

Decay of massive Dirac hair on a brane-world black hole

Gary W. Gibbons*

DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, United Kingdom

Marek Rogatko[†] and Agnieszka Szyplowska

Institute of Physics, Maria Curie-Skłodowska University, 20-031 Lublin, pl. Marii Curie-Skłodowskiej 1, Poland

(Received 29 January 2008; published 20 March 2008)

We investigate the intermediate and late-time behavior of the massive Dirac spinor field in the background of static spherically symmetric brane-world black hole solutions. The intermediate asymptotic behavior of the massive spinor field exhibits a dependence on the field's parameter mass as well as the multiple number of the wave mode. On the other hand, the late-time behavior power-law decay has a rate which is independent of those factors.

DOI: [10.1103/PhysRevD.77.064024](https://doi.org/10.1103/PhysRevD.77.064024)

PACS numbers: 04.50.-h

I. INTRODUCTION

Nowadays it is widely believed that extra dimensions play a significant role in the construction of a unified theory of the four fundamental forces of nature. In such models it is often the case that our Universe can be treated as a submanifold to which the standard model is confined, embedded in a higher dimensional spacetime. If one takes the volume of the extra dimensions spacetime to be sufficiently large, one is able to lower the fundamental quantum gravity scale to the electrovac scale of the order of a TeV. It is thus of interest to construct black hole solutions in such brane-world models. The difficulties arising in such attempts stem from the fact that, in general, brane dynamics generates Weyl curvatures which in turn backreact on the brane dynamics. We can look at the problem in question by projecting the Einstein equations onto the brane. This approach was introduced in Refs. [1,2]. It is also of interest to think of a four-dimensional brane-world black hole solution as a slice that intersects a bulk black hole [3–5]. In Ref. [6] the possibility was raised of finding a regular Randall-Sundrum (RS) brane-world on which a static spherically symmetric black hole, surrounded by realistic matter, is located. This was achieved by slicing a fixed five-dimensional bulk black hole spacetime. On the other hand, studies of spherically symmetric brane-world solutions with induced gravity were extended to include nonlocal bulk effects [7]. The scalar as well as the axial gravitational perturbations of what we shall call “brane-world black holes” were studied in Ref. [8].

An important question for black hole physics is the investigation of how various fields decay in the spacetime outside a collapsing body. The importance arises from the fact that, regardless of the details of the gravitational collapse and features of the collapsing body, the outcome of this process, i.e. the resultant black hole is characterized

by just a few parameters such as mass, charge, and angular momentum. The first researches in this direction were carried by Price in Ref. [9] while the scalar perturbations on the Reissner-Nordström (RN) background were considered in [10]. It was found that charged scalar hair decayed more slowly than neutral hair [11–13], while the late-time tails in the gravitational collapse of a massive field in the background of the Schwarzschild solution were reported by Burko [14] and in the Reissner-Nordström solution at intermediate late-time were considered in Ref. [15]. The very late-time tails of the massive scalar fields in the Schwarzschild and nearly extremal Reissner-Nordström black holes were obtained in Refs. [16,17]. It was shown that the oscillatory tail of scalar field decays like $t^{-5/6}$ at late-time. The power-law tails in the evolution of a charged massless scalar field around the fixed background of a dilaton black hole were studied in Ref. [18], while the case of a massive scalar field was treated in [19]. The analytical proof of the intermediate and late-time behavior of that in the case of dilaton gravity with arbitrary coupling constant was provided in Ref. [20]. On the other hand, the problem of the late-time behavior of massive Dirac fields were studied, respectively, in the spacetime of the Schwarzschild, Reissner-Nordström, and Kerr-Newman black hole [21–23]. Reference [24] was devoted to the analytical studies of the intermediate and late-time decay pattern of massive Dirac hair on a spherically symmetric dilaton black hole, in dilaton gravity theory with arbitrary coupling constant α .

The growth of interest in unification scheme such as superstring/M theory triggered in turn an interest in the decay of hair in the spacetimes of n -dimensional black holes. The *no-hair* and uniqueness property for static holes is by now quite well established [25]. The decay mechanism for massless scalar hair in the n -dimensional Schwarzschild spacetime was given in Ref. [26]. The decay pattern of scalar massive fields in the spacetime of the n -dimensional static charged black hole was discussed in Ref. [27]. It was shown that the intermediate asymptotic behavior of the hair in question was of the form

*g.w.gibbons@damtp.cam.ac.uk

[†]rogat@tytan.umcs.lublin.pl;

E-mail: rogat@kft.umcs.lublin.pl

$t^{-(l+n/2-1/2)}$. A numerical experiment for $n = 5$ and $n = 6$ confirmed these results. In Ref. [28] the authors obtained fermion quasinormal modes for the massless Dirac fermion in the background of the higher dimensional Schwarzschild black hole.

As far as the brane-world black holes are concerned, Ref. [29] was devoted to studies of the intermediate and late-time behavior of the massive scalar field in the background of a static spherically symmetric brane-world black hole. Among other things, it was shown that the late-time power-law decay rate is proportional to $t^{-5/6}$. The massless fermion excitations on a tensional 3-brane embedded in six-dimensional spacetime were studied in [30].

The main aim of our paper will be to clarify what kind of mass-induced behavior plays the dominant role in the asymptotic late-time tails as a result of decaying massive Dirac spinor hair in the background of the brane-world black hole.

The paper is organized as follows. In Sec. II we gave the analytic arguments concerning the decay of massive Dirac hair in the background of the considered black hole. Section III will be devoted to a summary and discussion.

II. THE DECAY OF DIRAC HAIR IN THE BACKGROUND OF THE BLACK HOLE BRANE SOLUTION

A. Spinor fields

We shall begin our analysis by recalling the general properties of the massive Dirac equation in an n -dimensional spherically symmetric background [24]. Namely, we shall study the massive Dirac equation given by the relation

$$(\gamma^\mu \nabla_\mu \psi - m)\psi = 0, \quad (1)$$

where ∇_μ is the covariant derivative $\nabla_\mu = \partial_\mu + \frac{1}{4} \omega_\mu^{ab} \gamma_a \gamma_b$, μ and a are tangent and spacetime indices. There are related by e_μ^a , a basis of orthonormal one-forms. The quantity $\omega_\mu^{ab} \equiv \omega^{ab}$ are the associated connection one-forms satisfying $de^a + \omega_b^a \wedge e^b = 0$. On the other hand, γ^μ are Dirac matrices fulfilling the relation $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$.

If a metric takes the product form:

$$g_{\mu\nu} dx^\mu dx^\nu = g_{ab}(x) dx^a dx^b + g_{mn}(y) dy^m dy^n, \quad (2)$$

then Dirac operator \not{D} satisfies a direct sum decomposition

$$\not{D} = \not{D}_x + \not{D}_y. \quad (3)$$

If one defines a Weyl conformally rescaled metric by $g_{\mu\nu} = \Omega^2 \tilde{g}_{\mu\nu}$ one finds that

$$\not{D}\psi = \Omega^{-(1/2)(n+1)} \tilde{\not{D}} \tilde{\psi}, \quad \psi = \Omega^{-(1/2)(n-1)} \tilde{\psi}. \quad (4)$$

Because a spherically symmetric line element is necessarily conformally flat, a static metric, spherically sym-

metric metric of the form

$$ds^2 = -A^2 dt^2 + B^2 dr^2 + C^2 d\Sigma_{n-2}^2, \quad (5)$$

where $A = A(r)$, $B = B(r)$, $C = C(r)$ are functions only of the radial variable r . The *transverse* metric $d\Sigma_{n-2}^2$ depends neither on t nor r . It is conformal to an ultrastatic metric, the factor of which is conformally flat. This allows us to solve the Dirac equation by a succession of conformal transformations and direct sum decompositions. The assumption that Ψ is a spinor eigenfunction on the $(n-2)$ -dimensional *transverse* manifold Σ , leads to the equation:

$$\not{D}_\Sigma \Psi = \lambda \Psi. \quad (6)$$

In the case of the $(n-2)$ -dimensional sphere the eigenvalues for spinor Ψ were found in Ref. [31]. They imply

$$\lambda^2 = \left(l + \frac{n-2}{2}\right)^2, \quad (7)$$

where $l = 0, 1, \dots$

Having in mind the properties given above, one may suppose that

$$\not{D}\psi = m\psi, \quad (8)$$

and take the form of the spinor ψ to be

$$\psi = \frac{1}{A^{1/2}} \frac{1}{C^{(n-2)/2}} \chi \otimes \Psi. \quad (9)$$

If one carries out the explicit calculations it turns out that

$$(\gamma^0 \partial_t + \gamma^1 \partial_x) \chi = A \left(m - \frac{\lambda}{C}\right) \chi, \quad (10)$$

where we have denoted by

$$dy = \frac{B}{A} dr, \quad (11)$$

the radial optical distance (i.e., the Regge-Wheeler radial coordinate). On the other hand, the gamma matrices γ^0, γ^1 satisfy the Clifford algebra in two spacetime dimensions. One should remark that having in mind a Yang-Mills gauge field A_μ , an identical result can be provided on the *transverse* manifold Σ . Namely, we have

$$\not{D}_{\Sigma, A_\mu} \Psi = \lambda \Psi, \quad (12)$$

where \not{D}_{Σ, A_μ} is the Dirac operator twisted by the connection A_μ .

Finally, if we take into account that ψ has the form as $\psi \propto e^{-i\omega t}$ one obtains a second order equation for χ , that is

$$\frac{d^2 \chi}{dy^2} + \omega^2 \chi = A^2 \left(m - \frac{\lambda}{C}\right)^2 \chi. \quad (13)$$

B. Dadhich-Maartens-Papadopoulos-Rezania (DMPR) brane-world black hole solution

We treat first the case of the static spherically symmetric black hole localized on a three-brane in five-dimensional gravity in the Randall-Sundrum model [32]. Having in mind the effective field equations on the brane one gets the following brane-world black hole metric [1]:

$$ds^2 = -\left(1 - \frac{2M}{M_p^2 r} + \frac{q^2}{\tilde{M}_p^2 r^2}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{M_p^2 r} + \frac{q^2}{\tilde{M}_p^2 r^2}\right)} + r^2 d\Omega^2, \quad (14)$$

where q is a dimensionless tidal parameter arising from the projection onto the brane of the gravitational field in the bulk, \tilde{M}_p is a fundamental five-dimensional Planck mass while M_p is the effective Planck mass in the brane-world. Typically, one has $\tilde{M}_p \ll M_p$. In what follows we shall concentrate on the negative tidal charge which is claimed [1] to be the more natural case. Thus, the roots of $g_{00} = 0$ are respectively r_+ and r_- . Namely, they imply

$$r_{\pm} = \frac{M}{\tilde{M}_p^2} \left(1 \pm \sqrt{1 - \frac{qM_p^4}{M^2 \tilde{M}_p^2}}\right). \quad (15)$$

Expressing the negative charge as Q , for simplicity, we can rewrite the roots as follows:

$$r_{\pm} = M \left(1 \pm \sqrt{1 + \frac{Q}{M^2}}\right). \quad (16)$$

Our main aim will be to analyze the time evolution of a massive Dirac spinor field in the background of the brane-world black hole by means of the spectral decomposition method. In Refs. [15,33] it was argued that the asymptotic massive tail is due to the existence of a branch cut placed along the interval $-m \leq \omega \leq m$. Thus, an oscillatory inverse power-law behavior of the massive spinor field arises from the integral of the Green function $\tilde{G}(y, y'; \omega)$ around the branch cut. Consider, next, the time evolution of the massive Dirac spinor field provided by the relation

$$\chi(y, t) = \int dy' [G(y, y'; t) \chi_t(y', 0) + G_t(y, y'; t) \chi(y', 0)], \quad (17)$$

for $t > 0$, where the Green's function $G(y, y'; t)$ implies

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial y^2} + V\right] G(y, y'; t) = \delta(t) \delta(y - y'). \quad (18)$$

By means of the Fourier transform [33] $\tilde{G}(y, y'; \omega) = \int_0^\infty dt G(y, y'; t) e^{i\omega t}$, Eq. (18) can be reduced to an ordinary differential equation. The Fourier transform is well defined for $\text{Im } \omega \geq 0$, while the corresponding inverse transform yields

$$G(y, y'; t) = \frac{1}{2\pi} \int_{-\infty+i\epsilon}^{\infty+i\epsilon} d\omega \tilde{G}(y, y'; \omega) e^{-i\omega t}, \quad (19)$$

for some positive number ϵ . By virtue of the above the Fourier component of the Green's function $\tilde{G}(y, y'; \omega)$ can be rewritten in terms of two linearly independent solutions for the homogeneous equation. Namely, it reduces to

$$\left(\frac{d^2}{dy^2} + \omega^2 - \tilde{V}\right) \chi_i = 0, \quad i = 1, 2, \quad (20)$$

where $\tilde{V} = A^2(m - \frac{\lambda}{r})^2$.

The boundary conditions for χ_i are described by purely ingoing waves crossing the outer horizon H_+ of the static black hole $\chi_1 \simeq e^{-i\omega y}$ as $y \rightarrow -\infty$, while χ_2 should be damped exponentially at i_+ . Thus, $\chi_2 \simeq e^{-\sqrt{m^2 - \omega^2} y}$ at $y \rightarrow \infty$.

Suppose now that the observer and the initial data are situated far away from the considered brane black hole. Let us rewrite Eq. (20) in the more convenient form using the change of variables

$$\chi_i = \frac{\xi}{(1 - \frac{r_+}{r})^{1/2} (1 - \frac{r_-}{r})^{1/2}}, \quad (21)$$

where $i = 1, 2$. Then, we expand Eq. (20) as a power series of r_{\pm}/r neglecting terms of order $O((\omega/r)^2)$ and higher. We obtain the following:

$$\frac{d^2}{dr^2} \xi + \left[\omega^2 - m^2 + \frac{2\omega^2(r_+ + r_-) - m^2(r_+ + r_-) + 2\lambda m(1 + r_+)}{r} - \frac{\lambda^2 - 2\lambda m r_- + m^2 r_+ r_-}{r^2} \right] \xi = 0. \quad (22)$$

The solution of Eq. (22) may be obtained in terms of Whittaker functions. Two basic solutions are needed to construct the Green function, with the condition that $|\omega| \geq m$, i.e., $\tilde{\chi}_1 = M_{\delta, \tilde{\mu}}(2\tilde{\omega}r)$ and $\tilde{\chi}_2 = W_{\delta, \tilde{\mu}}(2\tilde{\omega}r)$. The parameters of them imply

$$\begin{aligned} \tilde{\mu} &= \sqrt{1/4 + \lambda^2 - 2\lambda m r_- + m^2 r_+ r_-}, \\ \delta &= \frac{\omega^2(r_+ + r_-) + \lambda m(1 + r_+) - \frac{m^2}{2}(r_+ + r_-)}{\tilde{\omega}}, \\ \tilde{\omega}^2 &= m^2 - \omega^2. \end{aligned} \quad (23)$$

Having all this in mind, we reach to the following form of the spectral Green function:

$$\begin{aligned}
G_c(x, y; t) &= \frac{1}{2\pi} \int_{-m}^m dw \left[\frac{\tilde{\chi}_1(x, \tilde{\omega} e^{\pi i}) \tilde{\chi}_2(y, \tilde{\omega} e^{\pi i})}{W(\tilde{\omega} e^{\pi i})} \right. \\
&\quad \left. - \frac{\tilde{\chi}_1(x, \tilde{\omega}) \tilde{\chi}_2(y, \tilde{\omega})}{W(\tilde{\omega})} \right] e^{-iwt} \\
&= \frac{1}{2\pi} \int_{-m}^m dw f(\tilde{\omega}) e^{-iwt}, \tag{24}
\end{aligned}$$

where $W(\tilde{\omega})$ stands for Wronskian.

Let us analyze first, the intermediate asymptotic behavior of the massive spinor field with the range of parameters $M \ll r \ll t \ll M/(mM)^2$. The intermediate asymptotic contribution to the Green function integral gives the frequency equal to $\tilde{\omega} = \mathcal{O}(\sqrt{m/t})$, which in turns implies that $\delta \ll 1$. Using the fact that δ results from the $1/r$ term in the massive spinor field equation of motion, it illustrates the effect of backscattering off the spacetime curvature. In the case under consideration the backscattering is negligible. Thus, we find the following:

$$\begin{aligned}
f(\tilde{\omega}) &= \frac{2^{2\tilde{\mu}-1} \Gamma(-2\tilde{\mu}) \Gamma(\frac{1}{2} + \tilde{\mu})}{\tilde{\mu} \Gamma(2\tilde{\mu}) \Gamma(\frac{1}{2} - \tilde{\mu})} [1 + e^{(2\tilde{\mu}+1)\pi i}] \\
&\quad \times (rr')^{(1/2)+\tilde{\mu}} \tilde{\omega}^{2\tilde{\mu}}, \tag{25}
\end{aligned}$$

where one applied the fact that $\tilde{\omega} r \ll 1$. We also have in mind that $f(\tilde{\omega})$ can be approximated using the fact that $M(a, b, z) = 1$ as z tends to zero. Consequently, the resulting spectral Green function reduces to the form as

$$\begin{aligned}
f(\tilde{\omega}) &= \frac{\Gamma(1 + 2\tilde{\mu}) \Gamma(1 - 2\tilde{\mu})}{2\tilde{\mu}} (rr')^{1/2} [J_{2\tilde{\mu}}(\sqrt{8\delta\tilde{\omega}r}) J_{-2\tilde{\mu}}(\sqrt{8\delta\tilde{\omega}r'}) - I_{2\tilde{\mu}}(\sqrt{8\delta\tilde{\omega}r}) I_{-2\tilde{\mu}}(\sqrt{8\delta\tilde{\omega}r'})] \\
&\quad + \frac{(\Gamma(1 + 2\tilde{\mu}))^2 \Gamma(-2\tilde{\mu}) \Gamma(\frac{1}{2} + \tilde{\mu} - \delta)}{2\tilde{\mu} \Gamma(2\tilde{\mu}) \Gamma(\frac{1}{2} - \tilde{\mu} - \delta)} (rr')^{1/2} \kappa^{-2\tilde{\mu}} [J_{2\tilde{\mu}}(\sqrt{8\delta\tilde{\omega}r}) J_{2\tilde{\mu}}(\sqrt{8\delta\tilde{\omega}r'}) \\
&\quad + e^{(2\tilde{\mu}+1)\pi i} I_{2\tilde{\mu}}(\sqrt{8\delta\tilde{\omega}r}) I_{2\tilde{\mu}}(\sqrt{8\delta\tilde{\omega}r'})], \tag{28}
\end{aligned}$$

where we used the limit $M_{\delta, \tilde{\mu}}(2\tilde{\omega}r) \approx \Gamma(1 + 2\tilde{\mu}) \times (2\tilde{\omega}r)^{1/2} \delta^{-\tilde{\mu}} J_{\tilde{\mu}}(\sqrt{8\delta\tilde{\omega}r})$. One should notice that the first part of Eq. (28), the late-time tail, is proportional to t^{-1} . It occurs that we shall concentrate on the second term of the right-hand side of Eq. (28). For the case under consideration it can be brought to the form:

$$G_{c(2)}(r, r'; t) = \frac{M}{2\pi} \int_{-m}^m dw e^{i(2\pi\delta - wt)} e^{i\varphi}, \tag{29}$$

where the phase φ is defined by the relation

$$e^{i\varphi} = \frac{1 + (-1)^{2\tilde{\mu}} e^{-2\pi i \delta}}{1 + (-1)^{2\tilde{\mu}} e^{2\pi i \delta}}, \tag{30}$$

while M is given by

$$\begin{aligned}
G_c(r, r'; t) &= \frac{2^{3\tilde{\mu}-(3/2)} \Gamma(-2\tilde{\mu}) \Gamma(\frac{1}{2} + \tilde{\mu}) \Gamma(\tilde{\mu} + 1)}{\tilde{\mu} \sqrt{\pi} \tilde{\mu} \Gamma(2\tilde{\mu}) \Gamma(\frac{1}{2} - \tilde{\mu})} \\
&\quad \times (1 + e^{(2\tilde{\mu}+1)\pi i}) (rr')^{(1/2)+\tilde{\mu}} \left(\frac{m}{t}\right)^{(1/2)+\tilde{\mu}} \\
&\quad \times J_{(1/2)+\tilde{\mu}}(mt). \tag{26}
\end{aligned}$$

Taking into account the limit when $t \gg 1/m$ we conclude that the spectral Green function yields

$$\begin{aligned}
G_c(r, r'; t) &= \frac{2^{3\tilde{\mu}-1} \Gamma(-2\tilde{\mu}) \Gamma(\frac{1}{2} + \tilde{\mu}) \Gamma(\tilde{\mu} + 1)}{\tilde{\mu} \sqrt{\pi} \tilde{\mu} \Gamma(2\tilde{\mu}) \Gamma(\frac{1}{2} - \tilde{\mu})} \\
&\quad \times (1 + e^{(2\tilde{\mu}+1)\pi i}) (rr')^{(1/2)+\tilde{\mu}} m^{\tilde{\mu}} t^{-1-\tilde{\mu}} \\
&\quad \times \cos\left(mt - \frac{\pi}{2}(\tilde{\mu} + 1)\right). \tag{27}
\end{aligned}$$

We remark that Eq. (27) exhibits an oscillatory inverse power-law behavior. In our case the intermediate times of the power-law tail depends only on $\tilde{\mu}$ which in turn is a function of the multiple number of the wave modes.

The other pattern of decay of massive spinor Dirac hair is expected when $\delta \gg 1$, for the late-time behavior. Namely, when the backscattering off the curvature is taken into account. Under the assumption that $\delta \gg 1$, $f(\tilde{\omega})$ may be rewritten in the form as

$$\begin{aligned}
M &= \frac{(\Gamma(1 + 2\tilde{\mu}))^2 \Gamma(-2\tilde{\mu})}{2\tilde{\mu} \Gamma(2\tilde{\mu})} (rr')^{1/2} \\
&\quad \times [J_{2\tilde{\mu}}(\sqrt{8\delta\tilde{\omega}r}) J_{2\tilde{\mu}}(\sqrt{8\delta\tilde{\omega}r'}) \\
&\quad + I_{2\tilde{\mu}}(\sqrt{8\delta\tilde{\omega}r}) I_{2\tilde{\mu}}(\sqrt{8\delta\tilde{\omega}r'})]. \tag{31}
\end{aligned}$$

The saddle point integration allows one to find accurately the asymptotic behavior. This method is applicable in our case because of the fact that at very late time both terms e^{iwt} and $e^{2\pi\delta}$ are rapidly oscillating, which in turn means that the spinor waves are mixed states consisting of the states with multipole phases backscattered by spacetime curvature. Most of them cancel with each other which have the inverse phase. The saddle point is found to exist at the following value:

$$a_0 = \left[\frac{\pi(\omega^2(r_+ + r_-) + \lambda m(1 + r_-) - \frac{m^2}{2}(r_+ + r_-))}{\sqrt{2}m} \right]^{1/3}. \quad (32)$$

Then, the resultant form of the spectral Green function yields

$$G_c(r, r'; t) = \frac{2\sqrt{2}}{3} m^{2/3} (\pi)^{5/6} [2m^2(r_+ + r_-) + 2\lambda m(1 + r_+) - m^2(r_+ + r_-)]^{1/3} (mt)^{-5/6} \times \sin(mt) \tilde{\chi}(r, m) \tilde{\chi}(r', m). \quad (33)$$

The above form of the spectral Green function concludes our investigations of the late-time behavior of massive Dirac hair in the background of the DMPR brane-world black hole. The form of it envisages the fact that the late-time behavior of the fields in question is independent on the field parameter mass as well as the number of the wave mode. The late-time pattern of decay is proportional to $-5/6$.

C. Casadio-Fabbri-Mazzacurati (CFM) brane black hole solution

Our next task will be to consider a general class of the spherically symmetric static solution to five-dimensional equations of motion by considering the general form of the line element provided by the metric

$$ds^2 = -A(r)dt^2 + \frac{1}{B(r)} dr^2 + r^2 d\Omega^2. \quad (34)$$

Casadio *et al.* [2] obtained two types of analytic solutions by fixing either $A(r)$ or $B(r)$. The solution will be given in terms of the Arnowitt-Deser-Misner (ADM) mass M and the parametrized post-Newtonian (PPN) parameter β which affects the perihelion shift and the Nordtvedt effect [34]. The momentum constraints are identically satisfied by the metric coefficients and the *Hamiltonian* constraints can be written out [2]. Setting $A(r) = (1 - \frac{2M}{r})$ the resulting metric yields

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{(1 - \frac{3M}{2r})}{(1 - \frac{2M}{r})(1 - \frac{\gamma M}{2r})} dr^2 + r^2 d\Omega^2, \quad (35)$$

where $\gamma = 4\beta - 1$. A convenient form of the equation of motion for a massive Dirac field can be obtained by the transformation:

$$\chi_i = \frac{(1 - \frac{3M}{2r})^{1/4}}{(1 - \frac{2M}{r})^{1/2} (1 - \frac{\gamma M}{2r})^{1/4}} \xi, \quad (36)$$

where $i = 1, 2$. As in the preceding section, let us expand Eq. (18) as a power series of M/r neglecting terms of order $\mathcal{O}((\omega/r)^2)$ and higher. It then follows directly that one has

$$\frac{d^2}{dr^2} \xi + \left[\omega^2 - m^2 + \frac{\omega^2 \tilde{a} - m^2 \tilde{b} + 2\lambda m}{r} - \frac{\lambda^2 + \lambda M m (3 - \gamma) - \frac{3}{4} M^2 \gamma m^2}{r^2} \right] \xi = 0, \quad (37)$$

where $\tilde{a} = \frac{M}{2}(5 + \gamma)$ and $\tilde{b} = \frac{m}{2}(\gamma - 3)$.

Thus, the two basic solutions which are needed to construct the Green function, with the condition that $|\omega| \geq m$ are given by $\tilde{\chi}_1 = M_{\delta, \tilde{\mu}}(2\tilde{\omega}r)$ and $\tilde{\chi}_2 = W_{\delta, \tilde{\mu}}(2\tilde{\omega}r)$, with the following parameters:

$$\tilde{\mu} = \sqrt{1/4 + \lambda^2 + \lambda M m (3 - \gamma) - \frac{3}{4} M^2 \gamma m^2}, \quad (38)$$

$$\delta = \frac{\omega^2 \tilde{a} - m^2 \tilde{b} + 2\lambda m}{2\tilde{\omega}}, \quad \tilde{\omega}^2 = m^2 - \omega^2.$$

The preceding section arguments can be repeated. The conclusion is that the spectral Green function of the intermediate late-time behavior of massive Dirac spinor fields with the new parameters of the Whittaker functions given by the relation (38). Consequently, the next step will be to calculate the late-time behavior of the considered field. It can be verified that the stationarity of the integral will be achieved for the parameter

$$a_0 = \left[\frac{\pi(\omega^2 \tilde{a} - m^2 \tilde{b} + 2\lambda m)}{2\sqrt{2}m} \right]^{1/3}. \quad (39)$$

By virtue of the saddle point method, on evaluating the adequate expressions, we find that the spectral Green function provides the following:

$$G_c(r, r'; t) = \frac{2\sqrt{2}}{3} m^{2/3} (\pi)^{5/6} [4Mm^2 + 2\lambda m]^{1/3} (mt)^{-5/6} \times \sin(mt) \tilde{\psi}(r, m) \tilde{\psi}(r', m). \quad (40)$$

One can observe that the dominant role in the late-time behavior is played by the term proportional to $-5/6$. On the other hand, let us consider that $B(r) = (1 - \frac{2\gamma M}{r})$ for the other model of the brane-world black hole. It implies the following line element:

$$ds^2 = \frac{1}{\gamma} \left(\gamma - 1 + \sqrt{1 - \frac{2\gamma M}{r}} \right)^2 dt^2 + \frac{dr^2}{(1 - \frac{2\gamma M}{r})} + r^2 d\Omega^2. \quad (41)$$

Next, let us change coordinates as follows:

$$\chi_i = \frac{\gamma^{1/2} \xi}{(\gamma - 1 + \sqrt{1 - \frac{2\gamma M}{r}})^{1/2} (1 - \frac{2\gamma M}{r})^{1/2}}, \quad (42)$$

where $i = 1, 2$. Then, expand Eq. (18) as a power series of M/r neglecting terms of order $\mathcal{O}((\omega/r)^2)$ and higher. It yields

$$\frac{d^2}{dr^2}\xi + \left[\omega^2 \gamma^2 \rho^2 - m^2 + \frac{4\gamma M(\omega^2(1 + \rho^2) - m^2) + 2\lambda m}{r} - \frac{\lambda^2 - 8mM\lambda\gamma - 4M^2 m^2 \gamma^2}{r^2} \right] \xi = 0, \quad (43)$$

where $\rho^2 = (\gamma - 1)^2 + 3$.

Equation (43) can be brought to the form of Whittaker's equation. Two basic solutions are needed to construct the Green function. The additional requirement that $|\omega| \geq m$, implies that they are of the form $\tilde{\chi}_1 = M_{\delta, \tilde{\mu}}(2\tilde{\omega}r)$ and $\tilde{\chi}_2 = W_{\delta, \tilde{\mu}}(2\tilde{\omega}r)$. The parameters of the Whittaker functions are given by

$$\begin{aligned} \tilde{\mu} &= \sqrt{1/4 + \lambda^2 - 8mM\lambda\gamma - 4M^2 m^2 \gamma^2}, \\ \delta &= \frac{4\gamma M[\omega^2(1 + \rho^2) - m^2] + 2\lambda m}{2\tilde{\omega}}, \end{aligned} \quad (44)$$

$$\tilde{\omega}^2 = m^2 - \omega^2 \gamma^2 \rho^2.$$

On the other hand, from the considerations presented in the preceding case, the stationarity of $2\pi\delta - \omega t$ can be obtained for the parameter equal to

$$a_0 = \left[\frac{\pi(4\gamma M(\omega^2(1 + \rho^2) - m^2) + 2\lambda m)}{2\sqrt{2}m} \right]^{1/3}. \quad (45)$$

Summing it all up, one obtains the asymptotic late-time spectral Green function in the form

$$\begin{aligned} G_c(r, r'; t) &= \frac{2\sqrt{2}}{\sqrt{3}\gamma^2[(\gamma - 1)^2 + 3]} m^{2/3} (\pi)^{5/6} \\ &\times [4\gamma M m^2((\gamma - 1)^2 + 3) + 2\lambda m]^{1/3} \\ &\times (mt)^{-(5/6)} \sin(mt) \tilde{\chi}(r, m) \tilde{\chi}(r', m). \end{aligned} \quad (46)$$

As in the previous cases the dominant role in the asymptotic late-time decay of massive Dirac hair in the spacetime

of the CFM brane black hole plays the oscillatory tail with the decay rate proportional to $t^{-5/6}$.

III. CONCLUSIONS

In our paper we have considered the problem of the asymptotic tail behavior of massive Dirac hair in the spacetime of various brane-world black hole solutions. Our main aim was to reveal what type of mass-induced behaviors play the main role in the asymptotic intermediate and late-time decay pattern of the black hole hair in question. In our considerations we took into account two brane-world black hole solutions given in Refs. [1,2]. It was shown that in the case of intermediate asymptotic behavior one gets the oscillatory power-law dependence which varied with the multiple number of the wave mode l as well as with the mass of the Dirac fields. As in the case of *ordinary* Einstein static spherically symmetric black hole spacetimes this pattern of decay is not the final one. At very late-times the resonance backscattering off the spacetime curvature emerges, which in turn is independent on the angular momentum parameter and the field parameter m . The late-time asymptotic pattern of the decay is of the form $t^{-5/6}$. It should be interesting to find a general proof of this pattern of decay for massive Dirac fields in the spacetime of the static spherically symmetric black object. The investigations in this direction are in progress and will be published elsewhere.

ACKNOWLEDGMENTS

M. R. is grateful for the hospitality of DAMTP, Center for Mathematical Sciences, Cambridge, where part of the research was started. This work was partially financed by the Polish budget funds in the 2007 year as the research project.

-
- [1] N. Dadhich, R. Maartens, P. Papadopoulos, and V. Rezanian, *Phys. Lett. B* **487**, 1 (2000).
 - [2] R. Casadio, A. Fabbri, and L. Mazzacurati, *Phys. Rev. D* **65**, 084040 (2002); C. Germani and R. Maartens, *ibid.* **64**, 124010 (2001).
 - [3] H. Kodama, *Prog. Theor. Phys.* **108**, 253 (2002).
 - [4] T. Tanaka, *Prog. Theor. Phys. Suppl.* **148**, 307 (2002).
 - [5] S. Seahra, *Phys. Rev. D* **71**, 084020 (2005).
 - [6] C. Galfard, C. Germani, and A. Ishibashi, *Phys. Rev. D* **73**, 064014 (2006).
 - [7] G. Kofinas, E. Papantonopoulos, and V. Zamarias, *Phys. Rev. D* **66**, 104028 (2002).
 - [8] E. Abdalla, B. Cuadros-Melgar, A.B. Pavan, and C. Molina, *Nucl. Phys.* **B752**, 40 (2006).
 - [9] R.H. Price, *Phys. Rev. D* **5**, 2419 (1972).
 - [10] J. Bicak, *Gen. Relativ. Gravit.* **3**, 331 (1972).
 - [11] S. Hod and T. Piran, *Phys. Rev. D* **58**, 024017 (1998).
 - [12] S. Hod and T. Piran, *Phys. Rev. D* **58**, 024018 (1998).
 - [13] S. Hod and T. Piran, *Phys. Rev. D* **58**, 024019 (1998).
 - [14] L.M. Burko, Abstracts of plenary talks and contributed papers, 15th International Conference on General Relativity and Gravitation, Pune, 1997 (unpublished), p. 143.
 - [15] S. Hod and T. Piran, *Phys. Rev. D* **58**, 044018 (1998).
 - [16] H. Koyama and A. Tomimatsu, *Phys. Rev. D* **63**, 064032 (2001).
 - [17] H. Koyama and A. Tomimatsu, *Phys. Rev. D* **64**, 044014 (2001).
 - [18] R. Moderski and M. Rogatko, *Phys. Rev. D* **63**, 084014 (2001).
 - [19] R. Moderski and M. Rogatko, *Phys. Rev. D* **64**, 044024 (2001).

- [20] M. Rogatko, Phys. Rev. D **75**, 104006 (2007).
- [21] J.L. Jing, Phys. Rev. D **70**, 065004 (2004).
- [22] J.L. Jing, Phys. Rev. D **72**, 027501 (2005).
- [23] X. He and J.L. Jing, Nucl. Phys. **B755**, 313 (2006).
- [24] G. W. Gibbons and M. Rogatko, Phys. Rev. D **77**, 044034 (2008).
- [25] G. W. Gibbons, D. Ida, and T. Shiromizu, Prog. Theor. Phys. Suppl. **148**, 284 (2003); Phys. Rev. Lett. **89**, 041101 (2002); Phys. Rev. D **66**, 044010 (2002); M. Rogatko, Classical Quantum Gravity **19**, L151 (2002); Phys. Rev. D **67**, 084025 (2003); **70**, 044023 (2004); **71**, 024031 (2005); **73**, 124027 (2006).
- [26] V. Cardoso, S. Yoshida, and O. J. C. Dias, Phys. Rev. D **68**, 061503 (2003).
- [27] R. Moderski and M. Rogatko, Phys. Rev. D **72**, 044027 (2005).
- [28] H. T. Cho, A. S. Cornell, J. Doukas, and W. Naylor, Phys. Rev. D **75**, 104005 (2007).
- [29] M. Rogatko and A. Szyplowska, Phys. Rev. D **76**, 044010 (2007).
- [30] H. T. Cho, A. S. Cornell, J. Doukas, and W. Naylor, Phys. Rev. D **77**, 041502 (2008).
- [31] R. Camporesi and A. Higuchi, J. Geom. Phys. **20**, 1 (1996).
- [32] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
- [33] E. W. Leaver, Phys. Rev. D **34**, 384 (1986).
- [34] C.M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, England, 1993).