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Massive neutrino in noncommutative space-time

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We consider the noncommutative standard model based on $SU(3) \times SU(2) \times U(1)$. We study the gauge transformation of right-handed neutrinos and its direct interaction with photons in the noncommutative space-time. We show that the massive Dirac neutrinos, through the Higgs mechanism, cannot accommodate this extension of the standard model, while the massive Majorana neutrinos are consistent with the gauge symmetry of the model. The electromagnetic properties and the dispersion relations for the neutrino in the noncommutative standard model is examined. We also compare the results with the noncommutative standard model based on $U(3) \times U(2) \times U(1)$.

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I. INTRODUCTION

By now there are convincing evidences which suggest neutrinos have finite masses, such as the problem of the solar and atmospheric neutrinos and so on [1]. Therefore, the extension of the standard model to include massive neutrinos is natural. But theoretically there are two different types of neutrinos that are both neutral particles with different properties. While the Dirac neutrinos like the other charged fermions are Dirac particles, with the distinct antiparticles and the conservation of lepton number, this is not the case for the Majorana neutrinos which are the same as their antiparticles. Though the Majorana masses violate the lepton number, the smallness of the neutrino masses prevent us from confirming or excluding the Majorana property by using the data of the processes sensitive to the lepton number violation, whose typical example is the neutrinoless double β-decay. However, the violation of the total lepton number is predicted by many extensions of the standard model (SM); therefore, the neutrinos can be Majorana as well. Consequently, the next question is whether the existing neutrinos are Dirac or Majorana particles. Or, in other words, do the neutrino interactions within the SM or beyond the SM lead us to distinguish them experimentally or even suppress one of them theoretically? Indeed the solar and atmospheric neutrino oscillations lead to the finite masses for the neutrinos and their Δm^2 values and mixing angles are known in such experiments with fair accuracy [2]. But, neutrino oscillation does not distinguish between Dirac and Majorana neutrinos, because, for example, the equation for the neutrino oscillations in a vacuum is as follows:

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U\begin{pmatrix}\frac{m_1^2}{2E} & 0 & 0\\ 0 & \frac{m_2^2}{2E} & 0\\ 0 & 0 & \frac{m_3^2}{2E}\end{pmatrix}U^\dagger\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \frac{1}{2E}m_\nu m_\nu^\dagger\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}, \tag{1}$$

where this expression holds for both pure Dirac and the neutrinos appearing in the seesaw scenarios. In fact the most popular scenario for the neutrino mass is the seesaw mechanism [3] in which the small neutrino masses arise because of a large hierarchy between a Dirac mass of the order of other fermions in the standard model and a Majorana mass of the order of the grand unified mass scale or at least several orders of magnitude larger than the standard model scale. There are, however, many attempts to find a signal of new physics which can discriminate between Dirac and Majorana neutrinos [4].

On the other hand, the appearance of noncommutative (NC) field theories in a definite limit of string theory [5] has provided a strong motivation to investigate such theories. In recent years, NC-field theories and their phenomenological aspects have been explored by many authors [6–10]. In fact, new interactions in the NC-space-time seem to be potentially important to particle physics and cosmology. In the canonical version of the NC-space-time, the coordinates are operators and satisfy the following relation:

$$\theta^{\mu\nu} = -i[\hat{x}^\mu, \hat{x}^\nu], \tag{2}$$

where a hat indicates a noncommutative coordinate and $\theta^{\mu\nu}$ is a real, constant, and antisymmetric matrix. To construct the noncommutative field theory, according to the Weyl-Moyal correspondence, an ordinary function can be used instead of the corresponding noncommutative one by

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replacing the ordinary product with the star product as follows:

$$f * g(x) = f(x)\exp(i/2\overleftarrow{\partial}_{\mu}\theta^{\mu\nu}\overrightarrow{\partial}_{\nu})g(x). \quad (3)$$

Using this correspondence, however, there are two approaches to construct the gauge theories in the noncommutative space. In the first one the gauge group is restricted to $U(n)$ and the symmetry group of the standard model is achieved by the reduction of $U(3)\times U(2)\times U(1)$ to $SU(3)\times SU(2)\times U(1)$ by an appropriate symmetry breaking [10]. In the second approach, the noncommutative gauge theory can be constructed for $SU(n)$ gauge group via Seiberg-Witten map [9]. In the both versions of the NC-field theories among many new interactions there is a direct interaction between a neutral particle and a photon. For example in the minimal extension of the standard model in the noncommutative space there is neutrino-photon vertex which leads to neutrino-photon interaction at the tree level [7]. In this paper, our aim is to study the effects of such interactions on the properties of neutrino in vacuum.

In Sec. II we briefly review both versions of the noncommutative standard model (NCSM). In Sec. III we study the models for neutrino mass generation and show that the Dirac massive neutrino through the Higgs mechanism is forbidden in the minimal version of NCSM (mNCSM) which is based on the gauge group $SU(3)\times SU(2)\times U(1)$. Meanwhile the massive Majorana neutrino is consistent with both of the versions of NCSM. We also show that to describe the electromagnetic current for the neutrino, besides the usual form factors, new ones are needed. In Sec. IV we consider the dispersion relation for the neutrino in the NC-space-time and show that the neutrino and its antiparticle have generally different dispersion relations in the mNCSM. Finally, we summarize our results.

II. NONCOMMUTATIVE STANDARD MODEL

There are two approaches to construct the standard model in the noncommutative space. In the minimal extension of the standard model in the noncommutative space, the gauge group is $SU(3)_c\times SU(2)_L\times U(1)_Y$ in which the number of gauge fields, couplings, and particles are the same as the ordinary one [9]. Although in this extension new interactions will appear due to the star product and the SW map, the photon-neutrino vertex is absent. We denote the fermion content of the theory as

$$\hat{L} = \begin{pmatrix}\hat{\Psi}_{L_{\nu_l}}\\ \hat{\Psi}_{L_l}\end{pmatrix}, \begin{pmatrix}\hat{\Psi}_{L_u}\\ \hat{\Psi}_{L_d}\end{pmatrix} \quad (4)$$

and

$$\hat{R} = \hat{\Psi}_{R_l}, \qquad \hat{\Psi}_{R_u}, \qquad \hat{\Psi}_{R_d} \qquad \hat{\nu}_R = \hat{\Psi}_{R_{\nu_l}}, \quad (5)$$

where for three generations, l stands for e, μ, and τ meanwhile subscript u refers to up-type quarks and subscript d to down-type quarks. The fields with a hat are the noncommutative fields which can be written as a function of ordinary fields using appropriate SW-maps. Let us now consider an infinitesimal noncommutative local gauge transformation of these fields as follows:

$$\delta\Psi = i\rho_{\Psi}(\Lambda)\star\Psi, \quad (6)$$

where Λ is a gauge parameter and $\rho_{\Psi}(\Lambda)$ is a representation which is carried by the matter or Higgs fields according to the Table I. In mNCSM, under the infinitesimal gauge transformation $\hat{L}$ and $\hat{H}$ are transformed as follows:

$$\delta\hat{L} = i\hat{\Lambda}\star\hat{L}, \qquad \delta\hat{R} = i\hat{\Lambda}'\star\hat{R}, \qquad \delta\hat{H} = i\hat{\Lambda}\star\hat{H} - i\hat{H}\star\hat{\Lambda}', \quad (7)$$

in which the symbol ρ_{Ψ} is omitted. Meanwhile $\hat{\nu}_R$ as a neutral-hyper-charged particle in the standard model transforms as

$$\delta\hat{\nu}_R = i\hat{\Lambda}'\star\hat{\nu}_R - i\hat{\nu}_R\star\hat{\Lambda}'. \quad (8)$$

In fact, in a noncommutative setting the noncommutative gauge boson B_{μ}, compatible with the noncommutative gauge transformation, couples to a neutral matter field Ψ^0 as

$$\hat{\mathcal{D}}_{\mu}\Psi^0 = \partial_{\mu}\Psi^0 - i[B_{\mu}, \Psi^0], \quad (9)$$

with

$$\delta\Psi^0 = i[\Lambda \overset{\star}{,} \Psi^0]. \quad (10)$$

In the second approach the gauge group is $U_{\star}(3)\times U_{\star}(2)\times U_{\star}(1)$ which is reduced to $SU(3)_c\times SU(2)_L\times U(1)_Y$ by reducing the two extra $U(1)$ factors through the appropriate Higgs mechanism and Higgs particles (*Higgsac*) [10]. In this approach the number of possible particles in each family (which are six: left-handed lepton, right-handed charged lepton, left-handed quark, right-handed up quark, right-handed down quark, and Higgs boson) as well as their hypercharges are naturally fixed. Hence, in contrast to the mNCSM, the right-handed neu-

TABLE I. Matter and Higgs field content of the extended standard model and their representations.

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
e_R	1	1	-2
$L_l = \begin{pmatrix}\nu_L\\ e_L\end{pmatrix}$	1	2	-1
u_R	3	1	$\frac{4}{3}$
d_R	3	1	$\frac{-2}{3}$
$L_q = \begin{pmatrix}u_L\\ d_L\end{pmatrix}$	3	2	$\frac{1}{3}$
$\Phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$	1	2	1
ν_R	1	1	0

trino could only be a sterile neutrino, i.e. a singlet under all gauge groups. The gauge transformations for the first generation of the leptonic part of the matter fields are as follows:

(1) *Right-handed leptons.*—In this group there is only the right-handed electron and its right-handed neutrino, which transform as

$$e_R(x) \to e_R(x)\upsilon^{-1}(x), \tag{11}$$

and

$$\nu_{eR}(x) \to \upsilon(x)\nu_{eR}(x)\upsilon^{-1}(x), \tag{12}$$

in which υ is the element of $U_\star(1)$.

(2) *Left-handed leptons.*—Here we have the left-handed electron and its neutrino, in a doublet:

$$\Psi^l_L(x) = \begin{pmatrix} \nu(x) \\ e(x) \end{pmatrix}_L. \tag{13}$$

Under the gauge transformations, the doublet transforms as

$$\Psi^l_L(x) \to V(x)\Psi^l_L(x)\upsilon^{-1}(x), \tag{14}$$

where $V(x)$ is the element $U_\star(2)$.

(3) *Higgs doublet.*—

$$\Phi(x) = \begin{pmatrix} \Phi^+(x) \\ \Phi^0(x) \end{pmatrix}, \tag{15}$$

which transforms as

$$\Phi(x) \to V(x)\Phi(x), \tag{16}$$

while the charge conjugated field of Φ transforms as

$$\Phi^c(x) \to V(x)\Phi^c(x)\upsilon^{-1}(x). \tag{17}$$

III. MASSIVE NEUTRINO IN NONCOMMUTATIVE STANDARD MODEL

There are several indications for nonzero neutrino masses of which the most stringent ones come from the solar and atmospheric neutrino experiments. It is obvious from the observed mass scale of the neutrinos that they cannot be treated exactly the same way as the other basic fermions. There must be some reasons for them being almost massless. In the standard model they are precisely massless because there are no right-handed neutrino states. In fact, as the most straightforward way, one can construct Dirac mass terms for neutrinos through the Higgs mechanism and Yukawa coupling. Here one encounters the smallness of the Yukawa coupling in comparison with the other charged fermions. Indeed there is no natural explanation for the smallness of the neutrino mass. If we consider the right-handed neutrinos, the Yukawa terms for neutrinos can be written in the standard model similar to the other fermions as follows:

$$Y_{\nu ij}\bar{L}_i H^c \nu_{Rj} + \text{H.c.}, \tag{18}$$

in which H^c is the charged conjugate of the Higgs field. This term after the electroweak symmetry breaking leads to the usual Dirac fermion mass, i.e. $M_{\nu Dij} = Y_{\nu ij}\langle H\rangle$.

In the mNCSM, (18) leads to

$$Y_{\nu ij}\bar{\hat{L}}_i \star \hat{H}^c \star \hat{\nu}_{R_j} + \text{H.c.}, \tag{19}$$

that under the gauge transformations which are given in Eqs. (7) and (8) obviously violates the mNCSM gauge symmetry which is in contrast with the other fermions. Therefore, the massive Dirac neutrino through the Higgs mechanism is forbidden in the mNCSM. Meanwhile, it should be noted that in the nonminimal version of NCSM[1] based on the $U_\star(3)\times U_\star(2)\times U_\star(1)$ gauge group, (19) is conserved, see (12), (14), and (17). However, as a prediction of almost whole extensions of the standard model, the conservation of lepton number can be broken. Therefore neutrinos as neutral particles do not hold any additive internal quantum numbers and it is possible to introduce Majorana mass terms for them as well. However, the seesaw mechanism is not consistent with the mNCSM.

A. Majorana mass for neutrino

In both versions of the standard model, the noncommutative space neutrino can be a massive Majorana particle. For the right-handed neutrino, the Majorana mass term can be written as

$$M_R\nu_R^T C^{-1}\nu_R + \text{H.c.}, \tag{20}$$

where C is the Dirac charge conjugation matrix. Since ν_R under the gauge group transforms as a singlet, (20) can be considered as a bare mass term in the Lagrangian or, alternatively, can be generated by interactions with a singlet scaler field σ

$$f_\sigma \nu_R^T C^{-1}(\sigma + \langle\sigma\rangle)\nu_R, \tag{21}$$

where $\langle\sigma\rangle$ is the vacuum expectation value of σ. Under the mNCSM gauge group, (21) is invariant if σ transforms as

$$\delta\hat{\sigma} = i\hat{\Lambda}'\star\hat{\sigma} - i\hat{\sigma}\star\hat{\Lambda}', \tag{22}$$

while for the nonminimal version of NCSM it should be transformed as

$$\sigma \to \upsilon(x)\sigma\upsilon^{-1}(x). \tag{23}$$

[1]Hereafter nonminimal version of NCSM is used for NCSM based on the gauge group $U_\star(3)\times U_\star(2)\times U_\star(1)$ and should not be confused with the nonminimal versions of the NCSM based on the gauge group $SU(3)_c\times SU(2)_L\times U(1)_Y$ which can be constructed due to the freedom in the choice of the representation of the gauge group in the pure gauge action [9]. In this paper the minimal choice for the gauge group $SU(3)_c\times SU(2)_L\times U(1)_Y$ is introduced.

To generate the Majorana mass for the left-handed neutrinos, one can use the nonrenormalizable operator if the renormalizability of the theory is abandoned [11]. For this purpose one can write

$$\frac{\lambda_{ij}}{M}(L_iH)^T(L_jH), i, j = e, \mu, \tau, \tag{24}$$

where λ_{ij}'s are the dimensionless couplings and M is an appropriate cutoff. However, to preserve the mNCSM symmetry in (24), the Higgs field should be transformed from the right-hand side as a singlet i.e. $\hat{\Lambda}' = 0$ in the transformation of the Higgs field given in (7). One can easily see that in the nonminimal NCSM, (24) under the symmetry transformations of this group is not conserved.

The Majorana mass term for the left-handed neutrino violates lepton number by 2 units and has the Weak isospin $I = 1$; therefore, it can be generated due to coupling with the Higgs triplet Δ:

$$f_\Delta L^T L\Delta + \text{H.c.} \tag{25}$$

The nonzero vacuum expectation value of Δ then gives $m_L = f_\Delta\langle\Delta\rangle$. To conserve (25) under the mNCSM gauge symmetry, Δ should be transformed as

$$\delta\hat{\Delta} = -i\hat{\Lambda}\star\hat{\Delta} + i\hat{\Delta}\star\hat{\Lambda}. \tag{26}$$

Meanwhile (25) is not conserved under the gauge transformations of the nonminimal version of NCSM.

B. Neutrino dipole moment

Since neutrinos are neutral, they can couple with the electromagnetic field in the ordinary space-time through loop corrections. Therefore, the most general form for the electromagnetic current between Dirac neutrinos, consistent with the Lorentz covariance and the Ward identity, can be written as follows:

$$\begin{aligned}\langle\nu(p',\lambda')|J_\mu^{\text{em}}|\nu(p,\lambda)\rangle = \bar{\nu}(p',\lambda')\{&\gamma_\mu F_1(q^2) \\ &- \gamma_\lambda\gamma_5(g_\mu^\lambda q^2 - q^\lambda q_\mu)G_1(q^2) \\ &+ \sigma_{\mu\nu}q^\nu[F_2(q^2) - \gamma_5 G_2(q^2)]\} \\ &\times \nu(p,\lambda),\end{aligned} \tag{27}$$

where $q = p' - p$ and F_1, G_1, F_2, and G_2 are electric charge, anapole (axial charge), magnetic and electric form factors, respectively. All of the form factors are nonzero for a massive Dirac neutrino if the CP violation is included. In fact even if the leptonic sector of the weak interaction conserved CP in the quark sector, CP is not conserved and at least at high order loops involving virtual quarks it will be induced. In the case of a massless neutrino, the matrix elements of electromagnetic current can be expressed in terms of only one form factor as [12]

$$\langle\nu(p',\lambda')|J_\mu^{\text{em}}|\nu(p,\lambda)\rangle = F(q^2)\bar{\nu}(p',\lambda')\gamma_\mu(1+\gamma_5)\nu(p,\lambda). \tag{28}$$

Meanwhile, for Majorana neutrinos which are the same as their antiparticles, the only nonzero form factor is the anapole. However the transition matrix elements relevant to $\nu_i \rightarrow \nu_j$ would still generally contain four form factors as given in (27) for both the Dirac and Majorana neutrinos.

In the noncommutative space-time the parameter $\theta^{\mu\nu}$ carries Lorentz indices but it does not mean that it is a Lorentz quantity under a general Lorentz transformation. In fact there are two distinct types of Lorentz transformation which are observer and particle Lorentz transformations [13]. Nonetheless, $\theta^{\mu\nu}$ is only a Lorentz tensor under the observer Lorentz transformation while it is a constant under the particle Lorentz transformation. Therefore the Lagrangian in the NC space is fully covariant under the observer Lorentz transformations. This means that the observer Lorentz symmetry is an invariance of the model, but the particle Lorentz group is broken. Hereafter Lorentz quantity means the Lorentz quantity under the observer Lorentz transformation. However, there is a new Lorentz quantity in the NC space beside the usual ones i.e. $\theta_{\mu\nu}$. Therefore for the Dirac neutrinos, the effective $\nu\nu\gamma$ vertex can be generally expanded in terms of the Lorentz vectors as follows:

$$\begin{aligned}\langle\nu(p',\lambda',\theta)|J_\mu^{\text{em}}|\nu(p,\lambda,\theta)\rangle = \bar{\hat{\nu}}(p',\lambda')\{&\gamma_\mu F_1 - \gamma_\lambda\gamma_5(g_\mu^\lambda q^2 - q^\lambda q_\mu)G_1 + \sigma_{\mu\nu}q^\nu[F_2 - \gamma_5 G_2] + \theta_{\mu\nu}\gamma^\nu(F_3 + \gamma_5 G_3) \\ &+ \theta_{\mu\nu}q^\nu(F_4 + \gamma_5 G_4) + F_5(\theta_{\mu\nu}\sigma^{\nu\rho} - \theta^{\rho\nu}\sigma_{\nu\mu})q_\rho + G_5(\theta_{\mu\nu}\sigma^{\nu\rho} - \theta^{\rho\nu}\sigma_{\nu\mu})\gamma_5 q_\rho\}\hat{\nu}(p,\lambda),\end{aligned} \tag{29}$$

where the form factors F_i's and G_i's depend on the Lorentz invariant quantities such as q^2 and $q^\mu\theta_{\mu\nu}p^\nu$. Here one has that

(1) the Ward identity leads to $F_3 = G_3 = 0$;
(2) up to the first order of $\theta_{\mu\nu}$ only F_i's and G_i's with $i = 1, 2$ have θ-dependent parts;
(3) since $\nu = \nu_L + \nu_R$ the current in (29) is left-left for F_1 and G_1 while it is left-right for F_i's and G_i's with $i = 2, 4, 5$. Therefore, the corresponding effective Lagrangian contains interactions which involve the neutrino bilinear $\bar{\nu}_L\Gamma_\mu\nu_L$ and $\bar{\nu}_L\Gamma'_\mu\nu_R$ multiplied by a functional of the hypercharge field.

In the mNCSM those terms which are proportional to $\bar{\nu}_L\Gamma'_\mu\nu_R$ obviously violate the mNCSM gauge symmetry, see Eqs. (7) and (8). Therefore, in contrast with the nonminimal version of NCSM, the current in (29) can be written only in terms of F_1 and G_1. In the mNCSM, the fermion fields depend on the parameter of the noncommutativity of space and time as follows:

$$\hat{\Psi} = \Psi^0 + \Psi^1 + \Psi^2 + \ldots, \tag{30}$$

where Ψ^0 is the commutative fermion field and the superscript $i = 1, 2, \ldots$ stands for the order of $\theta_{\mu\nu}$ in the expansion. For instance

$$\Psi^1 = -\frac{1}{2}\theta^{\mu\nu}A^0_\mu\partial_\nu\Psi^0 + \frac{i}{4}\theta^{\mu\nu}A^0_\mu A^0_\nu\Psi^0, \tag{31}$$

where A^0 is the ordinary gauge field [8]. Under charge conjugation operation ($\mathcal{C}$), a free fermion field in the noncommutative space-time transforms as

$$\mathcal{C}\hat{\Psi}\mathcal{C}^{-1} = \gamma_0 C\hat{\Psi}^*. \tag{32}$$

Since for the Majorana neutrino, $\hat{\Psi}^c = \hat{\Psi}$, we should have

$$\mathcal{C}^{-1}\theta_{\mu\nu}\mathcal{C} = -\theta_{\mu\nu}, \tag{33}$$

which is consistent with the transformation property of $\theta_{\mu\nu}$ under $\mathcal{C}$ for the both versions of NCSM which are given in [14,15]. One should note that the operator C in (32) contains the θ transformation in addition to the usual $i\gamma_2\gamma_0$. Therefore, in the current (29) for the Majorana neutrinos, up to the first order of $\theta_{\mu\nu}$, besides the anapole, the θ-dependent part of F_1, F_2, G_2, and F_5 and G_5 are nonzero. Here one should note that the all form factors would generally survive in the transition $\nu_i \to \nu_j$ for the Majorana neutrinos which are consistent with both versions of NCSM.

In ordinary space neutral particles can only interact with photons through the loop corrections which is in contrast with the noncommutative space where they can minimally couple to each other, even at the tree level, in the adjoint representation of the $U_\star(1)$ gauge group. For instance in the mNCSM only neutral particles with zero hypercharge can couple directly to the photon, therefore the left-handed neutrinos cannot couple to the hyperphoton. In fact if we consider the right-handed neutrino as a particle in the model, its interaction with the photon can be obtained from the following Lagrangian [7]:

$$\mathcal{L}_{\nu_R} = i\bar{\nu}_R\not{\partial}\nu_R + ie\theta^{\mu\nu}[\partial_\mu\bar{\nu}_R B_\nu\gamma^\rho(\partial_\rho\nu_R) - \partial_\rho\bar{\nu}_R B_\nu\gamma^\rho(\partial_\mu\nu_R) + \bar{\nu}_R(\partial_\mu B_\rho)\gamma^\rho(\partial_\nu\nu_R)], \tag{34}$$

where B in terms of the photon and the Z-gauge boson fields is

$$B = \cos\theta_W A - \sin\theta_W Z, \tag{35}$$

Which leads to the $\nu\nu\gamma$ vertex as follows:

$$\Gamma^\mu = i\tfrac{1}{2}\cos\theta_W(1+\gamma_5)(\theta^{\mu\nu}k_\nu\not{q} + \theta^{\rho\mu}q_\rho\not{k} + \theta^{\nu\rho}k_\nu q_\rho\gamma_\mu). \tag{36}$$

Meanwhile in the nonminimal version of NCSM the left-handed neutrinos, as well as the right-handed neutrinos, can be coupled to photons through the following interaction [10]:

$$\mathcal{L}_{\nu-\gamma} = -ie\bar{\nu}\gamma^\mu[\nu, A_\mu]_\star = -e\bar{\nu}\gamma^\mu(\theta_{\alpha\beta}\partial_\alpha A_\mu\partial_\beta\nu) + \mathcal{O}(\theta^2). \tag{37}$$

Therefore in contrast to the commutative space where the form factors F_1 and G_1 vanish for the arbitrary neutrino mass in the 't Hooft-Feynman gauge [16], these form factors which depend on the Lorentz invariant quantity $p.\theta.q$ are nonzero at the leading order even for the massless neutrinos. Moreover, comparing G_1 induced by the noncommutative space with the electroweak anapole defined through the $\nu_l l$ scattering at the one loop level [12], we find $\Lambda_{\rm NC} = \frac{1}{\sqrt{\theta^2}} \geq 10$ TeV.

IV. DISPERSION RELATION OF NEUTRINO IN VACUUM

In the previous section we showed that neutrino properties in the noncommutative space are different from the ordinary space. In this section we show how the new interaction of neutrino in the noncommutative space-time can affect its dispersion relation in comparison with the usual space. To this end we explore the pole of the neutrino-propagator in vacuum of the NC-space-time. In the ordinary space-time the Lagrangian of a free massless neutrino in the momentum space and in the vacuum can be written as follows:

$$\mathcal{L}_0 = \bar{\psi}_L(p)\not{p}\psi_L(p). \tag{38}$$

By including the interactions, the self-energy of the neutrino can be easily obtained as

$$\mathcal{L} = \bar{\psi}_L(p)(\not{p} - \Sigma)\psi_L(p), \tag{39}$$

where Σ, in the vacuum, is generally given by

$$\Sigma = a\not{p}, \tag{40}$$

in which a is a Lorentz scalar which depends on the Lorentz invariant quantities. However, in addition to the momentum of the neutrino, in terms of the parameter of the noncommutativity, new 4-vectors such as $\theta_{\mu\nu}p^\nu$ and $\theta_{\mu\nu}\theta^{\nu\rho}p_\rho$ can be constructed in the noncommutative space-time. Therefore, the most general form of the Σ up to the second order of θ is as follows:

$$\Sigma = a\not{p} + bp.\not{\theta} + cp.\theta.\not{\theta}, \tag{41}$$

where a, b, and c are Lorentz scalars and depend on the corresponding Lorentz invariant quantities in the NC-space-time and $p.\theta\!\!\!/$ and $p.\theta.\theta\!\!\!/$ stand for $p^\mu\theta_{\mu\nu}\gamma^\mu$ and $p^\mu\theta_{\mu\nu}\theta^{\nu\rho}\gamma_\rho$, respectively. The Lagrangian (39) can be rewritten in the form

$$\mathcal{L} = \bar{\psi}_L(p)\frac{V\!\!\!/}{V^2}\psi_L(p), \qquad (42)$$

where

$$V_\mu = (1-a)p_\mu - bp.\theta_\mu - cp.\theta.\theta_\mu. \qquad (43)$$

The dispersion relation can be easily obtained by solving the equation $V^2 = 0$, i.e.

$$(1-a)^2p^2 - (2c(1-a)+b^2)p.\theta.\theta.p = 0. \qquad (44)$$

It is clear from (44) that the solutions for the positive and negative energy and therefore the dispersion relations for the neutrino and antineutrino are different only if $\theta^{i0} \neq 0$. In fact if we construct a consistent field theory on the noncommutative space-time, the neutrino and its antiparticle can be distinguished in the vacuum by the nonzero value of θ^{i0} which is in contrast with the ordinary space-time.

Now we calculate the explicit form of a, b, and c for both versions of the NCSM. The relevant diagrams for the neutrino self-energy in the NC space are shown in Fig. 1. By expanding the propagator of the W-gauge boson as a power series in $1/M_W^2$

$$D_{\alpha\beta}(q^2) = \frac{-g_{\alpha\beta}+\frac{q_\alpha q_\beta}{M_W^2}}{q^2-M_W^2} = \frac{g_{\alpha\beta}}{M_W^2} + \frac{q^2g_{\alpha\beta}-q_\alpha q_\beta}{M_W^4} + \ldots, \qquad (45)$$

and using the dimensional regularization which leads to

$$\int d^dq(q^2)^\nu = 0; \quad \text{for any } \nu, \qquad (46)$$

one can easily see that the contributions from the diagrams a_1, a_2, a_3, a_5, and a_6 are zero. The diagram a_4 which is absent in the ordinary space can be obtained in the mNCSM by using the neutrino-hypercharge vertex given in (34) as

$$\Sigma = fp^2(2p^2p.\theta.\theta.\gamma + (p.\theta.\theta.p - \tfrac{1}{2}\theta^{\mu\nu}\theta_{\mu\nu}p^2)p\!\!\!/)R, \qquad (47)$$

where $R = \frac{1+\gamma_5}{2}$ and

$$f = \frac{ie^2}{12(4\pi)^2}\left(1 - \frac{1}{\varepsilon} - \ln\left(\frac{\mu^2}{p^2}\right) - \gamma - \ln 4 - \psi\left(\frac{5}{2}\right)\right). \qquad (48)$$

Comparing (47) with (41) leads to $b = 0$ and

$$a = fp^2(p.\theta.\theta.p - \tfrac{1}{2}\theta^{\mu\nu}\theta_{\mu\nu}p^2)R, \qquad (49)$$

and

$$c = 2fp^4R. \qquad (50)$$

Meanwhile the diagram a_4 in the nonminimal version of NCSM can be easily calculated by considering the vertex given in (37) which leads to

$$\Sigma = -fp.\theta.\theta.pp^2p\!\!\!/, \qquad (51)$$

where $b = c = 0$ and $a = -fp.\theta.\theta.pp^2$. Since a can be absorbed in the field redefinition, the effects of the noncommutativity on the dispersion relation can be detected only in the mNCSM.

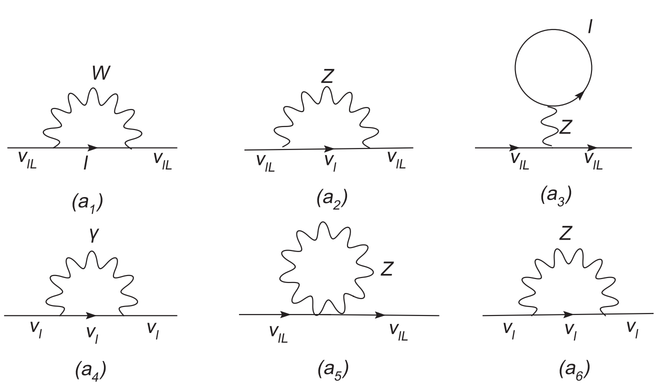

FIG. 1. 1-loop diagrams for neutrino self-energy in noncommutative space.

V. CONCLUSION

In this paper we have studied the effects of the photon-neutrino interaction in NCSM on the neutrino properties in the vacuum. In the minimal version of the NCSM we have found:

(1) The Yukawa terms for neutrinos in the mNCSM, see (19), violates the mNCSM gauge symmetry, therefore the massive Dirac neutrino through the Higgs mechanism is forbidden and the seesaw mechanism is not consistent with the mNCSM.

(2) The mass term for Majorana neutrinos can be defined directly, see (20), generated by interactions with a singlet scaler field σ, see (21), through coupling with the Higgs triplet Δ, see (25), or even through the nonrenormalizable operator, see (24).

(3) The effective $\nu\nu\gamma$ vertex, in the NCSM, for Dirac neutrinos can be generally expanded in terms of 8 form factors in contrast with 4 in the ordinary space, see (29). Those terms in the current which correspond to the interactions proportional to $\bar{\nu}_L\Gamma'_\mu\nu_R$ in the effective Lagrangian violate the mNCSM gauge symmetry. Therefore, the current in the mNCSM can be written only in terms of two form factors F_1 and G_1, see (7), (8), and (29).

(4) For the Majorana neutrino to be its antiparticle, the parameter of noncommutativity under charge conjugation operation $\mathcal{C}$ should transform as $\mathcal{C}^{-1}\theta_{\mu\nu}\mathcal{C} = -\theta_{\mu\nu}$, see (33). In fact in the NC space

the charge conjugation operator contains the θ transformation in addition to the usual $i\gamma_2\gamma_0$. Therefore, the electromagnetic current for the Majorana neutrino should be described in terms of 5 form factors F_1, F_2, G_2, F_5, and G_5 in contrast with the one, the anapole, in the commutative space.

(5) In contrary to the commutative space in both version of the NCSM, F_1 and G_1 are nonzero at the leading order even for the massless neutrinos and depend on the Lorentz invariant quantity $p.\theta.q$, see (34) and (37), which leads to the upper bound $\Lambda_{\rm NC} = \frac{1}{\sqrt{\theta^2}} \geq$ 10 TeV.

(6) The dispersion relations for the neutrino and antineutrino in the NC-space-time are generally different only if $\theta^{i0} \neq 0$, see (44). The self-energy for the neutrino can be expanded in terms of the new θ-dependent four vectors up to the second order of $\theta_{\mu\nu}$ as $\Sigma = a\not{p} + bp.\not{\theta} + cp.\theta.\not{\theta}$ where for the mNCSM $c \neq 0$, $a \neq 0$, and $b = 0$, see (41) and (47). In other words, the neutrino and its antiparticle have different dispersion relations in the mNCSM.

Meanwhile for the nonminimal version of NCSM we found:

(1) The Yukawa term for neutrino in the nonminimal version of NCSM, (19), is consistent with the gauge symmetry given in (12), (14), and (17); therefore, the massive Dirac neutrino through the Higgs mechanism is allowed.

(2) In the nonminimal version of NCSM the mass term for Majorana neutrinos can be defined directly, see (20), or generated by interactions with a singlet scaler field σ, see (21).

(3) Eight form factors are needed to describe the current for the Dirac neutrinos in the nonminimal version of NCSM and there is not any restriction by the gauge symmetry.

(4) The dispersion relations for the neutrino and its antiparticle in the nonminimal version of NCSM are the same because $c = b = 0$ and $a \neq 0$, see (41) and (51). One should note that at our disposal we expanded the nonlocal Lagrangian of the NCSM based on the gauge group $U_\star(3) \times U_\star(2) \times U_\star(1)$, to obtain the result. However, the expansion of the Lagrangian in terms of $\theta^{\mu\nu}$ destroys the nonlocality of the model, and it should be shown if the nonlocality of the model has any contribution on the obtained result or not.

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