ud-du)u, \tag{46}$$

$$\Phi_\lambda=-\frac{1}{\sqrt{6}}(udu+duu-2uud). \tag{47}$$

We can also simplify the amplitude (44) to

$$\begin{aligned}\mathcal{M}[\mathcal{B}_c\to D(\mathbf{q})p]=&\Big[Gq-\sqrt{\tfrac{2}{3}}q'_\lambda h\Big]\langle p\vert\sigma_{3z}\phi' I_3\vert\mathcal{B}_c\rangle\\&+i\sqrt{\tfrac{2}{3}}h\langle p\vert(\boldsymbol{\sigma}_3\cdot\vec{\nabla}_\lambda\\&-\alpha_\lambda^2\boldsymbol{\sigma}_3\cdot\vec{\lambda})\phi' I_3\vert\mathcal{B}_c\rangle,\end{aligned} \tag{48}$$

with

$$G\equiv-\frac{\omega_D}{E_f+M_f}-1,\qquad h\equiv\omega_D\frac{m+m'}{2m'm}, \tag{49}$$

TABLE VII. The decay amplitudes for all the states $\vert\Sigma_c{}^{2S+1}L_\sigma J^P\rangle$ up to the $N = 2$ shell in the $\Lambda_c\pi$ channel (a factor 2 is omitted). F, as the decay form factor, is defined in Eq. (43).

Initial state	Amplitude
$\vert\Sigma_c{}^2S\frac{1}{2}^+\rangle$	$g_1^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}F$
$\vert\Sigma_c{}^4S\frac{3}{2}^+\rangle$	$g_3^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}F$
$\vert\Sigma_c{}^2P_\lambda\frac{1}{2}^-\rangle$	$i\frac{\sqrt{6}}{6}g_1^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F + i\frac{1}{6}(g_1^\Sigma + 2g_2^\Sigma)g_I h\alpha_\lambda F$
$\vert\Sigma_c{}^2P_\lambda\frac{3}{2}^-\rangle$	$-i\frac{\sqrt{3}}{3}g_1^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F - i\frac{\sqrt{2}}{6}(g_1^\Sigma - g_2^\Sigma)g_I h\alpha_\lambda F$
$\vert\Sigma_c{}^4P_\lambda\frac{1}{2}^-\rangle$	$i\frac{\sqrt{6}}{6}g_3^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F + i\frac{1}{6}(g_3^\Sigma + g_4^\Sigma - \sqrt{3}g_5^\Sigma)g_I h\alpha_\lambda F$
$\vert\Sigma_c{}^4P_\lambda\frac{3}{2}^-\rangle$	$-i\frac{\sqrt{30}}{30}g_3^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F - i\frac{\sqrt{5}}{30}(g_3^\Sigma + 2\sqrt{3}g_5^\Sigma + 4g_4^\Sigma)g_I h\alpha_\lambda F$
$\vert\Sigma_c{}^4P_\lambda\frac{5}{2}^-\rangle$	$-i\frac{\sqrt{30}}{10}g_3^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F - i\frac{\sqrt{15}}{30}(\sqrt{3}g_3^\Sigma - \sqrt{3}g_4^\Sigma + g_5^\Sigma)g_I h\alpha_\lambda F$
$\vert\Sigma_c{}^2P_\rho\frac{1}{2}^-\rangle$	Forbidden
$\vert\Sigma_c{}^2P_\rho\frac{3}{2}^-\rangle$	Forbidden
$\vert\Sigma_c{}^2S_A\frac{1}{2}^+\rangle$	Forbidden
$\vert\Sigma_c{}^2P_A\frac{1}{2}^-\rangle$	Forbidden
$\vert\Sigma_c{}^2P_A\frac{3}{2}^-\rangle$	Forbidden
$\vert\Sigma_c{}^2D_A\frac{3}{2}^+\rangle$	Forbidden
$\vert\Sigma_c{}^2D_A\frac{5}{2}^+\rangle$	Forbidden
$\vert\Sigma_c{}^2D_{\rho\rho}\frac{3}{2}^+\rangle$	$\frac{\sqrt{30}}{30}g_1^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F + \frac{\sqrt{15}}{30}(2g_1^\Sigma + 3g_2^\Sigma)g_I hq_\rho F$
$\vert\Sigma_c{}^2D_{\rho\rho}\frac{5}{2}^+\rangle$	$-\frac{\sqrt{5}}{10}g_1^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F - \frac{\sqrt{10}}{10}(g_1^\Sigma - g_2^\Sigma)g_I hq_\rho F$
$\vert\Sigma_c{}^4D_{\rho\rho}\frac{1}{2}^+\rangle$	$-\frac{\sqrt{15}}{30}g_3^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F - \frac{\sqrt{30}}{60}(2g_3^\Sigma + 3g_4^\Sigma - \sqrt{3}g_5^\Sigma)g_I hq_\rho F$
$\vert\Sigma_c{}^4D_{\rho\rho}\frac{3}{2}^+\rangle$	$\frac{\sqrt{15}}{30}g_3^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F + \frac{\sqrt{30}}{30}(g_3^\Sigma - \sqrt{3}g_5^\Sigma)g_I hq_\rho F$
$\vert\Sigma_c{}^4D_{\rho\rho}\frac{5}{2}^+\rangle$	$\frac{\sqrt{35}}{70}g_3^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F + \frac{\sqrt{70}}{140}(2g_3^\Sigma + 5g_4^\Sigma + 3\sqrt{3}g_5^\Sigma)g_I hq_\rho F$
$\vert\Sigma_c{}^4D_{\rho\rho}\frac{7}{2}^+\rangle$	$-\frac{\sqrt{210}}{70}g_3^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F - \frac{\sqrt{35}}{35}(\sqrt{3}g_3^\Sigma - \sqrt{3}g_4^\Sigma + g_5^\Sigma)g_I hq_\rho F$
$\vert\Sigma_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$	$\frac{\sqrt{30}}{30}g_1^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F + \frac{\sqrt{5}}{30}(2g_1^\Sigma + 3g_2^\Sigma)g_I hq_\lambda F$
$\vert\Sigma_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$	$-\frac{\sqrt{5}}{10}g_1^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F - \frac{\sqrt{30}}{30}(g_1^\Sigma - g_2^\Sigma)g_I hq_\lambda F$
$\vert\Sigma_c{}^4D_{\lambda\lambda}\frac{1}{2}^+\rangle$	$-\frac{\sqrt{15}}{30}g_3^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F - \frac{\sqrt{10}}{60}(2g_3^\Sigma + 3g_4^\Sigma - \sqrt{3}g_5^\Sigma)g_I hq_\lambda F$
$\vert\Sigma_c{}^4D_{\lambda\lambda}\frac{3}{2}^+\rangle$	$\frac{\sqrt{15}}{30}g_3^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F + \frac{\sqrt{10}}{30}(g_3^\Sigma - \sqrt{3}g_5^\Sigma)g_I hq_\lambda F$
$\vert\Sigma_c{}^4D_{\lambda\lambda}\frac{5}{2}^+\rangle$	$\frac{\sqrt{35}}{70}g_3^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F + \frac{\sqrt{210}}{420}(2g_3^\Sigma + 5g_4^\Sigma + 3\sqrt{3}g_5^\Sigma)g_I hq_\lambda F$
$\vert\Sigma_c{}^4D_{\lambda\lambda}\frac{7}{2}^+\rangle$	$-\frac{\sqrt{210}}{70}g_3^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F - \frac{\sqrt{105}}{105}(\sqrt{3}g_3^\Sigma - \sqrt{3}g_4^\Sigma + g_5^\Sigma)g_I hq_\lambda F$
$\vert\Sigma_c{}^2S_{\rho\rho}\frac{1}{2}^+\rangle$	$\frac{\sqrt{6}}{12}g_1^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F + \frac{\sqrt{3}}{6}g_1^\Sigma g_I hq_\rho F$
$\vert\Sigma_c{}^4S_{\rho\rho}\frac{3}{2}^+\rangle$	$\frac{\sqrt{6}}{12}g_3^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F + \frac{\sqrt{3}}{6}g_3^\Sigma g_I hq_\rho F$
$\vert\Sigma_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$	$\frac{\sqrt{6}}{12}g_1^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F + \frac{1}{6}g_1^\Sigma g_I hq_\lambda F$
$\vert\Sigma_c{}^4S_{\lambda\lambda}\frac{3}{2}^+\rangle$	$\frac{\sqrt{6}}{12}g_3^\Sigma g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F + \frac{1}{6}g_3^\Sigma g_I hq_\lambda F$

TABLE VIII. The decay amplitudes for the first orbital and radial excitations $\vert\Sigma_c{}^{2S+1}L_\sigma J^P\rangle$ in the $\Sigma_c\pi$ channel (a factor 2 is omitted). F, as the decay form factor, is defined in Eq. (43).

Initial state	Amplitude
$\vert\Sigma_c{}^2P_\lambda\frac{1}{2}^-\rangle$	$i\frac{\sqrt{6}}{6}g_5 g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F + i\frac{1}{6}(g_5 + 2g_6)g_I h\alpha_\lambda F$
$\vert\Sigma_c{}^2P_\lambda\frac{3}{2}^-\rangle$	$-i\frac{\sqrt{3}}{3}g_5 g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F - i\frac{\sqrt{2}}{6}(g_5 - g_6)g_I h\alpha_\lambda F$
$\vert\Sigma_c{}^4P_\lambda\frac{1}{2}^-\rangle$	$i\frac{\sqrt{6}}{6}g_7 g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F + i\frac{1}{6}(g_7 + g_4 - \sqrt{3}g_3)g_I h\alpha_\lambda F$
$\vert\Sigma_c{}^4P_\lambda\frac{3}{2}^-\rangle$	$-i\frac{\sqrt{30}}{30}g_7 g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F - i\frac{\sqrt{5}}{30}(g_7 + 2\sqrt{3}g_3 + 4g_4)g_I h\alpha_\lambda F$
$\vert\Sigma_c{}^4P_\lambda\frac{5}{2}^-\rangle$	$-i\frac{\sqrt{30}}{10}g_7 g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F - i\frac{\sqrt{15}}{30}(\sqrt{3}g_7 - \sqrt{3}g_4 + g_3)g_I h\alpha_\lambda F$
$\vert\Sigma_c{}^2S_{\rho\rho}\frac{1}{2}^+\rangle$	$\frac{\sqrt{6}}{12}g_5 g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F + \frac{\sqrt{3}}{6}g_5 g_I hq_\rho F$
$\vert\Sigma_c{}^4S_{\rho\rho}\frac{3}{2}^+\rangle$	$\frac{\sqrt{6}}{12}g_7 g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F + \frac{\sqrt{3}}{6}g_7 g_I hq_\rho F$
$\vert\Sigma_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$	$\frac{\sqrt{6}}{12}g_5 g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F + \frac{1}{6}g_5 g_I hq_\lambda F$
$\vert\Sigma_c{}^4S_{\lambda\lambda}\frac{3}{2}^+\rangle$	$\frac{\sqrt{6}}{12}g_7 g_I\{Gq - (\frac{\sqrt{6}}{12}q_\lambda + \frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F + \frac{1}{6}g_7 g_I hq_\lambda F$

TABLE IX. The decay amplitudes for the first orbital and radial excitations $|\Sigma_c{}^{2S+1}L_\sigma J^P\rangle$ in $\Sigma_c(2520)\pi$ (a factor 2 is omitted). F, as the decay form factor, is defined in Eq. (43).

Initial state	J_z	Amplitude
$\vert\Sigma_c{}^2P_{\lambda}\frac{1}{2}^-\rangle$	$\frac{1}{2}$	$i\frac{\sqrt{6}}{6}g_2^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F+i\frac{1}{6}(g_2^*+2g_3^*)g_Ih\alpha_\lambda F$
$\vert\Sigma_c{}^2P_{\lambda}\frac{3}{2}^-\rangle$	$\frac{1}{2}$	$-i\frac{\sqrt{3}}{3}g_2^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F-i\frac{\sqrt{2}}{6}(g_2^*-g_3^*)g_Ih\alpha_\lambda F$
	$\frac{3}{2}$	$i\frac{1}{\sqrt{6}}g_1^*g_Ih\alpha_\lambda F$
$\vert\Sigma_c{}^4P_{\lambda}\frac{1}{2}^-\rangle$	$\frac{1}{2}$	$i\frac{\sqrt{6}}{6}g_7^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F+i\frac{1}{6}(g_7^*+g_8^*-\sqrt{3}g_6^*)g_Ih\alpha_\lambda F$
$\vert\Sigma_c{}^4P_{\lambda}\frac{3}{2}^-\rangle$	$\frac{1}{2}$	$-i\frac{\sqrt{30}}{30}g_7^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F-i\frac{\sqrt{5}}{30}(g_7^*+2\sqrt{3}g_6^*+4g_8^*)g_Ih\alpha_\lambda F$
	$\frac{3}{2}$	$-i\frac{\sqrt{30}}{10}g_4^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F-i\frac{\sqrt{15}}{30}(\sqrt{3}g_4^*+2g_5^*)g_Ih\alpha_\lambda F$
$\vert\Sigma_c{}^4P_{\lambda}\frac{5}{2}^-\rangle$	$\frac{1}{2}$	$-i\frac{\sqrt{30}}{10}g_7^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F-i\frac{\sqrt{15}}{30}(\sqrt{3}g_7^*+\sqrt{3}g_8^*+g_6^*)g_Ih\alpha_\lambda F$
	$\frac{3}{2}$	$-i\frac{\sqrt{5}}{10}g_4^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}\frac{q_\lambda}{\alpha_\lambda}F-i\frac{\sqrt{30}}{30}(g_4^*-\sqrt{3}g_5^*)g_Ih\alpha_\lambda F$
$\vert\Sigma_c{}^2S_{\rho\rho}\frac{1}{2}^+\rangle$	$\frac{1}{2}$	$\frac{\sqrt{6}}{12}g_2^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F+\frac{\sqrt{3}}{6}g_2^*g_Ihq_\rho F$
$\vert\Sigma_c{}^4S_{\rho\rho}\frac{3}{2}^+\rangle$	$\frac{1}{2}$	$\frac{\sqrt{6}}{12}g_7^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F+\frac{\sqrt{3}}{6}g_7^*g_Ihq_\rho F$
	$\frac{3}{2}$	$\frac{\sqrt{6}}{12}g_4^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\rho}{\alpha_\rho})^2F+\frac{\sqrt{3}}{6}g_4^*g_Ihq_\rho F$
$\vert\Sigma_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$	$\frac{1}{2}$	$\frac{\sqrt{6}}{12}g_2^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F+\frac{1}{6}g_2^*g_Ihq_\lambda F$
$\vert\Sigma_c{}^4S_{\lambda\lambda}\frac{3}{2}^+\rangle$	$\frac{1}{2}$	$\frac{\sqrt{6}}{12}g_7^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F+\frac{1}{6}g_7^*g_Ihq_\lambda F$
	$\frac{3}{2}$	$\frac{\sqrt{6}}{12}g_4^*g_I\{Gq-(\frac{\sqrt{6}}{12}q_\lambda+\frac{\sqrt{2}}{4}q_\rho)h\}(\frac{q_\lambda}{\alpha_\lambda})^2F+\frac{1}{6}g_4^*g_Ihq_\lambda F$

and

$$q'_\lambda=\frac{2}{\sqrt{6}}\frac{3m}{2m+m'}q, \qquad \phi'=\exp(iq'_\lambda\lambda_z). \tag{50}$$

In Eq. (48), the first term comes from the c.m. motion of the system and the last term attributes to the λ-mode orbital excitations of the charmed baryons, respectively.

There exist selection rules for the D^0p decay channel of Λ_c excitations, in which only $|\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$, $|\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$, and $|\Lambda_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$ can decay into D^0p. Their decay amplitudes are listed in Table X. States of $|\Lambda_c{}^2P_{\lambda}\frac{1}{2}^-\rangle$ and $|\Lambda_c{}^2P_{\lambda}\frac{3}{2}^-\rangle$ are likely below the D^0p threshold, while others are forbidden by the spin-isospin selection rule.

V. CALCULATION AND ANALYSIS

With the resonance decay amplitudes, one can calculate the width

$$\Gamma=\left(\frac{\delta}{f_m}\right)^2\frac{(E_f+M_f)|\mathbf{q}|}{4\pi M_i}\frac{1}{2J_i+1}\sum_{J_{iz},J_{fz}}|\mathcal{M}_{J_{iz},J_{fz}}|^2, \tag{51}$$

where J_i and J_f are the total angular momenta of the initial and final baryons, respectively. A dimensionless constant, δ, is introduced to take into account uncertainties arising from the model and to be determined by experimental data. In the calculation, the standard parameters of the quark model are adopted. For the oscillator parameters, we use $\alpha_\rho^2=0.16\ \mathrm{GeV}^2$. The u, d constituent quark masses are $m=350$ MeV, and the charm quark mass is $m'=1700$ MeV. The decay constants for π and D mesons are $f_\pi=132$ MeV and $f_D=226$ MeV, which are taken from the Particle Data Group (PDG) [39]. All the charmed baryon masses are also adopted from PDG [39].

A. Σ_c and $\Sigma_c(2520)$

Σ_c and $\Sigma_c(2520)$ are the two lowest states in the Σ_c-type charmed baryons. They are assigned to the two S-wave states, $|\Sigma_c{}^2S\frac{1}{2}^+\rangle$ and $|\Sigma_c{}^4S\frac{3}{2}^+\rangle$, respectively [11,39]. We use the measured width for $\Sigma_c^{++}(2520)\to\Lambda_c^+\pi^+$ as an input (i.e. $\Gamma=14.9$ MeV) to determine parameter δ in Eq. (51), which gives

$$\delta=0.557. \tag{52}$$

Applying this value for δ, we can predict the other strong decay widths. In particular, the decay widths of

TABLE X. The amplitudes of Λ_c-type charmed baryons decay into D^0p. $F(q'_\lambda)=\exp(q'^2_\lambda/4\alpha_\lambda^2)$ is the form factor.

Initial state	Amplitude
$\vert\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$	$\frac{\sqrt{15}}{30}(Gq-\frac{\sqrt{6}}{6}hq'_\lambda)(\frac{q'_\lambda}{\alpha_\lambda})^2F(q'_\lambda)+\frac{\sqrt{10}}{6}hq'_\lambda F(q'_\lambda)$
$\vert\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$	$-\frac{\sqrt{10}}{20}(Gq-\frac{\sqrt{6}}{6}hq'_\lambda)(\frac{q'_\lambda}{\alpha_\lambda})^2F(q'_\lambda)$
$\vert\Lambda_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$	$\frac{\sqrt{3}}{12}(Gq-\frac{\sqrt{6}}{6}hq'_\lambda)(\frac{q'_\lambda}{\alpha_\lambda})^2F(q'_\lambda)+\frac{\sqrt{2}}{6}hq'_\lambda F(q'_\lambda)$

$\Sigma_c \to \Lambda_c\pi$, $\Sigma_c^+(2520) \to \Lambda_c^+\pi^0$, and $\Sigma_c^0(2520) \to \Lambda_c^+\pi^-$ are calculated. The results are listed in Table XI, from which we find that our predictions are in good agreement with the experimental data [39], and compatible with other theoretical predictions [5,6,9,23,26,27]. We also see that the decay width of $\Sigma_c(2520)$ is larger than that of Σ_c by a factor of ~7, though their decay amplitudes have the same form (see Table VII). The reasons are as follows: (i) the spin factor g_3^Σ for $\Sigma_c(2520)$ is larger than g_1^Σ for Σ_c by a factor $\sqrt{2}$; (ii) the momentum of the pion, $|\mathbf{q}|$ in the $\Sigma_c(2520) \to \Lambda_c\pi$, is about 2 times larger than that in the $\Sigma_c \to \Lambda_c\pi$. This leads to larger values for quantities G and h. This feature was also mentioned in Ref. [5].

B. $\Lambda_c(2593)$ and $\Lambda_c(2625)$

$\Lambda_c(2593)$ and $\Lambda_c(2625)$ have $J^P = 1/2^-$ and $J^P = 3/2^-$, respectively, and can be naturally assigned to the $N = 1$ shell with one unit of orbital angular-momentum excitation. They can be excited via either the P_λ mode or the P_ρ mode. For the former assignment, their spatial wave functions are $|\Lambda_c{}^2P_{\lambda}\frac{1}{2}^-\rangle$ and $|\Lambda_c{}^2P_{\lambda}\frac{3}{2}^-\rangle$, from which the decay widths can be calculated. As shown in Table XI, the results are in good agreement with the experimental data [39] and consistent with the classification of Ref. [11] in the quark model.

Assuming $\Lambda_c(2593)$ and $\Lambda_c(2625)$ are P_ρ-mode excitations, we also calculate their widths. In contrast with the P_ρ mode, they turn out to be much broader than the P_λ mode. For $\Lambda_c(2593)$ it is possible that the physical state is a mixture of the P_λ and P_ρ modes within the uncertainties of the present data, though the determination of the mixing angle will also rely on the mass of the second state.

For $\Lambda_c(2625)$ the P_ρ-mode excitation turns out to overestimate the data significantly. The experimental upper limit is about 2 orders of magnitude smaller than the predictions from the P_ρ-mode excitation, while the P_λ-mode results are consistent with the data. This could be a sign that the mixing between the P_λ- and P_ρ modes in $\Lambda_c(2625)$ should be small. Concerning the possible mixings between the P_λ- and P_ρ-mode excitations, the search for the second heavier $1/2^-$ and $3/2^-$ states in experiment should be interesting.

Comparing $\Lambda_c(2593)$ with $\Lambda_c(2625)$ shows that the decay width of $\Lambda_c(2593)$ is much narrower than that of $\Lambda_c(2625)$, which can be well understood in our model. In the decay amplitude of $\Lambda_c(2625) \to \Sigma_c\pi$ (see Table V), only c.m. motion contributions are present, which leads to the small decay width. We should also emphasize that the partial decay width of $\Lambda_c(2593) \to \Sigma_c\pi$ is sensitive to the mass of the π meson due to $\Lambda_c(2593)$ being close to the $\Sigma_c\pi$ threshold. Thus, the decay width of the $\Sigma_c\pi^0$ channel is about 2 times larger than those of $\Sigma_c\pi^\pm$. Interestingly, experimental data for $\Lambda_c(2593)$ and $\Lambda_c(2625) \to \Sigma_c^+\pi^0$ are still not available.

Since the well-determined S- and P-wave charmed baryon strong decay widths are successfully described in our chiral quark model, we extend this approach in the next subsections to investigate the strong decays of other newly observed charmed baryons, such as $\Lambda_c(2880)$ and $\Lambda_c(2940)$.

TABLE XI. The decay widths for the low-lying charmed baryons. $\Lambda_c(2593)$ and $\Lambda_c(2625)$, assigned as both P_λ- and P_ρ-mode excitations, are listed. The partial decay widths for Σ_c and $\Sigma_c(2520) \to \Lambda_c\pi$ are also listed. They serve as experimental input for the determination of the parameter δ in this approach.

	Notation	Channel	Γ_{exp} (MeV)	Γ_{th} (MeV)
$\Lambda_c(2593)$	$\lvert\Lambda_c{}^2P_{\lambda}\frac{1}{2}^-\rangle$	$\Sigma_c^{++}\pi^-$	$0.65^{+0.41}_{-0.31}$	0.37
		$\Sigma_c^+\pi^0$		0.73
		$\Sigma_c^0\pi^+$	$0.67^{+0.41}_{-0.31}$	0.40
$\Lambda_c(2625)$	$\lvert\Lambda_c{}^2P_{\lambda}\frac{3}{2}^-\rangle$	$\Sigma_c^{++}\pi^-$	<0.10	1.47×10^{-2}
		$\Sigma_c^+\pi^0$		2.08×10^{-2}
		$\Sigma_c^0\pi^+$	<0.09	1.50×10^{-2}
$\Lambda_c(2593)$	$\lvert\Lambda_c{}^2P_{\rho}\frac{1}{2}^-\rangle$	$\Sigma_c^{++}\pi^-$	$0.65^{+0.41}_{-0.31}$	1.02
		$\Sigma_c^+\pi^0$		2.08
		$\Sigma_c^0\pi^+$	$0.67^{+0.41}_{-0.31}$	1.09
$\Lambda_c(2625)$	$\lvert\Lambda_c{}^2P_{\rho}\frac{3}{2}^-\rangle$	$\Sigma_c^{++}\pi^-$	<0.10	10.00
		$\Sigma_c^+\pi^0$		10.50
		$\Sigma_c^0\pi^+$	<0.09	10.05
$\Sigma_c(2455)$	$\lvert\Sigma_c{}^2S\frac{1}{2}^+\rangle$	$\Lambda_c\pi^+$	2.23 ± 0.30	1.89
		$\Lambda_c\pi^0$	<4.6	2.18
		$\Lambda_c\pi^-$	2.2 ± 0.4	1.86
$\Sigma_c(2520)$	$\lvert\Sigma_c{}^4S\frac{3}{2}^+\rangle$	$\Lambda_c\pi^+$	14.9 ± 1.9	input
		$\Lambda_c\pi^0$	<17	15.53
		$\Lambda_c\pi^-$	16.1 ± 2.1	14.92

C. $\Lambda_c(2880)$

$\Lambda_c(2880)$ was observed in $\Lambda_c^+\pi^+\pi^-$ by CLEO [4], in the D^0p channel by *BABAR* [1], and in $\Sigma_c\pi$, $\Sigma_c(2520)\pi$ by Belle [2]. It has a narrow decay width of less than 8 MeV [2,39], based on which it was proposed to be a $\tilde{\Lambda}_{c0}^+(\frac{1}{2}^-)$ state in Ref. [4]. In the heavy hadron chiral perturbation theory, Cheng *et al.* made a conjecture that $\Lambda_c(2880)$ is an admixture of $\Lambda_{c2}(\frac{5}{2}^+)$ with $\tilde{\Lambda}''_{c3}(\frac{5}{2}^+)$ [5], which are both $L = 2$ orbitally excited states. Chen *et al.* suggested that $\Lambda_c(2880)$ favors $\tilde{\Lambda}_{c3}^2(\frac{5}{2}^+)$ within the ${}^3P_0$ model [6]. According to the quark model predictions, the mass for $J^P = 3/2^+$ is around 2.9 GeV, which indicates $\Lambda_c(2880)$ may favor $J^P = 3/2^+$ as well [21,41]. The other suggestions about the quantum number of $\Lambda_c(2880)$ also can be found in Ref. [24].

Meanwhile, the Belle measurement [2] shows contributions from intermediate Σ_c^* states in $\Lambda_c^+(2880) \to \Sigma_c^*\pi \to \Lambda_c^+\pi^+\pi^-$, and the ratio of the partial decay widths for the intermediate $\Sigma_c(2520)$ and Σ_c is extracted:

$$\mathcal{R} = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c\pi)} = 0.225 \pm 0.062 \pm 0.025. \quad (53)$$

With the analysis of the angular distributions in $\Lambda_c(2880) \to \Sigma_c^{0,++}\pi^{+,-}$ decays, the $\Lambda_c(2880)$ spin-parity assignment is favored to be $J^P = 5/2^+$ over the others.

In the quark model the masses of the $N = 1$ shell Λ_c excitations are at the order of 2.5–2.6 GeV, which is much less than 2.88 GeV. Hence, we only consider the possible assignment of $\Lambda_c(2880)$ in the $N = 2$ shell. As shown by Table XII, only $|\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$ can produce results that fit in the three experimental observations: (i) with a narrow decay width; (ii) decaying into D^0p; and (iii) with the ratio $\mathcal{R} = \Gamma(\Sigma_c(2520)\pi)/\Gamma(\Sigma_c\pi) \simeq 0.25$. This is an orbital excitation with $l_\lambda = 2$ and $l_\rho = 0$. Note that the Capstick-Isgur quark model [41] and the relativistic quark model [21] predict the lowest $J^P = 3/2^+$ Λ_c excitation at 2910 MeV and 2874 MeV, respectively, which are consistent with the experimental value within the model accuracies. In this sense the assignment of $\Lambda_c(2880)$ as $|\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$ turns out to possible.

Interestingly, the experimental analysis of the decay angular distribution [2] indicates a preference of $J^P = 5/2^+$ over $3/2^+$ at a level of more than 4.5 standard deviations. By assigning $5/2^+$ to $\Lambda_c(2880)$, we find that only the state $|\Lambda_c{}^2D_{A}\frac{5}{2}^+\rangle$ is close to the experimental measurements. However, its D^0p decay channel is forbidden and the ratio $\mathcal{R} = 0.5$ turns out to be too large compared with the Belle data. This controversy may suggest that $\Lambda_c(2880)$ is neither a pure $|\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$ nor $|\Lambda_c{}^2D_{A}\frac{5}{2}^+\rangle$. We expect that more accurate measurements of the decay angular distributions will clarify its nature. In contrast, the calculations of Refs. [5,6] seem to agree with the data. The details of our model calculations are listed in Table XII.

D. $\Lambda_c(2940)$

$\Lambda_c(2940)$ was first seen in its decay into D^0p by *BABAR* [1], and then confirmed by Belle in $\Sigma_c^{0,++}\pi^{+,-}$ [2]. Its spin-parity has not yet been determined. In this mass region, it can be $J^P = 5/2^+$, $J^P = 3/2^+$, $J^P = 1/2^+$, or $J^P = 5/2^-$, as suggested by the quark model [41]. The ${}^3P_0$ model [6] suggests that its configuration favors $\check{\Lambda}_{c1}^0(\frac{1}{2}^+)$ or $\check{\Lambda}_{c1}^0(\frac{3}{2}^+)$, while a molecular state with $J^P = 1/2^-$ is also proposed [42].

Our analysis shows that only three states, $|\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$, $|\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$, and $|\Lambda_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$, can decay into D^0p. If we assign $\Lambda_c(2880)$ to be $|\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$, $\Lambda_c(2940)$ could thus be either $|\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$ or $|\Lambda_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$. In the quark model, the mass of $|\Lambda_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$ should be less than that of $|\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$ [i.e. $\Lambda_c(2880)$]. This leaves $\Lambda_c(2940)$ to be assigned as $|\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$.

In Table XIII, the calculation results are listed. The vanishing D^0p channel will eliminate most of those states, especially those with antisymmetric spatial wave functions

TABLE XII. The decay widths of $\Lambda_c(2880)$ for all the possible states in the $N = 2$ shell (in MeV). The ratio $\mathcal{R}$ is defined as $\mathcal{R} = \Gamma(\Sigma_c(2520)\pi^\pm)/\Gamma(\Sigma_c\pi^\pm)$.

Assignment	$\Sigma_c^+\pi^0$	$\Sigma_c^{0,++}\pi^{+,-}$	$\Sigma_c(2520)\pi^\pm$	$\mathcal{R}$	D^0p
$\|\Lambda_c{}^2S_{A}\frac{1}{2}^+\rangle$	0.45	0.47	0.29	0.62	0
$\|\Lambda_c{}^4S_{A}\frac{3}{2}^+\rangle$	0.11	0.12	0.40	3.33	0
$\|\Lambda_c{}^2P_{A}\frac{1}{2}^-\rangle$	0.41	0.40	0.03	0.08	0
$\|\Lambda_c{}^2P_{A}\frac{3}{2}^-\rangle$	0.10	0.10	0.06	0.60	0
$\|\Lambda_c{}^4P_{A}\frac{1}{2}^-\rangle$	0.21	0.20	0.01	0.05	0
$\|\Lambda_c{}^4P_{A}\frac{3}{2}^-\rangle$	0.13	0.13	0.05	0.38	0
$\|\Lambda_c{}^4P_{A}\frac{5}{2}^-\rangle$	0	0	0.12		0
$\|\Lambda_c{}^2D_{A}\frac{3}{2}^+\rangle$	4.46	4.32	0.90	0.21	0
$\|\Lambda_c{}^2D_{A}\frac{5}{2}^+\rangle$	3.85	3.84	1.93	0.50	0
$\|\Lambda_c{}^4D_{A}\frac{1}{2}^+\rangle$	3.35	3.33	1.06	0.32	0
$\|\Lambda_c{}^4D_{A}\frac{3}{2}^+\rangle$	1.86	1.85	3.12	1.69	0
$\|\Lambda_c{}^4D_{A}\frac{5}{2}^+\rangle$	0.11	0.11	4.86	44.18	0
$\|\Lambda_c{}^4D_{A}\frac{7}{2}^+\rangle$	0.40	0.38	0.36	0.95	0
$\|\Lambda_c{}^2D_{\rho\rho}\frac{3}{2}^+\rangle$	4.31	4.27	0.75	0.18	0
$\|\Lambda_c{}^2D_{\rho\rho}\frac{5}{2}^+\rangle$	0.45	0.43	5.18	12.05	0
$\|\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$	1.58	1.58	0.46	0.29	1.77
$\|\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$	0.48	0.46	1.33	2.89	1.44
$\|\Lambda_c{}^2S_{\rho\rho}\frac{1}{2}^+\rangle$	0.46	0.47	0.86	1.83	0
$\|\Lambda_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$	0.02	0.02	0.15	7.50	0.65

TABLE XIII. The decay widths of $\Lambda_c(2940)$ for all the possible states in the $N = 2$ shell (in MeV). The ratio $\mathcal{R}$ is defined as $\mathcal{R} = \Gamma(\Sigma_c(2520)\pi^\pm)/\Gamma(\Sigma_c\pi^\pm)$.

Assignment	$\Sigma_c^+\pi^0$	$\Sigma_c^{0,++}\pi^{+,-}$	$\Sigma_c(2520)\pi^\pm$	$\mathcal{R}$	D^0p
$\|\Lambda_c{}^2S_{A}\frac{1}{2}^+\rangle$	0.16	0.18	0.25	1.39	0
$\|\Lambda_c{}^4S_{A}\frac{3}{2}^+\rangle$	0.04	0.05	0.35	7.00	0
$\|\Lambda_c{}^2P_{A}\frac{1}{2}^-\rangle$	0.65	0.64	0.47	0.73	0
$\|\Lambda_c{}^2P_{A}\frac{3}{2}^-\rangle$	0.16	0.16	0.12	0.75	0
$\|\Lambda_c{}^4P_{A}\frac{1}{2}^-\rangle$	0.32	0.32	0.02	0.06	0
$\|\Lambda_c{}^4P_{A}\frac{3}{2}^-\rangle$	0.20	0.20	0.09	0.45	0
$\|\Lambda_c{}^4P_{A}\frac{5}{2}^-\rangle$	0	0	0.21		0
$\|\Lambda_c{}^2D_{A}\frac{3}{2}^+\rangle$	8.95	8.73	2.04	0.23	0
$\|\Lambda_c{}^2D_{A}\frac{5}{2}^+\rangle$	4.79	4.80	3.13	0.65	0
$\|\Lambda_c{}^4D_{A}\frac{1}{2}^+\rangle$	4.27	4.27	1.61	0.38	0
$\|\Lambda_c{}^4D_{A}\frac{3}{2}^+\rangle$	2.49	2.48	4.64	1.87	0
$\|\Lambda_c{}^4D_{A}\frac{5}{2}^+\rangle$	0.26	0.25	7.98	31.92	0
$\|\Lambda_c{}^4D_{A}\frac{7}{2}^+\rangle$	0.90	0.88	1.02	1.16	0
$\|\Lambda_c{}^2D_{\rho\rho}\frac{3}{2}^+\rangle$	5.97	5.94	1.51	0.25	0
$\|\Lambda_c{}^2D_{\rho\rho}\frac{5}{2}^+\rangle$	1.02	0.99	8.61	8.70	0
$\|\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$	1.98	1.98	1.04	0.53	4.05
$\|\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$	1.09	1.06	2.15	2.03	1.08
$\|\Lambda_c{}^2S_{\rho\rho}\frac{1}{2}^+\rangle$	0.46	0.36	0.95	2.64	0
$\|\Lambda_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$	0.01	0.008	0.06	7.50	1.38

and those of mixed $\rho\rho$-type. The states which have non-vanishing decays into $\Sigma_c\pi$, $\Sigma_c(2520)\pi$, and $D^0 p$ are $|\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}{}^+\rangle$ and $|\Lambda_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$. Based on the argument made in the last paragraph, we see that it is natural to assign $\Lambda_c(2940)$ as $|\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$. Note that the Capstick-Isgur quark model predicts the lowest $J^P = 5/2^+$ state at 2910 MeV [41], which will enhance the above assignment.

It should be noted that there are no Λ_c excitation states around 2940 MeV in the relativistic quark model predictions [21]. In [21], $\Lambda_c(2940)$ was assigned to be the first radial excited state with $J^P = 3/2^+$, of which the predicted mass was slightly below the experimental value. As the decay of the radial excited state into the $D^0 p$ channel is forbidden in the nonrelativistic limit, a more elaborate estimate of the relativistic corrections should be necessary.

E. $\Lambda_c(2765)$

Experimental information about $\Lambda_c(2765)$ is much poorer than for $\Lambda_c(2880)$ and $\Lambda_c(2940)$. Thus, we leave it to be discussed here.

$\Lambda_c(2765)$ was first observed in $\Lambda_c\pi\pi$ by the CLEO Collaboration [4,39] with a rather broad width of about 50 MeV, and appeared to resonate through $\Sigma_c\pi$ and probably also $\Sigma_c(2520)\pi$. In observations by the Belle Collaboration, its broad structure stands out clearly in the $\Lambda_c\pi\pi$ invariant mass spectrum [2]. However, almost nothing about its quantum numbers is known, including whether it is a Λ_c or a Σ_c excitation. Cheng *et al.* suggest that $\Lambda_c(2765)$ could be the first excited state of Λ_c with positive parity, according to the predictions of the Skyrme model [43] and the quark model [41]. It was also proposed that the $\Lambda_c(2765)$ could be either the first radial ($1S$) excitation of the Λ_c ($J^P = 1/2^+$) with a light scalar diquark component, or the first orbital excitation ($1P$) of the Σ_c ($J^P = 3/2^-$) with a light axial vector diquark [21].

Interestingly, our calculation shows that $\Lambda_c(2765)$ is very likely to be a P_ρ-mode excitation of Λ_c. The reason is that the masses of the two $^2P_\lambda$-mode excitations, $\Lambda_c(2593)$ and $\Lambda_c(2625)$, are about 2600 MeV, and according to the quark model the energies of the P_ρ mode are $\sim$140 MeV higher than those of the P_λ mode [11]. An implication from this is that the mass of the P_ρ-mode excitation is around 2740 MeV, which seems to fit into the mass spectrum well. We calculate the widths of all possible configurations, and the results are listed in Table XIV. Comparing with the experiment data, we find that $\Lambda_c(2765)$ as a $|\Lambda_c{}^2P_{\rho}\frac{1}{2}^-\rangle$ or $|\Lambda_c{}^2P_{\rho}\frac{3}{2}^-\rangle$ state is excluded due to the much broader widths. The first radial excitation of the Λ_c with $J^P = 1/2^+$ is also excluded for its extremely narrow width.

TABLE XIV. The decay widths (in MeV) of $\Lambda_c(2765)$ for the possible excitation modes. $\Gamma_{\rm sum} = \Gamma_{\Sigma_c\pi} + \Gamma_{\Sigma_c^*\pi}$, where Σ_c^* stands for $\Sigma_c(2520)$.

Assignment	$\Sigma_c\pi^{+,-,0}$	$\Sigma_c^*\pi^{+,-,0}$	$\Gamma_{\rm sum}$	$\Gamma_{\rm total}^{\rm exp}$
$\vert\Lambda_c{}^4P_{\rho}\frac{1}{2}^-\rangle$	7.0	0.2	21.6	~50–73
$\vert\Lambda_c{}^4P_{\rho}\frac{3}{2}^-\rangle$	0.4	20.6	63	
$\vert\Lambda_c{}^4P_{\rho}\frac{5}{2}^-\rangle$	2.2	0.6	8.4	
$\vert\Lambda_c{}^2P_{\rho}\frac{1}{2}^-\rangle$	41.4	3.0	133.2	
$\vert\Lambda_c{}^2P_{\rho}\frac{3}{2}^-\rangle$	29.8	11.4	123.6	
$\vert\Lambda_c{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$	0.08	0.1	0.5	
$\vert\Lambda_c{}^2S_{\rho\rho}\frac{1}{2}^+\rangle$	0.3	0.3	1.8	

Note that the $\Lambda_c(2765)$ was observed in a similar decay channel as $\Lambda_c(2880)$, i.e. in $\Lambda_c\pi\pi$, and via $\Sigma_c\pi/\Sigma_c(2520)\pi$. We hence assume that the decay modes of $\Lambda_c(2765)$ have a similar behavior as those of $\Lambda_c(2880)$, except that the $D^0 p$ channel is forbidden since $\Lambda_c(2765)$ is below the $D^0 p$ threshold. One thus expects that $[\Gamma(\Sigma_c\pi) + \Gamma(\Sigma_c(2520)\pi)]/\Gamma(\Lambda_c\pi\pi) \sim 0.4$ [39], which is similar to the experimental value of $\Lambda_c(2880)$. We can then calculate the partial decay width of $\Lambda_c(2765) \to \Sigma_c\pi$ and $\Sigma_c(2520)\pi$ for different configurations, and predict its width into $\Lambda_c\pi\pi$. For $\Lambda_c(2765)$ being a $|\Lambda_c{}^4P_{\rho}\frac{1}{2}^-\rangle$ state, we obtain $[\Gamma(\Sigma_c\pi) + \Gamma(\Sigma_c(2520)\pi)] \simeq 21.6$ MeV. Thus, $\Gamma_{\rm total}^{\rm exp} \simeq 21.6/0.4 = 53.4$ MeV is obtained and agrees well with the experimental value $\Gamma^{\rm exp} \simeq 50 \sim 73$ MeV [2,4].

As $|\Lambda_c{}^4P_{\rho}\frac{3}{2}^-\rangle$ shows, the partial decay width for $\Lambda_c(2765) \to \Sigma_c(2520)\pi$ is much larger than for $\Sigma_c\pi$ by about a factor of 50. If this is the case, one would expect that $\Sigma_c(2520)\pi$ is the dominant decay channel, which, however, is not consistent with the data. For $|\Lambda_c{}^4P_{\rho}\frac{5}{2}^-\rangle$, the extracted decay widths are rather small to compare with its total width. The above results make $\Lambda_c(2765)$ a good candidate for the $|\Lambda_c{}^4P_{\rho}\frac{1}{2}^-\rangle$ state, which also agrees with the quark model prediction.

We also check the possibility of $\Lambda_c(2765)$ being a Σ_c-type state. As the masses of the D-wave Σ_c-type states in the $N = 2$ shell are generally larger than 2.8 GeV in the quark model [11,41], and the decay channel $\Lambda_c\pi$ of the P-wave states in the $N = 2$ shell is forbidden due to the quark model selection rules (see Table VII), only the P-wave states in the $N = 1$ shell and radial excitations are possible.

We calculate the decay widths for those possible states, and the results are listed in Table XV. It shows that the radial excitation should be excluded since the decay width is extremely narrow. The negative parity states, except $|\Sigma_c^+{}^2P_{\lambda}\frac{3}{2}^-\rangle$, can produce widths at the same order of magnitude as the data when all the decay channels are summed up. However, note that the dominant channel of Σ_c-type charmed states is $\Lambda_c\pi$. The assignment of $\Lambda_c(2765)$ to a Σ_c excitation will lead to apparent contradictions to the experimental observations, and thus can be ruled out.

F. $\Sigma_c(2800)$

The observation of $\Sigma_c^{++,+,0}(2800)$ by Belle in the $\Lambda_c\pi$ channel enriches the spectrum of Σ_c excitation states [3].

TABLE XV. The decay widths (in MeV) of $\Lambda_c(2765)$ as a Σ_c-type excitation. $\Gamma_{\text{sum}} = \Gamma_{\Lambda_c\pi} + \Gamma_{\Sigma_c\pi} + \Gamma_{\Sigma_c^*\pi}$, where Σ_c^* stands for $\Sigma_c(2520)$.

Assignment	$\Lambda_c\pi^0$	$\Sigma_c^{++,0}\pi^{-,+}$	$\Sigma_c^{*++,0}\pi^{-,+}$	Γ_{sum}	$\Gamma_{\text{total}}^{\text{exp}}$
$\|\Sigma_c^+{}^2P_{\lambda}\frac{3}{2}^-\rangle$	0.02	1.98	1.40	6.78	~50–73
$\|\Sigma_c^+{}^2P_{\lambda}\frac{1}{2}^-\rangle$	36.90	9.66	1.06	58.34	
$\|\Sigma_c^+{}^4P_{\lambda}\frac{1}{2}^-\rangle$	6.68	4.20	0.03	15.14	
$\|\Sigma_c^+{}^4P_{\lambda}\frac{3}{2}^-\rangle$	3.36	0.13	8.67	20.96	
$\|\Sigma_c^+{}^4P_{\lambda}\frac{5}{2}^-\rangle$	20.16	0.75	0.60	22.86	
$\|\Sigma_c^+{}^2S_{\rho\rho}\frac{1}{2}^+\rangle$	0.37	0.41	0.08	1.35	
$\|\Sigma_c^+{}^4S_{\rho\rho}\frac{3}{2}^+\rangle$	0.37	0.33	0.22	1.47	
$\|\Sigma_c^+{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$	0.003	0.11	0.03	0.283	
$\|\Sigma_c^+{}^4S_{\lambda\lambda}\frac{3}{2}^+\rangle$	0.003	0.03	0.07	0.203	

However, the present experimental information still cannot determine its quantum numbers. Theoretical studies are strongly model dependent where the spin-parity $J^P = 1/2^-$, $3/2^-$, or $5/2^-$ seems possible [5,6,21,22,24].

Almost all the recent theoretical predictions suggest that $\Sigma_c(2800)$ could be the first orbital excitations; however, its quantum numbers are different in different models. Its spin-parity could be $J^P = 3/2^-$ in the heavy hadron chiral perturbation theory predictions [5], $J^P = 3/2^-$ or $J^P = 5/2^-$ in the $^3P_0$ model [6], $J^P = 5/2^-$ in the relativistic quark model [22], $J^P = 1/2^-$ or $3/2^-$ in the Faddeev studies [24], and $J^P = 1/2^-$, $3/2^-$, or $5/2^-$ in the latest calculations with the relativistic quark model [21].

Again, taking the quark model guidance that the masses of the D-wave Σ_c excitations in the $N = 2$ shell are much larger than 2800 MeV [11,41], while the decay channel $\Lambda_c\pi$ of the P-wave states in the $N = 2$ shell is forbidden (see Table VII), we classify $\Sigma_c(2880)$ as a P-wave state in either the $N = 1$ shell (i.e., the first orbital excitation) or the radial excitation. The decay widths of $\Lambda_c\pi$, $\Sigma_c\pi$, and $\Sigma_c(2520)\pi$ are calculated, and the results are listed in Table XVI.

TABLE XVI. The decay widths (in MeV) of $\Sigma_c(2800)$ for the possible excitation modes. $\Gamma_{\text{sum}} = \Gamma_{\Lambda_c\pi} + \Gamma_{\Sigma_c\pi} + \Gamma_{\Sigma_c^*\pi}$, where Σ_c^* stands for $\Sigma_c(2520)$.

Assignment	$\Lambda_c\pi^+$	$\Sigma_c^{++,+}\pi^{0,+}$	$\Sigma_c^{*++,+}\pi^{0,+}$	Γ_{sum}	$\Gamma_{\text{total}}^{\text{exp}}$
$\|\Sigma_c^{++}{}^2P_{\lambda}\frac{3}{2}^-\rangle$	0.36	1.56	2.07	7.62	75^{+22}_{-17}
$\|\Sigma_c^{++}{}^2P_{\lambda}\frac{1}{2}^-\rangle$	46.59	14.66	1.05	78.01	
$\|\Sigma_c^{++}{}^4P_{\lambda}\frac{1}{2}^-\rangle$	4.36	4.23	0.34	13.5	
$\|\Sigma_c^{++}{}^4P_{\lambda}\frac{3}{2}^-\rangle$	4.51	0.22	10.79	26.53	
$\|\Sigma_c^{++}{}^4P_{\lambda}\frac{5}{2}^-\rangle$	27.08	1.34	1.41	32.58	
$\|\Sigma_c^{++}{}^2S_{\rho\rho}\frac{1}{2}^+\rangle$	0.26	0.52	0.13	1.56	
$\|\Sigma_c^{++}{}^4S_{\rho\rho}\frac{3}{2}^+\rangle$	0.26	0.42	0.37	1.84	
$\|\Sigma_c^{++}{}^2S_{\lambda\lambda}\frac{1}{2}^+\rangle$	0.06	0.11	0.05	0.38	
$\|\Sigma_c^{++}{}^4S_{\lambda\lambda}\frac{3}{2}^+\rangle$	0.06	0.03	0.11	0.34	

The radial excitations can be excluded easily due to the extremely small predictions of the widths compared with the experimental data. Furthermore, the $\Lambda_c\pi$ channel may dominate over other channels since $\Sigma_c(2800)$ was only seen there. Thus, $|\Sigma_c^{++}{}^2P_{\lambda}\frac{3}{2}^-\rangle$, $|\Sigma_c{}^4P_{\lambda}\frac{3}{2}^-\rangle$, and $|\Sigma_c^{++}{}^4P_{\lambda}\frac{1}{2}^-\rangle$ should be ruled out due to the dominance of either $\Sigma_c(2520)\pi$ or $\Sigma_c\pi$. After this, only two possible states, $|\Sigma_c^{++}{}^2P_{\lambda}\frac{1}{2}^-\rangle$ and $|\Sigma_c^{++}{}^4P_{\lambda}\frac{5}{2}^-\rangle$, can be assigned to $\Sigma_c(2800)$. This leads to the same starting point as other works [5,6,21,22,24], and indicates how little we know about this state.

In these two states, $\Sigma_c(2800)$ as a $|\Sigma_c{}^2P_{\lambda}\frac{1}{2}^-\rangle$ state (i.e. a first P-wave orbital Σ_c excitation) is favored if there are no other decay channels to contribute significantly to the total width. Considering there might exist other decay channels and the uncertainties of the model, $|\Sigma_c^{++}{}^4P_{\lambda}\frac{5}{2}^-\rangle$ is favored since its decays into $\Lambda_c\pi$ are the dominant channel, while those into $\Sigma_c\pi$ and $\Sigma_c(2520)\pi$ are relatively small. The sum of these three channels, though smaller than the experimental total width, is acceptable, taking into account the uncertainties. To determine the quantum number of $\Sigma_c(2800)$, a measurement of the ratio of $\Lambda_c\pi/\Sigma_c(2520)\pi$ or $\Sigma_c\pi/\Sigma_c(2520)\pi$, or the $\Lambda_c\pi$ angular distributions should be useful.

VI. SUMMARY

In the framework of the nonrelativistic quark model, the strong decays of charmed baryons are analyzed with an effective chiral Lagrangian for the pseudoscalar-meson-quark coupling. This framework is successful in reproducing the strong decay widths of $\Sigma_c \to \Lambda_c\pi$, $\Lambda_c(2593) \to \Sigma_c\pi$, and $\Lambda_c(2625) \to \Sigma_c\pi$. It allows us to fix an additional parameter δ which is introduced to account for model uncertainties arising from the pseudoscalar-meson-quark coupling constants. We then carry out calculations for those newly observed states by assuming their possible configurations in the quark model. By comparing the theoretical results with the experimental measurement, we extract information about the classification of those states and their possible quantum numbers.

To be more specific, our results show that both $\Lambda_c(2880)$ and $\Lambda_c(2940)$ are consistent with being internal D-wave states. For $\Lambda_c(2880)$, its narrow widths, visible decays into D^0p, and the measured ratio $\mathcal{R} = \Gamma_c(2520)\pi/\Gamma_c(2455)\pi$ suggest a favored configuration $|\Lambda_c{}^2D_{\lambda\lambda}\frac{3}{2}^+\rangle$ with $l_\lambda = 2$ and $l_\rho = 0$. Considering the decay width and decay channel of $\Lambda_c(2940)$, our results indicate that $\Lambda_c(2940)$ could be a $|\Lambda_c{}^2D_{\lambda\lambda}\frac{5}{2}^+\rangle$ state. Our predictions are different from the suggestions of Refs. [5,6] that $\Lambda_c(2880)$ is a $l_\lambda = l_\rho = 1$ orbital excitation state with $J^P = 5/2^+$. Although the angular distribution fit for $\Lambda_c(2880) \to \Sigma_c\pi$ favors $J = 5/2$ [2], the data still possess large uncertainties and more precise measurements are desired.

We propose that $\Lambda_c(2765)$ is most likely a ρ-mode P-wave excitation in the $N = 1$ shell. In those multiplets, the most possible state is $|\Lambda_c{}^4P_{\rho}\frac{1}{2}^-\rangle$, which also turns out to be consistent with the quark model predictions.

For $\Sigma_c(2800)$, the present experimental information seems insufficient for its classification in our approach. Assuming that no other sizable decay channels apart from $\Lambda_c\pi$, $\Sigma_c\pi$, and $\Sigma_c(2520)\pi$ contribute to its total width, it is most likely a $|\Sigma_c{}^2P_{\lambda}\frac{1}{2}^-\rangle$ state. Otherwise, the possibility of its being a $|\Sigma_c^{++4}P_{\lambda}\frac{5}{2}^-\rangle$ state cannot be excluded. Measurements of the ratio of $\Lambda_c\pi/\Sigma_c(2520)\pi$ and/or $\Sigma_c\pi/\Sigma_c(2520)\pi$ are recommended to clarify its spin-parity.

ACKNOWLEDGMENTS

This work is supported, in part, by the National Natural Science Foundation of China (Grant No. 10675131 and No. 10775145), Chinese Academy of Sciences (No. KJCX3-SYW-N2), the U.K. EPSRC (Grant No. GR/S99433/01), the Post-Doctoral Programme Foundation of China, and the K. C. Wong Education Foundation, Hong Kong.

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