

**Dynamics of false vacuum bubbles with nonminimal coupling**Bum-Hoon Lee,<sup>1,3,\*</sup> Chul H. Lee,<sup>4,†</sup> Wonwoo Lee,<sup>2,‡</sup> Siyoung Nam,<sup>3,§</sup> and Chanyong Park<sup>3,||</sup><sup>1</sup>*Department of Physics, Sogang University, Seoul 121-742, Korea*<sup>2</sup>*Research Institute for Basic Science, Sogang University, Seoul 121-742, Korea*<sup>3</sup>*Center for Quantum Spacetime, Sogang University, Seoul 121-742, Korea*<sup>4</sup>*Department of Physics, and BK21 Division of Advanced Research and Education in Physics, Hanyang University, Seoul 133-791, Korea*

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We study the dynamics of false vacuum bubbles. A nonminimally coupled scalar field gives rise to the effect of negative tension. The mass of a false vacuum bubble from an outside observer's point of view can be positive, zero, or negative. The interior false vacuum has de Sitter geometry, while the exterior true vacuum background can have geometry depending on the vacuum energy. We show that there exist expanding false vacuum bubbles without the initial singularity in the past.

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**I. INTRODUCTION**

Can a false vacuum bubble expand within the true vacuum background? Or, is an expanding false vacuum bubble always inside the horizon of a black hole from an outside observer's point of view? If there was a dynamical spacetime foam structure in the very early universe [1], the detailed structure and evolution will depend on the cosmological constant. In the context of a bubble nucleation and dynamics, this phenomenon may be described as follows. A true vacuum bubble can always be nucleated somewhere within the false vacuum background as well as a false vacuum bubble nucleated within the true vacuum background. Some bubbles may expand while some bubbles collapse. Some of them may be connected by wormholes. Then the whole spacetime may have a complicated vacuum or spacetime structure due to the above processes. Earlier works [2–5] show that the unbound solution representing an expanding false vacuum bubble does not exist because the solution for the junction equation cannot cover all ranges of  $r$ . To obtain an expanding false vacuum bubble, the mass of the bubble should be over some critical value. To the observer in the exterior spacetime, the expanding false vacuum bubble will be inside the black hole horizon. Only to the observer inside the bubble will it appear to expand from a very small size to infinity. However, these bubbles start from an initial singularity. Moreover, there is a puzzle that the entropy of the expanding false vacuum region is greater than the entropy of the black hole surrounding it [5–7]. Are these descriptions always true? With nonminimal coupling we will show that the unbound solutions representing expanding false vacuum bubbles can exist. On the other hand, the idea of

the string theory landscape has a vast number of metastable vacua [8]. One of the intriguing features of this landscape is to understand a de Sitter universe or tunneling processes in the landscape [9]. Our motivation is an attempt to solve some of these questions within the framework of the classical theory of gravity.

The dynamics of the boundary wall of a spherically shaped false vacuum bubble surrounded by true vacuum regions was originally studied in Refs. [10] at the final stage of the true vacuum bubble nucleation in old inflation [11], and was studied systematically in Refs. [2,3] as an attempt to create a universe in the laboratory by quantum tunneling. They considered the case of interior de Sitter spacetime and exterior Schwarzschild spacetime divided by a thin wall (or domain wall). In Ref. [12] the dynamics of matter distribution that may contaminate a false vacuum bubble was considered. The case of interior de Sitter and exterior Schwarzschild–de Sitter spacetime was studied in Ref. [4], where they examined the instability of false vacuum bubbles. The case of interior de Sitter spacetime and exterior Schwarzschild–anti-de Sitter spacetime in relation to AdS/CFT correspondence was considered in Ref. [5]. The case of a charged false vacuum bubble with interior de Sitter spacetime and exterior Reissner–Nordström–anti de Sitter spacetime with arbitrary dimension was considered in Ref. [13]. The possibility of the creation of a universe out of a monopole in the laboratory was investigated in Ref. [14], where they have considered the classical and quantum thin-wall dynamics of a magnetic monopole. They have examined the stability of a spherically symmetric self-gravitating magnetic monopole in the thin-wall approximation, modeling the interior false vacuum as de Sitter spacetime and the exterior as the Reissner–Nordström spacetime as in Ref. [15]. Recently the classification scheme for the possible evolution of a vacuum wall in the Schwarzschild–de Sitter geometry was constructed [16]. In addition, there have been studies on the attempts to create a universe in the laboratory [17].

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As for the false vacuum bubble formation, Lee and Weinberg [18] have shown that gravitational effects make it possible for bubbles of a higher-energy false vacuum to nucleate if the vacuum energies are greater than zero. The oscillating bounce solutions, another type of Euclidean solution, have been studied in Refs. [6,19,20]. On the other hand, Kim *et al.* [21] have shown that there exists another decay channel which is described by the false vacuum region of the global monopole formed at the center of a bubble in the high temperature limit. The Hawking-Moss transition [22], as another way of vacuum decay, describes the scalar field jumping simultaneously at the top of the potential barrier. Recently this process has been interpreted in terms of a thermal transition [19,23]. It has been shown that the false vacuum bubble can be nucleated within the true vacuum background due to a nonminimally coupled scalar field or other similar coupling terms [24]. The quantum nucleation of the vacuum bubble was also studied [25]. In Ref. [26], they found an analytic expression for the tunneling amplitude and studied the tunneling between arbitrary (anti-)de Sitter spacetimes in arbitrary spacetime dimensions. The interesting models for bubble collisions in the very early universe are also discussed [27].

In this paper we use the metric junction conditions, which were developed in Ref. [28], to analyze the dynamics of a false vacuum bubble in the theory with nonminimal coupling. There are two types of boundaries related to this formalism. One is a ‘‘boundary surface’’ which is a surface that has the stress-energy tensor  $S_{\mu\nu} = 0$  [29]. The process of star collapsing is a well-known example involving a boundary surface [30]. The other is a ‘‘surface layer’’ which is a thin layer of matter where  $S_{\mu\nu} \neq 0$  [31]. In this case  $S_{\mu\nu}$  is related to the discontinuity of the extrinsic curvature of the surface. In this framework we classify the cosmological behaviors from the viewpoint of an observer on the domain wall and find a solution with multiple accelerations in five dimensions in the Einstein theory of gravity [32]. In the context of the brane cosmology, after Randall and Sundrum’s interesting proposal [33], the junction conditions have become one of the methods describing the inflationary cosmology on the brane [34].

Our approach to obtain junction conditions is based on the method of variational principle by Chamblin and Reall

[35]. Barcelo and Visser have obtained the generalized junction conditions using a different approach [36].

The plan of this paper is as follows. In Sec. II we present the formalism for the junction conditions in the Einstein theory of gravity with a nonminimally coupled scalar field. In Sec. III we study the dynamics of false vacuum bubbles using the junction conditions. In Sec. IV the bubble wall trajectories in the exterior bulk spacetime are analyzed according to the mass of the false vacuum bubble. Finally, we summarize and discuss our results in Sec. V.

## II. THE JUNCTION CONDITIONS WITH A NONMINIMALLY COUPLED SCALAR FIELD

In this section we consider a thin wall as a surface layer partitioning bulk spacetime into two distinct four-dimensional manifolds,  $\mathcal{M}^+$  and  $\mathcal{M}^-$ , with boundaries  $\Sigma^+$  and  $\Sigma^-$ , respectively. To obtain the single glued manifold  $\mathcal{M} = \mathcal{M}^+ \cup \mathcal{M}^-$ , we demand that the boundaries are identified as follows:

$$\Sigma^+ = \Sigma^- = \Sigma, \quad (1)$$

where the thin-wall boundary  $\Sigma$  is a timelike hypersurface with unit normal  $n^\lambda$ .

Let us consider the action

$$S = \int_{\mathcal{M}} \sqrt{-g} d^4x \left[ \frac{R}{2\kappa} (1 - \xi \kappa \Phi^2) - \frac{1}{2} \nabla^\alpha \Phi \nabla_\alpha \Phi - U(\Phi) \right] + \oint_{\Sigma} \sqrt{-h} d^3x \left[ \frac{K}{\kappa} (1 - \xi \kappa \Phi^2) + S_{tw} \right], \quad (2)$$

where  $S_{tw}$  is a Nambu-Goto-type action on the wall given by  $-\oint_{\Sigma} \sqrt{-h} d^3x \hat{U}(\Phi)$ ,  $\kappa \equiv 8\pi G$ , and  $g \equiv \det g_{\mu\nu}$ . The second term on the right-hand side of the above equation is the boundary term [37] with a nonminimally coupled scalar field.  $U(\Phi)$  is the scalar field potential,  $R$  denotes the Ricci curvature of spacetime in  $\mathcal{M}$ ,  $K$  is the trace of the extrinsic curvature of  $\Sigma$ , the term  $-\xi R \Phi^2/2$  describes the nonminimal coupling of the field  $\Phi$  to the Ricci curvature, and  $\xi$  is a dimensionless coupling constant.

We now vary the action to obtain Israel junction conditions. The case of minimal coupling has been considered in Ref. [35].

Varying the nonminimal coupling term in the bulk  $\mathcal{M}$ , we get

$$\begin{aligned} \int_{\mathcal{M}} d^4x \delta[\sqrt{-g} \xi R \Phi^2] &= \oint_{\Sigma} \sqrt{-h} d^3x \xi n^\lambda (\Phi^2 g^{\mu\nu} \nabla_\mu \delta g_{\lambda\nu} - \Phi^2 g^{\mu\nu} \nabla_\lambda \delta g_{\mu\nu}) \\ &+ 2 \oint_{\Sigma} \sqrt{-h} d^3x \xi n_\lambda \Phi [(\nabla^\lambda \Phi) g^{\mu\nu} - (\nabla^\mu \Phi) g^{\lambda\nu}] \delta g_{\mu\nu} \\ &+ \int_{\mathcal{M}} \sqrt{-g} d^4x \xi \Phi^2 \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) \delta g^{\mu\nu} + 2 \int_{\mathcal{M}} \sqrt{-g} d^4x \xi \Phi R \delta \Phi, \end{aligned} \quad (3)$$

and varying the scalar field action, we get

$$\int_{\mathcal{M}} d^4x \delta[\sqrt{-g} g^{\alpha\beta} \nabla_\alpha \Phi \nabla_\beta \Phi] = 2 \oint_{\Sigma} \sqrt{-h} d^3x n^\lambda (\nabla_\lambda \Phi) \delta\Phi - 2 \int_{\mathcal{M}} \sqrt{-g} d^4x \nabla^\lambda \nabla_\lambda \Phi \delta\Phi + \int_{\mathcal{M}} \sqrt{-g} d^4x \left( \nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} \nabla^\alpha \Phi \nabla_\lambda \Phi g_{\mu\nu} \right) \delta g^{\mu\nu}. \quad (4)$$

The variation of the boundary term with a nonminimal coupling gives

$$\oint_{\Sigma} \sqrt{-h} d^3x K (1 - \xi \kappa \Phi^2) = \oint_{\Sigma} \sqrt{-h} d^3x (1 - \xi \kappa \Phi^2) \left[ \frac{K}{2} h^{\mu\nu} \delta g_{\mu\nu} - K^{\alpha\lambda} \delta g_{\alpha\lambda} - h^{\mu\nu} n^\lambda \nabla_\mu \delta g_{\lambda\nu} + \frac{1}{2} h^{\mu\nu} n^\lambda \nabla_\lambda \delta g_{\mu\nu} + \frac{K}{2} n^\mu n^\nu \delta g_{\mu\nu} \right] - 2 \oint_{\Sigma} \sqrt{-h} d^3x K \xi \kappa \Phi \delta\Phi, \quad (5)$$

and the variation of the wall action gives

$$\oint_{\Sigma} d^3x \delta(\sqrt{-h} \hat{U}) = \oint_{\Sigma} \sqrt{-h} d^3x h^{\mu\nu} \frac{\hat{U}}{2} \delta g_{\mu\nu} + \oint_{\Sigma} \sqrt{-h} d^3x \frac{\partial \hat{U}}{\partial \Phi} \delta\Phi. \quad (6)$$

The bulk Einstein equations are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}, \quad (7)$$

where  $R_{\mu\nu}$  is the Ricci tensor and  $T_{\mu\nu}$  is the matter energy-momentum tensor,

$$T_{\mu\nu} = \frac{1}{1 - \xi \Phi^2 \kappa} \left[ \nabla_\mu \Phi \nabla_\nu \Phi - g_{\mu\nu} \left( \frac{1}{2} \nabla^\alpha \Phi \nabla_\alpha \Phi + U(\Phi) \right) + \xi (g_{\mu\nu} \nabla^\alpha \nabla_\alpha \Phi^2 - \nabla_\mu \nabla_\nu \Phi^2) \right]. \quad (8)$$

The corresponding scalar field equation on the bulk is written by

$$\frac{1}{\sqrt{-g}} \partial_\mu [\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi] - \xi R \Phi - \frac{dU}{d\Phi} = 0, \quad (9)$$

with a boundary condition at the thin wall,

$$n^\lambda (\nabla_\lambda \Phi) + 2K \xi \kappa \Phi = - \frac{d\hat{U}}{d\Phi}. \quad (10)$$

Here we adopt the notations and sign conventions of Misner, Thorne, and Wheeler [38].

The modified Lanczos equation due to a nonminimally coupled scalar field is given by

$$(1 - \xi \kappa \Phi_\pm^2) ([K_{\mu\nu}] - [K] h_{\mu\nu}) - 2\xi \kappa \Phi n^\lambda (\nabla_\lambda \Phi) h_{\mu\nu} = \kappa \hat{U} h_{\mu\nu} \quad (11)$$

where

$$[K] \equiv \lim_{\epsilon \rightarrow 0} K^+(\eta = \bar{\eta} + \epsilon) - K^-(\eta = \bar{\eta} - \epsilon). \quad (12)$$

The sign arises because we have chosen the convention that  $n^\lambda$  points towards the region of increasing  $\eta$ . The  $\bar{\eta}$  is the location of the hypersurface. The signs (+) and (-) represent exterior and interior spacetime, respectively.

After Eq. (10) is substituted in Eq. (11), the junction conditions become

$$(1 - \xi \kappa \Phi_\pm^2) K_{\mu\nu}^\pm - (1 - \xi \kappa \Phi_\mp^2) K_{\mu\nu}^\mp = - \frac{\kappa}{2} \hat{U} h_{\mu\nu} - \xi \kappa \left( \Phi_+ \frac{d\Phi_+}{d\eta} - \Phi_- \frac{d\Phi_-}{d\eta} \right) h_{\mu\nu}. \quad (13)$$

Actually, the second term on the right-hand side of Eq. (13) vanishes because  $\frac{d\Phi_+}{d\eta}$  and  $\frac{d\Phi_-}{d\eta}$  vanish in the exterior and in the interior spacetime of the wall, respectively.

In order to find the gravitational field and the motion of a wall, we must first find two sets, both inside and outside of the wall, of solutions of the bulk Einstein equation and scalar field equation. So, if the bulk solutions are given, we only need to match the junction conditions to determine the motion of the wall. In the next section, we will only consider the junction equations because bulk solutions are easily given; the bulk solution of  $M = 0$  is already known in Ref. [24], while the case for  $M \neq 0$  corresponds to the Schwarzschild solution because of Birkhoff's theorem [39]. For the case of  $M < 0$ , the mass of a false vacuum becomes effectively negative, which is possible due to nonminimal coupling.

### III. DYNAMICS OF A FALSE VACUUM BUBBLE

For applications of the modified junction equations on the false vacuum bubble, the bulk spacetime geometry for the inside (-) and outside (+) of the wall have a spherically symmetric spacetime,

$$ds^2 = -H_\pm(R) dT^2 + \frac{dR^2}{H_\pm(R)} + R^2 d\Omega^2, \quad (14)$$

where

$$H_- = 1 - A_- R^2 \quad \text{and} \quad H_+ = 1 - A_+ R^2 - \frac{2GM}{R}, \quad (15)$$

and  $M$  is the mass or the total energy of a false vacuum bubble. The constant  $A$  is related to a cosmological constant;  $A = +\frac{\Lambda}{3} = +\frac{8\pi G}{3} \rho$  for de Sitter spacetime,  $A = 0$  for Minkowski spacetime, and  $A = -\frac{\Lambda}{3} = -\frac{8\pi G}{3} \rho$  for

anti-de Sitter spacetime. Since we consider a false vacuum bubble,  $\rho_- > \rho_+$ .

We take the energy-momentum tensor  $T^{\mu\nu}$  as the form

$$T^{\mu\nu} = S^{\mu\nu} \delta(\eta) + (\text{regular terms}), \quad (16)$$

where  $S^{\mu\nu} = -\sigma h^{\mu\nu}(x^i, \eta = \bar{\eta})$  and  $\sigma$  is the positive surface energy density, or surface tension, of the wall without nonminimal coupling. For bubble walls  $\sigma$  is a constant having the same value at all events on the timelike surface [2,40]. Note that the stress-energy tensor of the surface  $S_{\mu\nu}$  can be defined as the integral over the thickness,  $\epsilon$ , of the surface  $\Sigma$  in the limit as  $\epsilon$  goes to zero,

$$S_{\mu\nu} = \lim_{\epsilon \rightarrow 0} \int_{\bar{\eta}-\epsilon}^{\bar{\eta}+\epsilon} T_{\mu\nu} d\eta. \quad (17)$$

Then,  $\hat{U} = \sigma$  because the internal structure of the wall is neglected in the thin-wall limit and  $h^{\eta\eta} = h^{\eta i} = 0$ .

To keep the analysis as simple as possible, we take the position of a false vacuum in the potential as zero, that is,  $\Phi_- = 0$  (see Ref. [24]). In this case Eq. (13) becomes

$$(1 - \xi \kappa \Phi_+^2) K_j^{+i} - K_j^{-i} = -4\pi G \sigma \delta_j^i. \quad (18)$$

By spherical symmetry, the extrinsic curvature has only two components,  $K_\theta^\theta \equiv K_\phi^\phi$  and  $K_\tau^\tau$ . The junction equation is related to  $K_\theta^\theta$ , and the covariant acceleration in the normal direction is related to  $K_\tau^\tau$ .

We introduce the Gaussian normal coordinate system near the wall,

$$dS^2 = -d\tau^2 + d\eta^2 + \bar{r}^2(\tau, \eta) d\Omega^2, \quad (19)$$

where  $g_{\tau\tau} = -1$  and  $\bar{r}(\tau, \bar{\eta}) = r(\tau)$ . It must agree with the coordinate  $R$  of the interior and exterior coordinate systems. The angle variables can be taken to be invariant in all regions. In this coordinate system the induced metric on the wall is given by

$$dS_\Sigma^2 = -d\tau^2 + r^2(\tau) d\Omega^2, \quad (20)$$

where  $\tau$  is the proper time measured by an observer at rest

with respect to the wall and  $r(\tau)$  is the proper radius of  $\Sigma$ . The following relation is satisfied:

$$d\tau^2 = H_\pm(R) dT^2 - \frac{dR^2}{H_\pm(R)}. \quad (21)$$

In these treatments, the condition becomes

$$\sqrt{\dot{r}^2 + H_-} - (1 - \xi \kappa \Phi_+^2) \sqrt{\dot{r}^2 + H_+} = \frac{1}{2} \kappa \sigma r \quad (22)$$

or more generally

$$\epsilon_- \sqrt{\dot{r}^2 + H_-} - \epsilon_+ \sqrt{\dot{r}^2 + H_+} = \frac{1}{2} \kappa r (\sigma - \xi \bar{\sigma}), \quad (23)$$

where  $\bar{\sigma} = \frac{2c}{r} \sqrt{\dot{r}^2 + H_+}$ . Hereafter  $c$  denotes  $\Phi_+^2$ . We are using the dot notation to refer to derivatives with respect to  $\tau$ .  $\epsilon_\pm$  are  $+1$  if the outward normal to the wall is pointing towards increasing  $r$ , and  $-1$  if it is pointing towards decreasing  $r$  [13,41]. There are parameter regions where both  $\epsilon_-$  and  $\epsilon_+$  are positive in all ranges of  $r$ . This situation is similar to the case of the evolution of a true vacuum bubble. In earlier works, a sign change of  $\epsilon_\pm$  was needed to cover all ranges of  $r$  for the solution. This is because the positive signs for  $\epsilon_\pm$  covered only partial ranges of  $r$ ; thus the interesting unbound solutions were excluded. To obtain an expanding false vacuum bubble, the mass of the bubble should be greater than some critical value. These bubbles start from an initial singularity. The second term on the right-hand side of Eq. (23) can be interpreted as the negative tension of the wall due to a nonminimal coupling term.

After squaring twice, Eq. (22) can be written in the form

$$\frac{1}{2} \dot{r}^2 + V_{\text{eff}}(r) = 0, \quad (24)$$

where the effective potential is

$$V_{\text{eff}}(r) = \frac{T + \sqrt{T^2 - PQ}}{2P}, \quad (25)$$

with

$$\begin{aligned} T &= [1 - (1 - \xi \kappa c)^2]^2 + \frac{2GM}{r} [1 - (1 - \xi \kappa c)^2] (1 - \xi \kappa c)^2 \\ &\quad + \left\{ [1 - (1 - \xi \kappa c)^2] \left[ (1 - \xi \kappa c)^2 A_+ - A_- + \frac{1}{4} \kappa^2 \sigma^2 \right] - \frac{1}{2} \kappa^2 \sigma^2 \right\} r^2, \\ Q &= \left\{ \left[ (1 - \xi \kappa c)^2 A_+ - A_- + \frac{1}{4} \kappa^2 \sigma^2 \right]^2 + A_- \kappa^2 \sigma^2 \right\} r^4 + 2 \left\{ [1 - (1 - \xi \kappa c)^2] \right. \\ &\quad \times \left. \left[ (1 - \xi \kappa c)^2 A_+ - A_- + \frac{1}{4} \kappa^2 \sigma^2 \right] - \frac{1}{2} \kappa^2 \sigma^2 \right\} r^2 + \left[ (1 - \xi \kappa c)^2 A_+ - A_- + \frac{1}{4} \kappa^2 \sigma^2 \right] (1 - \xi \kappa c)^2 4GM r \\ &\quad + [1 - (1 - \xi \kappa c)^2]^2 + [1 - (1 - \xi \kappa c)^2] (1 - \xi \kappa c)^2 \frac{4GM}{r} + (1 - \xi \kappa c)^4 \frac{4G^2 M^2}{r^2}, \\ P &= [1 - (1 - \xi \kappa c)^2]^2. \end{aligned} \quad (26)$$

Equation (24) formally coincides with the equation describing one-dimensional motion of a unit-mass particle moving in the corresponding potential  $V_{\text{eff}}$  with zero total energy. The properties of the trajectory of the wall can be read off directly from the shape of  $V_{\text{eff}}$ . In the next section we will discuss the details of trajectories of the wall.

**IV. THE BUBBLE WALL TRAJECTORIES**

In this section we consider the bubble wall trajectories according to the mass of a false vacuum bubble in the exterior bulk spacetime. The modified junction equations with nonminimal coupling determine the trajectories. From the shape of  $V_{\text{eff}}$  we can obtain the behavior of solutions without solving the equations exactly. We consider only bubble solutions without black holes. In other words, we consider the size of a false vacuum bubble larger than the black hole horizon.

**A. The case of  $M = 0$**

This case is related to the results in Ref. [24]. If a false vacuum bubble can be nucleated within the true vacuum background without changing the exterior spacetime, the surface density becomes negative and/or the junction conditions themselves need to be modified. Our results show how the surface tension as well as junction conditions are modified by nonminimal coupling. From now on, we scale the dimension of a vacuum energy density to  $M^4$ , that of  $\sigma$  to  $M^3$ , that of  $G$  to  $M^{-2}$ , that of  $r$  to  $M^{-1}$ , and that of  $c$  to

$M^2$ , to make all terms in Eq. (24) dimensionless. For the sake of simplicity, we take  $\kappa c = 0.1$ ,  $\kappa\sigma = 0.033$ , and  $A_- - A_+ = 0.0025$ . In this case the effective potential function is  $V_{\text{eff}}(0) = \frac{1}{2}$  at  $r = 0$ , and there exist only “unbound” solutions. We consider three particular cases: case 1, where the interior as well as the exterior spacetime is de Sitter; case 2, with the interior false vacuum as de Sitter and the exterior as flat Minkowski spacetime; case 3, with the interior false vacuum as de Sitter and the exterior as anti-de Sitter spacetime. The shapes of the effective potential as a function of  $r$  are shown in Fig. 1. These figures indicate only unbound trajectories are possible. That is, the bound and monotonic trajectories do not appear as classical solutions.

We see that the allowed minimum size of a false vacuum bubble is diminished as  $\xi$  is decreased. So, if the radius of a nucleated false vacuum bubble is greater than the allowed minimum size, then the false vacuum bubble can expand within the true vacuum background. These expanding bubbles have no initial singularity, as we can see from Fig. 1. So it is possible to create a universe by an expanding false vacuum bubble nucleated by a proper mechanism.

**B. The case of  $M > 0$**

In this case the mass of the false vacuum bubble is taken as a constant parameter. The portion of the potential is inherently restricted since the allowed region of  $r$  from Eq. (25) is given by

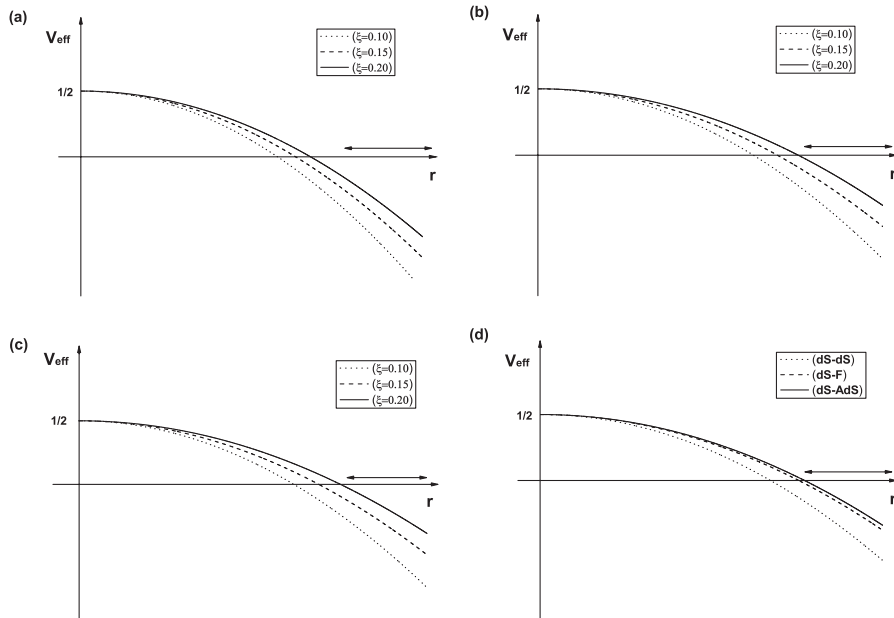


FIG. 1. The effective potential  $V_{\text{eff}}$  for various  $\xi$  in the case of  $M = 0$ . The time evolution of the wall can be interpreted as the motion of a particle coming from infinity, reflecting at the barrier, and then going back to infinity. The three curves are (i) the dotted curve:  $\xi = 0.10$ ; (ii) the dashed curve:  $\xi = 0.15$ ; and (iii) the solid curve:  $\xi = 0.20$ , in (a), (b), and (c). Figure (d) indicates the potential with a different background at  $\xi = 0.20$ . There exist only unbound solutions.

$$r > \left( \frac{2GM[1 - (1 - \xi\kappa c)^2](1 - \xi\kappa c)^2}{\frac{1}{4}\kappa^2\sigma^2(1 - \xi\kappa c)^2 - [1 - (1 - \xi\kappa c)^2][(1 - \xi\kappa c)^2A_+ - A_-]} \right)^{1/3}. \tag{27}$$

In the case of  $M > 0$ , we consider three particular cases: case 1, with the interior spacetime as de Sitter and the exterior as Schwarzschild–de Sitter; case 2, with the interior as de Sitter and the exterior as Schwarzschild spacetime; case 3, with the interior as de Sitter and the exterior as Schwarzschild anti–de Sitter spacetime. For these cases massive bubbles can be formed in the early universe. These cases have been studied by many authors in the pure Einstein theory of gravity [2–5]. However, there are different features between their models and ours. One is related to the sign of  $\epsilon_{\pm}$ . Unlike previous works, unbound solutions are allowed in our model since there are parameter regions where both  $\epsilon_-$  and  $\epsilon_+$  are positive in all ranges of  $r$ . Note that there is the restricted region of  $r$  as in Eq. (27). The other is related to the junction equation (24). Our approach is somewhat different from their work. We consider the case of positive as well as zero mass. It seems inappropriate that the case of zero mass is applied in their formalism. We can consider the junction equation regardless of the mass. The shapes of the effective potential as a function of  $r$  are shown in Fig. 2. In these cases, the false vacuum bubble can also expand within the true vacuum background.

### C. The case of $M < 0$

The case of a negative mass bubble is allowed in this framework. In this case we assume that the geometry of

outside spacetime with spherical symmetry is similarly written by Eq. (14). These objects are considered in different contexts in Refs. [42].

In the case of  $M < 0$ , we consider three particular cases: case 1, with the interior spacetime as de Sitter and the exterior as Schwarzschild, with negative mass, de Sitter; case 2, with the interior as de Sitter and the exterior as Schwarzschild spacetime; case 3, with the interior as de Sitter and the exterior as Schwarzschild–anti-de Sitter spacetime. Although the case of negative mass is physically unclear, since it does not satisfy the positive energy condition, it still gives rise to the solutions of the Einstein equations. So, we proceed to analyze the case of  $M < 0$ . The shapes of the effective potential as a function of  $r$  are shown in Fig. 3. There also exist expanding false vacuum bubbles.

Here we discuss again the term which can be interpreted as the negative tension of the wall. This effect is different from earlier works [2–5]. In Fig. 4, we see that the magnitude of the negative tension of the wall,  $\bar{\sigma}$ , due to a non-minimal coupling term is a constant only in the case of  $M = 0$ . In other cases, the magnitudes approach the value in the case of  $M = 0$  as  $r$  is increased. On the other hand, the magnitude is increased as  $r$  is decreased in the case of  $M < 0$  and decreased as  $r$  is decreased in the case of  $M > 0$ . Unlike other cases, there exist bound solutions in a narrow range of  $r$  in the case of  $M < 0$ , although we are not interested in this. In Fig. 4 we take only unbound

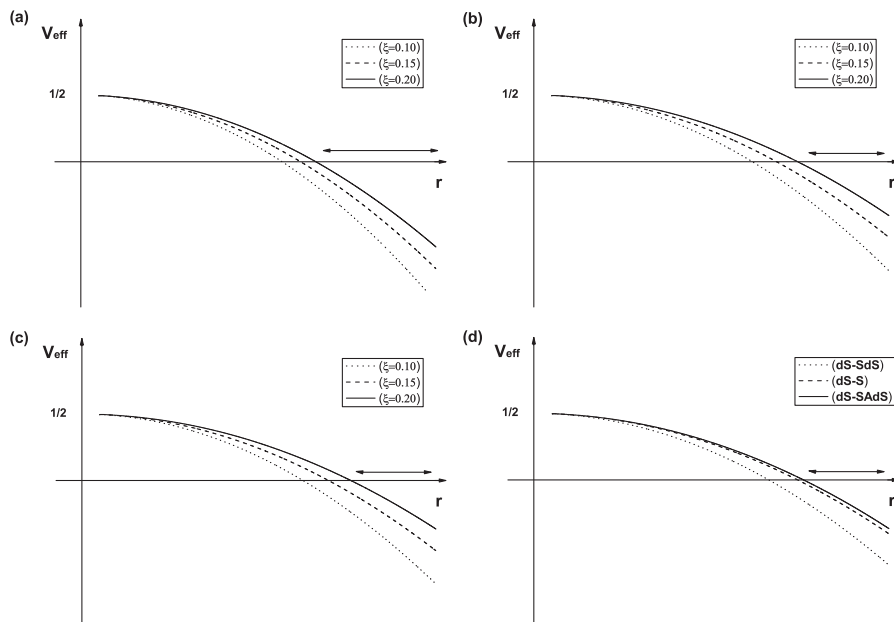


FIG. 2. The effective potential  $V_{\text{eff}}$  for various  $\xi$  in the case of  $M > 0$ . The three curves are (i) the dotted curve:  $\xi = 0.10$ ; (ii) the dashed curve:  $\xi = 0.15$ ; and (iii) the solid curve:  $\xi = 0.20$ , in (a), (b), and (c). There exist only unbound solutions.

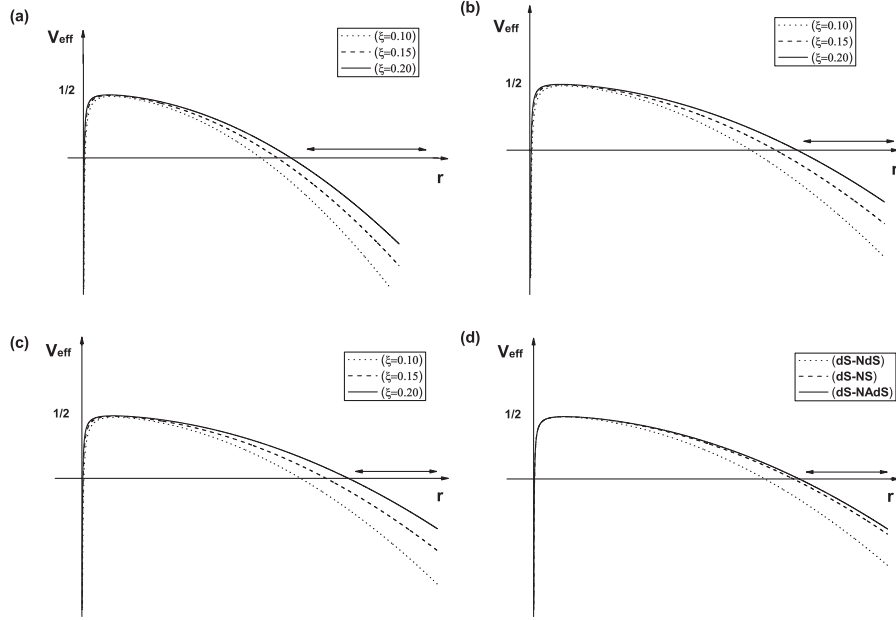


FIG. 3. The effective potential  $V_{\text{eff}}$  for various  $\xi$  in the case of  $M < 0$ . The three curves are (i) the dotted curve:  $\xi = 0.10$ ; (ii) the dashed curve:  $\xi = 0.15$ ; and (iii) the solid curve:  $\xi = 0.20$ , in (a), (b), and (c). There exist unbound solutions as well as “bound” solutions.

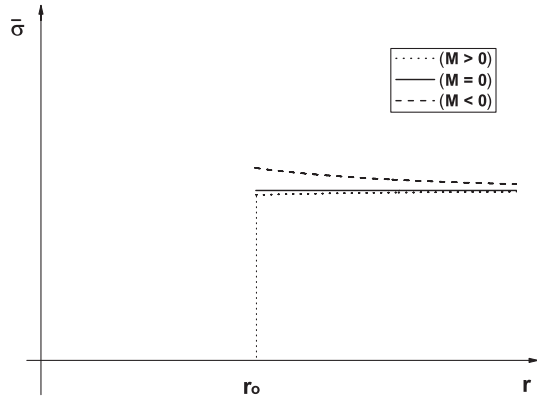


FIG. 4. The magnitude of the negative tension of the wall,  $\bar{\sigma}$ , due to a nonminimal coupling term in the spacetime with different mass sign. Here  $r_0$  denotes the allowed minimum size of a false vacuum bubble.

solutions.  $r_0$  denotes the allowed minimum size of a false vacuum bubble in this framework.

### V. SUMMARY AND DISCUSSIONS

In this paper we have shown that there can be an expanding false vacuum bubble within the true vacuum background. We have presented the formalism for the junction conditions with nonminimal coupling in Sec. II.

In Sec. III we have studied the dynamics of a false vacuum bubble using the modified junction conditions. The nonminimal coupling term can be interpreted as the negative tension of the wall in the junction conditions. In

this treatment the mass of a false vacuum bubble from an outside observer’s point of view can be positive, zero, or negative. The solutions in our model do not have black holes. In other words, the size of a false vacuum bubble is larger than the black hole horizon. The mass of a false vacuum bubble has been treated as a parameter in this work.

In Sec. IV, the bubble wall trajectories in the exterior bulk spacetime are analyzed according to the mass of false vacuum bubbles. We have obtained the behavior of solutions in various cases. In this framework there are parameter regions where both  $\epsilon_-$  and  $\epsilon_+$  are positive in all ranges of  $r$ . This situation is similar to the case of the evolution of a true vacuum bubble. In the case of  $M = 0$ , we have considered three particular cases: case 1, where the interior as well as the exterior spacetime is de Sitter; case 2, with the interior false vacuum as de Sitter and the exterior as flat Minkowski spacetime; case 3, with the interior false vacuum as de Sitter and the exterior as anti-de Sitter spacetime. In these cases only unbound trajectories are possible. In the case  $M > 0$ , we have considered three particular cases: case 1, with the interior spacetime as de Sitter and the exterior as Schwarzschild–de Sitter; case 2, with the interior as de Sitter and the exterior as Schwarzschild spacetime; case 3, with the interior as de Sitter and the exterior as Schwarzschild–anti de Sitter spacetime. In these cases also, only unbound trajectories are possible. The portion of the potential is inherently restricted since the allowed region of  $r$  is given by Eq. (27). For the case of  $M < 0$ , there also exist expanding false vacuum bubbles. These objects are considered in different contexts in Ref. [42].

In earlier works on the expanding false vacuum bubbles [2–5], they have the initial singularity in the past. In Ref. [24], it was shown that a false vacuum bubble can be nucleated within the true vacuum background due to a nonminimally coupled scalar field. In order to keep the outside geometry invariant, after a false vacuum bubble is nucleated, the junction conditions need to be modified. In our model the false vacuum bubbles with minimal coupling can expand within the true vacuum background with non-minimal coupling. It will be interesting if this solution can be related to tunneling from nothing to de Sitter space [43] or related to a kind of eternal inflation [44]. Our model is within the framework of the classical theory of gravity. It will be interesting if this framework can be embedded in the superstring theory.

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