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# Conformally invariant "massless" spin-2 field in the de Sitter universe

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A massless spin-2 field equation in de Sitter space, which is invariant under the conformal transformation, has been obtained. The framework utilized is the symmetric rank-2 tensor field of the conformal group. Our method is based on the group theoretical approach and six-cone formalism, initially introduced by Dirac. Dirac's six-cone is used to obtain conformally invariant equations on de Sitter space. The solution of the physical sector of massless spin-2 field (linear gravity) in de Sitter ambient space is written as a product of a generalized polarization tensor and a massless minimally coupled scalar field. Similar to the minimally coupled scalar field, for quantization of this sector, the Krein space quantization is utilized. We have calculated the physical part of the linear graviton two-point function. This two-point function is de Sitter invariant and free of pathological large-distance behavior.

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## I. INTRODUCTION

Quantum field theory in de Sitter (dS) space-time has evolved as an exceedingly important subject, studied by many authors over the course of the past decade. This is due to the fact that most recent astrophysical data indicate that our universe might currently be in a dS phase. The importance of dS space has been primarily ignited by the study of the inflationary model of the universe and the quantum gravity. In Minkowski space-time, it is well known that the massless fields propagate on the light-cone. These fields are invariant under the conformal group $SO(2, 4)$. For spin $s \geq 1$ they are invariant under the gauge transformation as well. In dS space, mass is not an invariant parameter for the set of observable transformations under the dS group $SO(1, 4)$. Concept of light-cone propagation, however, does exist and leads to the conformal invariance. "Massless" is used here in reference to the conformal invariance (propagation on the dS light-cone). The term "massive" fields is referred to the fields that in their Minkowskian limit (zero curvature) reduce to massive Minkowskian fields alone. Indeed, the principal series of unitary irreducible representations (UIRs) admits a massive Poincaré group UIR in the limit $H = 0$.

It has been shown that the massive and massless conformally coupled scalar fields in dS space correspond to the principal and complementary series representations of the dS group, respectively [1]. The massive vector field in dS space has been associated with the principal series, whereas massless field corresponds to the lowest representation of the vector discrete series representation in the dS group [2]. The massive and massless spin-2 fields in dS space have been also associated with the principal series and the lowest representation of the rank-2 tensor discrete series of the dS group, respectively [3–5]. The importance of the massless spin-2 field in the dS space is due to the fact that it plays the central role in quantum gravity and quantum cosmology.

In the previous paper, the conformally invariant (CI) wave equations for scalar and vector fields in dS space were obtained [6]. We are interested in the conformal invariance properties of the massless spin-2 field in dS space, i.e. dS linear gravity. In this paper, the massless spin-2 CI wave equation is obtained. The framework utilized here is the symmetric rank-2 tensor field. Our method is based on a group theoretical point of view and Dirac's six-cone formalism and the conformal space is used to obtain the CI equations. The concept of conformal space was used by Dirac [7] to demonstrate the field equations for spinor and vector fields in 1 + 3-dimensional space-time in manifestly CI form. He embedded Minkowski space as the hypersurface $\eta_{ab}u^a u^b = 0$, $(a, b = 0, 1, 2, 3, 4, 5)$, $\eta_{ab} = \mathrm{diag}(1, -1, -1, -1, -1, 1)$ in $\mathrm{IR}^6$. Then he extended the fields by homogeneity requirements to the whole of the space of homogeneous coordinates, namely, $\mathrm{IR}^6$. This formalism developed by Mack and Salam [8] and many others [9]. This approach to conformal symmetry leads to the best path to exploit the physical symmetry in contrast to approaches based on group theoretical treatment of state vector spaces associated with the group. We use this formalism to obtain CI wave equations in dS space. Many believe that conformal invariance may be the key to the solution of the problem of quantum gravity. The conformal invariance, and the light-cone propagation, constitutes the basis for construction of massless field in dS space. For $s \geq 1$, the gauge invariance provides an additional tool for analysis of this problem.

The organization of this paper and its brief outlook are as follows. Section II is devoted to a brief review of the dS massless spin-2 field equations in the ambient space.

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Section III introduces Dirac's manifestly covariant formalism of symmetric tensor fields on the six-cone; in this section, conditions for the existence of CI wave equations are given. Invariant subspace of fields are defined by means of subsidiary conditions: transversality, divergencelessness. Section IV is devoted to the solutions of the physical part of field equations. We show that this physical sector can be written in terms of a polarization tensor and a massless minimally coupled scalar field

$$\mathcal{K}_{\alpha\beta}(x) = \mathcal{D}_{\alpha\beta}(x, \partial)\phi(x).$$

A Krein space quantization [10,11] becomes necessary to circumvent the corresponding well-known anomalies. In Sec. V we calculate the two-point function $\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x')$ in ambient space notations. It is particularly shown that obtaining a covariant two-point function without infrared divergence necessitates the use a Krein space field quantization. Finally a brief conclusion and an outlook for further investigation will be presented. We will supplied some useful identities and mathematical details of calculations in the appendices, and in Appendix F, the two-point function is calculated in terms of the intrinsic coordinates from its ambient space counterpart.

## II. DE SITTER FIELD EQUATIONS

The dS metric is a solution of the cosmological Einstein's equation with positive constant $\Lambda$. It is conveniently described as a hyperboloid embedded in a five-dimensional Minkowski space

$$X_H = \left\{x \in \mathrm{IR}^5; x^2 = \eta_{\alpha\beta}x^\alpha x^\beta = -H^{-2} = -\frac{3}{\Lambda}\right\},$$
$$\alpha, \beta = 0, 1, 2, 3, 4, \tag{2.1}$$

where $\eta_{\alpha\beta} = \mathrm{diag}(1, -1, -1, -1, -1)$. The dS metrics reads

$$ds^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta = g^{\mathrm{dS}}_{\mu\nu}dX^\mu dX^\nu, \qquad \mu, \nu = 0, 1, 2, 3$$

where the $X^\mu$'s are 4 space-time intrinsic coordinates of the dS hyperboloid. Any geometrical object in this space can be written in terms of the four local coordinates $X^\mu$ (intrinsics) or in terms of the five global coordinates $x^\alpha$ (ambient space).

The linearized gravitational wave equation in intrinsic coordinates is [12,13]

$$\tfrac{1}{2}(\Box_H h_{\mu\nu} - \nabla_\mu\nabla^\rho h_{\nu\rho} - \nabla_\nu\nabla^\rho h_{\mu\rho} + \nabla_\mu\nabla_\nu h') + \tfrac{1}{2}g^{\mathrm{dS}}_{\mu\nu}(\nabla_\lambda\nabla_\rho h^{\lambda\rho} - \Box_H h') + H^2(h_{\mu\nu} + \tfrac{1}{2}h' g^{\mathrm{dS}}_{\mu\nu}) = 0, \tag{2.2}$$

where $\Box_H \equiv \nabla_\mu\nabla^\mu$ is the Laplace-Beltrami operator on dS space and $h' = h^\mu_\mu$ is the trace of $h_{\mu\nu}$ with respect to the background metric. Here, $\nabla^\nu$ is the covariant derivative, and the indices are raised and lowered by the background metric ($g_{\mu\nu} = g^{\mathrm{dS}}_{\mu\nu} + h_{\mu\nu}$). Not that the field Eq. (2.2) is invariant under the gauge transformation

$$h_{\mu\nu} \longrightarrow h^{gt}_{\mu\nu} = h_{\mu\nu} + \nabla_\mu\zeta_\nu + \nabla_\nu\zeta_\mu, \tag{2.3}$$

where $\zeta$ is an arbitrary vector field.

In the following the ambient space notations is used; in these notations, the relationship with UIRs of the dS group becomes straightforward because the Casimir operators are easy to identify [14]. There are two Casimir operators

$$Q^{(1)}_2 = -\tfrac{1}{2}L^{\alpha\beta}L_{\alpha\beta} = -\tfrac{1}{2}(M^{\alpha\beta} + S^{\alpha\beta})(M_{\alpha\beta} + S_{\alpha\beta}),$$
$$Q^{(2)}_2 = -W_\alpha W^\alpha, \tag{2.4}$$

where $M_{\alpha\beta} = -i(x_\alpha\partial_\beta - x_\beta\partial_\alpha) = -i(x_\alpha\bar\partial_\beta - x_\beta\bar\partial_\alpha)$ and $W_\alpha = -\frac{1}{8}\epsilon_{\alpha\beta\gamma\sigma\eta}L^{\beta\gamma}L^{\sigma\eta}$, in which the symbol $\epsilon_{\alpha\beta\gamma\sigma\eta}$ holds for the usual antisymmetric tensor. The action of the spin generator $S_{\alpha\beta}$ is defined by [14]

$$S_{\alpha\beta}\mathcal{K}_{\gamma\delta} = -i(\eta_{\alpha\gamma}\mathcal{K}_{\beta\delta} - \eta_{\beta\gamma}\mathcal{K}_{\alpha\delta} + \eta_{\alpha\delta}\mathcal{K}_{\beta\gamma} - \eta_{\beta\delta}\mathcal{K}_{\alpha\gamma}),$$

$\bar\partial_\alpha$ is the tangential (or transverse) derivative on dS space,

$$\bar\partial_\alpha = \theta_{\alpha\beta}\partial^\beta = \partial_\alpha + H^2x_\alpha x\cdot\partial, \quad \text{with } x\cdot\bar\partial = 0,$$

and $\theta_{\alpha\beta}$ is the transverse projector ($\theta_{\alpha\beta} = \eta_{\alpha\beta} + H^2x_\alpha x_\beta$).

It has been shown that the field Eq. (2.2) in the ambient space reads as ([14] and appendix B)

$$(Q^{(1)}_2 + 6)\mathcal{K}(x) + D_2\partial_2\cdot\mathcal{K} = 0, \tag{2.5}$$

where $\partial_2\cdot\mathcal{K}_\alpha = \bar\partial\cdot\mathcal{K}_\alpha - H^2x_\alpha\mathcal{K}' - \frac{1}{2}\bar\partial_\alpha\mathcal{K}'$ and the operator $D_2$ is the generalized gradient defined by

$$D_2K = H^{-2}\mathcal{S}(\bar\partial - H^2x)K. \tag{2.6}$$

Note that $\mathcal{S}$ is the symmetrizer operator and $K$ is a vector field. It is clear that the field Eq. (2.5) is invariant under the following gauge transformation:

$$\mathcal{K}_{\alpha\beta} \longrightarrow \mathcal{K}^{gt}_{\alpha\beta} = \mathcal{K}_{\alpha\beta} + D_2\Lambda_g. \tag{2.7}$$

The operator $Q^{(1)}_2$ commutes with the action of the group generators and, as a consequence, it is constant in each UIR. Thus the eigenvalues of $Q^{(1)}_2$ can be used to classify the UIRs i.e.,

$$(Q^{(1)}_2 - \langle Q^{(1)}_2\rangle)\mathcal{K}(x) = 0. \tag{2.8}$$

Following Dixmier [15], we get a classification scheme using a pair $(p, q)$ of parameters involved in the following possible spectral values of the Casimir operators:

$$Q^{(1)}_p = (-p(p+1) - (q+1)(q-2))I_d,$$
$$Q^{(2)}_p = (-p(p+1)q(q-1))I_d. \tag{2.9}$$

Three types of scalar, tensorial, or spinorial UIRs are distinguished for $SO(1, 4)$ according to the range of values of the parameters $q$ and $p$ [15,16], namely, the principal, the complementary and the discrete series. The flat limit indicates that for the principal and the complementary series the value of $p$ bears meaning of spin. For the discrete series case, the only representation which has a physically meaningful Minkowskian counterpart is the $p = q$ case. Mathematical details of the group contraction and the physical principles underlying the relationship between dS and Poincaré groups can be found in Refs. [17,18], respectively. The spin-2 tensor representations relevant to the present work are as follows:

(i) The UIRs $U^{2,\nu}$ in the principal series where $p = s = 2$ and $q = \frac{1}{2} + i\nu$ correspond to the Casimir spectral values:

$$\langle Q_2^\nu \rangle = \nu^2 - \tfrac{15}{4}, \qquad \nu \in \mathrm{IR}, \tag{2.10}$$

note that $U^{2,\nu}$ and $U^{2,-\nu}$ are equivalent.

(ii) The UIRs $V^{2,q}$ in the complementary series where $p = s = 2$ and $q - q^2 = \mu$, correspond to

$$\langle Q_2^\mu \rangle = q - q^2 - 4 \equiv \mu - 4, \qquad 0 < \mu < \tfrac{1}{4}. \tag{2.11}$$

(iii) The UIRs $\Pi^\pm_{2,q}$ in the discrete series where $p = s = 2$ corresponds to

$$\begin{aligned} \langle Q_2^{(1)} \rangle &= -4, \qquad q = 1(\Pi^\pm_{2,1}); \\ \langle Q_2^{(2)} \rangle &= -6, \qquad q = 2(\Pi^\pm_{2,2}). \end{aligned} \tag{2.12}$$

The massless spin-2 field in dS space corresponds to the $\Pi^\pm_{2,2}$ and $\Pi^\pm_{2,1}$ cases in which the sign $\pm$ stands for the helicity. In these cases, the two representations $\Pi^\pm_{2,2}$, in the discrete series with $p = q = 2$, have a Minkowskian interpretation. It should be noted that $p$ and $q$ do not bear the meaning of mass and spin. For discrete series in the limit $H \to 0$, $p = q = s$ are indeed none other than spin. The representation $\Pi^+_{2,2}$ has a unique extension to a direct sum of two UIRs $\mathcal{C}(3; 2, 0)$ and $\mathcal{C}(-3; 2, 0)$ of the conformal group $SO(2, 4)$ with positive and negative energies, respectively [17,19]. The latter restricts to the massless tensor Poincaré UIRs $\mathcal{P}^>(0, 2)$ and $\mathcal{P}^<(0, 2)$ with positive and negative energies, respectively. The following diagrams illustrate these connections:

$$\Pi^+_{2,2} \hookrightarrow \begin{matrix} \mathcal{C}(3,2,0) \\ \oplus \\ \mathcal{C}(-3,2,0) \end{matrix} \overset{H=0}{\longrightarrow} \begin{matrix} \mathcal{C}(3,2,0) \\ \oplus \\ \mathcal{C}(-3,2,0) \end{matrix} \begin{matrix} \hookleftarrow \\ \\ \hookleftarrow \end{matrix} \begin{matrix} \mathcal{P}^>(0,2) \\ \oplus \\ \mathcal{P}^<(0,2), \end{matrix} \tag{2.13}$$

$$\Pi^-_{2,2} \hookrightarrow \begin{matrix} \mathcal{C}(3,0,2) \\ \oplus \\ \mathcal{C}(-3,0,2) \end{matrix} \overset{H=0}{\longrightarrow} \begin{matrix} \mathcal{C}(3,0,2) \\ \oplus \\ \mathcal{C}(-3,0,2) \end{matrix} \begin{matrix} \hookleftarrow \\ \\ \hookleftarrow \end{matrix} \begin{matrix} \mathcal{P}^>(0,-2) \\ \oplus \\ \mathcal{P}^<(0,-2), \end{matrix} \tag{2.14}$$

where the arrows $\hookrightarrow$ designate unique extension; $\mathcal{P}^{\gtrless}(0, 2)$ [$\mathcal{P}^{\gtrless}(0, -2)$] are the massless Poincaré UIRs with positive and negative energies and positive (negative) helicity. It is important to note that the representations $\Pi^\pm_{2,1}$ do not have corresponding flat limits.

## III. DIRAC'S SIX-CONE, CONFORMALLY INVARIANT EQUATIONS

In the Minkowski space, for every massless representation of the Poincaré group there exists only one corresponding representation in the conformal group [19,20]. In the dS space, as mentioned, for the massless tensor field, only two representations in the discrete series ($\Pi^\pm_{2,2}$) have a Minkowskian interpretation. The signs $\pm$ correspond to two types of helicity for the massless tensor field. In this section, the conformal invariance of massless tensor field in dS space is studied. CI wave equations in dS space are best obtained by first establishing the wave equations in Dirac's null-cone in $\mathrm{IR}^6$, and then followed by the projection of these equations to the dS space.

Dirac's six-cone (or Dirac's projection cone) is defined by $u^2 \equiv u_0^2 - \vec{u}^2 + u_5^2 = 0$, where $u_a \in \mathrm{IR}^6$, and $\vec{u} \equiv (u_1, u_2, u_3, u_4)$. Reduction to four-dimensional space (physical space-time) is achieved by projection, that is by fixing the degrees of homogeneity of all fields. Wave equations, subsidiary conditions, etc., must be expressed in terms of operators that are defined intrinsically on the cone. These are well-defined operators that map tensor fields on tensor fields with the same rank on cone $u^2 = 0$ [6,21]. It is important to note that on the cone $u^2 = 0$, the second order Casimir operator of conformal group, $\mathcal{Q}_2$, is not a suitable operator to obtain CI wave equations. For example, for a symmetric tensor field of rank-2, we have [19,21,22]

$$\begin{aligned} \mathcal{Q}_2 \Psi^{cd} &= \tfrac{1}{2} L_{ab} L^{ab} \Psi^{cd} \\ &= (-u^2 \partial^2 + \hat{N}_5(\hat{N}_5 + 4) + 8)\Psi^{cd}, \end{aligned} \tag{3.1}$$

where $\hat{N}_5$ is the conformal-degree operator defined by

$$\hat{N}_5 \equiv u^a \partial_a. \tag{3.2}$$

On the cone this operator reduces to a constant, i.e. $\hat{N}_5(\hat{N}_5 + 4) + 8$. It is clear that this operator cannot lead to the wave equations on the cone. The well-defined operators exist only in exceptional cases. For tensor fields of degree $-1, 0, 1, \ldots,$ the intrinsic wave operators are $\partial^2, (\partial^2)^2, (\partial^2)^3, \ldots,$ respectively [21]. Thus the following CI system of equations, on the cone, has been used [6]:

$$\begin{cases} (\partial_a \partial^a)^n \Psi = 0, \\ \hat{N}_5 \Psi = (n-2)\Psi, \end{cases} \tag{3.3}$$

where $\Psi$ is a tensor field of a definite rank and of a definite symmetry.

Other CI conditions can be added to the above system in order to restrict the space of the solutions. The following conditions are introduced to achieve the above goal:

(a) transversality

$$u_a \Psi^{ab\ldots} = 0,$$

(b) divergencelessness

$$\text{Grad}_a \Psi^{ab\ldots} = 0,$$

(c) tracelessness

$$\Psi^a_{ab\ldots} = 0.$$

The operator $\text{Grad}_a$ unlike $\partial_a$ is intrinsic on the cone, and is defined by [21]

$$\text{Grad}_a \equiv u_a \partial_b \partial^b - (2\hat{N}_5 + 4)\partial_a. \tag{3.4}$$

In order to project the coordinates on the cone $u^2 = 0$, to the $1 + 4$ dS space we chose the following relation:

$$\begin{cases} x^\alpha = (Hu^5)^{-1} u^\alpha, \\ x^5 = Hu^5. \end{cases} \tag{3.5}$$

Note that $x^5$ becomes superfluous when we deal with the projective cone. It is easy to show that various intrinsic operators introduced previously now read as

(1) the conformal-degree operator ($\hat{N}_5$)

$$\hat{N}_5 = x_5 \frac{\partial}{\partial x_5}, \tag{3.6}$$

(2) the conformal gradient ($\text{Grad}_\alpha$)

$$\begin{aligned} \text{Grad}_\alpha = -x_5^{-1}\{&H^2 x_\alpha[Q_0 - \hat{N}_5(\hat{N}_5 - 1)] \\ &+ 2\bar{\partial}_\alpha(\hat{N}_5 + 1)\}, \end{aligned} \tag{3.7}$$

where $Q_p^{(1)} \equiv Q_p$, ,

(3) and the powers of d'Alembertian $(\partial_a \partial^a)^n$, which act intrinsically on a field of conformal-degree $(n - 2)$,

$$(\partial_a \partial^a)^n = -H^{2n} x_5^{-2n} \prod_{j=1}^{n} [Q_0 + (j + 1)(j - 2)]. \tag{3.8}$$

Considering the conformal invariance in the dS space, we classify the 21 degrees of freedom of the symmetric tensor field $\Psi^{ab}$ on the cone by (in the following for the brevity we take $H = 1$)

$$\begin{aligned} &\mathcal{K}_{\alpha\beta} = \Psi_{\alpha\beta} + \mathcal{S} x_\alpha \Psi_\beta \cdot x + x_\alpha x_\beta x \cdot \Psi \cdot x, \\ &K_\alpha = x \cdot \Psi_\alpha + x_\alpha x \cdot \Psi \cdot x, \qquad \phi = x \cdot \Psi \cdot x, \end{aligned} \tag{3.9}$$

where $\mathcal{K}_{\alpha\beta}$ and $K_\alpha$ are tensor and vector fields on dS space, respectively ($x^\alpha \mathcal{K}_{\alpha\beta} = x^\beta \mathcal{K}_{\alpha\beta} = 0 = x^\alpha K_\alpha$). The fields $\Psi_{55}$, $x.\Psi_5$, $x_\alpha x.\Psi_5 + \Psi_{\alpha 5}$ are auxiliary fields which are unnecessary to demonstrate on the dS space.

In the following CI wave equation for the symmetric rank-2 tensor field is considered. We have shown [6], for scalar and vector fields, the simplest CI system of equations is obtained through setting $n = 1$ in (3.3), i.e. the field with conformal-degree $-1$, resulting equations are the UIRs of $SO(1, 4)$. For a symmetric tensor field of rank-2, the CI system (3.3) with $n = 1$ leads to (Appendix B)

$$(Q_0 - 2)\mathcal{K}_{\alpha\beta} + \tfrac{2}{3}\mathcal{S}(\bar{\partial}_\beta + 2x_\beta)\bar{\partial} \cdot \mathcal{K}_\alpha - \tfrac{1}{3}\theta_{\alpha\beta}\bar{\partial} \cdot \bar{\partial} \cdot \mathcal{K} = 0. \tag{3.10}$$

It is clear that (3.10) does not correspond to any UIRs of the dS group. The intrinsic counterpart of (3.10) becomes (Appendix B)

$$(\Box + 4)h_{\mu\nu} - \tfrac{2}{3}\mathcal{S}\nabla_\mu \nabla \cdot h_\nu + \tfrac{1}{3} g^{\text{dS}}_{\mu\nu} \nabla \cdot \nabla \cdot h = 0, \tag{3.11}$$

in which the intrinsic field $h_{\mu\nu}$ is locally determined by the transverse tensor field $\mathcal{K}_{\alpha\beta}$ through

$$h_{\mu\nu}(X) = \frac{\partial x^\alpha}{\partial X^\mu} \frac{\partial x^\beta}{\partial X^\nu} \mathcal{K}_{\alpha\beta}(x(X)).$$

Taking the flat limit ($H \to 0$) of (3.11) we will gain the second order CI massless spin-2 wave equation in four-dimensional Minkowski space, which was found by Barut and Xu [23]. They have found the conformally covariant massless spin-2 field equation by varying the coefficients of various terms in the standard equation.

In order to obtain the CI wave equation for a massless spin-2 field which is the physical state of dS space, let us set $n = 2$ in (3.3), then we have

$$\begin{cases} (\partial_a \partial^a)^2 \Psi = 0, \\ \hat{N}_5 \Psi = 0. \end{cases} \tag{3.12}$$

The following CI conditions can be added to the above system to restrict the space of solutions:

(a) transversality $u_a \Psi^{ab} = 0$, that results in

$$x^5(\Psi_{5b} + x \cdot \Psi_b) = 0, \tag{3.13}$$

(b) divergencelessness

$$\text{Grad}_a \Psi^{ab} = 0. \tag{3.14}$$

It is easy to show that (Appendix C)

$$\bar{\partial} \cdot \mathcal{K}_\alpha = 4(x \cdot \Psi_\alpha + x_\alpha x \cdot \Psi \cdot x) \equiv 4K_\alpha, \tag{3.15}$$

then we get the following relation for the vector field $\bar{\partial} \cdot \mathcal{K}_\alpha$:

$$Q_1 \bar{\partial}.\mathcal{K}_\alpha + \tfrac{2}{3} D_{1\alpha} \bar{\partial}.\bar{\partial}.\mathcal{K} + \tfrac{1}{6} Q_1 D_{1\alpha} \bar{\partial}.\bar{\partial}.\mathcal{K} = 0, \tag{3.16}$$

where $D_1 = -\bar{\partial}$. This CI equation is similar to the gauge-fixed wave equation for the vector field $\bar{\partial} \cdot \mathcal{K}_\alpha$ [6,24]. We are now in a position to write a CI system for the dS field $\mathcal{K}_{\alpha\beta}$. In order to accomplish this, we first determine the CI equation that corresponds to UIRs of the dS group

(Appendix D):

$$(Q_2+4)[(Q_2+6)\mathcal{K}_{\alpha\beta}+D_{2\alpha}\partial_2.\mathcal{K}_\beta] + \tfrac{1}{3}D_{2\alpha}D_{1\beta}\bar\partial.\bar\partial.\mathcal{K}-\tfrac{1}{3}\theta_{\alpha\beta}(Q_0+6)\bar\partial.\bar\partial.\mathcal{K}=0. \tag{3.17}$$

Finally, a CI system is obtained from (3.12) with respect to (3.9) defined by

$$\begin{aligned}(Q_2+4)[(Q_2+6)\mathcal{K}_{\alpha\beta}+D_{2\alpha}\partial_2.\mathcal{K}_\beta]+\tfrac{1}{3}D_{2\alpha}D_{1\beta}\bar\partial.\bar\partial.\mathcal{K}&\\ -\tfrac{1}{3}\theta_{\alpha\beta}(Q_0+6)\bar\partial.\bar\partial.\mathcal{K}&=0,\\ Q_1\bar\partial.\mathcal{K}_\alpha+\tfrac{2}{3}D_{1\alpha}\bar\partial.\bar\partial.\mathcal{K}+\tfrac{1}{6}Q_1D_{1\alpha}\bar\partial.\bar\partial.\mathcal{K}&=0,\\ \mathcal{K}'&=0.\end{aligned} \tag{3.18}$$

It is important to note that by imposing the following conditions on the tensor field $\mathcal{K}_{\alpha\beta}$, (which are necessary for the UIRs of dS group)

$$\mathcal{K}'=0=\bar\partial.\mathcal{K},$$

the CI system (3.18) becomes

$$(Q_2+4)(Q_2+6)\mathcal{K}_{\alpha\beta}=0.$$

It is clear that this conformally invariant field corresponds to the two representations of discrete series, namely, $\Pi^\pm_{2,1}$ and $\Pi^\pm_{2,2}$ [Eq. (2.12)], in other words it is the physical representation of the dS group. At this point it is clear that the parameter $p$ does have a physical significance. It is indeed spin. In the following, we only consider the tensor field that corresponds to the representations of discrete series $\Pi^\pm_{2,2}$ which have the Minkowskian limit i.e.

$$(Q_2+6)\mathcal{K}_{\alpha\beta}=0,\qquad \bar\partial.\mathcal{K}=0=\mathcal{K}'. \tag{3.19}$$

## IV. DE SITTER FIELD SOLUTIONS

The general solution of Eq. (3.19) can be written in the following form [4,25]:

$$\mathcal{K}=\theta\phi_1+\mathcal{S}\bar Z_1K+D_2K_g, \tag{4.1}$$

where $Z_1$ is a constant five-dimensional vector, $\phi_1$ is a scalar field, and $K$ and $K_g$ are two vector fields. By using divergenceless and transversality conditions, we obtain $\mathcal{K}'=0$, which results in

$$2\phi_1+Z_1.K+\bar\partial.K_g=0. \tag{4.2}$$

Conditions $x.K=0=x.K_g$ are used to obtain the above equation. By substituting $\mathcal{K}_{\alpha\beta}$ in (3.19) we have [4]

$$\begin{cases}(Q_0+6)\phi_1=-4Z_1.K, & \text{(I)}\\ (Q_1+2)K=0, & \text{(II)}\\ (Q_1+6)K_g=2(x.Z_1)K & \text{(III)}.\end{cases} \tag{4.3}$$

Using conditions $x.K=0=\bar\partial.K$, Eq. (4.3)-II reduces to $Q_0K=0$. From this reduced form and Eq. (4.3)-I, we can write

$$\phi_1=-\tfrac{2}{3}Z_1.K,\qquad Q_0\phi_1=0, \tag{4.4}$$

and from Eq. (4.2), we have

$$\bar\partial.K_g=\tfrac{1}{3}Z_1.K. \tag{4.5}$$

We choose the following form for the vector field $K$ [the solution of Eq. (4.3)-II] [2,26]

$$K=\bar Z_2\phi_2+D_1\phi_3, \tag{4.6}$$

where $Z_2$ is another five-dimensional constant vector, $\phi_2$ and $\phi_3$ are two scalar fields. Substituting $K$ into Eq. (4.3)-II results in

$$Q_0\phi_2=0. \tag{4.7}$$

It is clear that $\phi_2$ is a massless minimally coupled scalar field. Using the divergenceless condition, $\phi_3$ can be written in terms of $\phi_2$

$$\phi_3=-\tfrac{1}{2}[Z_2.\bar\partial\phi_2+2x.Z_2\phi_2]. \tag{4.8}$$

So we can write

$$K=\bar Z_2\phi_2-\tfrac{1}{2}D_1[Z_2.\bar\partial\phi_2+2x.Z_2\phi_2], \tag{4.9}$$

and

$$\phi_1=-\tfrac{2}{3}Z_1.(\bar Z_2\phi_2-\tfrac{1}{2}D_1[Z_2.\bar\partial\phi_2+2x.Z_2\phi_2]). \tag{4.10}$$

According to the following identity (Appendix E):

$$(Q_1+6)^{-1}(x.Z_1)K=\tfrac{1}{6}[(x.Z_1)K+\tfrac{1}{9}D_1(Z_1.K)], \tag{4.11}$$

the Eq. (4.3)-III leads to

$$K_g=\tfrac{1}{3}[(x.Z_1)K+\tfrac{1}{9}D_1(Z_1.K)], \tag{4.12}$$

where $x.K_g=0$ and $\bar\partial.K_g=\frac{1}{3}Z_1.K$.

Using the Eqs. (4.9), (4.10), and (4.12), we can rewrite $\mathcal{K}_{\alpha\beta}$ as the following form:

$$\mathcal{K}_{\alpha\beta}(x)=\mathcal{D}_{\alpha\beta}(x,\partial,Z_1,Z_2)\phi_2, \tag{4.13}$$

where $\mathcal{D}$ is the projector tensor

$$\mathcal{D}(x,\partial,Z_1,Z_2)=(-\tfrac{2}{3}\theta Z_1.+\mathcal{S}\bar Z_1+\tfrac{1}{3}D_2[\tfrac{1}{9}D_1Z_1. + x.Z_1])(\bar Z_2-\tfrac{1}{2}D_1[Z_2.\bar\partial+2x.Z_2]), \tag{4.14}$$

and $\phi_2$ is a massless minimally coupled scalar field. We should briefly recall the Gupta-Bleuler quantization of the massless minimally coupled scalar field [10]

$$\Box_H\phi(X)=0.$$

As proved by Allen [27], the covariant canonical quantization procedure with positive norm states fails in this case. The Allen's result can be reformulated in the following way: the Hilbert space generated by a complete set of modes (named here the positive modes, including the zero mode) is not dS invariant,

$$\mathcal{H} = \left\{\sum_{k\geq 0} \alpha_k \phi_k; \sum_{k\geq 0} |\alpha_k|^2 < \infty\right\},$$

where $\phi_k$ is defined in [10]. This means that it is not closed under the action of the de Sitter group. Nevertheless, one can obtain a fully covariant quantum field by adopting a new construction [10,11]. In order to obtain a fully covariant quantum field, we add all the conjugate modes to the previous ones. Consequently, we have to deal with an orthogonal sum of a positive and negative inner product space, which is closed under an indecomposable representation of the de Sitter group. The negative values of the inner product are precisely produced by the conjugate modes: $\langle \phi_k^*, \phi_k^* \rangle = -1$, $k \geq 0$. We do insist on the fact that the space of solution should contain the unphysical states with negative norm. Now, the decomposition of the field operator into positive and negative norm parts reads

$$\phi(X) = \frac{1}{\sqrt{2}}[\phi_p(X) + \phi_n(X)], \tag{4.15}$$

where

$$\phi_p(X) = \sum_{k\geq 0} a_k \phi_k(X) + \text{H.c.},$$
$$\phi_n(X) = \sum_{k\geq 0} b_k \phi_k^*(X) + \text{H.c} \tag{4.16}$$

The positive mode $\phi_p(X)$ is the scalar field as was used by Allen. The crucial departure from the standard QFT based on canonical commutation relation lies in the following requirement on commutation relations:

$$a_k|\Omega> = 0, \qquad [a_k, a_{k'}^\dagger] = \delta_{kk'},$$
$$b_k|\Omega> = 0, \qquad [b_k, b_{k'}^\dagger] = -\delta_{kk'}, \tag{4.17}$$

where $|\Omega\rangle$ is the Gupta-Bleuler vacuum state. In the next section the Gupta-Bleuler vacuum state is used to calculate the two-point function of the physical part of linear gravity.

## V. TWO-POINT FUNCTION

In the course of intensive studies by various scientists the following modalities related to linear gravity have been suggested. In the mainstream approach, it has been found the graviton propagator in the linear approximation for largely separated points has a pathological behavior (infrared divergence) and violates the dS invariance [28–30]. Some authors have suggested that infrared divergence could be exploited in order to create the instability of the dS space [31,32]. Tsamis and Woodard have considered a field operator for linear gravity in dS space in terms of flat coordinates [33], these cover only one-half of the dS hyperboloid. They have examined the possibility of quantum instability and they have found a quantum field, which breaks dS invariance.

Antoniadis, Iliopoulos, and Tomaras [34] have shown that the pathological large-distance behavior of the graviton propagator on a dS background does not manifest itself in the quadratic part of the effective action in the one-loop approximation. This means that the pathological behavior of the graviton propagator may be gauge dependent and so should not appear in an effective way as a physical quantity. Recently this result has been also confirmed by several authors [12,13,35–38].

The important result of the method used in this paper, i.e. using the Gupta-Bleuler vacuum, is the calculation of the physical graviton two-point function that is dS invariant and free of any divergences. In Appendix F, the graviton two-point function is expressed in terms of the dS intrinsic coordinates, which is also dS invariant and free of any divergences. This two-point function can be used for calculation of quantum effects of gravity in the interaction cases.

Pursuing our method, the two-point function $\mathcal{W}$ is defined by [4]

$$\mathcal{W}_{\alpha\beta\alpha'\beta'}(x, x') = \langle \Omega | \mathcal{K}_{\alpha\beta}(x) \mathcal{K}_{\alpha'\beta'}(x') | \Omega \rangle, \tag{5.1}$$

where $x, x' \in X_H$. This function which is a solution of the wave Eq. (3.19) with respect to $x$ or $x'$, can be found simply in terms of the scalar two-point function. We consider the following possibility for the transverse two-point function

$$\mathcal{W}(x, x') = \theta\theta' \mathcal{W}_0(x, x') + \mathcal{S}\mathcal{S}'\theta.\theta' \mathcal{W}_1(x, x') + D_2 D_2' \mathcal{W}_g(x, x'), \tag{5.2}$$

where $D_2 D_2' = D_2' D_2$ and $\mathcal{W}_1$ and $\mathcal{W}_g$ are transverse functions. At this stage it is shown that calculation of $\mathcal{W}(x, x')$ could be initiated from either $x$ or $x'$ without any difference that means each choices result in the same equation for $\mathcal{W}(x, x')$. We first consider the choice $x$. In this case $\mathcal{W}(x, x')$ must satisfy the Eq. (3.19), therefor it is easy to show that

$$\begin{cases} (Q_0 + 6)\theta' \mathcal{W}_0 = -4\mathcal{S}'\theta'.\mathcal{W}_1, & \text{(I)} \\ (Q_1 + 2)\mathcal{W}_1 = 0, & \text{(II)} \\ (Q_1 + 6)D_2' \mathcal{W}_g = 2\mathcal{S}'(x.\theta')\mathcal{W}_1 & \text{(III).} \end{cases} \tag{5.3}$$

Using the condition $\partial.\mathcal{W}_1 = 0$, Eq. (5.3)-I leads to

$$\theta' \mathcal{W}_0(x, x') = -\tfrac{2}{3}\mathcal{S}'\theta'.\mathcal{W}_1(x, x'). \tag{5.4}$$

The solution of the Eq. (5.3)-II can be written as the combination of two arbitrary scalar two-point functions $\mathcal{W}_2$ and $\mathcal{W}_3$ in the following form:

$$\mathcal{W}_1 = \theta.\theta' \mathcal{W}_2 + D_1 D_1' \mathcal{W}_3.$$

Substituting this in Eq. (5.3)-II and using the divergenceless condition we have

$$D'_1\mathcal{W}_3 = -\tfrac{1}{2}[2(x.\theta')\mathcal{W}_2 + \theta'.\bar{\partial}\mathcal{W}_2], \qquad Q_0\mathcal{W}_2 = 0.$$

This means that $\mathcal{W}_2$ is the massless minimally coupled two-point function. Putting $\mathcal{W}_2 \equiv \mathcal{W}_{\rm mc}$, we have

$$\mathcal{W}_1(x,x') = (\theta.\theta' - \tfrac{1}{2}D_1[\theta'.\bar{\partial} + 2x.\theta'])\mathcal{W}_{\rm mc}(x,x'). \tag{5.5}$$

Similar to (4.11) using the following identity

$$(Q_0+6)^{-1}(x.\theta')\mathcal{W}_1 = \tfrac{1}{6}[(x.\theta')\mathcal{W}_1 + \tfrac{1}{9}D_1(\theta'.\mathcal{W}_1)],$$

the Eq. (5.3)-III leads to

$$D'_2\mathcal{W}_g(x,x') = \tfrac{1}{3}\mathcal{S}'(\tfrac{1}{9}D_1\theta'. + x.\theta')\mathcal{W}_1(x,x'). \tag{5.6}$$

According to Eqs. (5.4), (5.5), and (5.6) it turns out that the two-point function can be written in the following form:

$$\mathcal{W}_{\alpha\beta\alpha'\beta'}(x,x') = \Delta_{\alpha\beta\alpha'\beta'}(x,\partial,x',\partial')\mathcal{W}_{\rm mc}(x,x'), \tag{5.7}$$

where

$$\begin{aligned}\Delta_{\alpha\beta\alpha'\beta'}(x,\partial,x',\partial') = {}& -\tfrac{2}{3}\mathcal{S}'\theta\theta'.(\theta.\theta' - \tfrac{1}{2}D_1[\theta'.\bar{\partial} + 2\theta'.x]) \\ & + \mathcal{S}\mathcal{S}'\theta.\theta'(\theta.\theta' - \tfrac{1}{2}D_1[\theta'.\bar{\partial} + 2\theta'.x]) \\ & + \tfrac{1}{3}D_2\mathcal{S}'(\tfrac{1}{9}D_1\theta'. + x.\theta') \\ & \times(\theta.\theta' - \tfrac{1}{2}D_1[\theta'.\bar{\partial} + 2\theta'.x]).\end{aligned} \tag{5.8}$$

On the other hand with the choice $x'$, the two-point function (5.2) satisfies Eq. (3.19) (with respect to $x'$), and we obtain

$$\begin{cases}(Q'_0+6)\theta\mathcal{W}_0 = -4\mathcal{S}\theta.\mathcal{W}_1, & \text{(I)}\\ (Q'_1+2)\mathcal{W}_1 = 0, & \text{(II)}\\ (Q'_1+6)D_2\mathcal{W}_g = 2\mathcal{S}(x'.\theta)\mathcal{W}_1 & \text{(III)}.\end{cases}$$

Using the condition $\partial'.\mathcal{W}_1 = 0$, we have

$$\theta\mathcal{W}_0(x,x') = -\tfrac{2}{3}\mathcal{S}\theta.\mathcal{W}_1(x,x'),$$
$$D_2\mathcal{W}_g(x,x') = \tfrac{1}{3}\mathcal{S}(\tfrac{1}{9}D'_1\theta. + x'.\theta)\mathcal{W}_1(x,x'),$$
$$\mathcal{W}_1(x,x') = (\theta'.\theta - \tfrac{1}{2}D'_1[\theta.\bar{\partial}' + 2x'.\theta])\mathcal{W}_{\rm mc}(x,x'),$$

where the primed operators act on the primed coordinates only. In this case, the two-point function can be written in the following form:

$$\mathcal{W}_{\alpha\beta\alpha'\beta'}(x,x') = \Delta'_{\alpha\beta\alpha'\beta'}(x,\partial,x',\partial')\mathcal{W}_{\rm mc}(x,x'),$$

where

$$\begin{aligned}\Delta'_{\alpha\beta\alpha'\beta'}(x,\partial,x',\partial') = {}& -\tfrac{2}{3}\mathcal{S}\theta'\theta.(\theta'.\theta - \tfrac{1}{2}D'_1[\theta.\bar{\partial}' + 2\theta.x']) \\ & + \mathcal{S}\mathcal{S}'\theta.\theta'(\theta.\theta' - \tfrac{1}{2}D'_1[\theta.\bar{\partial}' + 2\theta.x']) \\ & + \tfrac{1}{3}D'_2\mathcal{S}(\tfrac{1}{9}D'_1\theta. + x'.\theta) \\ & \times(\theta.\theta' - \tfrac{1}{2}D'_1[\theta.\bar{\partial}' + 2\theta.x']).\end{aligned}$$

In a few steps ahead, it is shown that this equation is none other than Eq. (5.8).

The minimally coupled scalar field two-point function in the Gupta-Bleuler vacuum is [39]

$$\mathcal{W}_{\rm mc}(x,x') = \frac{i}{8\pi^2}\epsilon(x^0 - x'^0)[\delta(1 - \mathcal{Z}(x,x')) + \vartheta(\mathcal{Z}(x,x') - 1)], \tag{5.9}$$

with

$$\epsilon(x^0 - x'^0) = \begin{cases}1 & x^0 > x'^0,\\ 0 & x^0 = x'^0,\\ -1 & x^0 < x'^0.\end{cases} \tag{5.10}$$

Equations (5.4), (5.5), (5.6), and (5.9) after relatively simple and straightforward calculations can be written as (Appendix A)

$$\theta'_{\alpha'\beta'}\mathcal{W}_0(x,x') = \frac{1}{3}\mathcal{S}'\left[\theta'_{\alpha'\beta'} + \frac{4}{1-\mathcal{Z}^2}(x.\theta'_{\alpha'})\times(x.\theta'_{\beta'})\right]\mathcal{Z}\frac{d}{d\mathcal{Z}}\mathcal{W}_{\rm mc}(\mathcal{Z}), \tag{5.11}$$

$$\mathcal{W}_{1\beta\beta'}(x,x') = \frac{1}{2}\left[\frac{3+\mathcal{Z}^2}{1-\mathcal{Z}^2}(x'.\theta_\beta)(x.\theta'_{\beta'}) - \mathcal{Z}(\theta_\beta.\theta'_{\beta'})\right]\times\frac{d}{d\mathcal{Z}}\mathcal{W}_{\rm mc}(\mathcal{Z}), \tag{5.12}$$

$$\begin{aligned}D_{2\alpha}D'_{2\alpha'}\mathcal{W}_{g\beta\beta'}(x,x') = {}& -\frac{1}{54(1-\mathcal{Z}^2)^2}\mathcal{S}\mathcal{S}'\Big[\mathcal{Z}(1-\mathcal{Z}^2)(1+3\mathcal{Z}^2)\theta_{\alpha\beta}\theta'_{\alpha'\beta'} + \frac{12\mathcal{Z}}{1-\mathcal{Z}^2}(21 - 2\mathcal{Z}^2 - 3\mathcal{Z}^4)(x'.\theta_\alpha)(x'.\theta_\beta) \\ & \times(x.\theta'_{\alpha'})(x.\theta'_{\beta'}) + 12\mathcal{Z}(1+\mathcal{Z}^2)\theta'_{\alpha'\beta'}(x'.\theta_\alpha)(x'.\theta_\beta) + 24\mathcal{Z}(2-\mathcal{Z}^2)\theta_{\alpha\beta}(x.\theta'_{\alpha'})(x.\theta'_{\beta'}) \\ & + \mathcal{Z}(1-\mathcal{Z}^2)(17 - 9\mathcal{Z}^2)(\theta_\alpha.\theta'_{\alpha'})(\theta_\beta.\theta'_{\beta'}) - (79 + 62\mathcal{Z}^2 - 45\mathcal{Z}^4)(\theta_\alpha.\theta'_{\alpha'})(x.\theta'_{\beta'})(x'.\theta_\beta)\Big] \\ & \times\frac{d}{d\mathcal{Z}}\mathcal{W}_{\rm mc}(\mathcal{Z}),\end{aligned} \tag{5.13}$$

where

$$Q_0\mathcal{W}_{\rm mc}(\mathcal{Z}) = \frac{3i}{8\pi^2}\epsilon(x^0 - x'^0)[(1-\mathcal{Z})\delta(1-\mathcal{Z})] = 0.$$

Substituting Eqs. (5.11), (5.12), and (5.13) in (5.2) yields

$$\mathcal{W}_{\alpha\beta\alpha'\beta'}(x,x') = -\frac{2Z}{27(1-Z^2)^2}\mathcal{S}\mathcal{S}'\Big[(1-Z^2)(3Z^2-2)\theta_{\alpha\beta}\theta'_{\alpha'\beta'} + 3(1+Z^2)\theta'_{\alpha'\beta'}(x'.\theta_\alpha)(x'.\theta_\beta) + 3(1+Z^2)\theta_{\alpha\beta}(x.\theta'_{\alpha'})(x.\theta'_{\beta'}) + \frac{3}{1-Z^2}(21-2Z^2-3Z^4)(x'.\theta_\alpha)(x'.\theta_\beta)(x.\theta'_{\alpha'})(x.\theta'_{\beta'}) + (1-Z^2)(11-9Z^2)(\theta_\alpha.\theta'_{\alpha'})(\theta_\beta.\theta'_{\beta'}) - \frac{2}{Z}(20+Z^2-9Z^4)(\theta_\alpha.\theta'_{\alpha'})(x.\theta'_{\beta'})(x'.\theta_\beta)\Big]\frac{d}{dZ}\mathcal{W}_{\rm mc}(Z), \tag{5.14}$$

in which we have

$$\frac{d}{dZ}\mathcal{W}_{\rm mc}(Z) = \frac{i}{8\pi^2}\frac{Z-2}{Z-1}\epsilon(x^0-x'^0)\delta(Z-1). \tag{5.15}$$

Equation (5.14) is the explicit form of the two-point function in ambient space notations. This equation satisfies the traceless and divergenceless conditions

$$\bar\partial.\mathcal{W} = \bar\partial'.\mathcal{W} = 0 \quad \text{and}$$

$$\mathcal{W}^{\alpha'}_{\alpha\beta\alpha'}(x,x') = \mathcal{W}^{\alpha}_{\alpha\alpha'\beta'}(x,x') = 0.$$

The two-point function (5.14) is obviously dS-invariant and free of any divergences. The ambient space notation clearly exhibits this fact that the gravitational field, $\mathcal{K}_{\alpha\beta}$, can be written in terms of the minimally coupled scalar field directly Eq. (4.13). It should be noted that the Gupta-Bleuler quantization of the minimally coupled scalar field, irrespective of choice of ambient space notation, does completely eliminate the infrared divergence in the scalar two-point function [10]. In Appendix F, the intrinsic counterpart of (5.14) is calculated.

## VI. CONCLUSION

It was pointed out that Einstein's theory of gravitation, in the background field method, $g_{\mu\nu} = g^{\rm BG}_{\mu\nu} + h_{\mu\nu}$, can be considered as a massless symmetric tensor field of rank-2 on a fixed background, such as dS space. Contrary to the Maxwell equation, the Einstein's equation of gravitation, as well as equation of $h_{\mu\nu}$, is not conformally invariant.

In this paper we used Dirac's six-cone formalism to obtain CI massless spin-2 wave equation in dS space which corresponds to UIRs of the dS group [$n = 2$ in (3.3)]. It was shown that the intrinsic counterpart of CI wave equation with conformal-degree $-1$ [$n = 1$ in (3.3)] is similar to what Barut and Xu have obtained in Minkowski space. This, however, is not a physical state of the dS group. Barut and Böhm [19] have shown that for the physical representation of the conformal group, the value of the conformal Casimir operator is 9. But according to (3.1) for the tensor field of rank-2 and conformal-degree 0, this value becomes 8 on the cone. Therefore, the tensor field of rank-2 does not correspond to the UIRs of the conformal group (physical state of group). In other words, the tensor field that carries the physical representations of the conformal group must be a tensor field of higher rank. In the forthcoming paper we will consider a mixed symmetry rank-3 tensor field $\Psi^{abc}$ with degree 0 that transforms simultaneously under the action of dS and conformal groups.

In addition to obtaining the CI wave equation in dS space, we have shown that the physical part of the linear gravity, in ambient space notations, can be written as the product of a generalized symmetric rank-2 polarization tensor and a massless minimally coupled scalar field. Using the Gupta-Bleuler quantization we have calculated the physical graviton two-point function, which is dS-invariant and free of any divergences. This two-point function can be used to calculate the quantum effects of gravity in the interaction cases, which will be considered in forthcoming papers.

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## APPENDIX A: SOME USEFUL RELATIONS

In this appendix, some useful relations are given. The action of the Casimir operators $Q_1$ and $Q_2$ can be written in the more explicit form

$$Q_1K_\alpha = (Q_0-2)K_\alpha + 2x_\alpha\partial.K - 2\partial_\alpha x\cdot K, \tag{A1}$$

$$Q_2\mathcal{K}_{\alpha\beta} = (Q_0-6)\mathcal{K}_{\alpha\beta} + 2\mathcal{S}x_\alpha\partial\cdot\mathcal{K}_\beta - 2\mathcal{S}\partial_\alpha x\cdot\mathcal{K}_\beta + 2\eta_{\alpha\beta}\mathcal{K}' \tag{A2}$$

$$Q_1D_1 = D_1Q_0, \tag{A3}$$

$$(Q_0-2)x_\alpha = x_\alpha Q_0 - 6x_\alpha - 2\bar\partial_\alpha, \tag{A4}$$

$$\bar\partial_\alpha(Q_0-2) = Q_0\bar\partial_\alpha - 8\bar\partial_\alpha - 2Q_0x_\alpha - 8x_\alpha, \tag{A5}$$

$$x_\alpha Q_0(Q_0-2) = (Q_0-2)(Q_0x_\alpha + 4x_\alpha + 4\bar\partial_\alpha), \tag{A6}$$

$$[Q_0Q_2, Q_2Q_0]\mathcal{K}_{\alpha\beta} = 4\mathcal{S}(x_\alpha - \bar\partial_\alpha)\bar\partial.\mathcal{K}_\beta, \tag{A7}$$

the transverse divergence $\bar\partial_\alpha$ can be written with respect to $\partial_\alpha$ as the following:

$$\bar\partial_\alpha \equiv \partial_\alpha + x_\alpha x\cdot\partial = \partial_\alpha - x_\alpha + x\cdot\partial x_\alpha. \tag{A8}$$

To obtain the two-point function, the following identities are used:

$$\bar\partial_{\alpha} f(\mathcal{Z}) = -(x'.\theta_\alpha)\frac{df(\mathcal{Z})}{d\mathcal{Z}}, \tag{A9}$$

$$\theta^{\alpha\beta}\theta'_{\alpha\beta} = \theta..\theta' = 3 + \mathcal{Z}^2, \tag{A10}$$

$$(x.\theta'_{\alpha'})(x.\theta'^{\alpha'}) = \mathcal{Z}^2 - 1, \tag{A11}$$

$$(x.\theta'_{\alpha})(x'.\theta^{\alpha}) = \mathcal{Z}(1 - \mathcal{Z}^2), \tag{A12}$$

$$\bar\partial_{\alpha}(x.\theta'_{\beta'}) = \theta_\alpha.\theta'_{\beta'}, \tag{A13}$$

$$\bar\partial_{\alpha}(x'.\theta_\beta) = x_\beta(x'.\theta_\alpha) - \mathcal{Z}\theta_{\alpha\beta}, \tag{A14}$$

$$\bar\partial_{\alpha}(\theta_\beta.\theta'_{\beta'}) = x_\beta(\theta_\alpha.\theta'_{\beta'}) + \theta_{\alpha\beta}(x.\theta'_{\beta'}), \tag{A15}$$

$$\theta'^{\beta}_{\alpha'}(x'.\theta_\beta) = -\mathcal{Z}(x.\theta'_{\alpha'}), \tag{A16}$$

$$\theta'^{\gamma}_{\alpha'}(\theta_\gamma.\theta'_{\beta'}) = \theta'_{\alpha'\beta'} + (x.\theta'_{\alpha'})(x.\theta'_{\beta'}), \tag{A17}$$

$$Q_0 f(\mathcal{Z}) = (1 - \mathcal{Z}^2)\frac{d^2 f(\mathcal{Z})}{d\mathcal{Z}^2} - 4\mathcal{Z}\frac{df(\mathcal{Z})}{d\mathcal{Z}}. \tag{A18}$$

## APPENDIX B: CI WAVE EQUATION WITH $n = 1$

We show that the CI wave equation for the tensor field $\Psi_{ab}$ with $n = 1$ does not transform according to the UIRs of the dS and conformal groups.

The CI system (3.3) with $n = 1$, i.e. for the tensor field with degree $-1$, reads as

$$(Q_0 - 2)\Psi_{\alpha\beta} = 0, \tag{B1}$$

using the transversality condition, $u_a \Psi^{ab} = 0$, we get

$$(Q_0 - 2)x.\Psi_\beta = 0, \tag{B2}$$

$$(Q_0 - 2)x.\Psi.x = 0. \tag{B3}$$

Note that the relation (3.3) is used.

Multiplying (B1) and (B2) by $x_\beta$ results in

$$2x.\Psi_\alpha = -\bar\partial \cdot \Psi_\alpha, \tag{B4}$$

$$2x.\Psi.x = -\bar\partial \cdot \Psi.x. \tag{B5}$$

The divergence of $\mathcal{K}_{\alpha\beta}$ leads to

$$\bar\partial \cdot \mathcal{K}_\beta = \bar\partial \cdot \Psi_\beta + 5x.\Psi_\beta + 5x_\beta x.\Psi.x + x_\beta \bar\partial \cdot \Psi \cdot x. \tag{B6}$$

Combining the Eq. (B4)–(B6) leads to

$$\bar\partial \cdot \mathcal{K}_\beta = 3(x.\Psi_\beta + x_\beta x.\Psi.x). \tag{B7}$$

After some calculations one finds

$$(Q_0 - 2)\mathcal{K}_{\alpha\beta} + 2(\bar\partial_\alpha + 2x_\alpha)x.\Psi_\beta + 2(\bar\partial_\beta + 2x_\beta)x.\Psi_\alpha + 2(\bar\partial_\alpha + 2x_\alpha)x_\beta x.\Psi.x + 2x_\alpha(\bar\partial_\beta + 2x_\beta)x \cdot \Psi \cdot x = 0. \tag{B8}$$

Substituting Eq. (B7) into Eq. (B8) leads exactly to (3.10). In order to express Eq. (3.9) in terms of the intrinsic coordinates the following relation is used [35]:

$$\nabla_\mu \nabla_\nu \cdots \nabla_\rho h_{\lambda_1\cdots\lambda_l} = \frac{\partial x^\alpha}{\partial X^\mu}\frac{\partial x^\beta}{\partial X^\nu}\cdots\frac{\partial x^\gamma}{\partial X^\rho}\frac{\partial x^{\eta_1}}{\partial X^{\lambda_1}}\cdots \times \frac{\partial x^{\eta_l}}{\partial X^{\lambda_l}}\mathrm{Tr}pr\bar\partial_\alpha \times \mathrm{Tr}pr\bar\partial_\beta \cdots \mathrm{Tr}pr\bar\partial_\gamma \mathcal{K}_{\eta_1\cdots\eta_l}$$

where the transverse projection defined by

$$(\mathrm{Tr}pr\mathcal{K})_{\lambda_1\cdots\lambda_l} \equiv \theta^{\eta_1}_{\lambda_1}\cdots\theta^{\eta_l}_{\lambda_l}\mathcal{K}_{\eta_1\cdots\eta_l}$$

guarantees the transversality in each index. Applying this procedure to a transverse second rank, symmetric tensor field, leads to

$$\nabla_\mu\nabla_\nu h_{\rho\lambda} = \frac{\partial x^\alpha}{\partial X^\mu}\frac{\partial x^\beta}{\partial X^\nu}\frac{\partial x^\gamma}{\partial X^\rho}\frac{\partial x^\eta}{\partial X^\lambda}\mathrm{Tr}pr\bar\partial_\alpha \mathrm{Tr}pr\bar\partial_\beta \mathcal{K}_{\gamma\eta},$$

where we have

$$\begin{aligned}\mathrm{Tr}pr\bar\partial_\alpha \mathrm{Tr}pr\bar\partial_\beta \mathcal{K}_{\gamma\eta} = {} & \bar\partial_\alpha(\bar\partial_\beta \mathcal{K}_{\gamma\eta} - x_\gamma \mathcal{K}_{\beta\eta} - x_\eta \mathcal{K}_{\gamma\beta}) \\ & - x_\beta(\bar\partial_\alpha \mathcal{K}_{\gamma\eta} - x_\gamma \mathcal{K}_{\alpha\eta} - x_\eta \mathcal{K}_{\gamma\alpha}) \\ & - x_\gamma(\bar\partial_\beta \mathcal{K}_{\alpha\eta} - x_\alpha \mathcal{K}_{\beta\eta} - x_\eta \mathcal{K}_{\alpha\beta}) \\ & - x_\eta(\bar\partial_\beta - x_\gamma \mathcal{K}_{\beta\alpha} - x_\alpha \mathcal{K}_{\gamma\beta}).\end{aligned}$$

Thus we can write

$$\nabla_\lambda \nabla^\lambda h_{\mu\nu} \equiv \Box h_{\mu\nu} \rightarrow \bar\partial_\alpha \bar\partial^\alpha \mathcal{K}_{\gamma\eta} - 2\mathcal{K}_{\gamma\eta} - 2\mathcal{S}x_\gamma \bar\partial \cdot \mathcal{K}_\eta, \tag{B9}$$

$$\nabla_\lambda \nabla \cdot h_\mu \rightarrow \bar\partial_\eta \bar\partial \cdot \mathcal{K}_\gamma - x_\gamma \bar\partial \cdot \mathcal{K}_\eta, \qquad g^{dS}_{\mu\nu} \rightarrow \theta_{\gamma\eta}. \tag{B10}$$

Using the above statements and $Q_0 = -\bar\partial_\alpha \bar\partial^\alpha$ the intrinsic counterpart of (3.10) can be easily derived.

## APPENDIX C: SOME DETAILS ABOUT EQS. (3.15) AND (3.16)

The condition (3.14) for the tensor field with degree zero leads to

$$\partial.\Psi_\beta = -x \cdot \partial x \cdot \Psi_\beta, \tag{C1}$$

$$\partial.\Psi_5 = -x \cdot \partial x \cdot \Psi_5. \tag{C2}$$

Combining (3.13) and (C2) results in

$$\partial.\Psi.x + x \cdot \partial x.\Psi.x = 0. \tag{C3}$$

In this case we rewrite (B6) in the following form:

$$\bar{\partial}.\mathcal{K}_\beta = 4(\Psi_\beta.x + x_\beta x.\Psi.x) + (\bar{\partial}.\Psi_\beta + x.\Psi_\beta + x_\beta x.\Psi.x + x_\beta\bar{\partial}.\Psi.x). \tag{C4}$$

According to relations (A8), (C1), and (C3), the second parenthesis vanishes, and therefore we get Eq. (3.15).

Finally according to (3.15) and (A1), we can write the following relations for the vector field $\bar{\partial}.\mathcal{K}_\alpha$:

$$Q_1\bar{\partial}.\mathcal{K}_\alpha = (Q_0 - 2)\bar{\partial}.\mathcal{K}_\alpha + 2x_\alpha\bar{\partial}.\bar{\partial}.\mathcal{K}, \tag{C5}$$

$$(Q_0 - 2)\bar{\partial}.\mathcal{K}_\alpha = 4((Q_0 - 2)x.\Psi_\alpha + (Q_0 - 2)x_\alpha x.\Psi.x). \tag{C6}$$

After some calculation it is easy to show that

$$(Q_0 - 2)(x.\Psi_\alpha + x_\alpha x.\Psi.x) = -\tfrac{1}{6}(D_{1\alpha}Q_0 + 4D_{1\alpha} + 12x_\alpha) \times(\bar{\partial}.\Psi.x + 4x.\Psi.x). \tag{C7}$$

Note that

$$\bar{\partial}.\bar{\partial}.\mathcal{K} = 4(\bar{\partial}.\Psi.x + 4x.\Psi.x). \tag{C8}$$

By substituting (3.15) and (C8) into (C7), we get (3.16).

## APPENDIX D: DETAILS OF CALCULATION OF EQ. (3.17)

For symmetric rank-2 field $\Psi_{ab}$, the CI system (3.12) results in

$$Q_0(Q_0 - 2)\Psi_{\alpha\beta} = 0, \qquad Q_0(Q_0 - 2)\Psi_{55} = 0. \tag{D1}$$

Using conditions $\mathcal{K}' = 0$ and (3.13), we get

$$Q_0(Q_0 - 2)x.\Psi.x = 0, \tag{D2}$$

$$Q_0(Q_0 - 2)x.\Psi_\beta = 0. \tag{D3}$$

Taking the divergence of (3.16) leads to

$$Q_0(Q_0 - 2)\bar{\partial}.\Psi.x = 0. \tag{D4}$$

The action of operator $Q_0(Q_0 - 2)$ on the dS field can be written in more explicit form:

$$Q_0(Q_0 - 2)\mathcal{K}_{\alpha\beta} = Q_0(Q_0 - 2)\mathcal{S}x_\alpha\Psi_\beta\cdot x + Q_0(Q_0 - 2)x_\alpha x_\beta x\cdot\Psi\cdot x. \tag{D5}$$

According to (A4) and (A5), the above equation can be written as follows:

$$\begin{aligned}Q_0(Q_0 - 2)\mathcal{K}_{\alpha\beta} = {}& -4(3x_\alpha + \bar{\partial}_\alpha)(Q_0 - 2)\Psi_\beta\cdot x\\ &- 4(3x_\beta + \bar{\partial}_\beta)(Q_0 - 2)\Psi_\alpha\cdot x\\ &- 4x_\alpha(3x_\beta + \bar{\partial}_\beta)(Q_0 - 2)x\cdot\Psi\cdot x\\ &- 4(3x_\alpha + \bar{\partial}_\alpha)(Q_0 - 2)x_\beta x\cdot\Psi\cdot x,\end{aligned} \tag{D6}$$

or we can write

$$\begin{aligned}Q_0(Q_0 - 2)\mathcal{K}_{\alpha\beta} = {}& -4(3x_\beta + \bar{\partial}_\beta)(Q_0 - 2)\Psi_\alpha\cdot x\\ &- (3x_\alpha + \bar{\partial}_\alpha)(Q_0 - 2)\bar{\partial}\cdot\mathcal{K}_\beta\\ &- 4x_\alpha(3x_\beta + \bar{\partial}_\beta)(Q_0 - 2)x\cdot\Psi\cdot x.\end{aligned} \tag{D7}$$

Note that identity (3.15) is used.

Multiplying (D3) by $x_\beta$ results in

$$(Q_0 - 2)(x.\Psi.x + \bar{\partial}.\Psi.x) = 0. \tag{D8}$$

Substituting the divergence of (3.15) into the above equation leads to

$$(Q_0 - 2)x.\Psi.x = \tfrac{1}{12}(Q_0 - 2)\bar{\partial}\cdot\bar{\partial}\cdot\mathcal{K}. \tag{D9}$$

So we can rewrite (D7) as follows:

$$\begin{aligned}(Q_0 - 2)Q_0\mathcal{K}_{\alpha\beta} = {}& -4(3x_\beta + \bar{\partial}_\beta)(Q_0 - 2)\Psi_\alpha\cdot x\\ &- (3x_\alpha + \bar{\partial}_\alpha)(Q_0 - 2)\bar{\partial}\cdot\mathcal{K}_\beta\\ &- \tfrac{1}{3}x_\alpha(3x_\beta + \bar{\partial}_\beta)(Q_0 - 2)\bar{\partial}.\bar{\partial}.\mathcal{K}.\end{aligned} \tag{D10}$$

Multiplying the above equation by $x_\beta$ leads to

$$\begin{aligned}(Q_0 - 2)\Psi_\alpha\cdot x = {}& \tfrac{1}{4}(Q_0 - 2)\bar{\partial}\cdot\mathcal{K}_\alpha + \tfrac{1}{3}x_\alpha\bar{\partial}\cdot\bar{\partial}\cdot\mathcal{K}\\ &- \tfrac{1}{12}x_\alpha(Q_0 - 2)\bar{\partial}\cdot\bar{\partial}\cdot\mathcal{K}\\ &+ \tfrac{1}{6}\bar{\partial}_\alpha\bar{\partial}\cdot\bar{\partial}\cdot\mathcal{K}.\end{aligned} \tag{D11}$$

Finally combining (D10) and (D11) leads to

$$\begin{aligned}&(Q_0 - 2)Q_0\mathcal{K}_{\alpha\beta} + Q_0\mathcal{S}x_\beta\bar{\partial}\cdot\mathcal{K}_\alpha + Q_0\mathcal{S}\bar{\partial}_\alpha\bar{\partial}\cdot\mathcal{K}_\beta\\ &\quad - 2\mathcal{S}x_\alpha\bar{\partial}\cdot\mathcal{K}_\beta - 2\mathcal{S}\bar{\partial}_\alpha\bar{\partial}\cdot\mathcal{K}_\beta + 4x_\alpha x_\beta\bar{\partial}.\bar{\partial}.\mathcal{K}\\ &\quad + \tfrac{1}{3}\mathcal{S}\bar{\partial}_\beta\bar{\partial}_\alpha\bar{\partial}.\bar{\partial}.\mathcal{K} + \tfrac{5}{3}\mathcal{S}x_\alpha\bar{\partial}_\beta\bar{\partial}.\bar{\partial}.\mathcal{K}\\ &\quad + 2\theta_{\alpha\beta}\bar{\partial}.\bar{\partial}.\mathcal{K} - \tfrac{1}{3}\theta_{\alpha\beta}Q_0\bar{\partial}.\bar{\partial}.\mathcal{K} = 0.\end{aligned} \tag{D12}$$

It is easy to show that if we rewrite Eq. (3.17) in terms of $Q_0$, we will get back exactly to Eq. (D12). Note that for this calculation the following relations have been used:

$$\begin{aligned}(Q_2 + 4)(Q_2 + 6)\mathcal{K}_{\alpha\beta} = {}& Q_0(Q_0 - 2)\mathcal{K}_{\alpha\beta}\\ &+ 4\mathcal{S}[Q_0x_\alpha\bar{\partial}.\mathcal{K}_\beta + 3x_\alpha\bar{\partial}.\mathcal{K}_\beta\\ &+ \bar{\partial}_\alpha\bar{\partial}.\mathcal{K}_\beta + x_\alpha x_\beta\bar{\partial}.\bar{\partial}.\mathcal{K}],\end{aligned} \tag{D13}$$

$$\begin{aligned}(Q_2 + 4)D_{2\alpha}\partial_2.\mathcal{K}_\beta = {}& \mathcal{S}[-3Q_0x_\alpha\bar{\partial}.\mathcal{K}_\beta + Q_0\bar{\partial}_\alpha\bar{\partial}.\mathcal{K}_\beta\\ &- 6\bar{\partial}_\alpha\bar{\partial}.\mathcal{K}_\beta - 14x_\alpha\bar{\partial}.\mathcal{K}_\beta\\ &- 2x_\alpha x_\beta\bar{\partial}.\bar{\partial}.\mathcal{K} + 2\theta_{\alpha\beta}\bar{\partial}.\bar{\partial}.\mathcal{K}\\ &+ 2x_\beta\bar{\partial}_\alpha\bar{\partial}.\bar{\partial}.\mathcal{K}],\end{aligned} \tag{D14}$$

$$D_{2\alpha}D_{1\beta}\bar{\partial}.\bar{\partial}.\mathcal{K} = \mathcal{S}[\bar{\partial}_\alpha\bar{\partial}_\beta - x_\alpha\bar{\partial}_\beta]\bar{\partial}.\bar{\partial}.\mathcal{K}. \tag{D15}$$

## APPENDIX E: DETAILS ON EQ. (4.12)

Using (A1), it is easy to show that

$$D_1(Z_1.K) = \tfrac{1}{6}(Q_1+6)[D_1(Z_1.K)], \tag{E1}$$

$$x(Z_1.K) = \tfrac{1}{6}(Q_1+6)[x(Z_1.K)], \tag{E2}$$

$$Z_1.\bar{\partial}K = \tfrac{1}{6}(Q_1+6)[Z_1.\bar{\partial}K - \tfrac{1}{3}D_1(Z_1.K)], \tag{E3}$$

$$(Q_1+6)[(x.Z_1)K] = 2[x(Z_1.K) - Z_1.\bar{\partial}K]. \tag{E4}$$

The conditions $x.K = \bar{\partial}.K = 0$ and $Q_0K = 0$, are used to obtain the above equations.

Substituting Eqs. (E2) and (E3) in (E4) we have

$$(Q_1+6)[(x.Z_1)K] = \tfrac{1}{3}(Q_1+6)[\tfrac{1}{3}D_1(Z_1.K) + x(Z_1.K) - Z_1.\bar{\partial}K], \tag{E5}$$

or

$$(x.Z_1)K = \tfrac{1}{3}[\tfrac{1}{3}D_1(Z_1.K) + x(Z_1.K) - Z_1.\bar{\partial}K]. \tag{E6}$$

Finally according to Eqs. (E1) and (E4), we obtain

$$(x.Z_1)K = \tfrac{1}{6}(Q_1+6)[\tfrac{1}{9}D_1(Z_1.K) + (x.Z_1)K]. \tag{E7}$$

This automatically leads to Eq. (4.12).

## APPENDIX F: TWO-POINT FUNCTION IN DS INTRINSIC COORDINATES

In order to compare our results with the work of the other authors [12,13], we write the two-point function in dS space (maximally symmetric) in terms of bitensors. These are functions of two points $(x, x')$ and behave like tensors under coordinate transformations at each points.

As mentioned in [4], any maximally symmetric bitensor can be expressed as a sum of products of three basic tensors. The coefficients in this expansion are functions of the geodesic distance $\sigma(x, x')$, that is the distance along the geodesic connecting the points $x$ and $x'$ [note that $\sigma(x, x')$ can be defined by an unique analytic extension also when no geodesic connects $x$ and $x'$]. In this sense, these fundamental tensors form a complete set. They can be obtained by differentiating the geodesic distance:

$$n_\mu = \nabla_\mu \sigma(x, x'), \qquad n_{\mu'} = \nabla_{\mu'}\sigma(x, x'),$$

and the parallel propagator

$$g_{\mu\nu'} = -c^{-1}(\mathcal{Z})\nabla_\mu n_{\nu'} + n_\mu n_{\nu'}.$$

The geodesic distance is implicitly defined for $\mathcal{Z} = -x\cdot x'$, by (1) $\mathcal{Z} = \cosh(\sigma)$ if $x$ and $x'$ are timelike separated, (2) $\mathcal{Z} = \cos(\sigma)$ if $x$ and $x'$ are spacelike separated. The basic bitensors in ambient space notations are found through

$$\bar{\partial}_\alpha \sigma(x, x'), \qquad \bar{\partial}'_{\beta'}\sigma(x, x'), \qquad \theta_\alpha.\theta'_{\beta'},$$

restricted to the hyperboloid by

$$\mathcal{T}_{\mu\nu'} = \frac{\partial x^\alpha}{\partial X^\mu}\frac{\partial x'^{\beta'}}{\partial X'^{\nu'}} T_{\alpha\beta'}.$$

For $\mathcal{Z} = \cos(\sigma)$, one can find

$$n_\mu = \frac{\partial x^\alpha}{\partial X^\mu}\bar{\partial}_\alpha \sigma(x, x') = \frac{\partial x^\alpha}{\partial X^\mu}\frac{(x'\cdot\theta_\alpha)}{\sqrt{1-\mathcal{Z}^2}},$$

$$n_{\nu'} = \frac{\partial x'^{\beta'}}{\partial X'^{\nu'}}\bar{\partial}_{\beta'}\sigma(x, x') = \frac{\partial x'^{\beta'}}{\partial X'^{\nu'}}\frac{(x\cdot\theta'_{\beta'})}{\sqrt{1-\mathcal{Z}^2}},$$

$$\nabla_\mu n_{\nu'} = \frac{\partial x^\alpha}{\partial X^\mu}\frac{\partial x'^{\beta'}}{\partial X'^{\nu'}}\theta^\varrho_\alpha \theta'^{\gamma'}_{\beta'}\bar{\partial}_\varrho \bar{\partial}_{\gamma'}\sigma(x, x')$$

$$= c(\mathcal{Z})\left[n_\mu n_{\nu'}\mathcal{Z} - \frac{\partial x^\alpha}{\partial X^\mu}\frac{\partial x'^{\beta'}}{\partial X'^{\nu'}}\theta_\alpha\cdot\theta'_{\beta'}\right],$$

with $c^{-1}(\mathcal{Z}) = -\frac{1}{\sqrt{1-\mathcal{Z}^2}}$. For $\mathcal{Z} = \cosh(\sigma)$, $n_\mu$ and $n_\nu$ are multiplied by $i$ and $c(\mathcal{Z})$ becomes $-\frac{i}{\sqrt{1-\mathcal{Z}^2}}$. In both cases we have

$$g_{\mu\nu'} + (\mathcal{Z}-1)n_\mu n_{\nu'} = \frac{\partial x^\alpha}{\partial X^\mu}\frac{\partial x'^{\beta'}}{\partial X'^{\nu'}}\theta_\alpha\cdot\theta'_{\beta'},$$

and the two-point functions are related through

$$Q_{\mu\nu\mu'\nu'} = \frac{\partial x^\alpha}{\partial X^\mu}\frac{\partial x^\beta}{\partial X^\nu}\frac{\partial x'^{\alpha'}}{\partial X'^{\mu'}}\frac{\partial x'^{\beta'}}{\partial X'^{\nu'}}\mathcal{W}_{\alpha\beta\alpha'\beta'}.$$

Considering the above expressions the two-point function (5.14) takes the following form:

$$\begin{aligned} Q_{\mu\nu\mu'\nu'}(X, X') = &-\frac{2}{27(1-\mathcal{Z}^2)}\mathcal{S}\mathcal{S}'[\mathcal{Z}(3\mathcal{Z}^2-2)g_{\mu\nu}g'_{\mu'\nu'} \\ &+ 3\mathcal{Z}(1+\mathcal{Z}^2)(g'_{\mu'\nu'}n_\mu n_\nu + g_{\mu\nu}n_{\mu'}n_{\nu'}) \\ &+ \mathcal{Z}(11-9\mathcal{Z}^2)g_{\mu\mu'}g_{\nu\nu'} + (40+32\mathcal{Z} \\ &- 20\mathcal{Z}^2 - 6\mathcal{Z}^3 + 9\mathcal{Z}^4 - 9\mathcal{Z}^5)n_\mu n_\nu n_{\mu'}n_{\nu'} \\ &+ (-40+9\mathcal{Z}^2+9\mathcal{Z}^4)g_{\mu\mu'}n_\nu n_{\nu'}] \\ &\times \frac{d}{d\mathcal{Z}}\mathcal{W}_{\rm mc}(\mathcal{Z}). \end{aligned} \tag{F1}$$

The two-point function (F1) is obviously dS-invariant, and appearance of the factors $\mathcal{Z}\delta(\mathcal{Z}-1)$, $\mathcal{Z}^2\delta(\mathcal{Z}-1)$, $\mathcal{Z}^3\delta(\mathcal{Z}-1)$ make it free of any divergences.

[1] J. Bros and U. Moschella, Rev. Math. Phys. **8**, 327 (1996).
[2] J. P. Gazeau and M. V. Takook, J. Math. Phys. (N.Y.) **41**, 5920 (2000).
[3] C. Gabriel and P. Spindel, J. Math. Phys. (N.Y.) **38**, 622 (1997).
[4] T. Garidi, J. P. Gazeau, and M. V. Takook, J. Math. Phys. (N.Y.) **44**, 3838 (2003).
[5] E. Angelopoulos and M. Laoues, Rev. Math. Phys. **10**, 1079 (1998).
[6] S. Behroozi, S. Rouhani, M. V. Takook, and M. R. Tanhayi, Phys. Rev. D **74**, 124014 (2006).
[7] P. A. M. Dirac, Annals of mathematical logic **36**, 657 (1935).
[8] G. Mack and A. Salam, Ann. Phys. (N.Y.) **53**, 174 (1969).
[9] H. A. Kastrup, Phys. Rev. **150**, 1183 (1966); C. R. Preitschopf and M. A. Vasiliev, Nucl. Phys. **B549**, 450 (1999).
[10] J. P. Gazeau, J. Renaud, and M. V. Takook, Classical Quantum Gravity **17**, 1415 (2000).
[11] S. de Bièvre and J. Renaud, Phys. Rev. D **57**, 6230 (1998).
[12] A. Higuchi and S. S. Kouris, Classical Quantum Gravity **17**, 3077 (2000).
[13] A. Higuchi and S. S. Kouris, Classical Quantum Gravity **20**, 3005 (2003).
[14] J. P. Gazeau, Lett. Math. Phys. **8**, 507 (1984).
[15] J. Dixmier, Bulletin de la Societe mathematique de France **89**, 9 (1961).
[16] B. Takahashi, Bulletin de la Societe mathematique de France **91**, 289 (1963).
[17] M. Levy-Nahas, J. Math. Phys. (N.Y.) **8**, 1211 (1967).
[18] H. Bacry and J. M. Levy-Leblond, J. Math. Phys. (N.Y.) **9**, 1605 (1968).
[19] A. O. Barut and A. Böhm, J. Math. Phys. (N.Y.) **11**, 2938 (1970).
[20] E. Angelopoulos and M. Laoues, Rev. Math. Phys. **10**, 271 (1998).
[21] C. Fronsdal, Phys. Rev. D **20**, 848 (1979).
[22] B. Binegar, C. Fronsdal, and W. Heidenreich, Phys. Rev. D **27**, 2249 (1983).
[23] A. O. Barut and B. W. Xu, J. Phys. A **15**, L207 (1982).
[24] J. P. Gazeau, M. Hans, and R. Murenzi, Classical Quantum Gravity **6**, 329 (1989).
[25] J. P. Gazeau, S. Rouhani, and M. V. Takook (unpublished).
[26] T. Garidi, J. P. Gazeau, S. Rouhani, and M. V. Takook, J. Math. Phys. (N.Y.) **49**, 032501 (2008).
[27] B. Allen, Phys. Rev. D **32**, 3136 (1985).
[28] B. Allen and M. Turyn, Nucl. Phys. **B292**, 813 (1987).
[29] E. G. Floratos, J. Iliopoulos, and T. N. Tomaras, Phys. Lett. B **197**, 373 (1987).
[30] I. Antoniadis and E. Mottola, J. Math. Phys. (N.Y.) **32**, 1037 (1991).
[31] H. L. Ford, Phys. Rev. D **31**, 710 (1985).
[32] I. Antoniadis, J. Iliopoulos, and T. N. Tomaras, Phys. Rev. Lett. **56**, 1319 (1986).
[33] N. C. Tsamis and R. P. Woodard, Phys. Lett. B **292**, 269 (1992); Commun. Math. Phys. **162**, 217 (1994).
[34] I. Antoniadis, J. Iliopoulos, and T. N. Tomaras, Nucl. Phys. **B462**, 437 (1996).
[35] M. V. Takook, Ph.D. thesis, Universit Paris P. M. Curie (Paris 6),, 1997.
[36] H. J. de Vega, J. Ramirez, and N. Sanchez, Phys. Rev. D **60**, 044007 (1999).
[37] M. V. Takook, Proceeding of the Wigsym6, August, 1999, Istanbul, Turkey.
[38] S. W. Hawking, T. Hertog, and N. Turok, Phys. Rev. D **62**, 063502 (2000).
[39] M. V. Takook, Mod. Phys. Lett. A **16**, 1691 (2001).