Scalar resonance contributions to the dipion transition rates of $\Upsilon(4S, 5S)$ in the rescattering model

Ce Meng[1] and Kuang-Ta Chao[1,2]

[1]*Department of Physics, Peking University, Beijing 100871, People's Republic of China*
[2]*Center for High Energy Physics, Peking University, Beijing 100871, People's Republic of China*

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In order to explain the observed unusually large dipion transition rates of $\Upsilon(10870)$, the scalar resonance contributions in the rescattering model to the dipion transitions of $\Upsilon(4S)$ and $\Upsilon(5S)$ are studied. Since the imaginary part of the rescattering amplitude is expected to be dominant, the large ratios of the transition rates of $\Upsilon(10870)$, which is identified with $\Upsilon(5S)$, to that of $\Upsilon(4S)$ can be understood as mainly coming from the difference between the p values in their decays into open bottom channels, and the ratios are estimated numerically to be about 200–600 with reasonable choices of parameters. The absolute and relative rates of $\Upsilon(5S) \to \Upsilon(1S, 2S, 3S)\pi^+\pi^-$ and $\Upsilon(5S) \to \Upsilon(1S)K^+K^-$ are roughly consistent with data. We emphasize that the dipion transitions observed for some of the newly discovered Y states associated with charmonia may have similar features to the dipion transitions of $\Upsilon(5S)$. Measurements on the dipion transitions of $\Upsilon(6S)$ could provide further tests for this mechanism.

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I. INTRODUCTION

Hadronic transitions of heavy quarkonia are important for understanding both the heavy quarkonium dynamics and the formation of light hadrons. Because heavy quarkonium is expected to be compact and nonrelativistic, at least for the lower-lying states, QCD multiple expansion (QCDME) approach [1] can be used in analysis of these transitions, where the heavy quarkonium system serves as a compact color source and emits soft gluons which are hadronized into pions or other mesons.

Applying factorization and using the measurement of $\psi(2S) \to J/\psi\pi\pi$ as input, the widths of dipion transitions of the Υ system were successfully predicted [2] (see Ref. [3] for an extensive review and the updates; see also Ref. [4] for a comprehensive review on charmonium hadronic transitions). However, the situation became more complicated when comparing the predicted $M_{\pi\pi}$ distribution, which is peaked at the large $M_{\pi\pi}$ region, with the double-peaked one measured by CLEO [5,6] for $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$. A similar shape was also found in the $M_{\pi\pi}$ distribution of the $\Upsilon(4S) \to \Upsilon(2S)\pi\pi$ transition [7]. Lots of attempts (see [3] and references therein) have been made to improve the QCDME approach. In particular, a study of the $\Upsilon(4S)$ dipion transitions with $\pi\pi$ interactions was made in Ref. [8].

More strikingly, the widths of $\Upsilon(10870) \to \Upsilon(1S, 2S)\pi^+\pi^-$ recently measured by the Belle Collaboration [9] are about 2–3 orders in magnitude larger than those of $\Upsilon(nS) \to \Upsilon(mS)\pi^+\pi^-$ [7,10,11], where $n = 4, 3, 2$ and $m < n$, with even more complex structures in the $M_{\pi\pi}$ distributions. If the resonance $\Upsilon(10870)$ is indeed the $\Upsilon(5S)$ (note that the measured mass and leptonic width are consistent with this assignment), then its dipion transitions to lower-lying states can evidently not be described by the simple multiple expansion approach. The large rates of $\Upsilon(5S) \to \Upsilon(1S, 2S)\pi^+\pi^-$ are puzzling, and new mechanisms seem to be needed to explain them. [In this paper we will focus on the possibility that $\Upsilon(10870)$ is the $\Upsilon(5S)$, and leave discussions on other possible assignments e.g. $b\bar{b}g$ hybrids or $b\bar{b}q\bar{q}$ tetraquarks for $\Upsilon(10870)$ (see, e.g. [12]) elsewhere.]

In general, for higher-excited states of charmonium and bottomonium, the radius becomes larger, and can even be larger than the range of the soft gluon field if they are high enough. Then, for these exited heavy quarkonia, justification of the QCDME scenario becomes problematic. Particularly, when the excited state lies above the open flavor thresholds, the coupled-channel effects will change the QCDME scenario markedly and add new mechanisms to the analysis of its dipion transitions. Some of these effects were studied in Ref. [13], but the effects were found to be tiny for $\Upsilon(3S, 2S) \to \Upsilon(2S, 1S)\pi\pi$. This is probably due to the fact that $\Upsilon(3S, 2S)$ are too far below the open flavor threshold of $B\bar{B}$. However, the case should be changed for $\Upsilon(5S)$ and $\Upsilon(4S)$, since open bottom channels, such as $B^{(*)}\bar{B}^{(*)}$ and $B_s^{(*)}\bar{B}_s^{(*)}$, can be open and contribute to their transition rates significantly.

One important feature of coupled decay channels for $\Upsilon(5S)$ and $\Upsilon(4S)$ is the final-state interaction, i.e., in the decay the $B^{(*)}$ and $\bar{B}^{(*)}$ can interact with each other at long distances and then convert into a lower Υ plus light mesons. For simplification, in this paper we will use the rescattering model [14–17] to study the scalar resonance contributions to the dipion transition rates of $\Upsilon(5S)$ as well as $\Upsilon(4S)$. In this picture, the higher Υ decays into $B^{(*)}\bar{B}^{(*)}$ first, and then through one $B^{(*)}$ meson exchange turns into another lower Υ and a scalar resonance, such as σ or $f_0(980)$ [perhaps also $f_0(1370)$], which couples to the dipion. Experimentally, for both charmonium dipion transitions (see, e.g., [18]) and bottomonium dipion transitions (see, [6]), the dipion systems are found to be dominated by the S-wave; therefore, the scalar resonances could play an essential role in these transitions. In fact, the scalar resonance, i.e. the σ dominance approach, has been used to fit the $\psi(2S)$ data [18]. In addition, including the contributions from scalar resonances could be helpful to explain the $M_{\pi\pi}$ distributions in $\Upsilon(3S, 2S) \to \Upsilon(2S, 1S)\pi\pi$ [19,20], especially the double-peaked structure of mass distribution in $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$. It would also be interesting to further examine the complex $M_{\pi\pi}$ distributions in the dipion transitions of $\Upsilon(4S)$ [7] and $\Upsilon(5S)$ [9].

In this paper we will assume that in the $\Upsilon(4S, 5S)$ dipion transitions the two pions are produced mainly via scalar resonances coupled to intermediate $B^{(*)}$ mesons due to the long-distance final-state interactions. We will discuss the model and calculate the transition rates of $\Upsilon(4S, 5S)$. A summary will be given in the last section.

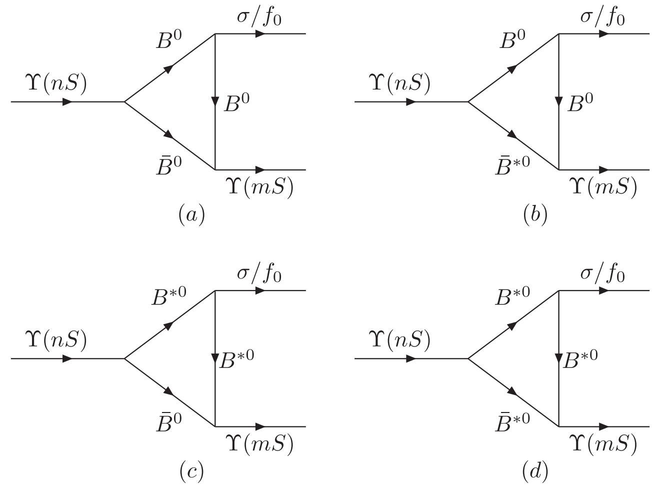

FIG. 1. The diagrams for $\Upsilon(nS) \to B^{(*)0}\bar{B}^{(*)0} \to \Upsilon(mS)\mathcal{S}$.

II. THE MODEL

In the rescattering model, the transitions $\Upsilon(4S, 5S) \to \Upsilon(1S, 2S)\mathcal{S}$ can arise from scattering of intermediate state $B^{(*)}\bar{B}^{(*)}$ by exchange of another B meson. Here, $\mathcal{S}$ denotes scalar resonance σ or $f_0(980)$ [perhaps also $f_0(1370)$], which will decay to $\pi\pi(K\bar{K})$ eventually. The typical diagrams are shown in Fig. 1, and the other ones can be related to those in Fig. 1 by charge conjugation transformation $B \leftrightarrow \bar{B}$ and isospin transformation $B^0 \leftrightarrow B^+$ and $\bar{B}^0 \leftrightarrow B^-$. Therefore, the amplitudes of Figs. 1(a)–1(d) should be multiplied by a factor of 4, respectively.

To evaluate the amplitudes, we need the following effective Lagrangians:

$$\mathcal{L}_{\Upsilon BB} = g_{\Upsilon BB}\Upsilon_\mu(\partial^\mu B B^\dagger - B\partial^\mu B^\dagger), \tag{1a}$$

$$\mathcal{L}_{\Upsilon B^* B} = \frac{g_{\Upsilon B^* B}}{m_\Upsilon}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\Upsilon_\nu(B^*_\alpha\overleftrightarrow{\partial}_\beta B^\dagger - B\overleftrightarrow{\partial}_\beta B^{*\dagger}_\alpha), \tag{1b}$$

$$\mathcal{L}_{\Upsilon B^* B^*} = g_{\Upsilon B^* B^*}(-\Upsilon^\mu B^{*\nu}\overleftrightarrow{\partial}_\mu B^{*\dagger}_\nu + \Upsilon^\mu B^{*\nu}\partial_\nu B^{*\dagger}_\mu - \Upsilon_\mu \partial_\nu B^{*\mu} B^{*\nu\dagger}), \tag{1c}$$

$$\mathcal{L}_{\mathcal{S}BB} = g_{\mathcal{S}BB}\mathcal{S}BB^\dagger, \tag{1d}$$

$$\mathcal{L}_{\mathcal{S}B^*B^*} = -g_{\mathcal{S}B^*B^*}\mathcal{S}B^*\cdot B^{*\dagger}, \tag{1e}$$

where $\overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial}$. In the heavy quark limit, the coupling constants in (1) can be related to each other by heavy quark symmetry as

$$g_{\Upsilon BB} = g_{\Upsilon B^* B} = g_{\Upsilon B^* B^*}, \tag{2}$$

$$g_{\mathcal{S}BB} = g_{\mathcal{S}B^*B^*}. \tag{3}$$

Particularly, the coupling constants for $\Upsilon(4S)$ and $\Upsilon(5S)$ can be determined by the observed values of their partial decay widths.

All the coupling constants will be determined in the next section. However, it is necessary to emphasize here that the determinations will not account for the off-shell effect of the exchanged $B^{(*)}$ meson, of which the virtuality cannot be ignored. Such effects can be compensated by introducing, e.g., the monopole [14] form factors for off-shell vertexes. Let q denote the momentum transferred and m_i the mass of exchanged meson, the form factor can be written as

$$\mathcal{F}(m_i, q^2) = \frac{(\Lambda + m_i)^2 - m_i^2}{(\Lambda + m_i)^2 - q^2}. \tag{4}$$

We will fix the cutoff $\Lambda = 660$ MeV [16] in our numerical analysis in the next section.

As emphasized in Ref. [17], the form factor suppression favors the production of a higher-excited heavy quarkonium state over that of the lower one because the mass of the former is closer to the open flavor threshold than the later. This effect will largely balance the final-state phase space factor, which favors the production of the lower state, and will give a reasonable relative rate between $\Upsilon(5S) \to \Upsilon(1S)\pi\pi$ and $\Upsilon(5S) \to \Upsilon(2S)\pi\pi$ transitions, as one can see in the next section.

We are now in a position to compute the contributions of diagrams in Fig. 1. If the $\Upsilon(nS)$ state lies above the $B^{(*)}\bar{B}^{(*)}$ threshold, the absorptive part (imaginary part) of the amplitude arising from Fig. 1 can be evaluated by the Cutkosky rule. For the process $\Upsilon(nS) \to B^{(*)}(p_1) + \bar{B}^{(*)} \to \Upsilon(mS) + \mathcal{S}$, the absorptive part of the amplitude reads

$$\mathbf{Abs}_i = \frac{|\vec{p}_1|}{32\pi^2 m_{\Upsilon(nS)}}\int d\Omega \mathcal{A}_i(\Upsilon(nS) \to B^{(*)}\bar{B}^{(*)}) \times \mathcal{C}_i(B^{(*)}\bar{B}^{(*)} \to \Upsilon(mS)\mathcal{S}), \tag{5}$$

where $i = (a, b, c, d)$, and $d\Omega$ and $\vec{p}_1$ denote the solid angle of the on-shell $B^{(*)}\bar{B}^{(*)}$ system and the 3-momentum of the on-shell $B^{(*)}$ meson in the rest frame of $\Upsilon(nS)$, respectively. Constrained by the heavy quark symmetry relation (2) and (3), the amplitude of Fig. 1(b) is equal to that of Fig. 1(c) in the heavy quark limit. Therefore, the total absorptive part of the rescattering amplitude can be given by

$$\mathbf{A\,bs} \approx 4\mathbf{Abs}_a + 8\mathbf{Abs}_b + 4\mathbf{Abs}_d. \tag{6}$$

The evaluation of the real part of the amplitude is difficult to be achieved and will bring large uncertainties inevitably. Fortunately, for the transitions $\Upsilon(4S, 5S) \to \Upsilon(1S, 2S)\mathcal{S}$, the contributions from the real part are expected to be small, because the masses of $\Upsilon(4S, 5S)$ are not very close to the open flavor thresholds as those of $X(3872)$ [16] and $Z(4430)$ [17]. Thus we assume that the contributions from the real part can be neglected and use (5) to determine the full amplitude in the calculations.

In the absorptive part, the intermediate states $B^{(*)}\bar{B}^{(*)}$ are on shell, and the amplitude in (5) is proportional to the phase space factor of decay $\Upsilon(nS) \to B^{(*)}\bar{B}^{(*)}$:

$$\frac{|\vec{p}_1|}{32\pi^2 m_{\Upsilon(nS)}}. \tag{7}$$

The amplitude $\mathcal{A}_i$ in (5) is also proportional to $|\vec{p}_1|$ since it involves an on-shell P-wave vertex $\Upsilon(nS)B^{(*)}B^{(*)}$. Furthermore, a hidden factor $|\vec{p}_1|$ will emerge after performing the integral in (5) explicitly. As a result, the amplitude $\mathbf{Abs}_i$ is proportional to $|\vec{p}_1|^3$. This fact is important in understanding the huge difference between the decay rates of $\Upsilon(5S) \to \Upsilon(1S, 2S)\pi\pi$ and $\Upsilon(4S) \to \Upsilon(1S, 2S)\pi\pi$, since for $\Upsilon(4S)$ only one decay channel $B\bar{B}$ is really open, and the corresponding p-value $|\vec{p}_1|$ is small, whereas for $\Upsilon(5S) \to \Upsilon(1S, 2S)\pi\pi$ three decay channels $B\bar{B}$, $B^*\bar{B}$ + c.c., $B^*\bar{B}^*$ are all open with rather large p values. Similar to this, the essential role played by the phase space factor in determining the absorptive part of the rescattering amplitude has been emphasized in the case of $X(3872)$ in Refs. [15,16], where the tiny phase space greatly suppresses the absorptive part of the rescattering amplitude.

Generally, to compare the result with the experimental measurement, one needs to describe the transition amplitude to $\pi\pi(K\bar{K})$ for a virtual scalar resonance $\mathcal{S}$ explicitly. However, since we will focus on the total rate, we treat $\mathcal{S}$ as narrow resonance and use the Breit-Wigner distribution

$$\mathcal{F}_{\mathcal{S}}(t) = \frac{1}{\pi}\frac{\sqrt{t}\Gamma_{\mathcal{S}}(t)}{(t - m_{\mathcal{S}}^2)^2 + m_{\mathcal{S}}^2\Gamma_{\mathcal{S}}(t)^2} \tag{8}$$

to describe the resonance in the calculation of cross sections, as the treatment of ρ resonance in Ref. [16]. In (8), the variable t denotes the momentum squared of $\mathcal{S}$, and the function $\Gamma_{\mathcal{S}}(t)$ is given by

$$\Gamma_{\mathcal{S}}(t) = \frac{p_\pi g_{\mathcal{S}\pi\pi}}{8\pi t} + \frac{p_K g_{\mathcal{S}KK}}{8\pi t}, \qquad p_\pi = \sqrt{\frac{t}{4} - m_\pi^2}, \qquad p_K = \sqrt{\frac{t}{4} - m_K^2}. \tag{9}$$

The resonance parameters in (8) and the coupling constants in (9) will be evaluated in the next section following Ref. [19].

III. NUMERICAL RESULTS AND DISCUSSIONS

Since the contribution from the absorptive part of the rescattering amplitude corresponds to the real decay process $\Upsilon(nS) \to B^{(*)}\bar{B}^{(*)}$, the coupling constants $g_{\Upsilon(nS)B^{(*)}B^{(*)}}$ should be determined by the measured values of the decay widths of $\Upsilon(4S, 5S) \to B^{(*)}\bar{B}^{(*)}$ [11], and the results are given by

$$g_{\Upsilon(4S)BB} = 24, \tag{10}$$

$$g_{\Upsilon(5S)BB} < 2.9, \tag{11}$$

$$g_{\Upsilon(5S)B^*B} = 1.4 \pm 0.3, \tag{12}$$

$$g_{\Upsilon(5S)B^*B^*} = 2.5 \pm 0.4. \tag{13}$$

The value of $g_{\Upsilon(4S)BB}$ in (10) is typical, and is comparable to the estimation using the vector meson dominance model [16] for $g_{\Upsilon(1S)BB}$:

$$g_{\Upsilon BB} \approx \frac{m_{\Upsilon(1S)}}{f_{\Upsilon(1S)}} \sim 15, \tag{14}$$

where the decay constant $f_{\Upsilon(1S)}$ can be determined by the leptonic width of $\Upsilon(1S)$. However, the values determined from the $\Upsilon(5S)$ data in (11)–(13) are small. This may be partly due to the fact that as a high-excited $b\bar{b}$ state, the wave function of $\Upsilon(5S)$ has a complicated node structure, and the coupling constants will be small if the p values of $B^{(*)}\bar{B}^{(*)}$ channels (1060–1270 MeV) are close to those corresponding to the zeros in the amplitude. The symmetry relation in (2) can also be violated by the same reason.

As for the coupling constants $g_{\Upsilon(mS)B^{(*)}B^{(*)}}$ ($m < 5$), we assume that the symmetry relations in (2) hold, and they are equal to each other, which is implied by comparison between (10) and (14).

Numerically, we find that the amplitude $\mathbf{Abs}_a$ is relatively small, and the amplitude $\mathbf{Abs}_b$ partly cancels $\mathbf{Abs}_d$ in (6). So we choose $g_{\Upsilon(5S)BB} = 2.5$ and focus on the sensitivities of the decay rates to the coupling constants $g_{\Upsilon(5S)B^*B}$ and $g_{\Upsilon(5S)B^*B^*}$ in (12) and (13).

The phenomenological coupling constants $g_{\mathcal{S}B^{(*)}B^{(*)}}$ are difficult to be determined. However, in the linear realization of chiral symmetry, one can relate them to the coupling constant [21]

$$g_{B^*B\pi} = \frac{2gm_B}{f_\pi}, \tag{15}$$

where $g \approx 0.6$ [16], and f_π is the decay constant of π. In general, the coupling constant $g_{SB^{(*)}B^{(*)}}$ could be obtained through scaling $g_{B^*B\pi}$ by a typical chiral scale like f_π. Thus, they are of order $\mathcal{O}(m_B)$, and we choose

$$g_{\sigma BB} = g_{\sigma B^*B^*} = 10 \text{ GeV}, \tag{16}$$

$$g_{f0BB} = g_{f0B^*B^*} = 10\sqrt{2} \text{ GeV}. \tag{17}$$

Here in (17) we introduce a numerical factor of $\sqrt{2}$, which is somewhat arbitrary, to roughly account for the contributions from other higher scalar resonances, such as $f_0(1370)$.

In the linear realization of chiral symmetry, there should exist a coupling of $BB\pi\pi$, which will cancel the one of $BB\sigma$ in the low-energy limit. However, the cancellation is no longer effective in processes with large energy release [22]. We will not take into account this cancellation in the present paper and leave it to be studied in the future.

The scalar resonance parameters, which are listed in Table I, are chosen following Ref. [19] (while $m_{f_0(980)}$ following Ref. [11]), where they are determined by fitting the $M_{\pi\pi}$ distributions in $\Upsilon(2S,3S) \to \Upsilon(1S,2S)\pi\pi$, $\psi(2S) \to J/\psi\pi\pi$, and $J/\psi \to \phi\pi\pi(KK)$.

Neglecting the interference between contributions from σ and that from $f_0(980)$, we can now evaluate the transition widths $\Upsilon(4S,5S) \to \Upsilon(1S,2S)\pi^+\pi^-$ using the parameters and the coupling constants determined above, and the results are listed in Table II. Since the transitions $\Upsilon(5S) \to \Upsilon(3S)\pi^+\pi^-$ and $\Upsilon(5S) \to \Upsilon(1S)K^+K^-$ are also observed [9] with large rates and quite high statistic significance (3.2σ and 4.9σ, respectively), we also evaluate the corresponding rates to be compared with the experimental data. Contributions from σ and $f_0(980)$ are listed in the second and third columns in Table II, respectively. The error bars in these two columns come from those of $m_\sigma(m_{f_0(980)})$, $g_{\Upsilon(5S)B^*B^*}$, and $g_{\Upsilon(5S)B^*B}$ in turn, and the signs "+" correspond to the smaller $m_\sigma(m_{f_0(980)})$, the larger $g_{\Upsilon(5S)B^*B^*}$, and the smaller $g_{\Upsilon(5S)B^*B}$, respectively. The only exception is that the width of $\Upsilon(5S) \to \Upsilon(1S)K^+K^-$ increases with $m_{f_0(980)}$. The sensitivity of the width of $\Upsilon(5S) \to \Upsilon(3S)\pi^+\pi^-$ to the parameter m_σ can be easily understood since the phase space is small and can only cover part of the distribution of σ resonance. On the other hand, the sensitivities of the widths of $\Upsilon(5S) \to \Upsilon(1S,2S,3S)\pi^+\pi^-$ and $\Upsilon(5S) \to \Upsilon(1S)K^+K^-$ to the coupling constant $g_{\Upsilon(5S)B^*B}$ are mainly due to the partial cancellation between amplitudes $\mathbf{Abs}_b$ and $\mathbf{Abs}_d$.

The dependence on the cutoff Λ, which is defined in (4), is not shown in Table II. In our evaluations, we have chosen $\Lambda = 660$ MeV following Ref. [16]. If the cutoff increases (decreases) by, say, 220 MeV, the rates listed in Table II will increase (decrease) by 2–3 times in magnitude correspondingly.

As mentioned in last section, the main difference between $\Upsilon(5S) \to \Upsilon(1S,2S,3S)\pi\pi$ and $\Upsilon(4S) \to \Upsilon(1S,2S)\pi\pi$ in this rescattering model is the number of open flavor channels involved and, essentially, the corresponding p values $|\vec{p}_1|$. Especially, the contribution to the rate from a given channel is proportional to $|\vec{p}_1|^6$, which can cause a big difference between the partial widths of $\Upsilon(5S)$ and $\Upsilon(4S)$ of order $\mathcal{O}(10^3 - 10^4)$ in magnitude. After other ingredients, such as the difference between coupling constants in (10)–(13), are taken into account, the difference becomes about 200–600 in magnitude and, although with large error bars, is in rough agreement with experimental data.

The above results are obtained with the assumption of the absorptive part dominance. If the real part of the rescattering amplitude cannot be neglected, the difference between the transition decay widths of $\Upsilon(5S)$ and $\Upsilon(4S)$

TABLE I. Resonance parameters of σ and $f_0(980)$ [11,19].

	m_S (MeV)	$g_{S\pi\pi}$ (GeV)	$\Gamma_{S\pi\pi}$ (MeV)	g_{SKK} (GeV)	Γ_{SKK}/MeV
σ	526 ± 30	3.06	302 ± 10		
$f_0(980)$	980 ± 10	1.77	61 ± 1	2.70	12 ± 1

TABLE II. Transition widths of $\Upsilon(nS) \to \Upsilon(mS)\pi^+\pi^-/K^+K^-$ in units of KeV. Respectively, the contributions from σ and $f_0(980)$ are listed in the second and the third columns, and the error bars come from those of $m_\sigma(m_{f_0(980)})$, $g_{\Upsilon(5S)B^*B^*}$, and $g_{\Upsilon(5S)B^*B}$ in turn. Experimental data of $\Upsilon(4S) \to \Upsilon(1S,2S)\pi^+\pi^-$ are taken from Ref. [7], and the others from Ref. [9].

	From σ	From $f_0(980)$	Total	Experimental data
$\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$	$0.54^{+0.00}_{-0.00}$	$0.93^{+0.03}_{-0.03}$	1.47 ± 0.03	1.8 ± 0.4
$\Upsilon(4S) \to \Upsilon(2S)\pi^+\pi^-$	$1.09^{+0.23}_{-0.21}$	$0.05^{+0.00}_{-0.01}$	$1.14^{+0.23}_{-0.21}$	2.7 ± 0.8
$\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-$	$102^{+1+42+21}_{-0-35-9}$	$225^{+1+93+47}_{-1-77-43}$	327^{+114}_{-97}	$590 \pm 40 \pm 90$
$\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$	$385^{+10+164+87}_{-11-135-78}$	$37^{+4+16+9}_{-3-13-7}$	422^{+187}_{-157}	$850 \pm 70 \pm 160$
$\Upsilon(5S) \to \Upsilon(3S)\pi^+\pi^-$	$306^{+78+133+73}_{-64-108-64}$	13^{+1+6+4}_{-1-4-2}	319^{+171}_{-141}	$520^{+200}_{-170} \pm 100$
$\Upsilon(5S) \to \Upsilon(1S)K^+K^-$		$32^{+5+13+5}_{-5-11-6}$	32^{+15}_{-13}	$67^{+17}_{-13} \pm 13$

will decrease, since the contributions from the real part do not obey the $|\vec{p}_1|^6$ rule. To clarify to what extent the absorptive part dominance assumption is sensible, we evaluated the real part by using the dispersion relation [16]. To evaluate the dispersive integral, e.g. for the $B\bar{B}$ decay channel, we take the upper limit of the integral to be $s_{\max} = (m_B + m_B + \Delta)^2$ and choose the cutoff Δ to be equal to the splitting $m_{B^*} - m_B$ following Ref. [16]. This choice of the cutoff will lead to a contribution to the rates of less than 1 KeV for all modes that we are interested in. However, if one chooses the cutoff $\Delta = 100$ MeV, the contributions of the real part will increase and result in rates of about 10 KeV for $\Upsilon(4S) \to \Upsilon(1S, 2S)\pi\pi$ decays. So, the $\Upsilon(4S)$ decays are sensitive to the cutoff in the real part. Nevertheless, this sensitivity does not affect the calculated large difference between the transition widths of $\Upsilon(5S)$ and $\Upsilon(4S)$, which is the main point addressed in this paper. In addition, the above results are obtained by using the couplings shown in (10)–(13), and if we choose a smaller coupling of $g_{\Upsilon(4S)BB}$ [say, to be equal to $g_{\Upsilon(5S)BB}$], with other parameters readjusted, then the real part contribution to $\Upsilon(4S)$ transitions will be much less than 1 KeV even with the cutoff $\Delta = 100$ MeV. So, despite the large uncertainties with the model and chosen parameters, the absorptive part dominance should be a reasonable assumption unless the resonance is very close to the open channel threshold (as in the case of $X(3872)$ [16], whose mass departs from the DD^* threshold by less than a few MeV). In any case, however, a more reliable approach for estimating the real part contribution in the rescattering model is needed and deserves further study.

As we mentioned in the last section, in the $\Upsilon(5S)$ decays the Υ form factor defined in (4) favors the production of the higher Υ state over the lower one, while the final-state phase space plays an opposite role. Thus, as one can see in Table II, the contributions from σ favor the production of $\Upsilon(2S, 3S)$ since the phase space differences are relatively small, while those from $f_0(980)$ [perhaps also $f_0(1370)$] favor the production of $\Upsilon(1S)$. This fact is important in obtaining a reasonable ratio between the widths of $\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$, and may also imply that the $M_{\pi\pi}$ spectrum of $\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-$ should be concentrated in higher mass regions, which is in agreement with the experimental measurement [9].

The $M_{\pi\pi}$ spectrum of $\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-$ in our model is also dependent on the ratio of g_{f_0BB} to $g_{\sigma BB}$, which is assumed to be $\sqrt{2}$ from (16) and (17). If we choose a relatively larger value for this ratio, the spectrum will be even more concentrated to higher masses. However, because the ratio is introduced to account for the contributions from higher scalar resonances in a rather arbitrary way, and the nonresonance contributions are neglected in our model, we are unable to fit the $M_{\pi\pi}$ spectrum at the quantitative level.

We can make a rough prediction for the dipion transitions of $\Upsilon(11020)$, if it is identified with the $\Upsilon(6S)$. Constraining the coupling constant $g_{\Upsilon B^{(*)}B^{(*)}}$ by the total width of $\Upsilon(11020)$, we find that the dominant transition mode of $\Upsilon(11020)$ could be $\Upsilon(3S)\pi\pi$ with a large partial width of about 1–2 MeV, while that for $\Upsilon(1S)\pi\pi$ is about 300 KeV. Of course, due to the large uncertainties from model parameters, this estimate only serves as a possible tendency that $\Upsilon(3S)\pi\pi$ and $\Upsilon(2S)\pi\pi$ will be favored over $\Upsilon(1S)\pi\pi$ in the $\Upsilon(6S)$ decays.

The situation in the $c\bar{c}$ system can be similar to but more complicated than the $b\bar{b}$ system. In fact in the initial state radiation process a number of Y states have been found [23,24] to decay to J/ψ or, but not "and," $\psi(2S)$ through dipion transitions with large decay rates, while there seem no "adequate" assignments in the conventional charmonium family for these states. However, the abnormal large rates of dipion transitions of these Y-states might indicate that they, or at least some of them, are indeed the conventional charmonium states that are coupled to the open charm meson channels, just as the $\Upsilon(10870)$ in the $b\bar{b}$ system discussed above. The coupled-channel effects are expected to be more complicated for higher $c\bar{c}$ states than for $b\bar{b}$, since more open charm channels, e.g. $D_1\bar{D}$, aside from $D^{(*)}\bar{D}^{(*)}$, are involved for higher charmonia. The large rates of dipion transitions of these charmonium states might be accounted for in the rescattering picture. The fact [24] that the lower Y states are only found in $J/\psi\pi\pi$ mode while the higher ones in $\psi(2S)\pi\pi$ might be understood as signals of the competition between the form factor, which favors $\psi(2S)$, and the final-state phase space, which favors J/ψ, in the rescattering model, especially when the contributions from resonances, such as σ and $f_0(980)$ [perhaps also $f_0(1370)$], are dominant.

IV. SUMMARY

In summary, we study the long-distance final-state interactions in $\Upsilon(4S, 5S)$ dipion transitions. We calculate the scalar resonance contributions in the rescattering model to the dipion transition rates of $\Upsilon(4S)$ and $\Upsilon(10870)$, which is identified with $\Upsilon(5S)$, in order to explain the observed unusually large dipion transition rates of $\Upsilon(10870)$. We assume that the $\Upsilon(4S, 5S)$ decay into $B^{(*)}\bar{B}^{(*)}$ first, and then through one $B^{(*)}$ meson exchange turn into another lower Υ and a scalar resonance, such as σ or $f_0(980)$ [perhaps also $f_0(1370)$], which couples to the dipion. Assuming the imaginary part of the rescattering amplitude dominates, the large ratios of the transition rates of $\Upsilon(5S)$ to that of $\Upsilon(4S)$ can be understood as mainly coming from the difference between the p values of their decays into open bottom channels and are estimated to be about 200–600 with reasonable choices for the parameters. Besides, the absolute and relative rates of $\Upsilon(5S) \to \Upsilon(1S, 2S, 3S)\pi^+\pi^-$ and $\Upsilon(5S) \to \Upsilon(1S)K^+K^-$ are also roughly compatible with experimental data. We find that the competition between the form factor and the final-state phase space plays an important role in the determination of these relative rates and the $M_{\pi\pi}$ distribution in $\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-$. We emphasize that the dipion transitions observed for some of the newly discovered Y states associated with charmonia may have similar features to the dipion transitions of $\Upsilon(5S)$ discussed here. Measurements on the dipion transitions of $\Upsilon(6S)$ could provide further tests for this mechanism.

Recently, in Ref. [25] the author suggested an approach to explain the dipion transitions of $\Upsilon(4S, 5S)$, which is similar, in some sense, to ours but without introducing scalar resonances. It will be interesting to compare our result with theirs when their result for $\Upsilon(4S, 5S)$ comes out.

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