Leptoquarks: Neutrino masses and related accelerator signals

D. Aristizabal Sierra* and M. Hirsch

AHEP Group, Instituto de Física Corpuscular–C.S.I.C./Universitat de València Edificio de Institutos de Paterna, Apartado 22085, E-46071 València, Spain

S. G. Kovalenko

Centro de Estudios Subatómicos (CES), Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile

(Received 20 November 2007; published 17 March 2008)

Leptoquark-Higgs interactions induce mixing between leptoquark (LQ) states with different chiralities once the electroweak symmetry is broken. In such LQ models Majorana neutrino masses are generated at 1-loop order. Here we calculate the neutrino mass matrix and explore the constraints on the parameter space enforced by the assumption that LQ-loops explain current neutrino oscillation data. LQs will be produced at the CERN LHC, if their masses are at or below the TeV scale. Since the fermionic decays of LQs are governed by the same Yukawa couplings, which are responsible for the nontrivial neutrino mass matrix, several decay branching ratios of LQ states can be predicted from measured neutrino data. Especially interesting is that large lepton flavor violating rates in muon and tau final states are expected. In addition, the model predicts that, if kinematically possible, heavier LQs decay into lighter ones plus either a standard model Higgs boson or a $Z^0/W^{\pm}$ gauge boson. Thus, experiments at the LHC might be able to exclude the LQ mechanism as an explanation of neutrino data.

DOI: 10.1103/PhysRevD.77.055011 PACS numbers: 12.60.−i, 13.15.+g, 13.90.+i, 14.60.St

I. INTRODUCTION

Leptoquarks (LQs) appear in many extensions of the standard model. First discussed in the classic papers by Pati and Salam [1] and Georgi and Glashow [2], LQs are a common ingredient to grand unified theories [3,4]. They can also appear in composite [6] as well as in technicolor models [7,8]. Also in supersymmetric models with R-parity violation, scalar quarks have leptoquarklike interactions [9]. From a low-energy point of view, however, LQs are best described in a ''model-independent'' way, using a LQ Lagrangian based only on the minimal assumptions of (a) renormalizability and (b) standard model (SM) gauge invariance [10]. An exhaustive list of limits on such LQs from low-energy experiments can be found, for example, in [11].

Direct searches so far have not turned up any evidence for LQs [12]. The best limits on pair produced LQs currently come from the D0 [13] and CDF [14] experiments at the Tevatron. These typically give limits on LQ masses in the ballpark of $m_{\rm LQ} \gtrsim$ (200–250) GeV, depending mainly on the final state decay branching ratios and on the lepton-quark generation, to which the LQ state couples. Considerably more stringent limits are expected from the CERN LHC experiments. Depending on the accumulated luminosity, the LHC should be able to find LQs up to masses of order of $m_{\rm LQ} \sim$ (1.2–1.5) TeV [15].

Solar [16], atmospheric [17], and reactor [18] neutrino oscillation experiments have firmly established that neutrinos have mass and nontrivial mixing between different generations. In the SM neutrinos are massless. However, nonzero neutrino masses can easily be generated and the literature is abounding in neutrino mass models [19]. Certainly the most popular way to generate neutrino masses is the seesaw mechanism [20–23]; countless variants exist. However, it is also conceivable that the scale of lepton number violation is near—or at—the electroweak scale. To mention a few examples, there are supersymmetric models with violation of R-parity [9,24], models with Higgs triplets [23], or a combination of both [25]. Also purely radiative models have been discussed in the literature, both with neutrino masses at 1-loop [26,27] or at 2-loop [28–31] order. Radiative mechanisms might be considered especially appealing, since they generate small neutrino masses automatically, essentially due to loop suppression factors.

In this paper, we study the generation of neutrino masses due to loops involving light leptoquarks, in a model with nonzero leptoquark-Higgs interactions [32]. LQ-Higgs interactions lead to mixing between LQs of different chiralities (and lepton number) once electroweak symmetry is broken and thus can contribute nontrivially to the Majorana neutrino mass matrix at 1-loop level [33]. As discussed below, the peculiar structure of leptonic mixing, observed in neutrino oscillation experiments, enforces a number of constraints on the LQ parameter space. The main result of our current work is that these constraints can be used to make definite predictions for different decay branching ratios of several LQ states. Therefore, the hypothesis that LQ-loops are responsible for the generation of neutrino mass is testable at the LHC, if LQs have masses of the order of $\mathcal{O}(1)$ TeV.

Before proceeding, a few more comments on LQs might be in order. First, for the LQ model to be able to explain

*Also at INFN, Laboratori Nazionali di Frascati, Italy

neutrino data, nonzero LQ-Higgs interactions are essential. Limits on these couplings, on the other hand, can be derived from low-energy data such as, for example, pion decay [32]. Especially stringent are limits from neutrinoless double beta decay [34] and from the decay $K^0 \to e^\pm \mu^\mp$ [35]. However, as we will discuss below, the small neutrino masses themselves are up to now the most sensitive low-energy probe of LQ-Higgs mixing terms.

Second, it should be mentioned that LQ-loops as a source of neutrino mass have been discussed previously in [36]. We will improve upon this work in several aspects: (i) We will present neutrino mass formulas containing all possible LQ-loops, while in [36] only down-type quark loops were considered. (ii) [36] concentrated on upper limits on LQ parameters, which can potentially be derived from observed neutrino masses. We, on the other hand, identify the regions of LQ parameters where the neutrino mass matrix is dominated by LQ-loops, thus providing a potential explanation of oscillation data. And, last but most importantly, (iii) we discuss possible accelerator tests of the LQ hypothesis of neutrino masses, to the best of our knowledge for the first time in the literature.

Finally, it should also be mentioned that LQs can be either scalar or vector particles. We consider only scalars in detail. However, we note that most of our results straightforwardly apply also for vector LQs.

The rest of this paper is organized as follows. In Sec. II we define the leptoquark interactions, both with quark-lepton pairs and with the SM Higgs boson, and discuss the LQ mass matrices. In Sec. III we calculate the 1-loop neutrino mass matrix in the LQ model. Some particularly simple and interesting limits are defined and discussed analytically. The typical ranges of LQ parameters, required to explain current neutrino data, are explored. We then turn to the phenomenology of LQs at future accelerators in Sec. IV. It is found that some fermionic LQ decays trace the measured neutrino angles and thus can serve, in principle, as a test of the LQ model. Next we discuss LQ decays to the SM Higgs and to gauge bosons. Higgs (and Z^0) decays should occur, if kinematically possible, due to the nonzero LQ mixing required to explain neutrino masses and thus form a particularly interesting signal of the LQ model. We then close the paper with a short summary.

II. LEPTOQUARK BASICS

A. Scalar leptoquark Lagrangian

The SM symmetries allow five scalar LQs. Table I shows their $SU(3)_c \times SU(2)_L \times U(1)_Y$ quantum numbers, as well as their standard baryon and lepton number assignments. LQs which couple nonchirally are strongly constrained by low-energy data [11]. Thus, the states S_0^L and S_0^R (as well as $S_{1/2}^L$ and $S_{1/2}^R$), which have the same SM quantum numbers, but couple to (quark) doublets and singlets, respectively, are usually assumed to be independent particles. Under these assumptions, the most general Yukawa interactions (LQ-lepton-quark) induced by the new scalar fields are given by [10]

TABLE I. Standard model quantum numbers of the scalar leptoquarks. The indices 0, 1/2, 1 indicate the weak isospin. The weak hypercharge is normalized according to $Y = 2(Q_{\rm em} - T_3)$.

LQ	$SU(3)_c$	$SU(2)_L$	Y	$Q_{\rm em}$	L	B
S_0	**3**	**1**	$-2/3$	$-1/3$	1	1/3
$\tilde{S}_0$	**3**	**1**	$-8/3$	$-4/3$	1	1/3
$S_{1/2}$	$\mathbf{3}^*$	**2**	$-7/3$	$(-2/3, -5/3)$	1	$-1/3$
$\tilde{S}_{1/2}$	$\mathbf{3}^*$	**2**	$-1/3$	$(1/3, -2/3)$	1	$-1/3$
S_1	**3**	**3**	$-2/3$	$(2/3, -1/3, -4/3)$	1	1/3

$$\begin{aligned}\mathcal{L}_{\mathrm{LQ}-l-q} = {} & \lambda_{S_0}^{(R)} \bar{u}^c P_R e S_0^{R\dagger} + \lambda_{\tilde{S}_0}^{(R)} \bar{d}^c P_R e \tilde{S}_0^{R\dagger} \\ & + \lambda_{S_{1/2}}^{(R)} \bar{u} P_L l S_{1/2}^{R\dagger} + \lambda_{\tilde{S}_{1/2}}^{(R)} \bar{d} P_L l \tilde{S}_{1/2}^{\dagger} \\ & + \lambda_{S_0}^{(L)} \bar{q}^c P_L i\tau_2 l S_0^{L\dagger} + \lambda_{S_{1/2}}^{(L)} \bar{q} P_R i\tau_2 e S_{1/2}^{L\dagger} \\ & + \lambda_{S_1}^{(L)} \bar{q}^c P_L i\tau_2 \tau \cdot S_1^\dagger l + \text{H.c.} \end{aligned} \tag{1}$$

Here we used the conventions of [11]. Note that Eq. (1) is written in one-generation notation. In general, all λ's are 3×3 matrices in generation space. q and l (u, d, and e) are the quark and lepton SM doublets (singlets), S_i^j are the scalar LQs with the weak isospin $i = 0, 1/2, 1$ coupled to left-handed ($j = L$) or right-handed ($j = R$) quarks, respectively. Thus, in total Eq. (1) contains seven LQ fields.

The most general renormalizable and gauge invariant scalar LQ interactions with the SM Higgs doublet (H) are described by the scalar potential [32]

$$\begin{aligned} V = {} & h_{S_0}^{(i)} H i\tau_2 \tilde{S}_{1/2} S_0^i + h_{S_1} H i\tau_2 \tau \cdot S_1 \tilde{S}_{1/2} \\ & + Y_{S_{1/2}}^{(i)} (H i\tau_2 S_{1/2}^i)(\tilde{S}_{1/2}^\dagger H) + Y_{S_1} (H i\tau_2 \tau \cdot S_1^\dagger H)\tilde{S}_0 \\ & + \kappa_S^{(i)} (H^\dagger \tau \cdot S_1 H) S_0^{i\dagger} - (M_\Phi^2 - g_\Phi^{(i_1 i_2)} H^\dagger H)\Phi^{i_1 \dagger}\Phi^{i_2} \\ & + \text{H.c.} \end{aligned} \tag{2}$$

Here Φ^i is a cumulative notation for all scalar LQ fields with $i = L, R$ (the same for $i_{1,2}$). The diagonal mass terms $M_\Phi^2 \Phi^\dagger \Phi$ can be generated by spontaneous breaking of the fundamental underlying symmetry down to the electroweak gauge group at some high-energy scale. The subsequent electroweak symmetry breaking produces nondiagonal LQ mass terms which, in addition to the diagonal terms given in Eq. (2), define the LQ squared-mass matrices. These will be discussed next.

It is important to note that the first two terms of the scalar potential in Eq. (2) violate total lepton number by two units $\Delta L = 2$ and, therefore, generate Majorana neutrino masses after electroweak symmetry breaking [37]. In the limit where $h_{S_0}^{(R)}$ and h_{S_1} vanish, neutrino masses vanish as well.

B. Scalar leptoquark mass spectrum

There are four squared-mass matrices which determine the masses of LQs with the same electric charge ($Q = -1/3, -2/3, -4/3, -5/3$). In the interaction eigenstate basis, defined by $S_{-1/3} = (S_0^L, S_0^R, \tilde{S}_{1/2}^\dagger, S_1)$, $S_{-2/3} = (\tilde{S}_{1/2}, S_{1/2}^L, S_{1/2}^R, S_1^\dagger)$, $S_{-4/3} = (\tilde{S}_0, S_1)$, and $S_{-5/3} = (S_{1/2}^L, S_{1/2}^R)$, the squared-mass matrices read

$$\mathcal{M}^2_{-1/3} = \begin{pmatrix} \bar{M}^2_{S_0^L} & g_{S_0}^{(LR)} v^2 & h_{S_0}^L v & \kappa_S^{(L)} v^2 \\ \cdot & \bar{M}^2_{S_0^R} & h_{S_0}^R v & \kappa_S^{(R)} v^2 \\ \cdot & \cdot & \bar{M}^2_{\tilde{S}_{1/2}} & h_{S_1} v \\ \cdot & \cdot & \cdot & \bar{M}^2_{S_1} \end{pmatrix}, \quad (3)$$

$$\mathcal{M}^2_{-2/3} = \begin{pmatrix} \bar{M}^2_{\tilde{S}_{1/2}} & Y_{S_{1/2}}^L v^2 & Y_{S_{1/2}}^R v^2 & \sqrt{2} h_{S_1} v \\ \cdot & \bar{M}^2_{S_{1/2}^L} & g_{S_{1/2}}^{(LR)} v^2 & 0 \\ \cdot & \cdot & \bar{M}^2_{S_{1/2}^R} & 0 \\ \cdot & \cdot & \cdot & \bar{M}^2_{S_1} \end{pmatrix}, \quad (4)$$

$$\mathcal{M}^2_{-4/3} = \begin{pmatrix} \bar{M}^2_{\tilde{S}_0} & \sqrt{2} Y_{S_1} v^2 \\ \cdot & \bar{M}^2_{S_1} \end{pmatrix}, \quad (5)$$

and

$$\mathcal{M}^2_{-5/3} = \begin{pmatrix} \bar{M}^2_{S_{1/2}^L} & -g_{S_{1/2}}^{(LR)} v^2 \\ \cdot & \bar{M}^2_{S_{1/2}^R} \end{pmatrix}. \quad (6)$$

Here $\bar{M}_\Phi^2 = M_\Phi^2 - g_\Phi v^2$ and only the elements above the diagonal have been written since the matrices are symmetric. v is the SM Higgs vacuum expectation value, $v^2 = (2\sqrt{2} G_F)^{-1}$. The mass eigenstate basis is defined as

$$(\hat{S}_Q)_i = R_{ij}^Q (S_Q)_j, \quad (7)$$

where R^Q is a rotation matrix. The diagonal squared-mass matrices are found in the usual way:

$$(\mathcal{M}_Q^2)_{\text{diag}} = R^Q \mathcal{M}_Q^2 (R^Q)^T. \quad (8)$$

Phenomenological implications of the LQ interactions given in Eqs. (1) and (2) have to be derived in terms of the mass eigenstates. For the LQs with charge $Q = -4/3$ and $Q = -5/3$, simple analytical expressions for the eigenvalues and rotation angle can be found. These are given by

$$M_{1,2}^2 = \frac{1}{2}(M_{11}^2 + M_{22}^2 - \sqrt{4M_{12}^4 + (M_{11}^2 - M_{22}^2)^2}), \quad (9)$$

and

$$\tan 2\theta_{12} = \frac{2M_{12}^2}{M_{11}^2 - M_{22}^2}. \quad (10)$$

Here, M_{11}^2, M_{22}^2, and M_{12}^2 stand symbolically for the corresponding entries in the mass matrices Eqs. (5) and (6). For LQs of charge $Q = -1/3, -2/3$ we will diagonalize the mass matrices numerically below. However, the following approximate expressions are useful for an analytical estimate of parameters. The rotation matrices which relate the interaction and mass eigenstates can be parametrized by six rotation angles, namely,

$$R^Q = R^Q(\theta_{34}) R^Q(\theta_{24}) R^Q(\theta_{14}) R^Q(\theta_{23}) R^Q(\theta_{13}) R^Q(\theta_{12}). \quad (11)$$

In the limit where the off-diagonal entries in the mass matrices Eqs. (3) and (4) are smaller than the difference between the corresponding diagonal ones, it is possible to find approximate expressions for the rotation angles also in this more complicated case. As discussed in the next section, for the neutrino masses the most relevant angles are $\theta_{34}^{Q=2/3}$, $\theta_{34}^{Q=1/3}$, and $\theta_{13}^{Q=1/3}$. For the angles in the $Q = 1/3$ case one can use Eq. (10) as an estimate, with obvious replacements of indices. For the angle $\theta_{34}^{Q=2/3}$, however, since the relevant $M_{34}^2 = 0$ in the mass basis, a more complicated expression results:

$$\theta_{34}^{Q=2/3} \simeq -\frac{\sqrt{2} Y_{S_{1/2}}^R h_{S_1} v^3}{(\bar{M}^2_{S_{1/2}^R} - \bar{M}^2_{\tilde{S}_{1/2}})(\bar{M}^2_{S_1} - \bar{M}^2_{S_{1/2}^R})}. \quad (12)$$

Equation (12) is exact in the limit $Y_{S_{1/2}}^L = g_{S_{1/2}}^{(LR)} = 0$. It remains a reasonable (factor-of-two) estimate as long as $Y_{S_{1/2}}^L, g_{S_{1/2}}^{(LR)} \leq Y_{S_{1/2}}^R h_{S_1}/v$, and all $Q = 2/3$ LQ-mixing angles are small numbers.

III. NEUTRINO MASSES FROM LEPTOQUARK LOOPS

A. Analytical formulas

LQ-$q\nu$ Yukawa interactions can be derived directly from the Lagrangian (1). In the interaction eigenstate basis, they have the following form:

$$\mathcal{L}_{\text{LQ}-u\nu} = \lambda_{S_{1/2}}^R \bar{u} P_L \nu (S_{1/2}^R)^\dagger_{-2/3} + \lambda_{S_1}^L \bar{u}^c P_L \nu (S_1)^\dagger_{-2/3} + \text{H.c.} \quad (13)$$

and

$$\mathcal{L}_{\text{LQ}-d\nu} = \lambda_{\tilde{S}_{1/2}}^R \bar{d} P_L \nu (\tilde{S}_{1/2}^R)^\dagger_{-1/3} - \lambda_{S_0}^L \bar{d}^c P_L \nu (S_0^L)^\dagger_{1/3} + \lambda_{S_1}^L \bar{d}^c P_L \nu (S_1)^\dagger_{1/3} + \text{H.c.} \quad (14)$$

Rotating to the mass eigenstate basis, the nontrivial mixing among LQs from different $SU(2)_L$ multiplets leads to neutrino Majorana masses at 1-loop order as shown in Fig. 1. A straightforward calculation of the Majorana

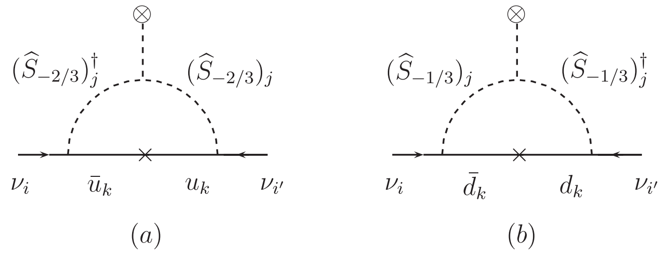

FIG. 1. Feynman diagrams for Majorana neutrino masses. Diagram (a) [(b)] gives contributions to the neutrino mass matrix from u-type [d-type] quark loops.

neutrino mass matrix from these diagrams gives

$$\mathcal{M}_\nu = \mathcal{M}_\nu^{\rm up} + \mathcal{M}_\nu^{\rm down} \tag{15}$$

where the matrix $\mathcal{M}_\nu^{\rm up}$ from diagram (a) reads

$$(\mathcal{M}_\nu^{\rm up})_{ii'} = \frac{3}{16\pi^2} \sum_{\substack{j=1\cdots 4\\ k=u,c,t}} m_k B_0(0, m_k^2, m_{S_j}^2) R_{j3}^{2/3} R_{j4}^{2/3} \times [(\lambda_{S_{1/2}}^R)_{ik}(\lambda_{S_1}^L)_{i'k} + (\lambda_{S_{1/2}}^R)_{i'k}(\lambda_{S_1}^L)_{ik}]. \tag{16}$$

Here $R^{2/3}$ is the rotation matrix that diagonalizes the mass matrix of $Q = -2/3$ LQs, Eq. (4), and $B_0(0, m_k^2, m_{S_j}^2)$ is a Passarino-Veltman function [39]. The matrix $\mathcal{M}_\nu^{\rm down}$ from diagram (b) can be written as

$$(\mathcal{M}_\nu^{\rm down})_{ii'} = \frac{3}{16\pi^2} \sum_{\substack{j=1\cdots 4\\ k=d,s,b}} m_k B_0(0, m_k^2, m_{S_j}^2) R_{j3}^{1/3} \times \{R_{j4}^{1/3}[(\lambda_{\tilde{S}_{1/2}}^R)_{ik}(\lambda_{S_1}^L)_{i'k} + (\lambda_{\tilde{S}_{1/2}}^R)_{i'k}(\lambda_{S_1}^L)_{ik}] + R_{j1}^{1/3}[(\lambda_{\tilde{S}_{1/2}}^R)_{ik}(\lambda_{S_0}^L)_{i'k} + (\lambda_{\tilde{S}_{1/2}}^R)_{i'k}(\lambda_{S_0}^L)_{ik}]\}. \tag{17}$$

Here $R^{1/3}$ is the rotation matrix that diagonalizes the mass matrix of the $Q = -1/3$ LQs given in Eq. (3). Note, that in the limit of unmixed LQs, i.e., $R_{ij} = \delta_{ij}$, the neutrino mass matrix vanishes.

The Passarino-Veltman function B_0 contains a finite and an infinite part. However, since the LQ model does not have a neutrino mass at tree-level there are no counter terms, which allow absorption of infinities. The infinite parts of the B_0 functions therefore must cancel among the different contributions in Eq. (16) and (17). Using the parametrization of the LQ rotation matrices given in Eq. (11), we have checked that this is indeed the case. The resulting formula can be expressed as a sum of *differences* of B_0 functions only, thus canceling all infinities. Since the coefficients in these formulas are rather lengthy (and of little use), we will not give them explicitly.

Diagonalizing Eq. (15) gives the neutrino masses and mixing angles,

$$U^T \mathcal{M}_\nu U = \mathcal{M}_\nu^{\rm diag}. \tag{18}$$

In standard parametrization U is written as

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{19}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ and δ is a CP-violating phase. Since we will consider only real parameters, $\delta = 0$, π and we have not written any Majorana phases in Eq. (19).

In general, the neutrino mass matrix receives contributions from diagrams involving up-type (u-loops) and down-type (d-loops) quarks. In order to find the eigensystem of Eq. (15) one has to solve a cubic equation. However, much simpler analytical formulas can be derived, if one particular loop dominates over all others. For example, in the limit where only the top loop contributes to $\mathcal{M}_\nu$ one finds $\mathrm{Det}[\mathcal{M}_\nu] = 0$, i.e., one of the three eigenvalues of the mass matrix goes to zero. Note that in this limit the model can produce only a (normal) hierarchical neutrino spectrum.

Analytical expressions for the two nonzero neutrino masses can be found easily in the limit $\mathrm{Det}[\mathcal{M}_\nu] = 0$. It is useful to define two vectors in parameter space,

$$\begin{aligned} \mathbf{R} &= [(\lambda_I^R)_1, (\lambda_I^R)_2, (\lambda_I^R)_3], \\ \mathbf{L} &= [(\lambda_{I'}^L)_1, (\lambda_{I'}^L)_2, (\lambda_{I'}^L)_3]. \end{aligned} \tag{20}$$

Here, $(\lambda_{I,I'}^{R,L})_j = (\lambda_{I,I'}^{R,L})_{jk}$, with j being the leptonic index, whereas we have suppressed for brevity the hadronic index k. The indices I and I' stand symbolically for $I = S_{1/2}$ and $I' = S_1$ if the top loop dominates, or $I = \tilde{S}_{1/2}$ and $I' = S_1$ or $I' = S_0$, if one of the bottom loops dominates. In terms of these vectors the two nonzero neutrino masses are given by

$$m_{\nu_{2,3}} = \mathcal{F}(|\mathbf{R}\cdot\mathbf{L}| \mp |\mathbf{R}||\mathbf{L}|), \tag{21}$$

where $\mathcal{F}$ is given by

$$\mathcal{F} = \frac{3}{16\pi^2} \sum_{j=1\cdots 4} m_k B_0(0, m_k^2, m_{S_j}^2) R_{j3}^Q R_{js}^Q, \tag{22}$$

$Q = 1/3, 2/3$ and $s = 1, 4$, depending on which contribution to $\mathcal{M}_\nu$ is most important. The ratio between the solar and the atmospheric scale is thus simply given by

$$R \equiv \frac{\Delta m_{12}^2}{\Delta m_{23}^2} \simeq \left(\frac{|\mathbf{R}\cdot\mathbf{L}| - |\mathbf{R}||\mathbf{L}|}{|\mathbf{R}\cdot\mathbf{L}| + |\mathbf{R}||\mathbf{L}|}\right)^2. \tag{23}$$

Note that R is independent of $\mathcal{F}$, i.e., independent of LQ masses and mixings. Relations among neutrino mixing angles and the Yukawa couplings can be found by using the eigenvalue equation for the massless neutrino [30,31],

$$\mathcal{M}_\nu v_0 = 0 \tag{24}$$

where the eigenvector v_0 is given by

$$v_0^T = \frac{(1, -\epsilon, \epsilon')}{\sqrt{\epsilon^2 + \epsilon'^2 + 1}}. \tag{25}$$

Solving Eq. (24) yields the result

$$\epsilon = \frac{m_{12}m_{33} - m_{13}m_{23}}{m_{22}m_{33} - m_{23}^2}, \qquad \epsilon' = \frac{m_{12}m_{23} - m_{13}m_{22}}{m_{22}m_{33} - m_{23}^2}, \tag{26}$$

where m_{ij} are the entries of the neutrino mass matrix $\mathcal{M}_\nu$. Interestingly, Eq. (26) can be expressed in terms of neutrino angles only. For a normal hierarchical spectrum, i.e., $m_{\nu_{1,2,3}} \simeq (0, m, M)$, where M (m) stands for the atmospheric (solar) mass scale, one obtains

$$\epsilon = \tan\theta_{12}\frac{\cos\theta_{23}}{\cos\theta_{13}} + \tan\theta_{13}\sin\theta_{23}, \tag{27}$$

$$\epsilon' = \tan\theta_{12}\frac{\sin\theta_{23}}{\cos\theta_{13}} - \tan\theta_{13}\cos\theta_{23}. \tag{28}$$

On the other hand, the expressions for ϵ and ϵ' in Eq. (26) depend on the entries in the neutrino mass matrix which are determined by the LQ Yukawa couplings,

$$\epsilon = \frac{(\lambda_I^R)_3(\lambda_{I'}^L)_1 - (\lambda_I^R)_1(\lambda_{I'}^L)_3}{(\lambda_I^R)_3(\lambda_{I'}^L)_2 - (\lambda_I^R)_2(\lambda_{I'}^L)_3}, \tag{29}$$

$$\epsilon' = \frac{(\lambda_I^R)_2(\lambda_{I'}^L)_1 - (\lambda_I^R)_1(\lambda_{I'}^L)_2}{(\lambda_I^R)_3(\lambda_{I'}^L)_2 - (\lambda_I^R)_2(\lambda_{I'}^L)_3}. \tag{30}$$

The above equations allow us to relate the Yukawa couplings directly to the measured neutrino angles. Note also, that current neutrino data require both, ϵ and ϵ', to be nonzero.

B. Neutrino data and parameter estimates

Before discussing the constraints on LQ parameter space imposed by neutrino physics, let us briefly recall that from neutrino oscillation experiments two neutrino mass squared differences and two neutrino angles are by now known quite precisely [40]. These are the atmospheric neutrino mass, $\Delta m^2_{\rm Atm} = (2.0\text{–}3.2)\ [10^{-3}\ {\rm eV}^2]$, and angle, $\sin^2\theta_{\rm Atm} = (0.34\text{–}0.68)$, as well as the solar neutrino mass $\Delta m^2_\odot = (7.1\text{–}8.9)\ [10^{-5}\ {\rm eV}^2]$, and angle, $\sin^2\theta_\odot = (0.24\text{–}0.40)$, all numbers at 3σ c.l. For the remaining neutrino angle, the so-called Chooz [41] or reactor neutrino angle θ_R, a global fit to all neutrino data [40] currently gives a limit of $\sin^2\theta_R \leq 0.04$ at 3σ c.l.

Neutrino oscillation experiments have no sensitivity on the absolute scale of neutrino masses, but the atmospheric data requires that at least one neutrino has a mass larger than $M \equiv m_\nu^{\rm Atm} \gtrsim 50$ meV. The minimal size of LQ Yukawa couplings and LQ-mixing, required to explain such a neutrino mass, can be estimated from Eq. (21). Parametrizing the rotation matrices as in Eq. (11) we can estimate $\mathcal{F}$ as

$$\mathcal{F} \simeq \frac{3}{16\pi^2} m_k \sin(2\theta_{3s})\Delta B_{3s}. \tag{31}$$

Here, $\sin(2\theta_{3s})$ stands symbolically for the largest LQ-mixing angle, and $\Delta B_{ij} = B_0(0, m_k^2, m_{S_i}^2) - B_0(0, m_k^2, m_{S_j}^2)$. The finite part B_0^f of B_0 is given by

$$B_0^f(0, m_k^2, m_{S_j}^2) = \frac{m_k^2\log(m_k^2) - m_{S_j}^2\log(m_{S_j}^2)}{m_k^2 - m_{S_j}^2}. \tag{32}$$

The maximum allowed value of $|\Delta B_{ij}|$ for $m_{\rm LQ} \leq 1.5$ TeV is $|\Delta B_{ij}| \simeq 3$ (3.5) for $m_k = m_t$ ($m_k = 0$). With the current central values for the quark masses [42] we then find the maximal value(s) of $\mathcal{F}$ as

$$\mathcal{F}^{\rm max} \simeq [4.9{:}0.14{:}4.1\times 10^{-2}{:}3\times 10^{-3}{:}1.7\times 10^{-4}{:}8 \times 10^{-5}]\ {\rm GeV}, \qquad \text{for } t{:}b{:}c{:}s{:}d{:}u \tag{33}$$

for maximal LQ-mixing, i.e., $\sin(2\theta_{3s}) = 1/2$. For this value of $\mathcal{F}$ the minimum values for the Yukawa couplings required to explain the atmospheric mass scale are then very roughly given by

$$\begin{aligned} (\lambda^R_{S_{1/2}})_{it}(\lambda^L_{S_1})_{i't} &\gtrsim 5.1\times 10^{-12}, \\ (\lambda^R_{\tilde{S}_{1/2}})_{ib}(\lambda^L_{S_{0,1}})_{i'b} &\gtrsim 1.8\times 10^{-10}, \\ (\lambda^R_{S_{1/2}})_{ic}(\lambda^L_{S_1})_{i'c} &\gtrsim 6.0\times 10^{-10}, \\ (\lambda^R_{\tilde{S}_{1/2}})_{is}(\lambda^L_{S_{0,1}})_{i's} &\gtrsim 8.0\times 10^{-9}, \\ (\lambda^R_{\tilde{S}_{1/2}})_{id}(\lambda^L_{S_{0,1}})_{i'd} &\gtrsim 1.5\times 10^{-7}, \\ (\lambda^R_{S_{1/2}})_{iu}(\lambda^L_{S_1})_{i'u} &\gtrsim 3.0\times 10^{-7}. \end{aligned} \tag{34}$$

Obviously, unless the λ_{ik} follow a hierarchy inversely proportional to the SM quark masses, third generation quark loops give the by far largest contribution to the neutrino mass matrix.

We should compare the minimal values of Eq. (34) with the constraints coming from low-energy phenomenology. The most stringent upper bounds for the first generation Yukawa couplings ($(\lambda^R_{S_{1/2}})_{i1}$ and $(\lambda^L_{S_1})_{i1}$) are currently found from the upper limit on the lepton flavor violating process $\mu{\rm Ti} \to e{\rm Ti}$ [11,43]:

$$\begin{aligned} (\lambda^R_{S_{1/2}})_{11}(\lambda^R_{S_{1/2}})_{21} &< 2.6\times 10^{-7}\left(\frac{m_{S_j}}{100\ {\rm GeV}}\right)^2, \\ (\lambda^L_{S_1})_{11}(\lambda^L_{S_1})_{21} &< 1.7\times 10^{-7}\left(\frac{m_{S_j}}{100\ {\rm GeV}}\right)^2. \end{aligned} \tag{35}$$

Here j labels the corresponding mass of the LQ eigenstate. Upper bounds for the second (and third) quark generation couplings come from the charged lepton flavor violating decay $\mu \to e\gamma$ and are given by

$$(\lambda^R_{S_{1/2}})_{12(13)}(\lambda^R_{S_{1/2}})_{22(23)} < 1.8\times 10^{-5}\left(\frac{m_{S_j}}{100\ \text{GeV}}\right)^2, \qquad (36)$$
$$(\lambda^L_{S_1})_{12(13)}(\lambda^L_{S_1})_{22(23)} < 1.8\times 10^{-5}\left(\frac{m_{S_j}}{100\ \text{GeV}}\right)^2.$$

Here, we have updated [11] with the current experimental upper limit on $\text{Br}(\mu \to e\gamma)$ [42].

Although Eq. (35) constrains a different combination of left- and right-LQ couplings than Eq. (34) we conclude that, barring cases where some fine-tuned cancellation between different LQ contributions occur, we expect that first generation quark loops cannot explain current neutrino data. Second and third generation LQ-loops, on the other hand, could both produce the observed neutrino masses, consistent with all phenomenological constraints. However, considering the hierarchy in $m_c/m_t \sim 8\times 10^{-3}$ and $m_s/m_b \sim 0.02$, from now on we will concentrate on third (quark) generation LQs. Note that, comparing Eq. (34) with Eq. (36) one finds that the atmospheric mass scale can be generated consistent with low-energy constraints for LQ-mixing as small as 10^{-6} (10^{-5}) in case of top-loops (bottom-loops). These numbers are significantly smaller than constraints derived from other low-energy processes [34,35].

The observed large mixing angles in the neutrino sector require certain ratios of Yukawa couplings to be nonzero. This can be most easily understood as follows. One can use Eqs. (18) and (19) to invert the problem and calculate the neutrino mass matrix in the "flavor basis" (in the basis where the charged lepton mass matrix is diagonal). The resulting $\mathcal{M}_\nu$ in the general case is a complicated function of the eigenvalues and mixing angles. However, as first observed in [44], the so-called tribimaximal mixing pattern,

$$U^{\text{HPS}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \qquad (37)$$

is a good first-order approximation to the observed neutrino angles. In case of hierarchical neutrinos $\mathcal{M}^{\text{diag}}_\nu = (0, m, M)$ it leads to

$$\mathcal{M}^{\text{HPS}}_\nu = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & M & -M \\ 0 & -M & M \end{pmatrix} + \frac{1}{3}\begin{pmatrix} m & m & m \\ m & m & m \\ m & m & m \end{pmatrix}. \qquad (38)$$

Comparing Eq. (38) with the index structure of Eq. (16) and (17), one expects that

$$(\lambda^L_I)_1(\lambda^R_I)_3 + (\lambda^L_I)_3(\lambda^R_I)_1 \ll (\lambda^L_I)_2(\lambda^R_I)_3 + (\lambda^L_I)_3(\lambda^R_I)_2, \qquad (39)$$

$$(\lambda^L_I)_2(\lambda^R_I)_3 + (\lambda^L_I)_3(\lambda^R_I)_2 \gtrsim (\lambda^L_I)_2(\lambda^R_I)_2 - (\lambda^L_I)_3(\lambda^R_I)_3 \qquad (40)$$

for the couplings which give the largest contribution to $\mathcal{M}_\nu$. Equation (39) is essentially due to smallness of the reactor angle, while Eq. (40) follows from the observed near-maximality of the atmospheric angle. Note that, if more than one loop contributes to $\mathcal{M}_\nu$ of Eq. (15), $m_{\nu_1} \neq 0$, but the "large" off-diagonal entry in the (2,3) element of $\mathcal{M}_\nu$ always requires $(\lambda^{L/R}_I)_2 \sim (\lambda^{L/R}_I)_3$, for at least one LQ state. Finally, it should also be mentioned that the smallness of solar versus atmospheric splitting requires that the vectors **R** and **L**, defined in Eq. (20), are nearly aligned for *all* vectors contributing to $\mathcal{M}_\nu$, compare Eq. (23).

There are three different contributions to the neutrino mass matrix; see Eqs. (16) and (17). The top loop is proportional to $\theta^{Q=2/3}_{34} \sim Y^R_{S_{1/2}} h_{S_1}$, while the bottom loop is either proportional to $\theta^{Q=1/3}_{34} \sim h_{S_1}$ or to $\theta^{Q=1/3}_{13} \sim h^L_{S_0}$. Lacking a theoretical ansatz for these parameters, it is not possible to predict which of these gives the dominant contribution to the neutrino mass matrix. However, since $m_b/m_t \sim 2\%$, the top loop will be most important, if all LQ-mixing angles (and Yukawa couplings) are of similar size. We will refer to this case, $\mathcal{M}_\nu = \mathcal{M}^t_\nu$, as scenario I. $\theta^{Q=2/3}_{34}$, on the other hand, can be much smaller than the corresponding angles in the down-type loops in those parts of parameter space where all relevant off-diagonal entries in the LQ mass matrices are small. In this case $\mathcal{M}_\nu \simeq \mathcal{M}^b_\nu$, and we will refer to this situation as scenario II (II.a: if $h^L_{S_0} \ll h_{S_1}/v$ and II.b: if $h_{S_1}/v \ll h^L_{S_0}$).

IV. LEPTOQUARKS: ACCELERATOR SIGNALS RELATED TO NEUTRINO PHYSICS

LQs, once produced, will decay almost instantaneously. There are two different sets of possible final states. In the current model, apart from the usual decays into a quark and a lepton there are also vector ($W^\pm$ and Z^0) and scalar (h^0) final states, if kinematically allowed. We will discuss first the fermionic decays.

A. Fermionic LQ decays

Fermionic decays of the LQ mass eigenstates are dictated by the Yukawa interactions given in the Lagrangian (1). Possible final states can be either $\bar{\ell}_i u_k$, $\bar{\ell}_i d_k$, $\bar{\nu}_i u_k$, or $\bar{\nu}_i d_k$. Most interesting, from the phenomenological point of view, are final states with charged leptons, since these allow one to tag the flavor. Partial widths for two-body final states can be calculated in a straightforward manner. For charged lepton final states these are given by

$$\Gamma[(\hat{S}_{-1/3})_j \to \ell_i \bar{t}^c] = n^j_{-1/3}\{[(\lambda^L_{S_1})_{i3} R^{1/3}_{j4}]^2 + [(\lambda^L_{S_0})_{i3} R^{1/3}_{j1}]^2 + [(\lambda^R_{S_0})_{i3} R^{1/3}_{j2}]^2\}, \qquad (41)$$

$$\Gamma[(\hat{S}_{-2/3})_j \to \ell_i \bar{b}] = n^j_{-2/3}\{[(\lambda^R_{\tilde{S}_{1/2}})_{i3} R^{2/3}_{j1}]^2 + [(\lambda^L_{S_{1/2}})_{i3} R^{2/3}_{j2}]^2\}, \qquad (42)$$

$$\Gamma[(\hat{S}_{-4/3})_j \to \ell_i \bar{b}^c] = n^j_{-4/3}\{[(\lambda^R_{\tilde{S}_0})_{i3} R^{4/3}_{j1}]^2 + [(\lambda^L_{S_1})_{i3} R^{4/3}_{j2}]^2\}, \quad (43)$$

$$\Gamma[(\hat{S}_{-5/3})_j \to \ell_i \bar{t}] = n^j_{-5/3}\{[(\lambda^L_{S_{1/2}})_{i3} R^{5/3}_{j1}]^2 + [(\lambda^R_{S_{1/2}})_{i3} R^{5/3}_{j2}]^2\}. \quad (44)$$

Here, n^j_Q is an overall constant given by

$$n^j_Q = \frac{3}{16\pi m_{S_j}}\left[1 - \frac{m_q^2 + m_l^2}{m^2_{S_j}}\right]\lambda^{1/2}(m^2_{S_j}, m_q^2, m_l^2) \quad (45)$$

with $\lambda^{1/2}(a, b, c)$ the usual phase space factor, $\lambda(a, b, c) = (a + b - c)^2 - 4ab$, and m_{S_j}, m_q, and m_l the corresponding LQ, quark, and lepton masses. Absolute values for the LQ widths cannot be predicted. However, minimal (maximal) values can be estimated from the atmospheric neutrino mass scale (low-energy bounds). Putting all parameters to their extreme values fermionic widths could be as small (large) as $\mathcal{O}$(eV) [$\mathcal{O}$(MeV)].

In the above equation, we have written only the partial widths to top and bottom quarks. Formulas for the lighter generations can be found with straightforward replacements of indices. However, since the widths, Eqs. (41)–(44), are not suppressed by quark masses, our assumption that 3rd generation quark loops give the dominant contribution to $\mathcal{M}_\nu$ can in principle be checked experimentally. For example if $\mathrm{Br}(\hat{S}_{-5/3} \to \sum_i t + l_i)/\mathrm{Br}(\hat{S}_{-5/3} \to \sum_i j + l_i)$ and $\mathrm{Br}(\hat{S}_{-4/3} \to \sum_i b + l_i)/\mathrm{Br}(\hat{S}_{-4/3} \to \sum_i j + l_i)$, where j stands for any non−b jet, are larger than m_c/m_t, charm (and up) loops are guaranteed to be subdominant. Similar tests can be devised for the case of bottom loops.

Since we do not have a theory for the Yukawas, absolute values for the branching ratios of LQ decays can not be predicted, only certain ratios of branching ratios are fixed by neutrino data, as we will show below. Nevertheless, Fig. 2 shows some examples for the decay of the lightest $Q = 4/3$ LQ state decaying to $b + \tau$ versus $(\lambda^L_{S_1})_{33}$, for three different choices of $(\lambda^L_{S_1})_{13} = (\lambda^L_{S_1})_{23}$ and $(\bar{\lambda}^L_{S_1}) = 0$ (left figure) and versus $(\bar{\lambda}^L_{S_1})$ for different choices of $(\lambda_{S_1})_{23}$, $(\lambda_{S_1})_{33}$ (to the right). Here, $(\bar{\lambda}^L_{S_1})$ represents the average coupling of the lightest $Q = 4/3$ LQ state to the first two generations of quarks. The range of variation between $(\lambda^L_{S_1})_{i3}$ for different i is motivated by the uncertainty in the determination of neutrino angles. A very similar behavior to the one shown in the figure is found also for the decays of $Q = 5/3$ LQ states. Observing $\mathrm{Br}(\hat{S}_{4/3} \to b + l) \gtrsim \frac{m_s}{m_b}$ (and for the decays of $\mathrm{Br}(\hat{S}_{5/3} \to t + l) \gtrsim \frac{m_c}{m_t}$) would serve as a demonstration that 3rd generation quark loops give the dominant contribution to $\mathcal{M}_\nu$.

Note that if the mixing between different LQs is small, as is generally expected, the decays of some of the LQ states $\hat{S}_Q$ are controlled by *the same Yukawa couplings that determine the nontrivial structure of the neutrino mass matrix*. This observation forms the basis of the different decay pattern predictions discussed below. However, one complication arises from the fact that we cannot predict if the decays of the lightest or one of the heavier of the LQ states is dictated by the Yukawas fixed by neutrino physics. Again, in the limit of small LQ-mixing, this question can be decided experimentally, in principle. Consider, for example, scenario I, $\mathcal{M}_\nu \simeq \mathcal{M}^t_\nu$. The decays controlled by neutrino physics are those governed by $\lambda^R_{S_{1/2}}$ and $\lambda^L_{S_1}$. These states couple mainly to lepton doublets. Their components have the same diagonal entries in the LQ mass matrices; we expect them to have similar masses. These states should have very roughly $\Gamma[(\hat{S}_{-5/3}) \to \sum \ell_i \bar{t}] \sim \Gamma[(\hat{S}_{-2/3}) \to \sum \nu_i \bar{t}]$ and $\Gamma[(\hat{S}_{-4/3}) \to \sum \ell_i b] \sim \Gamma[(\hat{S}_{-1/3}) \to \sum \nu_i b]$. The other $Q = 4/3$, $5/3$ mass eigenstates mainly couple to singlet leptons, i.e., these states do not decay to neutrinos. In what follows below, we will always assume that the small mixing limit is realized and the LQ states relevant for the experimental cross-checks can be identified.

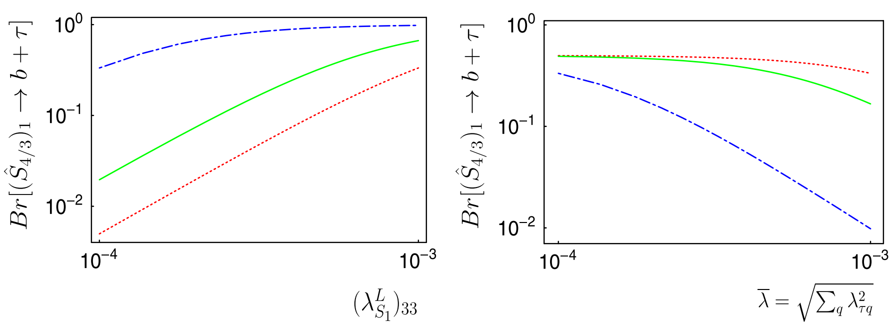

FIG. 2 (color online). Decay branching ratios for the lightest $Q = 4/3$ LQ. To the left: $\mathrm{Br}(\hat{S}_{4/3} \to b + \tau)$ versus $(\lambda^L_{S_1})_{33}$ for: dot-dashed line $(\lambda^L_{S_1})_{13} = (\lambda^L_{S_1})_{23} = 10^{-4}$, full line $(\lambda^L_{S_1})_{13} = (\lambda^L_{S_1})_{23} = 5 \times 10^{-4}$, and dotted line $(\lambda^L_{S_1})_{13} = (\lambda^L_{S_1})_{23} = 10^{-3}$, with first and second generation quark couplings assumed to be zero. To the right: $\mathrm{Br}(\hat{S}_{4/3} \to b + \tau)$ versus $(\bar{\lambda}^L_{S_1}) = \sqrt{\sum_{q=1}^2 \lambda_{lq}}$ for: dotted line $(\lambda_{S_1})_{23} = (\lambda_{S_1})_{33} = 10^{-3}$, full line $(\lambda_{S_1})_{23} = (\lambda_{S_1})_{33} = 5 \times 10^{-4}$, and dot-dashed line $(\lambda_{S_1})_{23} = (\lambda_{S_1})_{33} = 10^{-4}$. These plots are only illustrative examples, for a discussion see text.

Combining Eqs. (23) and (40) with the decay rates Eqs. (41)–(44), one can derive some qualitative expectations for some ratios of branching ratios of fermionic LQ decays. In general, the constraint from the large atmospheric angle, plus the smallness of $R = \Delta m^2_{\odot}/\Delta m^2_{\rm Atm}$ can only be fulfilled if there are (at least) two LQ states which have similar branching ratios to muonic and tau final states. At the same time, these LQ states should have less final states with electrons, essentially due to Eq. (39) and the upper limit on the reactor angle.

Much more detailed predictions for fermionic decays of LQs can be made in the explicit scenarios defined in the last section. We will first discuss in some detail the results for scenario I, $\mathcal{M}_\nu = \mathcal{M}^t_\nu$. For all figures presented in the following we have scanned the Yukawa parameter space randomly, in such a way that all low-energy bounds are obeyed. We then numerically diagonalized the resulting neutrino mass matrices and checked for consistency with current neutrino oscillation data. Different correlations among ratios of branching ratios with the different pieces of neutrino data are then found.

Figure 3 demonstrates that $\sqrt{\mathrm{Br}^{t\mu}_{5/3}\mathrm{Br}^{b\mu}_{4/3}}/\sqrt{\mathrm{Br}^{t\tau}_{5/3}\mathrm{Br}^{b\tau}_{4/3}}$ is correlated with the atmospheric mixing angle. For the best fit point value $\tan^2\theta_{23} = 1$ one expects $\sqrt{\mathrm{Br}^{t\mu}_{5/3}\mathrm{Br}^{b\mu}_{4/3}} \simeq \sqrt{\mathrm{Br}^{t\tau}_{5/3}\mathrm{Br}^{b\tau}_{4/3}}$. Using the current 3σ range for the atmospheric mixing angle this observable can be predicted to lie within the interval [0.4,4.7].

We have found that there exists an upper bound on the ratio of branching ratios

$$\frac{\sqrt{\mathrm{Br}^{te}_{5/3}\mathrm{Br}^{be}_{4/3}}}{\sqrt{\mathrm{Br}^{t\mu}_{5/3}\mathrm{Br}^{b\mu}_{4/3}} + \sqrt{\mathrm{Br}^{t\tau}_{5/3}\mathrm{Br}^{b\tau}_{4/3}}} \lesssim 9 \times 10^{-2} \qquad (46)$$

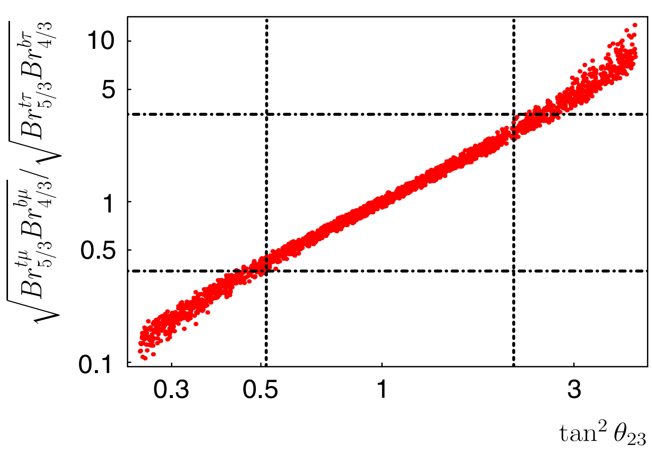

FIG. 3 (color online). Ratio of decay branching ratios $\sqrt{\mathrm{Br}^{t\mu}_{5/3}\mathrm{Br}^{b\mu}_{4/3}}/\sqrt{\mathrm{Br}^{t\tau}_{5/3}\mathrm{Br}^{b\tau}_{4/3}}$ versus $\tan^2\theta_{23}$. Vertical lines indicate the current 3σ range for $\tan^2\theta_{23}$ while horizontal lines determine the predicted range for this observable.

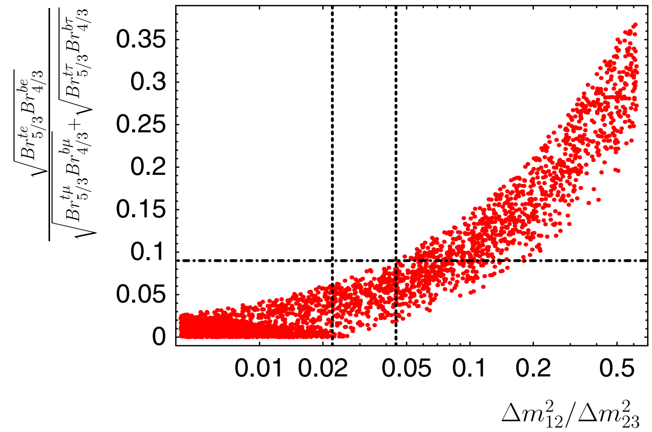

FIG. 4 (color online). Ratio of branching ratios $\sqrt{\mathrm{Br}^{te}_{5/3}\mathrm{Br}^{be}_{4/3}}/(\sqrt{\mathrm{Br}^{t\mu}_{5/3}\mathrm{Br}^{b\mu}_{4/3}} + \sqrt{\mathrm{Br}^{t\tau}_{5/3}\mathrm{Br}^{b\tau}_{4/3}})$ versus R. Vertical lines indicate current 3σ limits on R whereas the horizontal line shows the upper bound for this observable.

which can be derived from the ratio $R = \Delta m^2_{\odot}/\Delta m^2_{\rm Atm}$ as shown in Fig. 4. This bound shows that the product of branching ratios $\mathrm{Br}^{te}_{5/3}\mathrm{Br}^{be}_{4/3}$ is expected to be nearly 2 orders of magnitude smaller than the sum of $\mathrm{Br}^{t\mu}_{5/3}\mathrm{Br}^{b\mu}_{4/3}$ and $\mathrm{Br}^{t\tau}_{5/3}\mathrm{Br}^{b\tau}_{4/3}$.

Individual values for electron final state decay branching ratios are shown in Fig. 5. It can be seen that the smallness of $\mathrm{Br}^{te}_{5/3}\mathrm{Br}^{be}_{4/3}$ can be due to the smallness of either $\mathrm{Br}^{be}_{4/3}$ or $\mathrm{Br}^{te}_{5/3}$. This implies that for one of the two LQ eigenstates ($Q = 4/3$ and $Q = 5/3$) electron final states could be as large as ~20%, but only if the other LQ state shows a very much supressed branching ratio to electrons.

Numerically we have found that there is certain combination of ratios of branching ratios that is correlated with $\sin\theta_R = \sin\theta_{13}$ as shown in Fig. 6. With the current upper

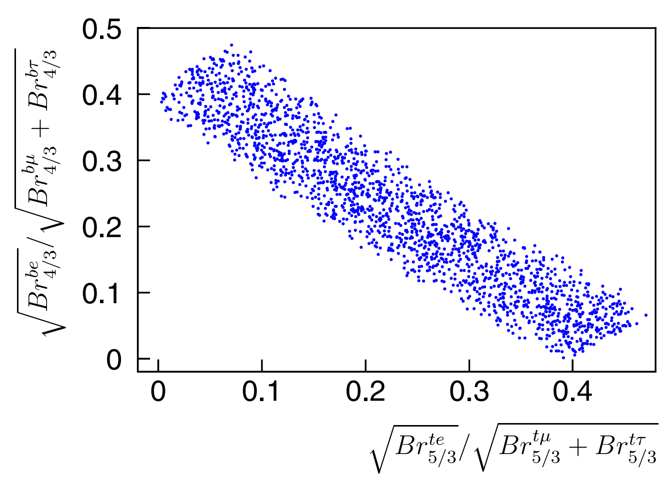

FIG. 5 (color online). Ratio of decay branching ratios $\sqrt{\mathrm{Br}^{be}_{4/3}}/\sqrt{\mathrm{Br}^{b\mu}_{4/3} + Br^{b\tau}_{4/3}}$ versus $\sqrt{\mathrm{Br}^{te}_{5/3}}/\sqrt{\mathrm{Br}^{t\mu}_{5/3} + Br^{t\tau}_{5/3}}$.

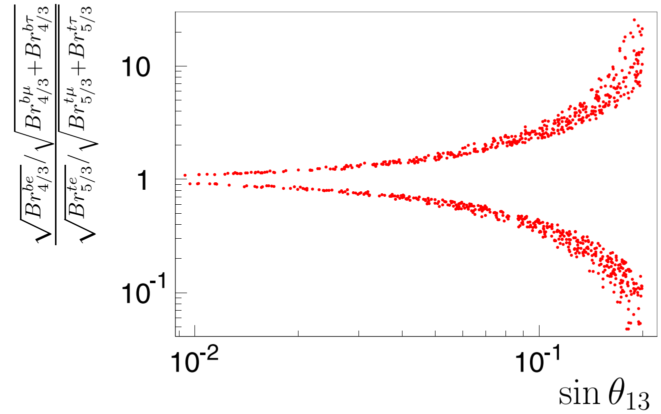

FIG. 6 (color online). Ratio of decay branching ratios $(\sqrt{\mathrm{Br}^{be}_{4/3}}/\sqrt{\mathrm{Br}^{b\mu}_{4/3}+Br^{b\tau}_{4/3}})/(\sqrt{\mathrm{Br}^{te}_{5/3}}/\sqrt{\mathrm{Br}^{t\mu}_{5/3}+Br^{t\tau}_{5/3}})$ versus $\sin\theta_{13}$.

limit on $\sin\theta_R$, this ratio is not very much constrained. However, a future measurement of $\sin\theta_R$, would confine this ratio to lie in a very small, albeit double-valued, interval and thus such a measurement could become a powerful experimental cross-check of the scenario discussed here. Note also that this ratio approaches 1 for small values of $\sin\theta_R$, thus also an improved upper limit on this angle will lead to an interesting constraint.

Finally, from Eq. (23) one expects that

$$R = \frac{\mathrm{Br}'_-}{\mathrm{Br}'_+} \equiv \frac{\sqrt{\sum_{i=e,\mu\tau}\mathrm{Br}^{ti}_{5/3}Br^{bi}_{4/3}} - \sqrt{\sum_{i,j=e,\mu\tau}\mathrm{Br}^{ti}_{5/3}\mathrm{Br}^{bj}_{4/3}}}{\sqrt{\sum_{i=e,\mu\tau}\mathrm{Br}^{ti}_{5/3}\mathrm{Br}^{bi}_{4/3}} + \sqrt{\sum_{i,j=e,\mu\tau}\mathrm{Br}^{ti}_{5/3}\mathrm{Br}^{bj}_{4/3}}}$$
$$\simeq \frac{\sqrt{\sum_{i=\mu\tau}\mathrm{Br}^{ti}_{5/3}\mathrm{Br}^{bi}_{4/3}} - \sqrt{\sum_{i,j=\mu\tau}\mathrm{Br}^{ti}_{5/3}\mathrm{Br}^{bj}_{4/3}}}{\sqrt{\sum_{i=\mu\tau}\mathrm{Br}^{ti}_{5/3}\mathrm{Br}^{bi}_{4/3}} + \sqrt{\sum_{i,j=\mu\tau}\mathrm{Br}^{ti}_{5/3}\mathrm{Br}^{bj}_{4/3}}}. \quad (47)$$

The neglection of electron final states in the 2nd equation above is motivated by Eq. (46). Numerical results are shown in Fig. 7. The spread of the points in the plot gives the precision with which the ratio $\mathrm{Br}_-/\mathrm{Br}_+$ can be predicted, neglecting electron final states and scanning over the allowed ranges of other neutrino physics observables. As demonstrated by Fig. 7 the observable $\mathrm{Br}_-/\mathrm{Br}_+$ is currently expected to lie within the range $[7.5 \times 10^{-3}, 2.9 \times 10^{-2}]$.

All results shown in Figs. 3–7 are based on the assumption that the top loop gives the most important contribution to the neutrino mass matrix. However, very similar results can be obtained if the bottom loop dominates in either scenario II.a or scenario II.b. We will not repeat the discussion in detail here. The results for scenario II.a can be obtained by the replacement of $\hat{S}_{5/3} \rightarrow \hat{S}_{2/3}$ and scenario

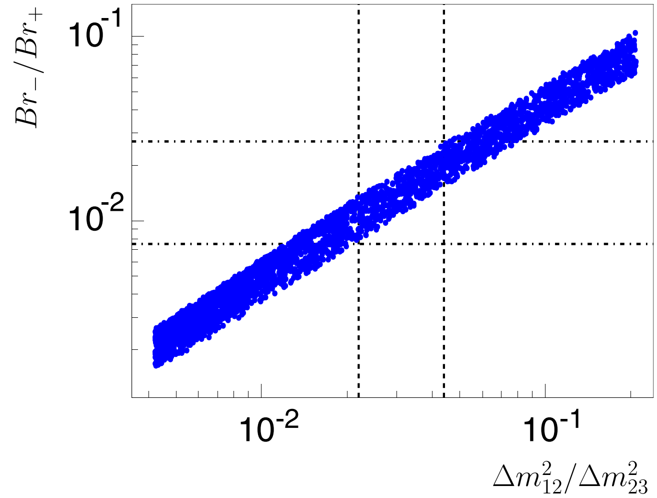

FIG. 7 (color online). Ratio of decay branching ratio $\mathrm{Br}_-/\mathrm{Br}_+$ versus R. Vertical lines indicate current 3σ range for R whereas horizontal lines show the predicted range for this observable. The spread of the points in the plot determines the uncertainty with which this ratio can currently be predicted.

II.b by the replacements $\hat{S}_{5/3} \rightarrow \hat{S}_{2/3}$ and $\hat{S}_{4/3} \rightarrow \hat{S}_{1/3}$ in all equations and figures above.

In summary, qualitative expectations for some ratios of branching ratios of fermionic LQ decays can be derived from the requirement that LQ-loops explain neutrino oscillation data. In general, lepton flavor violating decays with similar branching ratios to muonic and tau final states are expected for some specific LQ decays. Sharp predictions for various decay modes can be made, under the reasonable assumption that one LQ loop dominates over all others.

B. Leptoquark decays to Higgs and gauge boson final states

Since the current lower limit on the mass of a standard model like Higgs boson is $m_{h^0} \geq 114.4$ GeV [42], one expects that LQs can decay also to standard model gauge bosons, $W^\pm$ and Z^0, if the Higgs final state is kinematically possible. We will therefore discuss partial decay widths to Higgs, $W^\pm$, and Z^0 final states jointly in this subsection.

In the model discussed here, heavier LQs can decay to lighter LQs plus a standard model Higgs boson, i.e., $(\hat{S}_Q)_j \rightarrow h^0 + (\hat{S}_Q)_i$, due to the interactions given in Eq. (2). Partial decay widths can be written as

$$\Gamma[(\hat{S}_Q)_j \rightarrow h^0 + (\hat{S}_Q)_i] = \frac{1}{16\pi}\tilde{g}^2_Q m_{S_j}\lambda^{1/2}(1, r_{ij}, r_h). \quad (48)$$

Here, the arguments of $\lambda^{1/2}(a, b, c)$ have been defined dimensionless, $r_{ij} \equiv m^2_{S_i}/m^2_{S_j}$ and $r_h \equiv m^2_{h^0}/m^2_{S_j}$. The effective couplings $\tilde{g}_Q$ for the different values of $Q = -1/3$, $-2/3$, $-4/3$, $-5/3$ are defined as

$$\tilde{g}_{-1/3} = \frac{g_{S_0}^{(LR)}}{2}\frac{v}{m_{S_j}}R_{j1}^{1/3}R_{i2}^{1/3} + \frac{h_{S_0}^{(L)}}{\sqrt{2}m_{S_j}}R_{j1}^{1/3}R_{i3}^{1/3} + \frac{\kappa_S^{(L)}}{2}\frac{v}{m_{S_j}}R_{j1}^{1/3}R_{i4}^{1/3} + \frac{h_{S_0}^{(R)}}{\sqrt{2}m_{S_j}}R_{j2}^{1/3}R_{i3}^{1/3} + \frac{\kappa_S^{(R)}}{2}\frac{v}{m_{S_j}}R_{j2}^{1/3}R_{i4}^{1/3} + \frac{h_{S_1}}{\sqrt{2}m_{S_j}}R_{j3}^{1/3}R_{i4}^{1/3}, \quad (49)$$

$$\tilde{g}_{-2/3} = \frac{Y_{S_{1/2}}^L}{2}\frac{v}{m_{S_j}}R_{j1}^{2/3}R_{i2}^{2/3} + \frac{Y_{S_{1/2}}^R}{2}\frac{v}{m_{S_j}}R_{j1}^{2/3}R_{i3}^{2/3} + \frac{h_{S_1}}{m_{S_j}}R_{j1}^{2/3}R_{i4}^{2/3} + \frac{g_{S_{1/2}}^{(LR)}}{2}\frac{v}{m_{S_j}}R_{j2}^{2/3}R_{i3}^{2/3}, \quad (50)$$

$$\tilde{g}_{-4/3} = \frac{Y_{S_1}}{\sqrt{2}}\frac{v}{m_{S_j}}R_{j1}^{4/3}R_{i2}^{4/3}, \quad (51)$$

$$\tilde{g}_{-5/3} = \frac{g_{S_{1/2}}^{(LR)}}{2}\frac{v}{m_{S_j}}R_{j1}^{5/3}R_{i2}^{5/3}. \quad (52)$$

R_{ij}^Q are the rotation matrices, which diagonalize the LQ mass matrices. Note that the above couplings contain the same parameters which induce neutrino masses due to LQ-mixing.

For any given set of LQs of charge Q the couplings with the Z^0 can be written as

$$\frac{ig}{\cos\theta_W}Z^\mu\sum_l(T_3^l - Q\sin^2\theta_W)S_Q^l\overleftrightarrow{\partial}_\mu(S_Q^l)^\dagger. \quad (53)$$

Nondiagonal couplings of the Z^0 gauge boson to different LQ states of the same Q, but different T_3 appear, after rotation to the mass eigenstate basis. The partial decay width can be written as

$$\Gamma[(\hat{S}_Q)_j \to Z^0 + (\hat{S}_Q)_i] = \frac{1}{16\pi}\frac{g^2}{\cos\theta_W^2}\theta_Q^2\frac{M_{S_j}^3}{M_Z^2} \times \lambda^{3/2}(1, r_{ij}, r_Z), \quad (54)$$

where $r_Z = (m_{Z^0}/m_{S_j})^2$ and

$$\theta_{-1/3} = -\frac{1}{2}R_{j3}^{1/3}R_{i3}^{1/3},$$
$$\theta_{-2/3} = -\left(R_{j1}^{2/3}R_{i1}^{2/3} + \frac{3}{2}R_{j4}^{2/3}R_{i4}^{2/3}\right), \quad (55)$$
$$\theta_{-4/3} = -R_{j2}^{4/3}R_{i2}^{4/3}.$$

Note that $Q = -5/3$ LQs do not have any decays to Z^0 bosons, since their couplings to Z^0 are completely diagonal. Closer inspection of Eq. (55) reveals that the decays to Z^0 states can occur only if LQ-mixing (by the same parameters which govern the Higgs final states) is nonzero. Thus, also observation of Z^0 final states gives valuable information about the parameters in Eq. (2).

Heavier LQs can decay to a lighter one and a $W^\pm$ gauge boson, $S_Q \to W + S_{Q'}$, where S_Q and $S_{Q'}$ are members of the same doublet (triplet). Possible decays therefore are

$$(\hat{S}_{-5/3})_j \leftrightarrow W^- + (\hat{S}_{-2/3})_i, \quad (56)$$

$$(\hat{S}_{-2/3})_j \leftrightarrow W^- + (\hat{S}_{-1/3})_i^\dagger, \quad (57)$$

$$(\hat{S}_{-4/3})_j \leftrightarrow W^- + (\hat{S}_{-1/3})_i, \quad (58)$$

where the processes in (56)–(58) come from the decays of the members of the doublet $S_{1/2}$, $\tilde{S}_{1/2}$ and the triplet S_1, respectively, after rotation to the mass eigenstate basis. Note that the process in Eq. (57) can also come from the decay of the $T_3 = 1$ to the $T_3 = 0$ components of the triplet. The decay widths for the processes in (56)–(58) can be written as

$$\Gamma[(\hat{S}_Q)_j \to W^\pm + (\hat{S}_{Q'})_i] = \frac{g^2\theta_Q^2}{32\pi}\frac{m_{S_j}^3}{M_W^2}\lambda^{3/2}(1, r_{ij}, r_W). \quad (59)$$

Here $r_W \equiv M_W^2/m_{S_j}^2$ and the mixing factors are given by

$$\theta_{-5/3} = (R^{2/3})_{i2}(R^{5/3})_{j1} + (R^{2/3})_{i3}(R^{5/3})_{j2}, \quad (60)$$

$$\theta_{-2/3} = (R^{1/3})_{i3}(R^{2/3})_{j1} + \sqrt{2}(R^{1/3})_{i4}(R^{2/3})_{j4}, \quad (61)$$

$$\theta_{-4/3} = \sqrt{2}(R^{1/3})_{i4}(R^{4/3})_{j2}. \quad (62)$$

Our formula Eq. (59) agrees with the one calculated earlier in [45], once LQ-mixing is properly taken into account.

We now turn to a discussion of typical ranges for the branching ratios of bosonic final states. We will first discuss the example of decays of LQs with $Q = 4/3$, assuming the decay to $Q = 1/3$ LQs plus $W^\pm$ is kinematically closed.

Figure 8 shows a set of numerical examples of branching ratios of the heavier of the $Q = 4/3$ LQ mass eigenstate to fermionic, h^0 and Z^0 final states, for some typical choices of parameters; see figure caption. For Yukawa couplings of the order $\bar{\lambda} \sim 10^{-3}$ values of Y_{S_1} as small as $Y_{S_1} \simeq 10^{-2}$ can lead to observable branching ratios into bosonic final states.

While we cannot predict whether fermionic or bosonic final states will dominate, it is interesting to note that the current LQ model makes a definite prediction for the ratio of branching ratios of h^0 and Z^0 final states, if $m_2^{Q=4/3}$ is sufficiently larger than $m_1^{Q=4/3} + m_{h^0}$. This can be understood as follows. If $m_2^{Q=4/3}$ is much larger than $m_1^{Q=4/3}$, m_{h^0}, and m_{Z^0}, one can neglect the phase space correction factors, $\lambda(1, x, y)$, and approximate the $Q = 4/3$ mixing angle by $\theta_{Q=4/3} \simeq \sqrt{2}Y_{S_1}v^2/(m_2^{Q=4/3})^2$. The ratio of the

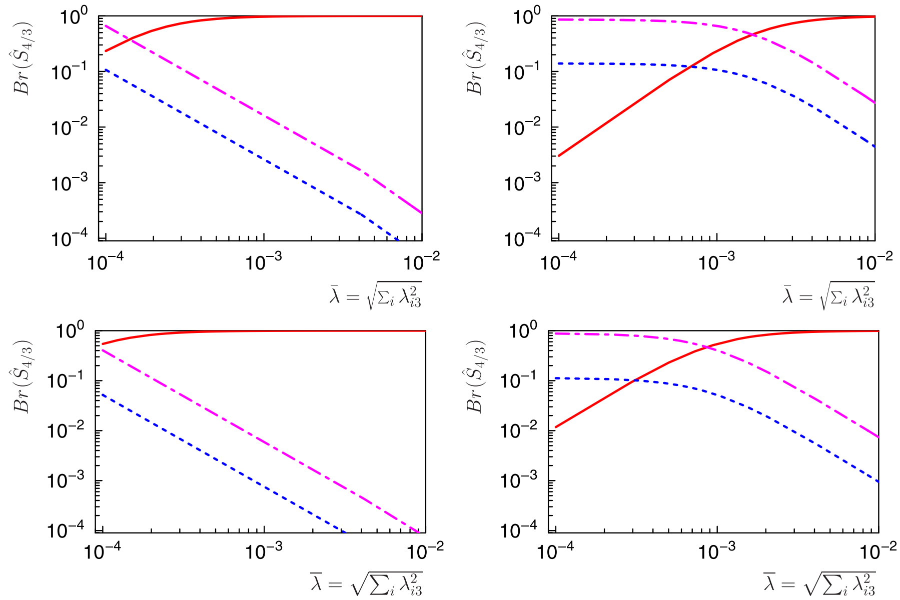

FIG. 8 (color online). Typical values for decay branching ratios for the heavier $Q = 4/3$ LQ, in case $W^{\pm}$ final states are kinematically closed. Branching ratios are plotted versus the "average" Yukawa coupling $\bar{\lambda} \equiv \sqrt{\sum_i \lambda_{i3}^2}$, for different values of Y_{S_1} and $m_2^{Q=4/3}$. Full line: fermionic final states, dashed line Higgs final state, dot-dashed line Z^0 final state. In all figures $m_1^{Q=4/3}$ has been set to $m_1^{Q=4/3} = 250$ GeV and we have chosen $m_{h^0} = 115$ GeV, motivated by the LEP limit. Top left: ($Y_{S_1} = 0.01$, $m_2^{Q=4/3} =$ 400 GeV); top right: ($Y_{S_1} = 0.1$, $m_2^{Q=4/3} = 400$ GeV); bottom left: ($Y_{S_1} = 0.01$, $m_2^{Q=4/3} = 800$ GeV); bottom right: ($Y_{S_1} = 0.1$, $m_2^{Q=4/3} = 800$ GeV).

partial widths to Higgs and Z^0 states is then simply given by

$$\frac{\Gamma[(\hat{S}_{4/3})_2 \to Z^0 + (\hat{S}_{4/3})_1]}{\Gamma[(\hat{S}_{4/3})_2 \to h^0 + (\hat{S}_{4/3})_1]} \simeq 4\frac{g^2}{c_W^2}\frac{v^2}{m_{Z^0}^2} \sim 8, \tag{63}$$

independent of all non-SM parameters. This explains the ratio observed in the numerical examples of Fig. 8 and constitutes a nice consistency test for the LQ model of neutrino masses.

We now turn to $W^{\pm}$ final states. In general, in electroweak symmetry breaking new mass terms for LQs, which are members of the same multiplet, could be generated by some non-SM scalars, potentially introducing large splitting *within* a given multiplet. In this case, LQ decays to $W^{\pm}$ states could occur independent of LQ-mixing between *different* multiplets. Then, since LQ-$W^{\pm}$ decays are of order g^2 they would easily become dominant once kinematically allowed. In the current model, however, mass splitting of LQs within the same multiplet comes only from LQ-mixing; see Eqs. (3)–(6). Thus LQ-$W^{\pm}$ final states should have widths similar to the Z^0 final states discussed above. Consider, for example, the decays of a $Q = 5/3$ LQ. The mass matrix of the $Q = 5/3$ LQs, see Eq. (5), contains the same parameters as a (2-by-2) submatrix of the $Q = 2/3$ mass matrix; compare to Eq. (3). If the other $Q = 2/3$ states are heavier than these states, we can give a similar estimate of the ratio of Higgs and $W^{\pm}$ final states, as has been derived above for Z^0 final states, see Eq. (63). Assuming again $m_2^{Q=5/3}$ being much heavier than final state particles, we find

$$\frac{\Gamma[(\hat{S}_{5/3})_2 \to h^0 + (\hat{S}_{5/3})_1]}{\Gamma[(\hat{S}_{5/3})_2 \to W^{\pm} + (\hat{S}_{2/3})_1]} \simeq \frac{1}{8g^2}\frac{m_W^2}{v^2} \simeq 0.063. \tag{64}$$

In the general situation, however, when all $Q = 2/3$ states are relatively light, the branching ratio to h^0 final states cannot be predicted accurately. Thus, $W^{\pm}$ final states cannot provide an as valuable test for the model as is the case for Z^0 decays.

In summary, heavier LQs will decay to bosonic final states, if kinematically allowed. Since in the current model all these decays are induced by the presence of the LQ-Higgs interaction parameters, observing such decays is an essential test of the LQ model of neutrino mass. Branching ratios for bosonic final states typically fall into the range $\mathcal{O}(10^{-4} - 1)$, for LQ-Higgs couplings of the order $\mathcal{O}(10^{-2} - 1)$. Although we have discussed only the cases $Q = 4/3$ and $Q = 5/3$, bosonic widths of LQs with other electric charges are expected to show a very similar parameter dependence (and therefore similar branching ratios).

V. SUMMARY

LQ fields with baryon number conserving Yukawa interactions can have masses at or near the electroweak scale.

If these LQ fields couple to the SM Higgs, the resulting model generates neutrino masses at the 1-loop level. In this work we have explored the phenomenological consequences of LQs as the origin of the observed neutrino masses for future accelerator experiments, such as the LHC.

Fermionic decays of (some of the) LQ states trace the neutrino angles, i.e., certain ratios of decay branching ratios can be predicted from current neutrino data. In general one expects that those LQs, which give the dominant contribution to the neutrino mass matrix, if (pair) produced at the LHC decay with sizeable flavor violation. For these states there should be a similar number of events with $\tau^{\pm}\mu^{\mp}$ final states, as there are final states with muon and tau pairs. One also expects a smaller number of events of the type $e^{\pm}\mu^{\mp}$ (and $e^{\pm}\tau^{\mp}$), although the details in this case are more involved, as discussed above.

In this context we would like to stress that one of the basic assumptions applied in practically all accelerator searches for LQs is that LQs couple only to one generation of leptons and quarks at a time. As discussed at length above, such completely generation diagonal couplings would *exclude* LQ-loops as an explanation of neutrino oscillation data. Extending the LQ search to lepton flavor violating decays thus should be considered seriously by experimentalists.

We have also discussed how, in some specific scenarios, much more detailed predictions can be made. Given the observed hierarchy of standard model quark masses, it seems reasonable to assume that contributions from 3rd generation quark loops dominate the neutrino mass matrix. For the case of top quark dominance, our results are summarized in Figs. 3–7. Similar results hold in case of pure bottom-loop dominance.

Finally, an important test of the hypothesis that LQs can generate Majorana neutrino masses is the search for decays of heavier LQs into lighter ones plus a standard model Higgs or gauge boson. Any observation of a nonzero branching ratio for the decay $S_i \to S_j + h^0/Z^0$ constitutes proof for LQ-mixing, which is the basic ingredient for the LQ explanation of neutrino masses. If LQs are found at the LHC, the search for such decays should be made a priority.

ACKNOWLEDGMENTS

D. A. S. wants to thank the Instituto de Física de la Universidad de Antioquia for their hospitality. This work was supported by Spanish Grant No. FPA2005-01269, by the European Commission Human Potential Program RTN network No. MRTN-CT-2004-503369 and by Chilean Grant CONICYT No. PBCT/No.285/2006.

[1] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); **11**, 703(E) (1975).
[2] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).
[3] P. Langacker, Phys. Rep. **72**, 185 (1981).
[4] For a recent example of a GUT model with light leptoquarks, see [5].
[5] I. Dorsner and P. F. Perez, Nucl. Phys. **B723**, 53 (2005).
[6] B. Schrempp and F. Schrempp, Phys. Lett. **153B**, 101 (1985); L. F. Abbott and E. Farhi, Phys. Lett. **101B**, 69 (1981).
[7] S. Dimopoulos and L. Susskind, Nucl. Phys. **B155**, 237 (1979).
[8] E. Eichten and K. D. Lane, Phys. Lett. **90B**, 125 (1980).
[9] L. J. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984); S. Dawson, Nucl. Phys. **B261**, 297 (1985).
[10] W. Buchmuller, R. Ruckl, and D. Wyler, Phys. Lett. B **191**, 442 (1987); **448**, 320(E) (1999).
[11] S. Davidson, D. C. Bailey, and B. A. Campbell, Z. Phys. C **61**, 613 (1994).
[12] G. Chiarelli, AIP Conf. Proc. **815**, 268 (2006).
[13] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **99**, 061801 (2007); Phys. Lett. B **636**, 183 (2006); See also web page: http://www-d0.fnal.gov/Run2Physics/WWW/results/np.htm.
[14] D. Acosta *et al.* (CDF Collaboration), Phys. Rev. D **72**, 051107 (2005); A. Abulencia *et al.* (CDF Collaboration), Phys. Rev. D **73**, 051102 (2006); See also web p.: http://www-cdf.fnal.gov/physics/exotic/exotic.html.
[15] M. Kramer, T. Plehn, M. Spira, and P. M. Zerwas, Phys. Rev. D **71**, 057503 (2005).
[16] Q. R. Ahmad *et al.* (SNO Collaboration), Phys. Rev. Lett. **89**, 011301 (2002).
[17] Y. Fukuda *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **81**, 1562 (1998).
[18] K. Eguchi *et al.* (KamLAND Collaboration), Phys. Rev. Lett. **90**, 021802 (2003).
[19] For a recent review see, for example J. W. F. Valle, arXiv:hep-ph/0608101.
[20] P. Minkowski, Phys. Lett. B **67**, 421 (1977).
[21] T. Yanagida, in KEK Lectures, edited by O. Sawada and A. Sugamoto (KEK, 1979); M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Niewenhuizen and D. Freedman (North-Holland, Amsterdam, 1979).
[22] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
[23] J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980).
[24] M. Hirsch *et al.*, Phys. Rev. D **62**, 113008 (2000); **65**, 119901(E) (2002); **68**, 013009 (2003); **71**, 059904(E) (2005).
[25] D. Aristizabal Sierra, M. Hirsch, J. W. F. Valle, and A. Villanova del Moral, Phys. Rev. D **68**, 033006 (2003).

[26] A. Zee, Phys. Lett. **93B**, 389 (1980); **95B**, 461 (1980).
[27] D. Aristizabal Sierra and D. Restrepo, J. High Energy Phys. 08 (2006) 036.
[28] J. F. Nieves, Nucl. Phys. **B189**, 182 (1981).
[29] A. Zee, Nucl. Phys. **B264**, 99 (1986); K. S. Babu, Phys. Lett. B **203**, 132 (1988).
[30] K. S. Babu and C. Macesanu, Phys. Rev. D **67**, 073010 (2003).
[31] D. Aristizabal Sierra and M. Hirsch, J. High Energy Phys. 12 (2006) 052.
[32] M. Hirsch, H. V. Klapdor-Kleingrothaus, and S. G. Kovalenko, Phys. Lett. B **378**, 17 (1996).
[33] This is not an entirely new subject. Majorana neutrino masses due to loops involving colored scalars (and vectors) have been discussed first in [28]. However, because in [28] it was assumed that these particles have masses $M \geq 10^8$ GeV, their contribution to $\mathcal{M}_\nu$ was deemed negligible.
[34] M. Hirsch, H. V. Klapdor-Kleingrothaus, and S. G. Kovalenko, Phys. Rev. D **54**, R4207 (1996).
[35] S. Kolb, M. Hirsch, and H. V. Klapdor-Kleingrothaus, Phys. Lett. B **391**, 131 (1997).
[36] U. Mahanta, Phys. Rev. D **62**, 073009 (2000).
[37] Note that an extension of the scalar potential in Eq. (2) to include the LQ trilinear self-interaction terms has been discused in [38]. Such terms, however, introduce violation of baryon number and, hence, proton decay; for details see [38]. Therefore we will not consider LQ self-interaction terms here.
[38] S. Kovalenko and I. Schmidt, Phys. Lett. B **562**, 104 (2003).
[39] G. Passarino and M. J. G. Veltman, Nucl. Phys. **B160**, 151 (1979).
[40] M. Maltoni, T. Schwetz, M. A. Tortola, and J. W. F. Valle, New J. Phys. **6**, 122 (2004). arXiv:hep-ph/0405172v5 provides updated numbers taking into account all relevant data as of June 2006.
[41] M. Apollonio *et al.*, Eur. Phys. J. C **27**, 331 (2003).
[42] W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006); partial 2007 update, http://pdg.lbl.gov.
[43] E. Gabrielli, Phys. Rev. D **62**, 055009 (2000).
[44] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B **530**, 167 (2002).
[45] M. A. Doncheski and R. W. Robinett, Phys. Lett. B **411**, 107 (1997).