M theory through the looking glass: Tachyon condensation in the E_8 heterotic string

Petr Hořava and Cynthia A. Keeler

Berkeley Center for Theoretical Physics and Department of Physics, University of California, Berkeley, California, 94720-7300, USA and Physics Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720-8162, USA

(Received 24 September 2007; published 19 March 2008)

We study the spacetime decay to nothing in string theory and M-theory. First we recall a nonsupersymmetric version of heterotic M-theory, in which bubbles of nothing—connecting the two E_8 boundaries by a throat—are expected to be nucleated. We argue that the fate of this system should be addressed at weak string coupling, where the nonperturbative instanton instability is expected to turn into a perturbative tachyonic one. We identify the unique string theory that could describe this process: The heterotic model with one E_8 gauge group and a singlet tachyon. We then use world sheet methods to study the tachyon condensation in the Neveu-Schwarz-Ramond formulation of this model, and show that it induces a world sheet super-Higgs effect. The main theme of our analysis is the possibility of making meaningful alternative gauge choices for world sheet supersymmetry, in place of the conventional superconformal gauge. We show in a version of unitary gauge how the world sheet gravitino assimilates the Goldstino and becomes dynamical. This picture clarifies recent results of Hellerman and Swanson. We also present analogs of R_ξ gauges, and note the importance of logarithmic conformal field theories in the context of tachyon condensation.

DOI: 10.1103/PhysRevD.77.066013 PACS numbers: 11.25.−w, 11.25.Yb

I. INTRODUCTION

The motivation for this paper is to further the studies of time-dependent backgrounds in string theory. In particular, we concentrate on the problem of closed-string tachyon condensation, and its hypothetical relation to the "spacetime decay to nothing."

Open-string tachyon condensation is now relatively well-understood (see, e.g., [1,2] for reviews), as a description of D-brane decay into the vacuum (or to lower-dimensional stable defects). On the other hand, the problem of the bulk closed-string tachyon condensation appears related to a much more dramatic instability in which the spacetime itself decays, or at least undergoes some other extensive change indicating that the system is far from equilibrium. In the spacetime supergravity approximation, this phenomenon has been linked to nonperturbative instabilities due to the nucleation of "bubbles of nothing" [3]. One of the first examples studied in the string and M-theory literature was the nonsupersymmetric version of heterotic M-theory [4], in which the two E_8 boundaries of 11-dimensional spacetime carry opposite relative orientation and consequently break complementary sets of 16 supercharges. At large separation between the boundaries, this system has an instanton solution that nucleates "bubbles of nothing." In 11 dimensions, the nucleated bubbles are smooth throats connecting the two boundaries; the "nothing" phase is thus the phase "on the other side" of the spacetime boundary.

In addition to this effect, the boundaries are attracted to each other by a Casimir force which drives the system to weak string coupling, suggesting some weakly coupled heterotic string description in ten dimensions. In the regime of weak string coupling, we expect the originally nonperturbative instability of the heterotic M-theory background to turn into a perturbative tachyonic one.

We claim that there is a unique viable candidate for describing this system at weak string coupling: The tachyonic heterotic string with one copy of E_8 gauge symmetry, and a singlet tachyon. In this paper, we study in detail the world sheet theory of this model—in the Neveu-Schwarz-Ramond (NSR) formalism with local world sheet (0, 1) supersymmetry—when the tachyon develops a condensate that grows exponentially along a light cone direction X^+. There is a close similarity between this background and the class of backgrounds studied recently by Hellerman and Swanson [5–9]. The main novelty of our approach is the use of alternative gauge choices for world sheet supersymmetry, replacing the traditional superconformal gauge. We show that the world sheet dynamics of spacetime tachyon condensation involves a super-Higgs mechanism, and its picture simplifies considerably in our alternative gauge.

Our main results were briefly reported in [12]; in the present paper, we elaborate on the conjectured connection to spacetime decay in heterotic M-theory, and provide more details of the world sheet theory of tachyon condensation, including the analysis of the super-Higgs mechanism and its compatibility with conformal invariance.

Section II reviews the nonsupersymmetric version of heterotic M-theory, as a simple configuration that exhibits the "spacetime decay to nothing." We argue that the dynamics of this instability should be studied at weak string coupling, and advocate the role of the tachyonic E_8 heterotic model as a unique candidate for this weakly coupled description of the decay. In Sec. III, we review some of the world sheet structure of the tachyonic E_8 heterotic string. In particular, we point out that the E_8

current algebra of the nonsupersymmetric (left-moving) world sheet sector is realized at level two and central charge $c_L = 31/2$; this is further supplemented by a single real fermion λ of $c_L = 1/2$.

Sections IV and V represent the core of the paper, and are in principle independent of the motivation presented in Sec. II. In Sec. IV, we specify the world sheet theory in the NSR formulation, before and after the tachyon condensate is turned on. The condensate is exponentially growing along a spacetime null direction X^+. Conformal invariance then also requires a linear dilaton along X^- if we are in ten spacetime dimensions. We point out that when the tachyon condensate develops, λ transforms as a candidate Goldstino, suggesting a super-Higgs mechanism in world sheet supergravity.

Section V presents a detailed analysis of the world sheet super-Higgs mechanism. Traditionally, world sheet supersymmetry is fixed by working in superconformal gauge, in which the world sheet gravitino is set to zero. We discuss the model briefly in superconformal gauge in Sec. V A, mainly to point out that tachyon condensation leads to logarithmic conformal field theories (CFT).

Since the gravitino is expected to take on a more important role as a result of the super-Higgs effect, in Sec. V B and V C we present a gauge choice alternative to superconformal. This alternative gauge choice is inspired by the "unitary gauge" known from the conventional Higgs mechanism in Yang-Mills theories. We show in this gauge how the world sheet gravitino becomes a dynamical propagating field, contributing $c_L = -11$ units of central charge. Additionally, we analyze the Faddeev-Popov determinant of this gauge choice, and show that instead of the conventional right-moving superghosts β, γ of superconformal gauge, we get *left-moving* superghosts $\tilde{\beta}$, $\tilde{\gamma}$ of spin 1/2. In addition, we show how the proper treatment of the path-integral measure in this gauge induces a shift in the linear dilaton. This shift is precisely what is needed for the vanishing of the central charge when the ghosts are included. Thus, this string background is described in our gauge by a world sheet conformal (but not superconformal) field theory. Section VI points out some interesting features of the world sheet theory in the late X^+ region, deeply in the condensed phase.

In Appendix A we list all of our needed world sheet supergravity conventions. Appendix B presents a detailed evaluation of the determinants relevant for the body of the paper.

II. SPACETIME DECAY TO NOTHING IN HETEROTIC M-THEORY

The anomaly cancellation mechanism that permits the existence of spacetime boundaries in M-theory works locally near each boundary component. The conventional realization, describing the strongly coupled limit of the $E_8 \times E_8$ heterotic string [13,14], assumes two boundary components, separated by fixed distance R_{11} along the 11th dimension y, each breaking the same 16 supercharges and leaving the 16 supersymmetries of the heterotic string.

In [4], a nonsupersymmetric variant of heterotic M-theory was constructed, simply by flipping the orientation of one of the boundaries. This flipped boundary breaks the complementary set of 16 supercharges, leaving no unbroken supersymmetry. The motivation behind this construction was to find in M-theory a natural analog of D-brane anti-D-brane systems whose study turned out to be so illuminating in superstring theories. Dp-branes differ from $\overline{\mathrm{D}p}$-branes only in their orientation. In analogy with Dp-$\overline{\mathrm{D}p}$ systems, we refer to the nonsupersymmetric version of heterotic M-theory as $E_8 \times \bar{E}_8$ to reflect this similarity [15].

A. The $E_8 \times \bar{E}_8$ heterotic M-theory

This model, proposed as an M-theory analog of brane-antibrane systems in [4], exhibits two basic instabilities. First, the Casimir effect produces an attractive force between the two boundaries, driving the theory towards weak coupling. The strength of this force per unit boundary area is given by (see [4] for details):

$$\mathcal{F} = -\frac{1}{(R_{11})^{11}} \frac{5}{2^{14}} \int_0^\infty dt t^{9/2} \theta_2(0|it), \tag{1}$$

where R_{11} is the distance between the two branes along the 11th dimension y.

Second, as was first pointed out in [4], at large separations the theory has a nonperturbative instability. This instanton is given by the Euclidean Schwarzschild solution

$$ds^2 = \left(1 - \left(\frac{4R_{11}}{\pi r}\right)^8\right) dy^2 + \frac{dr^2}{1 - (\frac{4R_{11}}{\pi r})^8} + r^2 d^2\Omega_9 \tag{2}$$

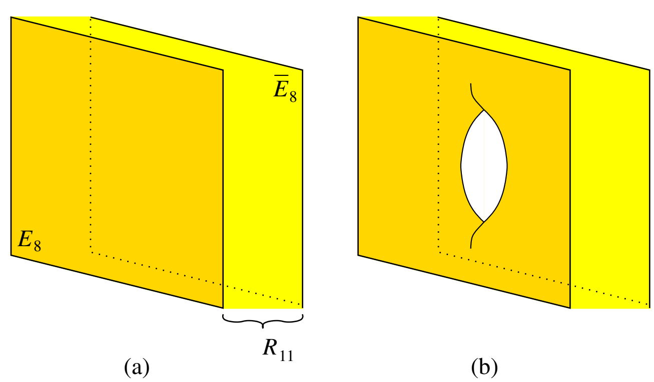

FIG. 1 (color online). (a) A schematic picture of the $E_8 \times \bar{E}_8$ heterotic M-theory. The two boundaries are separated by distance R_{11}, carry opposite orientations, and support one copy of E_8 gauge symmetry each. (b) A schematic picture of the instanton responsible for the decay of spacetime to "nothing." The instanton is a smooth throat connecting the two boundaries. Thus, the "bubble of nothing" is in fact a bubble of the hypothetical phase on the other side of the E_8 boundary.

under the $\mathbf{Z}_2$ orbifold action $y \rightarrow -y$. Here r and the coordinates in the S_9 are the other ten dimensions. This instanton is schematically depicted in Fig. 1(b).

The probability to nucleate a single "bubble of nothing" of this form is, per unit boundary area per unit time, of order

$$\exp\left(-\frac{4(2R_{11})^8}{3\pi^4 G_{10}}\right), \tag{3}$$

where G_{10} is the ten-dimensional effective Newton constant. As the boundaries are forced closer together by the Casimir force, the instanton becomes less and less suppressed. Eventually, there should be a crossover into a regime where the instability is visible in perturbation theory, as a string theory tachyon.

B. The other side of the E_8 wall

The strong-coupling picture of the instanton catalyzing the decay of spacetime to nothing suggests an interesting interpretation of this process. The instanton has only one boundary, interpolating smoothly between the two E_8 walls. Thus, the bubble of nothing that is being nucleated represents the bubble of a hypothetical phase on the other side of the boundary of 11-dimensional spacetime in heterotic M-theory. In the supergravity approximation, this phase truly represents "nothing," with no apparent spacetime interpretation. The boundary conditions at the E_8 boundary in the supergravity approximation to heterotic M-theory are reflective, and the boundary thus represents a perfect mirror. However, it is possible that more refined methods, beyond supergravity, may reveal a subtle world on the other side of the mirror. This world could correspond to a topological phase of the theory, with very few degrees of freedom (all of which are invisible in the supergravity approximation).

At first glance, it may seem that our limited understanding of M-theory would restrict our ability to improve on the semiclassical picture of spacetime decay at strong coupling. However, attempting to solve this problem at strong coupling could be asking the wrong question, and a change of perspective might be in order. Indeed, the theory itself suggests a less gloomy resolution: the problem should be properly addressed at weak string coupling, to which the system is driven by the attractive Casimir force. Thus, in the rest of the paper, our intention is to develop world sheet methods that lead to new insight into the hypothetical phase "behind the mirror," in the regime of the weak string coupling.

C. Heterotic string description at weak coupling

We conjecture that when the Casimir force has driven the E_8 boundaries into the weak coupling regime, the perturbative string description of this system is given by the little-studied tachyonic heterotic string model with one copy of E_8 gauge symmetry [16,17]. The existence of a unique tachyonic E_8 heterotic string theory in ten spacetime dimensions has always been rather puzzling. We suspect that its role in describing the weakly coupled stages of the spacetime decay in heterotic M-theory is the *raison d'être* of this previously mysterious model.

We intend to review the structure of this nonsupersymmetric heterotic string model in sufficient detail in Sec. III. Anticipating its properties, we list some preliminary evidence for this conjecture here:

(i) The E_8 current algebra is realized at level two. This is consistent with the anticipated Higgs mechanism $E_8 \times E_8 \rightarrow E_8$, analogous to that observed in brane-antibrane systems where $U(N) \times U(N)$ is first Higgsed to the diagonal $U(N)$ subgroup. (This analogy is discussed in more detail in [4].)

(ii) The nonperturbative "decay to nothing" instanton instability is expected to become—at weak string coupling—a perturbative instability, described by a tachyon which is a singlet under the gauge symmetry. The tachyon of the E_8 heterotic string is just such a singlet.

(iii) The spectrum of massless fermions is nonchiral, with each chirality of adjoint fermions present. This is again qualitatively the same behavior as in brane-antibrane systems.

(iv) The nonsupersymmetric $E_8 \times \bar{E}_8$ version of heterotic M-theory can be constructed as a $\mathbf{Z}_2$ orbifold of the standard supersymmetric $E_8 \times E_8$ heterotic M-theory vacuum. Similarly, the E_8 heterotic string is related to the supersymmetric $E_8 \times E_8$ heterotic string by a simple $\mathbf{Z}_2$ orbifold procedure.

The problem of tachyon condensation in the E_8 heterotic string theory is interesting in its own right and can be studied independently of any possible relation to instabilities in heterotic M-theory. Thus, our analysis in the remainder of the paper is independent of this conjectured relation to spacetime decay in M-theory. As we shall see, our detailed investigation of the tachyon condensation in the heterotic string at weak coupling provides further corroborating evidence in support of this conjecture.

III. THE FORGOTTEN E_8 HETEROTIC STRING

Classical Poincaré symmetry in ten dimensions restricts the number of consistent heterotic string theories to nine, of which six are tachyonic. These tachyonic models form a natural hierarchy, terminating with the E_8 model. We devote this section to a review of some of the salient aspects of the nearly forgotten heterotic E_8 theory. Most of these features have been known for quite some time but are scattered in the literature [16,19–23].

A. The free fermion language

The tachyonic E_8 string was first discovered in the free fermion description of the nonsupersymmetric left-movers [16]. The starting point of this construction is the same for all heterotic models in ten dimensions (including the better-known supersymmetric models): 32 real left-moving fermions λ^A, $A = 1, \dots, 32$, and ten right-moving superpartners ψ^μ_- of X^μ, described (in conformal gauge; see Appendix A for our conventions) by the free-field action

$$S_{\text{fermi}} = \frac{i}{2\pi\alpha'} \int d^2\sigma^\pm (\lambda^A_+ \partial_- \lambda^A_+ + \eta_{\mu\nu} \psi^\mu_- \partial_+ \psi^\nu_-). \quad (4)$$

The only difference between the various models is in the assignment of spin structures to various groups of fermions, and the consequent GSO projection. It is convenient to label various periodicity sectors by a 33-component vector whose entries take values in $\mathbf{Z}_2 = \{\pm\}$ [24],

$$\mathbf{U} = (\underbrace{\pm, \dots \pm}_{32} | \pm). \quad (5)$$

The first 32 entries indicate the (anti)periodicity of the Ath fermion λ^A, and the 33rd entry describes the (anti)periodicity of the right-moving superpartners ψ^μ of X^μ.

A specific model is selected by listing all the periodicities that contribute to the sum over spin structures. Modular invariance requires that the allowed periodicities $\mathbf{U}$ are given as linear combinations of n linearly independent basis vectors $\mathbf{U}_i$

$$\mathbf{U} = \sum_{i=1}^{n} \alpha_i \mathbf{U}_i \quad (6)$$

with $\mathbf{Z}_2$-valued coefficients α_i. Modular invariance also requires that in any given periodicity sector, the number of periodic fermions is an integer multiple of eight. All six tachyonic heterotic theories can be described using the following set of basis vectors:

$$\begin{aligned}
\mathbf{U}_1 &= (- | -),\\
\mathbf{U}_2 &= (+ + + + + + + + + + + + + + + + - - - - - - - - - - - - - - - - | -),\\
\mathbf{U}_3 &= (+ + + + + + + + - - - - - - - - + + + + + + + + - - - - - - - - | -),\\
\mathbf{U}_4 &= (+ + + + - - - - + + + + - - - - + + + + - - - - + + + + - - - - | -),\\
\mathbf{U}_5 &= (+ + - - + + - - + + - - + + - - + + - - + + - - + + - - + + - - | -),\\
\mathbf{U}_6 &= (+ - + - + - + - + - + - + - + - + - + - + - + - + - + - + - + - | -).
\end{aligned}$$

The theory which has only U_1 as a basis vector has 32 tachyons. Adding U_2 reduces the number of tachyons to 16. This process can be continued until the allowed periodicities are spanned by all six vectors $\mathbf{U}_i$; here only one bosonic tachyon is present. This most intricate of the tachyonic theories is the single-tachyon E_8 model we wish to discuss [25].

Thus, in our case there are $2^6 = 64$ different periodicity sectors. Note that there is a perfect permutation symmetry among the left-moving fermions, but this symmetry is lifted by the coupling to the spin structure of the right-moving fermions ψ^μ. There is precisely one left-moving fermion whose spin structure is always locked with the spin structure of the supersymmetric sector ψ^μ. Since this fermion λ^{32}_+ plays a special role, we shall denote it by λ_+ and refer to it as the "lone fermion" for brevity.

The tachyon in this theory is a singlet, and comes from the $(+ \dots + | +)$ sector in which all the fermions are Neveu-Schwarz. The left-moving vacuum is excited by the lowest oscillation mode $b_{-1/2}$ of the lone fermion λ_+; the right-moving vacuum is in the ground state. The vertex operator for the tachyon (in picture 0) is thus

$$\mathcal{V} = (F + \lambda p_\mu \psi^\mu) \exp(i p_\mu X^\mu). \quad (7)$$

The spectrum also contains 248 massless vector bosons. Their spacetime Yang-Mills group structure is rather obscure in the free fermion language, but they do form the adjoint of E_8. There is one family of adjoint massless fermions for each chirality; string loop effects are likely to combine these into one massive field. More information about the spectrum at higher levels can be extracted from the one-loop partition function calculated below, in (11).

The 31 free fermions $\lambda^A, A = 1 \dots 31$ realize a level-two E_8 current algebra. The E_8 current algebra at level k has central charge

$$c_{E_8,k} = \frac{k \dim E_8}{k + h} = \frac{248k}{k + 30}. \quad (8)$$

Here $h = 30$ is the dual Coxeter number of E_8. At level $k = 2$, this corresponds to the central charge of $31/2$, which agrees with the central charge of the 31 free fermions λ^A which comprise it. It is now convenient to switch to a more compact, mixed representation of the left-moving sector of the world sheet CFT. In this representation, the left-movers are succinctly described by the lone fermion $\lambda_+(\sigma^+)$ together with the algebra of E_8 currents $J^I(\sigma^+)$; here I is the adjoint index of E_8. The spin structure of λ_+ is locked with the spin structure of the right-moving superpartners ψ^μ_- of X^μ.

At level two, the E_8 current algebra has three integrable representations $|\mathbf{1}\rangle$, $|\mathbf{248}\rangle$ and $|\mathbf{3875}\rangle$, where $\mathbf{n}$ denotes the representation whose highest weight is in $\mathbf{n}$ of E_8. The conformal weights of the highest weight states in $|\mathbf{1}\rangle$, $|\mathbf{248}\rangle$

and $|\mathbf{3875}\rangle$ are 0, 15/16 and 3/2, respectively. In the spectrum of physical states, the NS and R sectors $|\pm\rangle$ of λ_+ and $\psi^\mu_\pm$ are intertwined with the representations of $(E_8)_2$, which leads to the following sectors of the physical Hilbert space,

$$|+\rangle \otimes (|\mathbf{1}\rangle \oplus |\mathbf{3875}\rangle), \qquad |-\rangle \otimes |\mathbf{248}\rangle.$$

One of the advantages of this representation is that the states charged under the E_8 symmetry are now generated by the modes of the currents J^I, making the E_8 symmetry manifest and the need for its realization via 31 free fermions with a complicated GSO projection obsolete.

B. The language of free bosons

The bosonization of this model is nontrivial. In order to rewrite a free fermion model in terms of bosonic fields, one typically associates a bosonic field with a pair of fermions. This is, however, impossible in the E_8 heterotic model: No two fermions carry the same spin structure in all sectors, due to the intricate interlacing of the spin structures reviewed above, and a twist of the conventional bosonization is needed.

As a result, the model cannot be constructed as a straight lattice compactification; however, it can be constructed as a $\mathbf{Z}_2$ orbifold of one [19]. In fact, the starting point can be the supersymmetric $E_8 \times E_8$ heterotic string in the bosonic language.

The $E_8 \times E_8$ lattice of left-moving scalars in the supersymmetric $E_8 \times E_8$ string has a nontrivial outer automorphism $\mathcal{I}$ that simply exchanges the two E_8 factors. One can use it to define a $\mathbf{Z}_2$ orbifold action on the CFT via

$$\mathcal{J} = \mathcal{I} \cdot \exp(\pi i F_s), \tag{9}$$

where F_s is the spacetime fermion number. Note that $\exp(\pi i F_s)$ can be conveniently realized as a 2π rotation in spacetime, say in the X^1, X^2 plane. The orbifold breaks supersymmetry completely, and yields the tachyonic E_8 heterotic string.

This surprisingly simple orbifold relation between the supersymmetric $E_8 \times E_8$ theory and the tachyonic E_8 model is possible because of some unique properties of the fusion algebra and the characters of E_8. The fusion rules for the three integrable highest-weight representations of $(E_8)_2$ are isomorphic to those of a free CFT of a single real fermion λ_+, with $|\mathbf{248}\rangle$ playing the role of the spin field, and $|\mathbf{3875}\rangle$ that of the fermion. This is related to the fact that the $c = 1/2$ CFT of λ_+ can be represented as a coset

$$\frac{(E_8)_1 \times (E_8)_1}{(E_8)_2}. \tag{10}$$

This explains why there is such a simple relation between the supersymmetric $E_8 \times E_8$ heterotic string and the tachyonic E_8 model: They can be viewed as two different ways of combining a sigle free-fermion theory (10) with the level-two E_8 current algebra.

In the bosonic form, the construction of the tachyonic E_8 heterotic string model is quite reminiscent of the string backgrounds studied by Chaudhuri, Hockney, and Lykken (CHL) [26]. In those models, a single copy of E_8 symmetry at level two is also obtained by a similar orbifold, but the vacua are spacetime supersymmetric [27]. It is conceivable that such supersymmetric CHL vacua in lower dimensions could represent endpoints for decay of the E_8 model when the tachyon profile is allowed an extra dependence on spatial dimensions, as in [5,6].

The one-loop partition function of the heterotic E_8 string theory can be most conveniently calculated in light cone gauge, by combining the bosonic picture for the left-movers with the Green-Schwarz representation of the right-movers. The one-loop amplitude is given by

$$\mathcal{Z} = \frac{1}{2}\int \frac{d^2\tau}{(\mathrm{Im}\tau)^2}\frac{1}{(\mathrm{Im}\tau)^4|\eta(\tau)|^{24}}\bigg\{16\Theta_{E_8}(2\tau)\frac{\theta^4_{10}(0,\bar\tau)}{\theta^4_{10}(0,\tau)} + \Theta_{E_8}(\tau/2)\frac{\theta^4_{01}(0,\bar\tau)}{\theta^4_{01}(0,\tau)} + \Theta_{E_8}((\tau+1)/2))\frac{\theta^4_{00}(0,\bar\tau)}{\theta^4_{00}(0,\tau)}\bigg\}. \tag{11}$$

For the remainder of the paper, we use the representation of the world sheet CFT in terms of the lone fermion λ and the level-two E_8 current algebra, represented by 31 free fermions.

IV. TACHYON CONDENSATION IN THE E_8 HETEROTIC STRING

A. The general philosophy

We wish to understand closed-string tachyon condensation as a dynamical spacetime process. Hence, we are looking for a time-dependent classical solution of string theory, which would describe the condensation as it interpolates between the perturbatively unstable configuration at early times and the endpoint of the condensation at late times. Classical solutions of string theory correspond to world sheet conformal field theories; thus, in order to describe the condensation as an on-shell process, we intend to maintain exact quantum conformal invariance on the world sheet. In particular, in this paper we are not interested in describing tachyon condensation in terms of an abstract RG flow between two different CFTs. In addition, we limit our attention to classical solutions, and leave the question of string loop corrections for future work.

B. The action

Before any gauge is selected, the E_8 heterotic string theory—with the tachyon condensate tuned to zero—is described in the NSR formalism by the covariant world sheet action

$$S_0 = -\frac{1}{4\pi\alpha'}\int d^2\sigma e(\eta_{\mu\nu}(h^{mn}\partial_m X^\mu \partial_n X^\nu + i\psi^\mu\gamma^m\partial_m\psi^\nu - i\kappa\chi_m\gamma^n\gamma^m\psi^\mu\partial_n X^\nu) + i\lambda^A\gamma^m\partial_m\lambda^A - F^A F^A), \tag{12}$$

where, as usual, $h_{mn} = \eta_{ab}e_m{}^a e_n{}^b$, $e = \det(e_m{}^a)$. We choose not to integrate out the auxiliary fields F^A from the action at this stage, thus maintaining its off-shell invariance under local supersymmetry, whose transformation rules on fields are given by (A8)–(A10). We have collected other useful formulas and our choices of conventions in Appendix A.

1. Linear dilaton

In order to obtain a description of tachyon condensation in terms of an exactly solvable CFT, we will consider the tachyon condensate that evolves along a null direction. Thus, our tachyon condensate will depend on a field, say X^+, which has trivial operator-product expansion (OPE) with itself. In order for such a condensate to maintain conformal invariance, we also need to turn on a linear dilaton background,

$$\Phi(X) = V_\mu X^\mu, \tag{13}$$

for some constant vector V_μ. If we wish to maintain the critical dimension equal to ten, the linear dilaton must be null, $V \cdot V = 0$. Hence, we can adjust our choice of spacetime coordinates $X^\pm$, X^i such that V is only nonzero in the X^- direction. Later on, when we turn on the tachyon profile, the linear dilaton will depend on the light cone direction X^+ instead [28].

In the presence of the linear dilaton, the covariant action of the heterotic model is $S = S_0 + S_V$, with S_0 given in (12) and

$$S_V = -\frac{1}{4\pi}\int d^2\sigma e V_\mu(X^\mu R(h) + i\kappa\chi_m\gamma^n\gamma^m D_n\psi^\mu). \tag{14}$$

Recall that we are in Minkowski signature both on the world sheet and in spacetime; this accounts for the negative sign in front of S_V. In the case of the null dilaton, $V \cdot V = 0$, both S_0 and S_V are separately Weyl invariant; the proof of this fact for S_V requires the use of the equations of motion that follow from varying X^+ in the full action. In addition, off-shell supersymmetry of $S_0 + S_V$ also requires a modification of the supersymmetry transformation rules in the supersymmetric matter sector, which now become

$$\delta X^\mu = i\epsilon\psi^\mu, \qquad \delta\psi^\mu = \gamma^m\partial_m X^\mu\epsilon + \alpha' V^\mu\gamma^m D_m\epsilon. \tag{15}$$

The remaining supersymmetry transformations (A9) and (A10) remain unmodified.

The first term in (14) produces the standard V-dependent term in the energy-momentum tensor, while the second term yields the well-known improvement term in the supercurrent of the linear dilaton theory. The second term also contributes a gravitino dependent term to the energy-momentum tensor, as we will show below.

2. Superpotential and the tachyon profile

At the classical level, the tachyon couples to the string as a world sheet superpotential. Classically, its coupling constant would be dimensionful. Additionally, the superpotential would be neither Weyl nor super-Weyl invariant: It would depend on the Liouville mode ϕ as well as its superpartner χ_{-+}.

We are only interested in adding superpotentials that are, in conformal gauge, exactly marginal deformations of the original theory. The leading-order condition for marginality requires the tachyon condensate $\mathcal{T}(X)$ to be a dimension $(1/2, 1/2)$ operator, and the quantum superpotential takes the following form,

$$S_W = -\frac{\mu}{\pi\alpha'}\int d^2\sigma(F\mathcal{T}(X) - i\lambda\psi^\mu\partial_\mu\mathcal{T}(X)); \tag{16}$$

μ is a dimensionless coupling.

With $\mathcal{T}(X) \sim \exp(k_\mu X^\mu)$ for some constant k_μ, the condition for $\mathcal{T}(X)$ to be of dimension $(1/2, 1/2)$ gives

$$-k^2 + 2V\cdot k = \frac{2}{\alpha'}. \tag{17}$$

If we wish to maintain quantum conformal invariance at higher orders in conformal perturbation theory in μ, the profile of the tachyon must be null, so that S_W stays marginal. Together with (17), this leads to

$$\mathcal{T}(X) = \exp(k_+X^+), \tag{18}$$

$$V_-k_+ = -\frac{1}{2\alpha'}. \tag{19}$$

Since our k_+ is positive, so that the tachyon condensate grows with growing X^+, this means that V_- is negative, and the theory is weakly coupled at late X^-.

From now on, we will only be interested in the specific form of the superpotential that follows from (18) and (19),

$$S_W = -\frac{\mu}{\pi\alpha'}\int d^2\sigma(F - ik_+\lambda_+\psi_-^+)\exp(k_+X^+). \tag{20}$$

Interestingly, the check of supersymmetry invariance of (20) requires the use of (19) together with the V-dependent supersymmetry transformations (15).

C. The lone fermion as a Goldstino

Under supersymmetry, the lone fermion λ_+ transforms in an interesting way,

$$\delta\lambda_+ = F\epsilon_+. \tag{21}$$

F is an auxiliary field that can be eliminated from the theory by solving its algebraic equation of motion. In the absence of the tachyon condensate, F is zero, leading to the standard (yet slightly imprecise) statement that λ_+ is a singlet under supersymmetry. In our case, when the tachyon condensate is turned on, F develops a nonzero vacuum expectation value, and λ_+ no longer transforms trivially under supersymmetry. In fact, the nonlinear behavior of λ_+ under supersymmetry in the presence of a nonzero condensate of F is typical of the Goldstino.

Traditionally, the Goldstino field η_+ is normalized such that its leading order transformation under supersymmetry is just $\delta\eta_+ = \epsilon + \ldots$, where "$\ldots$" indicates field-dependent corrections. In our case, choosing

$$\eta_+ = \frac{\lambda_+}{F} \tag{22}$$

gives the proper normalization for a Goldstino under supersymmetry. Classically, η_+ transforms as

$$\delta\eta_+ = \epsilon_+ - i\eta_+(\epsilon\gamma^m D_m \eta). \tag{23}$$

This is the standard nonlinear realization of supersymmetry on the Goldstino in the Volkov-Akulov sense. This realization of supersymmetry has also played a central role in the Berkovits-Vafa construction [29–33]. This construction has been directly linked by Hellerman and Swanson to the outcome of tachyon condensation, at least in the case of type 0 theory.

V. TACHYON CONDENSATION AND THE WORLD SHEET SUPER-HIGGS MECHANISM

Now that we have precisely defined the world sheet action in covariant form, we will show how alternative gauge choices for world sheet supersymmetry can elucidate the dynamics of the system, and, in particular, make the world sheet super-Higgs mechanism manifest.

Our alternative gauge choices will have one thing in common with superconformal gauge: For fixing the bosonic part of the world sheet gauge symmetry, we always pick the conventional conformal gauge, by setting (locally) $e_m{}^a = \delta_m{}^a$. This is logically consistent with the fact that we turn on the tachyon condensate as an exactly marginal deformation, maintaining world sheet conformal invariance throughout. In conformal gauge [and in world sheet light cone coordinates (σ^-, σ^+)], the full world sheet action becomes

$$\begin{aligned} S = \frac{1}{\pi\alpha'}\int d^2\sigma^{\pm}\Big(&\partial_+X^i\partial_-X^i + \frac{i}{2}\psi_-^i\partial_+\psi_-^i - \partial_+X^+\partial_-X^- - \frac{i}{2}\psi_-^+\partial_+\psi_-^- + \frac{i}{2}\lambda_+^A\partial_-\lambda_+^A - \mu^2\exp(2k_+X^+) \\ &- i\kappa\chi_{++}\psi_-^i\partial_-X^i + \frac{i}{2}\kappa\chi_{++}\psi_-^+\partial_-X^- + \frac{i}{2}\kappa\chi_{++}\psi_-^-\partial_-X^+ + i\kappa\alpha'V_-\chi_{++}\partial_-\psi_-^- + i\mu k_+\lambda_+\psi_-^+\exp(k_+X^+)\Big). \end{aligned} \tag{24}$$

In this action, we have integrated out the auxiliaries F^A using their algebraic equations of motion: All F^As are zero with the exception of the superpartner F of the lone fermion, which develops a nonzero condensate:

$$F = 2\mu\exp(k_+X^+). \tag{25}$$

The supersymmetry algebra now closes only on-shell, with the use of the λ_+ equation of motion. In the rest of the paper, F always refers to the composite operator in terms of X^+ as given by (25). We will use products of powers of F with other fields several times below. Because the OPE of F with any field other than X^- is trivial, these objects are quantum mechanically well-defined so long as X^- does not appear in them.

We also present the energy-momentum tensor and supercurrent, again in conformal gauge and world sheet light cone coordinates:

$$\begin{aligned} T_{++} = -\frac{1}{\alpha'}\Big\{&2i\kappa\chi_{++}\psi^\mu\partial_+X_\mu + \partial_+X^\mu\partial_+X_\mu \\ &- 2i\kappa\alpha'V_-\Big(\frac{3}{2}\chi_{++}\partial_+\psi_-^- + \frac{1}{2}(\partial_+\chi_{++})\psi_-^-\Big) \\ &+ \frac{i}{2}\lambda^A\partial_+\lambda^A - \alpha'V_-\partial_+\partial_+X^-\Big\}, \end{aligned} \tag{26}$$

$$\begin{aligned} T_{--} = -\frac{1}{\alpha'}\Big\{&\partial_-X^\mu\partial_-X_\mu + \alpha'V_-\partial_-\partial_-X^- \\ &+ \frac{i}{2}\psi^\mu\partial_-\psi_\mu\Big\}, \end{aligned} \tag{27}$$

$$\begin{aligned} G_{--} = -\frac{2}{\alpha'}\Big\{&\psi_-^i\partial_-X^i - \frac{1}{2}\psi_-^+\partial_-X^- - \frac{1}{2}\psi_-^-\partial_-X^+ \\ &- \alpha'V_-\partial_-\psi_-^-\Big\}. \end{aligned} \tag{28}$$

In the classical action (24), the condensate (25) induces a bosonic potential term $\sim\mu^2\exp(2k_+X^+)$. As shown in (26), this potential term does not contribute to the world sheet vacuum energy, since it is an operator of anomalous dimension (1, 1) and hence its integration over the world sheet does not require any dependence on the world sheet metric. Since this potential term does not contribute to the energy-momentum tensor, it will not contribute to the BRST charge either.

On the other hand, this bosonic potential does contribute to the equation of motion for $X^\pm$, which can be written locally as

$$\partial_+\partial_- X^+ = 0,$$
$$\partial_+\partial_- X^- = 2\mu^2 k_+ \exp(2k_+X^+) + \frac{i}{2}\kappa\partial_-(\chi_{++}\psi_-^-) - i\mu k_+^2 \lambda_+ \psi_-^+ \exp(k_+X^+). \tag{29}$$

These equations imply that the generic incoming physical excitations of the string are effectively shielded by the tachyon condensate from traveling too deeply into the bubble of nothing. Thus, fundamental string excitations are pushed away to infinity in the X^- direction by the walls of the "bubble of nothing." A similar phenomenon has been observed numerous times in the previous studies of closed-string tachyon condensation, see e.g. [5,6,34–36].

A. Superconformal gauge

In the conventional treatment of strings with world sheet supersymmetry, superconformal gauge is virtually always selected. In this gauge, the world sheet gravitino is simply set to zero:

$$\chi_{++} = 0. \tag{30}$$

In our background, however, we expect the gravitino to take on a prominent role as a result of the super-Higgs mechanism. For that reason, we will explore alternative gauge choices, friendlier to this more important role expected of the gravitino.

Before we introduce alternative gauge choices, however, we address some aspects of the theory in the conventional superconformal gauge. This exercise will reveal at least one intriguing feature of the model: The emergence of logarithmic CFT in the context of tachyon condensation.

Superconformal gauge leaves residual superconformal symmetry which should be realized on all fields by the action of the supercurrent G_{--}. Consider, in particular, the lone fermion λ_+. Before the tachyon condensate is turned on, the operator product of G_{--} with λ_+ is nonsingular, in accord with the fact that λ_+ transforms trivially under on-shell supersymmetry. As we have seen in Sec. IV, when the auxiliary field F develops a nonzero vacuum expectation value in the process of tachyon condensation, λ_+ transforms under supersymmetry nontrivially, as a candidate Goldstino. This raises an interesting question: How can this nontrivial transformation be reproduced by the action of G_{--} on λ_+, if, as we have seen in (28), the supercurrent G_{--} is unmodified by μ?

The resolution of this puzzle must come from nontrivial OPEs that develop at finite μ between the originally left-moving field λ_+ and the originally right-moving fields $\psi_-^\pm$. Here and in the following, it will be useful to introduce a rescaled version of the fields $\psi_-^\pm$,

$$\tilde{\psi}_-^- = \psi_-^-/F, \qquad \tilde{\psi}_-^+ = F\psi_-^+. \tag{31}$$

We will encounter this particular rescaled version of ψ_-^- again below, in another gauge. In terms of $\tilde{\psi}_-^-$, the supercurrent (28) simplifies to

$$G_{--} = -\frac{2}{\alpha'}\left\{\psi_-^i\partial_- X^i - \frac{1}{2}\psi_-^+\partial_- X^- - \alpha' V_- F\partial_-\tilde{\psi}_-^-\right\}. \tag{32}$$

The supersymmetry variations of fields are reproduced as follows. Consider, for example, the supermultiplet ψ_-^i and X^i. In superconformal gauge, these are free fields, satisfying standard OPEs such as

$$\psi_-^i(\sigma^\pm)\psi^j(\tau^\pm) \sim \frac{\alpha'\delta^{ij}}{\sigma^- - \tau^-}, \tag{33}$$

which imply

$$G_{--}(\sigma^\pm)\psi_-^i(\tau^\pm) \sim \frac{-2\partial_- X^i(\tau^\pm)}{\sigma^- - \tau^-}, \tag{34}$$

correctly reproducing the supersymmetry variation $\delta\psi_-^i = -2\partial_- X^i\epsilon$.

Similarly, the last term in (32) will reproduce the supersymmetry transformation $\delta\lambda_+ = F\epsilon$ if the following OPE holds,

$$2\alpha' V_-\partial_-\psi_-^-(\sigma^\pm)\lambda_+(\tau^\pm) \sim \frac{F(\tau^\pm)/F(\sigma^\pm)}{\sigma^- - \tau^-}. \tag{35}$$

This required OPE can be checked by an explicit calculation: Starting with the free-field OPEs

$$\psi_-^+(\sigma^\pm)\psi_-^-(\tau^\pm) \sim \frac{-2\alpha'}{\sigma^- - \tau^-}, \tag{36}$$

we get for the rescaled fields (31)

$$\tilde{\psi}_-^+(\sigma^\pm)\tilde{\psi}_-^-(\tau^\pm) \sim \frac{-2\alpha' F(\sigma^\pm)/F(\tau^\pm)}{\sigma^- - \tau^-} \sim \frac{-2\alpha'}{\sigma^- - \tau^-} \times \sum_{n=0}^{\infty}\frac{1}{n!}(\sigma^+ - \tau^+)^n(\partial_+^n F(\tau^\pm))/F(\tau^\pm). \tag{37}$$

Note that since X^+ satisfies locally the free equation of motion $\partial_+\partial_- X^+ = 0$, the coefficient of the $1/(\sigma^- - \tau^-)$ term in this OPE is only a function of σ^+ and τ^+. The OPE between $\partial_-\tilde{\psi}_-^-$ and λ_+ can then be determined from the equation of motion for λ_+:

$$\partial_-\lambda_+(\sigma) = -\mu k_+\psi_-^+\exp(k_+X^+) = -\frac{k_+}{2}\tilde{\psi}_-^+. \tag{38}$$

Combining these last two equations, we get

$$\partial_-\lambda_+(\sigma^\pm)\tilde{\psi}_-^-(\tau^\pm) = -\frac{k_+}{2}\tilde{\psi}_-^+(\sigma^\pm)\tilde{\psi}_-^-(\tau^\pm) = \alpha' k_+\frac{F(\sigma^\pm)/F(\tau^\pm)}{\sigma^- - \tau^-}. \tag{39}$$

Integrating the result with respect to σ^-, we finally obtain

$$\lambda_+(\sigma^\pm)\tilde{\psi}_-^-(\tau^\pm) \sim \alpha' k_+ \left\{ \sum_{n=0}^{\infty} \frac{1}{n!}(\sigma^+ - \tau^+)^n (\partial_+^n F(\tau^\pm))/F(\tau^\pm) \right\} \times \log(\sigma^- - \tau^-). \quad (40)$$

One can then easily check that this OPE implies (35) when (19) is invoked. This in turn leads to

$$G_{--}(\sigma^\pm)\lambda_+(\tau^\pm) \sim \frac{F(\tau^\pm)}{\sigma^- - \tau^-}, \quad (41)$$

and the supersymmetry transformation of λ_+ is correctly reproduced quantum mechanically, even in the presence of the tachyon condensate.

As is apparent from the form of (40), our theory exhibits—in superconformal gauge—OPEs with a logarithmic behavior. This establishes an unexpected connection between models of tachyon condensation and the branch of 2D CFT known as "logarithmic CFT" (or LCFT; see, e.g., [37,38] for reviews). The subject of LCFT has been vigorously studied in recent years, with applications to a wide range of physical problems, in particular, to systems with disorder. The logarithmic behavior of OPEs is compatible with conformal symmetry, but not with unitarity. Hence, it can only emerge in string backgrounds in Minkowski spacetime signature, in which the time dependence plays an important role (such as our problem of tachyon condensation). We expect that the concepts and techniques developed in LCFTs could be fruitful for understanding time-dependent backgrounds in string theory. In the present work, we will not explore this connection further.

B. Alternatives to superconformal gauge

In the presence of a super-Higgs mechanism, another natural choice of gauge suggests itself. In this gauge, one anticipates the assimilation of the Goldstone mode by the gauge field, by simply gauging away the Goldstone mode altogether. In the case of the bosonic Higgs mechanism, this gauge is often referred to as "unitary gauge" as it makes unitarity of the theory manifest.

Following this strategy, we will first try eliminating the Goldstino as a dynamical field, for example, by simply choosing

$$\lambda_+ = 0 \quad (42)$$

as our gauge fixing condition for local supersymmetry. Of course, this supplements the conformal gauge choice that we have made for world sheet diffeomorphisms. This gauge choice gives rise to nonpropagating bosonic superghosts, together with the usual propagating fermionic b, c system of central charge $c = -26$. We refer to this gauge as "unitary gauge" only due to its ancestry in the similar gauges that proved useful in the Higgs mechanism of Yang-Mills gauge theories. In fact, as we shall see, the proper implementation of this "unitary" gauge will lead to propagating superghosts, and therefore no manifest unitarity of the theory.

In addition to the conventional fermionic b, c ghosts from conformal gauge, this gauge choice would lead to nonpropagating bosonic ghosts, which might be integrated out algebraically. More importantly, this gauge choice still leaves complicated interaction terms, such as

$$\chi_{++}\psi_-^+\partial_- X^-, \quad (43)$$

in the gauge-fixed version of (24). Moreover, if the algebraic superghosts are integrated out, the equation of motion arising from variation of the action under λ_+,

$$\mu k_+ \psi_-^+ \exp(k_+ X^+) = 0, \quad (44)$$

needs to be imposed on physical states by hand. This could be accomplished by simply setting $\psi_-^+ = 0$. This leads to another constraint, imposing the equation of motion obtained from the variation of ψ_-^+ in the original action as a constraint on physical states.

Instead of resolving such difficulties, we will restart our analysis with $\psi_-^+ = 0$ as the gauge condition. As we will see below, this condition makes the gauge-fixing procedure transparent.

C. Liberating the world sheet gravitino in an alternative gauge

We will now explicitly consider the gauge that begins simply by setting

$$\psi_-^+ = 0. \quad (45)$$

If ψ_-^+ is eliminated from the action, the remaining χ_{++}, ψ_-^- system can be rewritten as a purely left-moving first-order system of conformal weights $(3/2, 0)$ and $(-1/2, 0)$ [39]. This can be seen as follows. Consider first the terms in the action that are bilinear in these two fields,

$$\tfrac{1}{2}\chi_{++}\psi_-^-\partial_- X^+ + \alpha' V_- \chi_{++}\partial_-\psi_-^-. \quad (46)$$

The presence of ∂_- here suggests χ_{++} and ψ_-^- ought to become purely left-moving. Additionally, these fields show up only in the energy-momentum tensor T_{++} in this gauge. However, their conformal weights do not reflect their left-moving nature. In order to obtain fields whose conformal weights are nonzero only in the left-moving sector, we can rescale

$$\tilde{\chi}_{++} = F\chi_{++}, \qquad \tilde{\psi}_-^- = \frac{\psi_-^-}{F}. \quad (47)$$

This rescaling leads to an additional benefit: The bilinear terms (46) in the classical action now assemble into the canonical kinetic term of a first-order system of spin 3/2 and $-1/2$,

$$\frac{i\kappa V_-}{\pi}\int d^2\sigma^\pm \tilde{\chi}_{++}\partial_-\tilde{\psi}_-^-, \quad (48)$$

with central charge $c_L = -11$.

The first-order system (48) describes the world sheet gravitino sector of the theory. In superconformal gauge in the absence of the tachyon condensate, the gravitino was nondynamical and led to a constraint. Here, instead, the gravitino has been liberated as a result of the super-Higgs mechanism: together with its conjugate $\tilde{\psi}_-^-$, at late times it appears to have formed a left-moving free-field massless system, of central charge $c = -11$. In this modified unitary gauge, the gravitino has been literally set free: It has become a free propagating massless field!

Our gauge choice (45) reduces the classical action significantly, to

$$S = \frac{1}{\pi\alpha'}\int d^2\sigma^{\pm}\Big(\partial_+ X^i\partial_- X^i + \frac{i}{2}\psi_-^i\partial_+\psi_-^i - \partial_+ X^+\partial_- X^- + \frac{i}{2}\lambda_+^A\partial_-\lambda_+^A - \mu^2\exp(2k_+X^+) - i\kappa\chi_{++}\psi_-^i\partial_- X^i + \frac{i}{2}\kappa\chi_{++}\psi_-^-\partial_- X^+ + i\kappa\alpha' V_-\chi_{++}\partial_-\psi_-^-\Big). \quad (49)$$

Note that unlike in the case of superconformal gauge, the gauge fixing condition (45) leaves no residual unfixed supersymmetries, at least at finite μ. Thus, in this gauge, the theory will be conformal, but not superconformal. In Section VI, we shall return to the issue of residual supersymmetry in this gauge, in the late-time limit of $\mu \to \infty$.

The action (49) will be corrected by one-loop effects. The first such correction is due to the Faddeev-Popov (super)determinant $\Delta_{\rm FP}$. As we will now see, the inherent X^+ dependence in our gauge fixing condition renders $\Delta_{\rm FP}$ dependent on X^+ as well.

Our full gauge fixing condition consists of the bosonic conformal gauge, $e_m{}^a = \delta_m{}^a$, as well as the fermionic condition (45). Note first that the corresponding Faddeev-Popov superdeterminant factorizes into the ratio of two bosonic determinants [40],

$$\Delta_{\rm FP} = J_{bc}/J_{\psi_-^+\epsilon}. \quad (50)$$

Here J_{bc} arises from the conformal gauge condition, and produces the standard set of fermionic b, c ghosts with central charge $c = -26$. On the other hand, $J_{\psi_-^+\epsilon}$, which comes from the change of variables between the gauge-fixing condition ψ_-^+ and the infinitesimal supersymmetry parameter ϵ, turns out to be more complicated. It will be useful to rewrite the variation of ψ_-^+ as

$$\delta\psi_-^+ = -2\partial_- X^+\epsilon + 4\alpha' V_- D_-\epsilon = \frac{4\alpha' V_-}{F}D_-(F\epsilon). \quad (51)$$

Here, we are only fixing world sheet diffeomorphisms and not Weyl transformations, in order to allow a systematic check of the vanishing of the Weyl anomaly. We are thus in a gauge where $h_{mn} = e^{2\phi}\hat{h}_{mn}$. The derivative D_- preserves information about the spin of the field it acts on. We can now see the denominator of (50) should be defined as

$$J_{\psi_-^+\epsilon} = \det\Big(\frac{1}{F}D_- F\Big). \quad (52)$$

As in the case of J_{bc}, we wish to represent this determinant as a path integral over a conjugate pair of bosonic fields β, γ: the bosonic ghosts associated with the fixing of local supersymmetry. Note, however, that the variation of our fixing condition is dependent on X^+. This fact leads to interesting additional subtleties, which we discuss in detail in Appendix B.

As a result, the Jacobian turns out to be given by a set of ghosts $\tilde{\beta}$, $\tilde{\gamma}$ of central charge $c = -1$ whose path integral measure is independent of X^+, plus an extra term that depends explicitly on X^+. As shown in Eq. (B17) of Appendix B, we find

$$\frac{1}{J_{\psi_-^+\epsilon}} = \exp\Big\{-\frac{i}{\pi\alpha'}\int d^2\sigma^{\pm}\alpha' k_+^2\partial_+X^+\partial_-X^+\Big\} \times \int \mathcal{D}\tilde{\beta}\mathcal{D}\tilde{\gamma}\exp(iS_{\tilde{\beta}\tilde{\gamma}}), \quad (53)$$

where $S_{\tilde{\beta}\tilde{\gamma}}$ is the ghost action for a free left-moving bosonic ghost system of spin $1/2$.

The Faddeev-Popov determinant thus renormalizes the spacetime metric. If this were the whole story, we could rediagonalize the spacetime metric to the canonical light cone form, by redefining the spacetime light cone coordinates

$$Y^+ = X^+, \qquad Y^- = X^- + \alpha' k_+^2 X^+. \quad (54)$$

In these new coordinates, the linear dilaton would acquire a shift:

$$\Phi = V_- X^- = V_- Y^- - V_-\alpha' k_+^2 X^+ = V_- Y^- + \frac{k_+}{2}Y^+. \quad (55)$$

The effective change in the central charge from this shift in the linear dilaton would be

$$c_{\rm dil} = 6\alpha' V^2 = -24\alpha' V_- V_+ = 6, \quad (56)$$

where we have again used (17).

However, understanding the Faddeev-Popov determinant is not the whole story. In addition, the X^+-dependent rescaling of χ_{++} and ψ_-^- as in (47) also produces a subtle Jacobian $\tilde{J}$. As shown in Appendix B, this Jacobian can be expressed in Minkowski signature as

$$\tilde{J} = \exp\left\{-\frac{i}{4\pi\alpha'}\int d^2\sigma\hat{e}(\alpha' k_+^2 \hat{h}^{mn}\partial_m X^+ \partial_n X^+ + \alpha' k_+ X^+ R)\right\}. \tag{57}$$

In this sense, the original χ_{++}, ψ_-^- system is equivalent to the canonical $\tilde{\chi}_{++}$, $\tilde{\psi}_-^-$ system when this additional renormalization of the linear dilaton term and the spacetime metric are taken into account. In light cone coordinates, the first factor in (57) becomes

$$\exp\left\{\frac{i}{\pi\alpha'}\int d^2\sigma^\pm \alpha' k_+^2 \partial_+ X^+ \partial_- X^+\right\}. \tag{58}$$

This contribution to the renormalization of the metric precisely cancels the contribution obtained from the Faddeev-Popov determinant (53). Thus, the combined effect of $\tilde{J}$ and $J_{\psi_-^+\epsilon}$ on the $X^\pm$ fields is just a simple shift of the linear dilaton in the original $X^\pm$ variables, as implied by the second term in (57). The contribution to the central charge due to this dilaton shift is $c = 12$, as $c_{\rm dil} = 6\alpha' V^2 = 12$.

The full action, still in conformal gauge, is thus given by:

$$\begin{aligned} S = \frac{1}{\pi\alpha'}\int d^2\sigma^\pm\Big(&\partial_+ X^i \partial_- X^i + \frac{i}{2}\psi_-^i \partial_+ \psi_-^i \\ &- \partial_+ X^+ \partial_- X^- + \frac{i}{2}\lambda_+^A \partial_- \lambda_+^A - \mu^2 \exp(2k_+ X^+) \\ &- i\kappa\frac{\tilde{\chi}_{++}}{F}\psi_-^i\partial_- X^i + i\kappa\alpha' V_- \tilde{\chi}_{++}\partial_- \tilde{\psi}_-^-\Big) + S_{bc} \\ &+ S_{\tilde{\beta}\tilde{\gamma}}. \end{aligned} \tag{59}$$

The information about the linear dilaton profile is absent in this action, but the energy-momentum tensor of the theory reveals that the dilaton is given by

$$V_\mu = (k_+, V_-, \vec{0}). \tag{60}$$

In this form, the action has only canonical kinetic terms, plus the potential term and an interaction term which will vanish at late times (since it goes as $1/F$). Specifically, we note that the fields $\tilde{\chi}_{++}$, $\tilde{\psi}_-^-$ have now become purely left-moving. This matches their conformal weights, which are $(3/2, 0)$ and $(1/2, 0)$ respectively. In addition, they contribute only to the left-moving energy-momentum tensor; their contribution results in -11 units of central charge for c_L, in the late X^+ limit when they decouple from the transverse degrees of freedom X^i, ψ^i.

Now, let us summarize the central charge contributions of each field present in this gauge, at late times. For comparison, we also present the central charge breakdown for the free theory in superconformal gauge.

Superconformal gauge

Field	c_L	c_R
X^+, X^-, X^i	10	10
linear dilaton	0	0
bc ghosts	-26	-26
ψ^i	0	4
ψ^+, ψ^-	0	1
β, γ ghosts	0	11
λ_+	1/2	0
$(E_8)_2$	31/2	0

The alternative gauge

Field	c_L	c_R
X^+, X^-, X^i	10	10
linear dilaton	12	12
bc ghosts	-26	-26
ψ^i	0	4
$\tilde{\chi}$, $\tilde{\psi}^-$	-11	0
$\tilde{\beta}$, $\tilde{\gamma}$ ghosts	-1	0
λ_+	1/2	0
$(E_8)_2$	31/2	0

(61)

Our new gauge choice has resulted in 12 units of central charge in the right-moving sector, from $\psi^\pm$ and the $\beta\gamma$ system, effectively moving to become -12 units of central charge on the left. These left-moving central charge units come from $\tilde{\chi}$, $\tilde{\psi}^-$, and the new $\tilde{\beta}\,\tilde{\gamma}$ ghosts. We also see that the shifted linear dilaton precisely compensates for this relocation of central charge, resulting in the theory in the alternative gauge still being exactly conformal at the quantum level.

Interestingly, the equation of motion that follows from varying ψ_-^+ in the original action allows us to make contact with the original unitary gauge. Classically, this equation of motion is

$$\partial_+ \psi_-^- + \kappa\chi_{++}\partial_- X^- + 2\mu k_+ \lambda_+ \exp(k_+ X^+) = 0. \tag{62}$$

This constraint can be interpreted and solved in a particularly natural way: Imagine solving the $X^\pm$ and χ_{++}, ψ_-^- sectors first. Then one can simply use the constraint to express λ_+ in terms of those other fields. Thus, the alternative gauge still allows the gravitino to assimilate the Goldstino in the process of becoming a propagating field. This is how the world sheet super-Higgs mechanism is implemented, in a way compatible with conformal invariance.

We should note that this classical constraint (62) could undergo a one-loop correction analogous to the one-loop shift in the dilaton. We might expect a term $\sim\partial_-\chi_{++}$, from varying a one-loop supercurrent term $\sim\chi_{++}\partial_-\psi_-^\pm$ in the full quantum action. Such a correction would not change the fact that λ_+ is determined in terms of the oscillators of other fields, it would simply change the precise details of such a rewriting.

D. R_ξ gauges

The history of understanding the Higgs mechanism in Yang-Mills theories was closely linked with the existence of a very useful family of gauge choices, known as R_ξ

gauges. R_ξ gauges interpolate—as one varies a control parameter ξ—between unitary gauge and one of the more traditional gauges (such as Lorentz or Coulomb gauge).

In string theory, one could similarly consider families of gauge fixing conditions for world sheet supersymmetry which interpolate between the traditional superconformal gauge and our alternative gauge. We wish to maintain conformal invariance of the theory in the new gauge, and will use conformal gauge to fix the bosonic part of the world sheet gauge symmetries. Because they carry disparate conformal weights $(1, -1/2)$ and $(0, 1/2)$ respectively, we cannot simply add the two gauge fixing conditions χ_{++} and ψ_-^+ with just a relative constant. In order to find a mixed gauge-fixing condition compatible with conformal invariance, we need a conversion factor that makes up for this difference in conformal weights. One could, for example, set

$$(\partial_- X^+)^2\chi_{++} + \xi F^2\psi_-^+ = 0, \tag{63}$$

with ξ a real constant (of conformal dimension 0). The added advantage of such a mixed gauge is that it interpolates between superconformal and alternative gauge not only as one changes ξ, but also at any fixed ξ as X^+ changes: At early light cone time X^+, the superconformal gauge fixing condition dominates, while at late X^+, the alternative gauge takes over, due to the relative factors of F between the two terms in (63).

Even though (63) is compatible with conformal invariance, it is highly nonlinear, and therefore impractical as a useful gauge fixing condition. Another, perhaps more practical, condition that incorporates both χ_{++} and ψ_-^+ is

$$\partial_-\chi_{++} + \xi\partial_+\psi_-^+ = 0. \tag{64}$$

Here the mismatch in conformal weights has been made up by inserting world sheet derivatives, rather than other composite operators. This condition can be written in the following covariant form,

$$\gamma^m\gamma^n D_n\chi_m - 2\xi\gamma^m D_m\psi^+ = 0, \tag{65}$$

demonstrating its compatibility with diffeomorphism invariance.

Note that (64) correctly anticipates the dynamics of the gravitino: If we solve for $\partial_+\psi_-^+$ using (64) in the kinetic term $\psi_-^-\partial_+\psi_-^+$, we get $\psi_-^-\partial_-\chi_{++}$. Thus, χ_{++} replaces ψ_-^+ as the conjugate partner of ψ_-^-, and turns it from a right-moving field into a left-moving one. Note also that both terms in (64) transform under supersymmetry as $\sim\partial_+\partial_-\epsilon + \ldots$, which implies that the bosonic superghosts $\hat\beta$, $\hat\gamma$ associated with this gauge fixing will have a second-order kinetic term,

$$\sim\int d^2\sigma^\pm(\hat\beta\partial_+\partial_-\hat\gamma + \ldots). \tag{66}$$

In the $\mu\to\infty$ limit, this gauge can be expected to leave a residual fermionic gauge symmetry.

Various other classes of R_ξ-type gauges can be considered. For example, one could study combinations of superconformal gauge with the naive unitary gauge in which λ_+ is set to zero, leading to gauge fixing conditions such as

$$F\chi_{++} + \xi k_+\lambda_+\partial_+X^+ = 0, \tag{67}$$

or

$$F\lambda_+ + \xi\chi_{++}k_+\partial_-X^+ = 0. \tag{68}$$

Another interesting possibility is

$$\chi_{++} + \xi\partial_+\left(\frac{\lambda_+}{F}\right) = 0, \tag{69}$$

which would interpolate between superconformal gauge and a weaker form of unitary gauge, in which the left-moving part of the Goldstino is set to zero.

We have not investigated the world sheet theory in these mixed gauges, but it would be interesting to see if any of them shed some new light on the dynamics of tachyon condensation.

VI. THE CONDENSED PHASE: EXPLORING THE WORLD SHEET THEORY AT $\mu = \infty$

The focus of the present paper has been on developing world sheet techniques that can elucidate the super-Higgs mechanism and the dynamics of the world sheet gravitino in the process of spacetime tachyon condensation. Here we comment briefly on the structure of the world sheet theory in the regime where the tachyon has already condensed.

This condensed phase corresponds to the system at late X^+. In the world sheet theory, a constant translation of X^+ rescales the value of the superpotential coupling μ, with the late X^+ limit mapping to $\mu\to\infty$. In that limit, the world sheet theory simplifies in an interesting way. First, we rescale the parameter of local supersymmetry transformation,

$$\tilde\epsilon_+ = F\epsilon_+. \tag{70}$$

The supersymmetry variations then reduce in the $\mu = \infty$ limit and in conformal gauge to

$$\begin{aligned} &\delta X^+ = 0, &&\delta\tilde\psi_-^+ = 4\alpha' V_-\partial_-\tilde\epsilon_+,\\ &\delta X^- = -i\tilde\psi_-^-\tilde\epsilon_+, &&\delta\tilde\psi_-^- = 0,\\ &\delta X^i = 0, &&\delta\tilde\psi_-^i = 0,\\ &\delta\lambda_+ = \tilde\epsilon_+, &&\delta\tilde\chi_{++} = \frac{2}{\kappa}(\partial_+\tilde\epsilon_+ - k_+\partial_+X^+\tilde\epsilon_+). \end{aligned} \tag{71}$$

Note that X^+ is now invariant under supersymmetry. Consequently, the terms in the action that originate from the superpotential,

$$-2\mu^2\exp(2k_+X^+) - ik_+\lambda_+\tilde\psi_-^+, \tag{72}$$

are now separately invariant under (71), in the strict $\mu = \infty$ limit. This in turn implies that we are free to drop the potential term $\sim\mu^2 \exp(2k_+X^+)$ without violating supersymmetry. The resulting model is then described, in the alternative gauge of Sec. V C, by a free field action:

$$S_{\mu=\infty} = \frac{1}{\pi\alpha'}\int d^2\sigma^{\pm}\Big(\partial_+X^i\partial_-X^i + \frac{i}{2}\psi^i_-\partial_+\psi^i_- - \partial_+X^+\partial_-X^- + \frac{i}{2}\lambda^A_+\partial_-\lambda^A_+ + i\kappa\alpha' V_-\tilde{\chi}_{++}\partial_-\tilde{\psi}^-_-\Big) + S_{bc} + S_{\tilde{\beta}\,\tilde{\gamma}}. \tag{73}$$

We argued in Sec. V C that at finite μ, our alternative gauge (45) does not leave any residual unfixed supersymmetry, making the theory conformal but not superconformal. This is to be contrasted with superconformal gauge, which leaves residual right-moving superconformal symmetry. At $\mu = \infty$, however, it turns out that the alternative gauge (45) does leave an exotic form of residual supersymmetry. Note first that at any μ, the fermionic gauge fixing condition is respected by $\tilde{\epsilon}$ that satisfy

$$\partial_-\epsilon + k_+\partial_-X^+\epsilon \equiv \frac{1}{F}\partial_-\tilde{\epsilon}_+ = 0. \tag{74}$$

At finite μ, these apparent residual transformations do not preserve the bosonic part of our gauge fixing condition, $e_m{}^a = \delta^a_m$. In the $\mu = \infty$ limit, however, the supersymmetry transformation of e^a_m is trivial, and all solutions of (74) survive as residual supersymmetry transformations.

Unlike in superconformal gauge, this residual supersymmetry is *left-moving*. Moreover, the generator $\tilde{\epsilon}_+$ of local supersymmetry is of conformal dimension (1/2, 0); hence, local supersymmetry cannot be expected to square to a world sheet conformal transformation. As is clear from (71), this local supersymmetry is in fact nilpotent.

This residual supersymmetry $\tilde{\epsilon}(\sigma^+)$ can be fixed by a supplemental gauge choice, setting a purely left-moving fermion to zero. The one that suggests itself is λ_+, which in this gauge satisfies the free equation of motion; moreover, λ_+ transforms very simply under $\tilde{\epsilon}$ supersymmetry, and setting it to zero fixes that symmetry completely. Once we add the condition $\lambda_+ = 0$ to the gauge choice $\psi^\pm_- = 0$, this gauge now becomes very similar to the naive unitary gauge of Sec. V B. This is another way of seeing that the Goldstino is not an independent dynamical field, since it has been absorbed into the dynamics of the other fields in the process of setting the gravitino free.

Alternatively, one can leave the residual supersymmetry unfixed, and instead impose the constraint (62) which reduces in the $\mu = \infty$ limit to

$$\lambda_+ = 2\alpha' V_-\partial_+\tilde{\psi}^-_-. \tag{75}$$

Depending on whether the coupling of the quantum bosonic potential $\exp(2k_+X^+)$ is tuned to zero or not, we have two different late-time theories: One whose equations of motion expel all degrees of freedom to future infinity along X^- for some constant X^+, the other allowing perturbations to reach large X^+. It is the latter theory which may represent a good set of variables suitable for understanding the physics at late X^+. If the potential is retained, the physical string modes are pushed away to infinity along X^- before reaching too deeply into the condensed phase, confirming that very few of the original stringy degrees of freedom are supported there.

VII. CONCLUSIONS

A. Overview

The main focus of this paper has been on examining the world sheet theory of tachyon condensation in the E_8 heterotic string model. We studied the theory with a linear dilaton $V = V_-X^-$, and a tachyon profile $\mathcal{T}(X^\mu) = 2\mu \exp(k_+X^+)$. At first glance, this tachyon profile produces a world sheet potential term which expels all degrees of freedom from the large X^+ region, indicating a possible topological phase, conjecturally related to the nothing phase in heterotic M-theory.

We found that the world sheet dynamics of tachyon condensation involves a super-Higgs mechanism, and that its analysis simplifies when local world sheet supersymmetry is fixed in a new gauge, specifically conformal gauge augmented by $\psi^\pm_- = 0$. Following a detailed analysis of one-loop measure effects, we found that exact quantum conformal invariance is maintained throughout in this gauge. At late times, the world sheet theory contains a free left-moving propagating gravitino sector (obscured in superconformal gauge, as there the gravitino is set to zero). The gravitino sector contributes -11 units of central charge to the left-movers. In addition, the gauge fixing leads to a set of left-moving ghosts with $c = -1$, and a spacelike shifted linear dilaton $V = V_-X^- + k_+X^+$.

In the process of making the gravitino dynamical, the world sheet Goldstino λ_+ has been effectively absorbed into the rest of the system; more precisely, the constraint generated by the alternative gauge can be solved by expressing λ_+ in terms of the remaining dynamical degrees of freedom.

B. Further analysis of the E_8 system

In this paper, we have laid the groundwork for an in-depth analysis of the late-time physics of the E_8 heterotic string under tachyon condensation. The emphasis here has been on developing the world sheet techniques, aimed, in particular, at clarifying the super-Higgs mechanism. The next step, which we leave for future work, would be to examine the spacetime physics in the regime of late X^+ where the tachyon has condensed. There are signs indicating that this phase contains very few conventional degrees of freedom; more work is needed to provide further evidence for the conjectured relation between tachyon condensation in the E_8 string and the spacetime decay to nothing in $E_8 \times \bar{E}_8$ heterotic M-theory.

It would be interesting to use the standard tools of string theory, combined with the new world sheet gauge, to study the spectrum and scattering amplitudes of BRST invariant states in this background, in particular, at late times. The use of mixed R_ξ gauges could possibly extend the range of such an analysis, by interpolating between the superconformal gauge and its alternative.

C. Towards nonequilibrium string theory

In the process of the world sheet analysis presented in this paper, we found two features which we believe may be of interest to a broader class of time-dependent systems in string theory: (1) in superconformal gauge, the spacetime tachyon condensate turns the world sheet theory into a logarithmic CFT; and (2) the world sheet dynamics of some backgrounds may simplify in alternative gauge choices for world sheet supersymmetry.

We have only explored the first hints of the LCFT story and its utility in the description of string solutions with substantial time dependence. The new gauge choices, however, are clearly applicable to other systems. As an example, consider the type 0 model studied in [7]. We can pick a gauge similar to our alternative gauge (45) by setting $\psi_-^+ = \psi_+^+ = 0$, again in addition to conformal gauge. We expect to simply double the gauge fixing procedure in Sec. V C, producing one copy of $c = -1$ superghosts and one copy of the propagating gravitino sector in both the left and right movers. When the one-loop determinant effects are included, the linear dilaton shifts by $2k_+$, resulting in additional 24 units of central charge. Together with the two $c = -11$ sectors and the two $c = -1$ ghost sectors, this shift again leads $c_{\rm tot} = 0$, similarly to the heterotic model studied in the present paper. Exact conformal invariance is again maintained at the quantum level. This is in accord with the results of [7]; however, our results are not manifestly equivalent to those of [7], as a result of a different gauge choice. In addition, our results also suggest an interpretation of the somewhat surprising appearance of fermionic first-order systems with $c = -11$ in [7]: It is likely that they represent the reemergence of the dynamical world sheet gravitino, in superconformal gauge.

This picture suggests a new kind of world sheet duality in string theory. Instead of viewing one CFT in two dual ways while in superconformal gauge, the new duality is between two different CFTs representing the same solution of string theory, but in two different world sheet gauges. The physics of BRST observables should of course be gauge independent, but this does not require the CFTs to be isomorphic. In fact, the alternative gauge studied in the body of this paper represents an example: It leads to a conformal, but not superconformal theory, yet it should contain the same physical information as the SCFT realization of the same string background in superconformal gauge.

We hope that this technique of using gauge choices other than superconformal gauge will be applicable to a wider class of time-dependent string backgrounds, beyond the case of models with tachyon condensation studied here, and that it will increase our understanding of nonequilibrium string theory.

ACKNOWLEDGMENTS

One of us (P. H.) wishes to thank Allan Adams, Michal Fabinger, Uday Varadarajan, and Bruno Zumino for useful discussions in the early stages of this project, in 2001-02. This work has been supported by NSF Grants PHY-0244900 and PHY-0555662, DOE Grant DE-AC03-76SF00098, and the Berkeley Center for Theoretical Physics. One of us (C.A.K.) received additional support from a NSF Graduate Student Grant.

APPENDIX A: SUPERSYMMETRY CONVENTIONS

In the NSR formalism, the tachyonic E_8 heterotic string is described by world sheet supergravity with (0, 1) supersymmetry [41]. Here we list our conventions for world sheet supergravity.

1. Flat world sheet

In the world sheet coordinates $\sigma^a = (\sigma^0 \equiv \tau, \sigma^1 \equiv \sigma)$, the flat Lorentz metric is

$$\eta_{ab} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{A1}$$

with gamma matrices given by

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{A2}$$

and satisfying $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$. We define the chirality matrix

$$\Gamma \equiv \gamma^0\gamma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A3}$$

The two-component spinor indices α are raised and lowered using the natural symplectic structure,

$$\xi^\alpha \equiv \varepsilon^{\alpha\beta}\xi_\beta, \qquad \varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{A4}$$

with $\varepsilon^{\alpha\beta}\varepsilon_{\beta\gamma} = \varepsilon_{\gamma\beta}\varepsilon^{\beta\alpha} = \delta^\alpha_\gamma$. This implies, for example, that $\xi^\alpha\zeta_\alpha = \zeta^\alpha\xi_\alpha = -\xi_\alpha\zeta^\alpha$ and $\xi\gamma^a\zeta = -\zeta\gamma^a\xi$, for any two real spinors ξ, ζ.

Any two-component spinor ξ_α can be decomposed into its chiral components, defined via

$$\xi_\alpha \equiv \begin{pmatrix} \xi_+ \\ \xi_- \end{pmatrix}, \qquad \xi_\pm = \frac{1}{2}(1 \pm \Gamma)\xi. \tag{A5}$$

2. Local world sheet supersymmetry

On curved world sheets, we will distinguish the world sheet index m from the internal Lorentz index a. The spacetime index μ runs over $0, \ldots D-1$, typically with $D = 10$. The heterotic string action with (0, 1) world sheet supersymmetry is given by

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma e(\eta_{\mu\nu}(h^{mn}\partial_m X^\mu \partial_n X^\nu + i\psi^\mu\gamma^m\partial_m\psi^\nu - i\kappa\chi_m\gamma^n\gamma^m\psi^\mu\partial_n X^\nu) + i\lambda^A\gamma^m\partial_m\lambda^A - F^A F^A). \tag{A6}$$

The fermions and gravitino satisfy the following chirality conditions:

$$\Gamma\psi^\mu_- = -\psi^\mu_-, \qquad \Gamma\lambda_+ = \lambda_+, \qquad \Gamma\chi_+ = \chi_+ \tag{A7}$$

The action (A6) is invariant under the supersymmetry transformations given by

$$\delta X^\mu = i\epsilon\psi^\mu, \qquad \delta\psi^\mu = \gamma^m\partial_m X^\mu\epsilon \tag{A8}$$

for the right-moving sector,

$$\delta\lambda^A = F^A\epsilon, \qquad \delta F^A = i\epsilon\gamma^m D_m\lambda^A \tag{A9}$$

for the left-movers, and

$$\delta e_m{}^a = i\kappa\epsilon\gamma^a\chi_m, \qquad \delta\chi_m = \frac{2}{\kappa}D_m\epsilon \tag{A10}$$

in the supergravity sector. Note that we set $\kappa = 2$ in [12]. Of course, $\gamma^m = e^m{}_a\gamma^a$, and the covariant derivative on spinors is

$$D_m\zeta = (\partial_m + \tfrac{1}{4}\omega_m{}^{ab}\gamma_{ab})\zeta, \tag{A11}$$

with $\gamma^{ab} = [\gamma^a, \gamma^b]/2$. In general, the spin connection $\omega_m{}^{ab}$ in supergravity contains the piece that depends solely on the vielbein, $\omega_m{}^{ab}(e)$, plus a fermion bilinear improvement term. In conformal (0, 1) supergravity in two dimensions as described by (A6), however, the improvement term vanishes identically, and we have $\omega_m{}^{ab} = \omega_m{}^{ab}(e)$, with

$$\omega_m{}^{ab} = \tfrac{1}{2}e^{na}(\partial_m e_n{}^b - \partial_n e_m{}^b) - \tfrac{1}{2}e^{nb}(\partial_m e_n{}^a - \partial_n e_m{}^a) - \tfrac{1}{2}e^{na}e^{pb}(\partial_n e_{pc} - \partial_p e_{nc})e_m{}^c. \tag{A12}$$

Note also that the susy variation of F^A is sometimes written in the literature as $\delta F^A = i\epsilon\gamma^m\hat{D}_m\lambda^A$, using the supercovariant derivative

$$\hat{D}_m\lambda^A_+ \equiv (\partial_m + \tfrac{1}{4}\omega_m{}^{ab}\gamma_{ab})\lambda^A_+ - \chi_{m+}F^A. \tag{A13}$$

This simplifies, however, in several ways. First of all, the gravitino drops out from (A13) if the (0, 1) theory is independent of the superpartner of the Liouville field, as is the case for our heterotic world sheet supergravity. Second, for terms relevant for the action, we get

$$\lambda^A\gamma^m\hat{D}_m\lambda^A_+ \equiv \lambda^A\gamma^m\partial_m\lambda^A_+. \tag{A14}$$

3. Light cone coordinates

The world sheet light cone coordinates are

$$\sigma^\pm = \tau \pm \sigma, \tag{A15}$$

in which the Minkowski metric becomes

$$\eta_{ab} = \begin{pmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}, \tag{A16}$$

resulting in the light cone gamma matrices

$$\gamma^+ = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}, \qquad \gamma^- = \begin{pmatrix} 0 & 0 \\ -2 & 0 \end{pmatrix}. \tag{A17}$$

On spin-vectors, such as the gravitino $\chi_{m\alpha}$, we put the world sheet index first, and the spinor index second when required. We will use $\pm$ labels for spinor indices as well as light cone world sheet and spacetime indices, as appropriate. The nature of a given index should be clear from context. In light cone coordinates, and in conformal gauge, the supersymmetry transformations of the matter multiplets are given by

$$\begin{aligned} \delta X^\mu &= i\epsilon_+\psi^\mu_-, & \delta\psi_- &= -2\partial_- X^\mu\epsilon_+, \\ \delta\lambda^A_+ &= F^A\epsilon_+, & \delta F^A &= -2i\epsilon_+\partial_-\lambda^A_+, \end{aligned} \tag{A18}$$

and the linearized supersymmetry transformations of the supergravity multiplet are

$$\delta e_{+-} = -2i\kappa\epsilon_+\chi_{++}, \qquad \delta\chi_{++} = \frac{2}{\kappa}\partial_+\epsilon_+. \tag{A19}$$

Once we have picked conformal gauge, we can meaningfully assign a conformal weight to each field. The chart below lists these conformal dimensions for all relevant objects:

Field	Symbol	Conformal Weight
gravitino	χ_{++}	$(1, -\frac{1}{2})$
goldstino	λ_+	$(\frac{1}{2}, 0)$
fermion	ψ^μ_-	$(0, \frac{1}{2})$
boson	X^μ	$(0, 0)$
aux. field	F	$(\frac{1}{2}, \frac{1}{2})$
SUSY parameter	ϵ	$(0, -\frac{1}{2})$
w.s. derivative	∂_-	$(0, 1)$
w.s. derivative	∂_+	$(1, 0)$

(A20)

In spacetime, similarly, we define the light cone coordinates thus:

$$X^\pm = X^0 \pm X^1, \tag{A21}$$

and denote the remaining transverse dimensions by X^i, so that the spacetime index decomposes as $\mu \equiv (+, -, i)$.

When we combine the spacetime light cone parametrization with the light cone coordinate choice on the world sheet, the heterotic action becomes

$$S = \frac{1}{\pi\alpha'} \int d^2\sigma^{\pm}\Big(\partial_+ X^i \partial_- X^i + \frac{i}{2}\psi_-^i \partial_+ \psi_-^i - \frac{1}{2}\partial_+ X^+ \partial_- X^- - \frac{1}{2}\partial_+ X^- \partial_- X^+ - \frac{i}{4}\psi_-^+ \partial_+ \psi_-^- - \frac{i}{4}\psi_-^- \partial_+ \psi_-^+ + \frac{i}{2}\lambda_+^A \partial_- \lambda_+^A + \frac{1}{4}F^A F^A - \frac{i}{2}\kappa\chi_{++}(2\psi_-^i \partial_- X^i - \psi_-^+ \partial_- X^- - \psi_-^- \partial_- X^+)\Big), \tag{A22}$$

where we have defined $d^2\sigma^{\pm} \equiv d\sigma^- \wedge d\sigma^+ = 2d\tau \wedge d\sigma$.

APPENDIX B: EVALUATION OF THE DETERMINANTS

We wish to calculate the determinant for the operator

$$D_F \equiv \frac{1}{F} D_- F \tag{B1}$$

as it acts on fields of arbitrary half-integer spin j. F here is a shorthand for the tachyon condensate $F = 2\mu \exp(k_+ X^+)$, as determined in (25); we will continue with this notation through this appendix. As D_F is a chiral operator, we will take the usual approach of finding its adjoint and calculating the determinant of the corresponding Laplacian. The actual contribution we are interested in will be the square root of the determinant of the Laplacian. Throughout this appendix, we work in Euclidean signature; we will Wick rotate our results back to Minkowski signature before adding the results to the body of the paper. Also, we gauge fix only world sheet diffeomorphisms by setting

$$h_{mn} = e^{2\phi}\hat{h}_{mn}, \tag{B2}$$

where ϕ is the Liouville field. For most calculations, we set the fiducial metric $\hat{h}_{mn}$ to be the flat metric. In this gauge, we find

$$D_- = e^{-2\phi}\bar{\partial}, \tag{B3}$$

independently of j.

We define the adjoint $D_F^\dagger$ of the Faddeev-Popov operator D_F via

$$\Big\langle T_1 \Big| \frac{1}{F} D_-(FT_2)\Big\rangle = \langle D_F^\dagger T_1 | T_2 \rangle, \tag{B4}$$

where T_1 is a world sheet tensor of spin $j-1$, T_2 is a tensor of spin j, and the inner product on the corresponding tensors is the standard one, independent of F [43].

We find the left-hand side of (B4) becomes

$$\begin{aligned}\Big\langle T_1 \Big| \frac{1}{F} D_- F T_2 \Big\rangle &= \int d^2z e^{2\phi(2-j)} T_1^* \Big(\frac{1}{F} D_- F\Big) T_2 \\ &= \int d^2z e^{2\phi(2-j)} T_1^* \frac{1}{F} e^{-2\phi}\bar{\partial}(FT_2) \\ &= -\int d^2z \bar{\partial}\Big(e^{-2\phi(j-1)}\frac{1}{F}T_1^*\Big) F T_2 \\ &= -\int d^2z e^{2\phi(1-j)}\Big(F D_+ \frac{1}{F} T_1\Big)^* T_2.\end{aligned} \tag{B5}$$

Thus, the adjoint operator is

$$D_F^\dagger = -F D_+ \frac{1}{F}, \tag{B6}$$

where

$$D_+ = e^{2\phi(j-1)}\partial e^{-2\phi(j-1)} \tag{B7}$$

when acting on a field of spin $j-1$. As above, our conventions are such that D_+ and ∂ act on everything to their right.

We are now interested in the determinant of the Laplace operator $D_F^\dagger D_F$,

$$\det\Big(-\frac{1}{F} D_+ F^2 D_- \frac{1}{F}\Big) = \det\Big(-\frac{1}{F^2} D_+ F^2 D_-\Big). \tag{B8}$$

Determinants of such operators were carefully evaluated in [45,46]; [46] Eq. (3.2) of that paper gives a general formula for the determinant of $f\partial g\bar{\partial}$. For our case,

$$f = e^{(2j-2)\phi + 2k_+X^+}, \qquad g = e^{-2j\phi - 2k_+X^+}, \tag{B9}$$

and we get

$$\log\det(D_F^\dagger D_F) = -\frac{1}{24\pi}\int d^2\sigma \hat{e}\{[3(2j-1)^2 - 1]\hat{h}^{mn}\partial_m\phi\partial_n\phi + 12(2j-1)k_+\hat{h}^{mn}\partial_m\phi\partial_n X^+ + 12k_+^2\hat{h}^{mn}\partial_m X^+\partial_n X^+\}. \tag{B10}$$

1. $J_{\psi_-^+\epsilon}$

The case of the Faddeev-Popov operator corresponds to spin $j = 1/2$. Since we are in fact interested in the inverse square root of the determinant of the Laplacian, we find that it contributes to the effective Euclidean world sheet action

$$-\frac{1}{48\pi}\int d^2\sigma \hat{e}(-\hat{h}^{mn}\partial_m\phi\partial_n\phi + 12k_+^2\hat{h}^{mn}\partial_m X^+\partial_n X^+). \tag{B11}$$

We can rewrite this contribution as a path integral over a bosonic ghost-antighost system,

$$\frac{1}{\det J_{\psi^{\pm}_{-}\epsilon}} = \int \mathcal{D}\beta\mathcal{D}\gamma \exp\left\{-\int d^2\sigma\beta\frac{1}{F}D_-(F\gamma)\right\}. \tag{B12}$$

The antighost β has the same conformal dimension as $\psi^{\pm}_{-}$, or $(0, 1/2)$, while the ghost field γ has the dimension of ϵ, namely $(0, -1/2)$. The path-integral measure of the ghost and antighost fields, $\mathcal{D}\beta\mathcal{D}\gamma$, is defined in the standard way, independently of X^+. More precisely, the standard measure on the fluctuations δf of a spin j field f is induced from the covariant norm

$$\|\delta f\|^2 = \int d^2\sigma e^{(2-2j)\phi}\delta f^*\delta f, \tag{B13}$$

written here in conformal gauge $h_{mn} = e^{2\phi}\delta_{mn}$.

The ghost fields β and γ have a kinetic term suggesting that they should be purely left-moving fields, but their conformal dimensions do not conform to this observation. It is natural to introduce the rescaled fields

$$\tilde{\beta} = \frac{\beta}{F}, \qquad \tilde{\gamma} = F\gamma. \tag{B14}$$

Because the conformal weight of F is $(1/2, 1/2)$, both $\tilde{\beta}$ and $\tilde{\gamma}$ are now fields of conformal dimension $(1/2, 0)$. Moreover, in terms of these rescaled fields, the classical ghost action takes the canonical form of a purely left-moving first-order system of central charge $c = -1$,

$$S_{\tilde{\beta}\tilde{\gamma}} = \int d^2\sigma\tilde{\beta}D_-\tilde{\gamma}. \tag{B15}$$

However, the rescaling (B14) has an effect on the measure in the path integral. In terms of the rescaled variables, the originally X^+ independent measure acquires a noncanonical, explicit X^+ dependence. The new measure on $\tilde{\beta}$ and $\tilde{\gamma}$ is induced from

$$\begin{aligned}\|\delta\tilde{\beta}\|^2 &= \int d^2\sigma e^{\phi}F^2\delta\tilde{\beta}^*\delta\tilde{\beta},\\ \|\delta\tilde{\gamma}\|^2 &= \int d^2\sigma e^{\phi}F^{-2}\delta\tilde{\gamma}^*\delta\tilde{\gamma}.\end{aligned} \tag{B16}$$

In order to distinguish it from the standard X^+ independent measure, we denote the measure induced from (B16) by $\mathcal{D}_{X^+}\tilde{\beta}\mathcal{D}_{X^+}\tilde{\gamma}$. It is convenient to replace this X^+ dependent measure by hand with the standard measure $\mathcal{D}\tilde{\beta}\mathcal{D}\tilde{\gamma}$ for left-moving fields $\tilde{\beta}$, $\tilde{\gamma}$ of spin $1/2$, defined with the use of the standard norm (B13) that is independent of X^+. In order to do so, we must correct for the error by including the corresponding Jacobian, which is precisely the X^+ dependent part of $J_{\psi^{\pm}_{-}\epsilon}$. Thus, we can now write

$$\begin{aligned}\frac{1}{\det J_{\psi^{\pm}_{-}\epsilon}} &= \int \mathcal{D}_{X^+}\tilde{\beta}\mathcal{D}_{X^+}\tilde{\gamma}\exp\left\{-\int d^2\sigma\tilde{\beta}D_-\tilde{\gamma}\right\}\\ &= \exp\left\{\frac{1}{48\pi}\int d^2\sigma\hat{e}(12k_+^2\hat{h}^{mn}\partial_m X^+\partial_n X^+)\right\}\\ &\quad\times\int\mathcal{D}\tilde{\beta}\mathcal{D}\tilde{\gamma}\exp\left\{-\int d^2\sigma\tilde{\beta}D_-\tilde{\gamma}\right\}.\end{aligned} \tag{B17}$$

The canonical $\tilde{\beta}$, $\tilde{\gamma}$ ghosts correctly reproduce the Liouville dependence of $J_{\psi^{\pm}_{-}\epsilon}$. Of course, we now have a contribution to the effective action. Written in Minkowski signature, this correction becomes

$$\frac{1}{4\pi}\int d^2\sigma\hat{e}(k_+^2\hat{h}^{mn}\partial_m X^+\partial_n X^+). \tag{B18}$$

2. $\tilde{J}$

The evaluation of $J_{\psi^{\pm}_{-}\epsilon}$ is not the only calculation that can contribute to the shift of the linear dilaton. We also need to analyze the determinant involved in the change of variables that turns the gravitino and its conjugate field into manifestly left-moving fields. As we shall now show, this transformation also contributes to the linear dilaton shift.

Consider the kinetic term between the conjugate pair of χ_{++} and ψ^{-}_{-}. The relevant part of the path integral, written here still in the Minkowski world sheet signature, is

$$\int\mathcal{D}\chi_{++}\mathcal{D}\psi^{-}_{-}\times\exp\left\{\frac{i\kappa V_-}{\pi}\int d^2\sigma^{\pm}(\chi_{++}\partial_-\psi^{-}_{-} - k_+\chi_{++}\partial_- X^+\psi^{-}_{-})\right\}. \tag{B19}$$

This path integral would give the determinant of the operator D_F acting on fields of spin $j = 3/2$. Wick rotating to Euclidean signature and using (B10), we obtain for this determinant

$$\exp\left\{-\frac{1}{48\pi}\int d^2\sigma e(11\hat{h}^{mn}\partial_m\phi\partial_n\phi + 24k_+\hat{h}^{mn}\partial_m\phi\partial_n X^+ + 12k_+^2\hat{h}^{mn}\partial_m X^+\partial_n X^+)\right\}. \tag{B20}$$

As in the calculation of the Faddeev-Popov determinant above, it is again useful to rescale the fields by an X^+ dependent factor. This rescaling was in fact introduced in Eq. (47), repeated here for convenience:

$$\tilde{\chi}_{++} = F\chi_{++}, \qquad \tilde{\psi}^{-}_{-} = \frac{\psi^{-}_{-}}{F}. \tag{B21}$$

The first term in the conformal anomaly can be interpreted as due to the purely left-moving conjugate pair of fields $\tilde{\chi}_{++}$ and $\tilde{\psi}^{-}_{-}$ with the standard X^+ independent measure induced from (B13). In order to reproduce correctly the full determinant (B20), we again have to compensate for the

X^+ dependence of the measure by including the rest of the gravitino determinant (B20) as an explicit conversion factor. Thus, the consistent transformation of fields includes the measure change

$$\mathcal{D}_{X^+}\tilde{\chi}_{++}\mathcal{D}_{X^+}\tilde{\psi}_-^- = \tilde{J}\mathcal{D}\tilde{\chi}_{++}\mathcal{D}\tilde{\psi}_-^-, \tag{B22}$$

with

$$\tilde{J} = \exp\bigg\{-\frac{1}{4\pi}\int d^2\sigma e(2k_+\hat{h}^{mn}\partial_m\phi\partial_n X^+ + k_+^2\hat{h}^{mn}\partial_m X^+\partial_n X^+)\bigg\}. \tag{B23}$$

Together with the contribution from the Faddeev-Popov determinant (B17), we see that the contributions to the $\partial_+X^+\partial_-X^+$ term in fact cancel, and we are left with the following one-loop correction to the Euclidean action due to the measure factors,

$$\Delta S_E = \frac{1}{2\pi}\int d^2\sigma e k_+ h^{mn}\partial_m\phi\partial_n X^+, \tag{B24}$$

together with the bosonic superghosts of spin $1/2$ (and with the canonical path integral measure) and central charge $c = -1$, plus the gravitino sector consisting of the canonical conjugate pair $\tilde{\chi}_{++}$, $\tilde{\psi}_-^-$ also with the canonical measure and central charge $c = -11$.

Integrating (B24) by parts and using the following expression for the world sheet scalar curvature in conformal gauge,

$$eR = -2\delta^{mn}\partial_m\partial_n\phi, \tag{B25}$$

we end up with

$$\Delta S_E = \frac{1}{4\pi}\int d^2\sigma e k_+ X^+ R. \tag{B26}$$

Rotating back to Minkowski signature, we find

$$\Delta S = -\frac{1}{4\pi}\int d^2\sigma e k_+ X^+ R. \tag{B27}$$

This term represents an effective shift in the dilaton by $V_+ = k_+$. The total central charge of the combined matter and ghost system, with the one-loop correction (B27) to the linear dilaton included, is zero.

[1] A. Sen, Int. J. Mod. Phys. A **20**, 5513 (2005).
[2] M. Headrick, S. Minwalla, and T. Takayanagi, Classical Quantum Gravity **21**, S1539 (2004).
[3] E. Witten, Nucl. Phys. **B195**, 481 (1982).
[4] M. Fabinger and P. Hořava, Nucl. Phys. **B580**, 243 (2000).
[5] S. Hellerman and I. Swanson, arXiv:hep-th/0611317.
[6] S. Hellerman and I. Swanson, arXiv:hep-th/0612051.
[7] S. Hellerman and I. Swanson, arXiv:hep-th/0612116.
[8] S. Hellerman and I. Swanson, Phys. Rev. Lett. **99**, 171601 (2007).
[9] Similar spacetime decay has also been seen in solutions of noncritical string theory in $1 + 1$ dimensions [10], and noncritical M-theory in $2 + 1$ dimensions [11].
[10] J. L. Karczmarek and A. Strominger, J. High Energy Phys. 05 (2004) 062.
[11] P. Hořava and C. A. Keeler, J. High Energy Phys. 07 (2007) 059.
[12] P. Hořava and C. A. Keeler, Phys. Rev. Lett. **100**, 051601 (2008).
[13] P. Hořava and E. Witten, Nucl. Phys. **B460**, 506 (1996).
[14] P. Hořava and E. Witten, Nucl. Phys. **B475**, 94 (1996).
[15] Actually, this heterotic M-theory configuration is an even closer analog of a more complicated unstable string theory system: A stack of D-branes together with an orientifold plane, plus anti-D-branes with an antiorientifold plane, such that each of the two collections is separately neutral. These collections are only attracted to each other quantum mechanically, due to the one-loop Casimir effect.
[16] H. Kawai, D. C. Lewellen, and S. H. H. Tye, Phys. Rev. D **34**, 3794 (1986).
[17] Another candidate perturbative description was suggested in [18].
[18] S. P. De Alwis and A. T. Flournoy, Phys. Rev. D **66**, 026005 (2002).
[19] L. J. Dixon and J. A. Harvey, Nucl. Phys. **B274**, 93 (1986).
[20] P. Forgács, Z. Horváth, L. Palla, and P. Vecsernyés, Nucl. Phys. **B308**, 477 (1988).
[21] D. C. Lewellen, Nucl. Phys. **B337**, 61 (1990).
[22] S. Elitzur and A. Giveon, Phys. Lett. B **189**, 52 (1987).
[23] P. Di Francesco, P. Mathieu, and D. Sénéchal, in *Conformal Field Theory* (Springer, New York, 1997).
[24] Here "+" and "−" correspond to the NS sector and the R sector, respectively. This choice is consistent with the grading on the operator product algebra of the corresponding operators. Hence, the sector labeled by + (or −) corresponds to an antiperiodic (or periodic) fermion on the cylinder.
[25] Adding other generators, for example $\mathbf{U}_0 = (- \ldots - |+)$ which would relax the lock between the spin structures of the left and right-moving fermions, will produce other heterotic models. In the case of $\mathbf{U}_0$ added to any portion of the basis $\mathbf{U}_1, \ldots, \mathbf{U}_6$, the familiar $SO(16) \times SO(16)$ model is produced. Adding generators which have only eight periodic fermions will produce the remaining supersymmetric theories, as well as extra copies of the tachyonic ones [16].
[26] S. Chaudhuri, G. Hockney, and J. D. Lykken, Phys. Rev. Lett. **75**, 2264 (1995).
[27] S. Chaudhuri and J. Polchinski, Phys. Rev. D **52**, 7168 (1995).

[28] The need for a nonzero dilaton gradient at weak string coupling is somewhat reminiscent of a similar phenomenon at strong coupling: the perturbative Casimir force in the $E_8 \times \bar{E}_8$ heterotic M-theory. In both cases, the ''decay-to-nothing'' instability plays out on top of a nontrivial spacetime dependence of the string coupling. Given the absence of a reliable interpolation between the weakly and strongly coupled regimes, it is not possible to determine whether these two phenomena are directly related.

[29] N. Berkovits and C. Vafa, Mod. Phys. Lett. A **9**, 653 (1994).

[30] F. Bastianelli, Phys. Lett. B **322**, 340 (1994).

[31] H. Kunitomo, Phys. Lett. B **343**, 144 (1995).

[32] I. N. McArthur, Phys. Lett. B **342**, 94 (1995).

[33] I. N. McArthur, Nucl. Phys. **B452**, 456 (1995).

[34] J. McGreevy and E. Silverstein, J. High Energy Phys. 08 (2005) 090.

[35] G. T. Horowitz and E. Silverstein, Phys. Rev. D **73**, 064016 (2006).

[36] O. Aharony and E. Silverstein, Phys. Rev. D **75**, 046003 (2007).

[37] M. Flohr, Int. J. Mod. Phys. A **18**, 4497 (2003).

[38] M. R. Gaberdiel, Int. J. Mod. Phys. A **18**, 4593 (2003).

[39] This system is still coupled to the transverse fields ψ^i, X^i but the strength of the interaction goes to zero with growing F.

[40] This is true because the operator whose superdeterminant is being calculated has a block-triangular form, as the action of diffeomorphisms on the fermionic gauge-fixing condition $\psi^{\pm}_{\pm}$ vanishes when (45) is satisfied.

[41] A useful source of information on two-dimensional supergravities is, e.g., [42].

[42] P. C. West, in *Introduction to Supersymmetry and Supergravity* (World Scientific, Singapore, 1990), 2nd ed.

[43] See Section II.E of [44] for more details on the corresponding inner products and the definition and properties of differential operators on Riemann surfaces.

[44] E. D'Hoker and D. H. Phong, Rev. Mod. Phys. **60**, 917 (1988).

[45] R. Kallosh and A. Y. Morozov, Int. J. Mod. Phys. A **3**, 1943 (1988).

[46] Similar determinants have played a central role in other areas of CFT, perhaps most notably in the free-field Wakimoto realizations of WZW models; see, e.g., [47].

[47] A. Gerasimov, A. Morozov, M. Olshanetsky, A. Marshakov, and S. L. Shatashvili, Int. J. Mod. Phys. A **5**, 2495 (1990).