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Electroweak chiral Lagrangian from one-doublet and topcolor-assisted technicolor models

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Based on previous studies deriving the chiral Lagrangian for pseudoscalar mesons from the first principle of QCD, we derive an electroweak chiral Lagrangian and build up a formulation for computing its coefficients from a one-doublet technicolor model and a schematic topcolor-assisted technicolor model. We find that coefficients of the electroweak chiral Lagrangian for the topcolor-assisted technicolor model are divided into three parts: direct TC2 interaction part, TC1 and TC2 induced effective Z' particle contribution part, and ordinary quarks contribution part. The first two parts are computed in this paper. We show that the direct TC2 interaction part is the same as that in the one-doublet technicolor model, while effective Z' contributions are at least proportional to the p^2 order parameter β_1 in the electroweak chiral Lagrangian. Typical features of the topcolor-assisted technicolor model are that it only allows positive T and U parameters, the T parameter varies in the range $0 \sim 1/(25\alpha)$, and the upper bound of T parameter will decrease as long as Z' mass become large. The S parameter can be either positive or negative depending on whether the Z' mass is large or small. The Z' mass is bounded above and the upper bound depends on the value of the T parameter. We obtain the values for all coefficients of the electroweak chiral Lagrangian up to the order of p^4.

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I. INTRODUCTION

The electroweak symmetry breaking mechanism (EWSBM) remains an intriguing puzzle for particle physics, although the standard model (SM) provides us with a version through introducing a Higgs doublet into the theory which suffers from triviality and unnaturalness problems. Beyond the SM, numerous new physics models are invented which exhibit many alternative EWSBMs. With the present situation that the Higgs particle is still not found in experiment, all new physics models at low energy region should be described by a theory which not only must match all present experiment data, but also have no Higgs. This theory is the well-known electroweak chiral Lagrangian (EWCL) [1–3] which offers the most general and economic description of electroweak interaction at a low energy region. With EWCL, new physics models at low energies can be parametrized by a set of coefficients. It universally describes all possible electroweak interactions among existing particles and offers a model independent platform for us to investigate various EWSBMs. Starting from this platform, further phenomenological research focuses on finding effective physical processes to fix the certain coefficients of EWCL [4–6], and theoretical studies concentrate on consistency of EWCL itself such as gauge invariance [7] and computing the values of the coefficients for SM with heavy Higgs [8]. We have not found systematic theoretical computations of the EWCL coefficients for various new physics models in the literature. The possible reasons are that for weakly coupled models, since one can perform perturbative computations, people prefer to directly discuss physics from the model, rather than make the extra effort to compute EWCL coefficients. While for strongly coupled models, nonperturbative difficulties for a long time prevented people from performing dynamical computations, only for some important coefficients such as S parameter, a special nonperturbative technique may be applied to perform calculations [9]. Or for special QCD-like technicolor models, in terms of their similarities with QCD, one can estimate coefficients of EWCL in terms of their QCD partners, which were fixed by experimental data. The estimation of EWCL coefficients for various models is of special importance in the sense that at present we already have some quantitative constraints on them, such as those for the S, T, U parameters and more generally for anomalous triple and quartic couplings [10]; along with the experimental progress, more constraints will be obtained. Once we know the values of the coefficients for detailed models, these constraints can directly be used to judge the correctness of the model. It is the purpose of this paper to develop a formulation to systematically compute coefficients of EWCL for strongly coupled new physics models. For simplicity, in this work we only discuss the bosonic part of EWCL and leave the matter part for future investigations. The basis of our formulation is the knowledge and experiences we obtained previously from the works of deriving the chiral Lagrangian for pseudoscalar mesons from QCD first principles [11] and calculating corresponding coefficients [12–14], which set up confidence and reliability of the present work. In fact, the formal derivation from a general

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underlying technicolor model to EWCL was already presented in Ref. [15] in which, in addition to deriving EWCL, coefficients of EWCL are formally expressed in terms of Green's functions in the underlying technicolor model. Once we know how to compute these Green's functions, we can obtain the corresponding EWCL coefficients. The pity is that the computation is nonperturbative, and therefore not easy to achieve. The aim of this paper is to solve this nonperturbative dynamical computation problems.

As the first step of dynamical computation, we especially care about the reliability of the formulation we will develop. We take the one-doublet technicolor model [16–19] as the prototype to build up our formulation. Although this model, as the earliest and simplest dynamical electroweak symmetry breaking model, was already denied by experiment in the sense that it results in too large a value for the S parameter, due to the following reasons we still start our investigations from it. First, it is similar as QCD in dynamics, which enable us to generalize techniques developed in dealing with the QCD chiral Lagrangian to this model easily—we call this generalized formulation the dynamical computation prescription. Second, due to its similarity with conventional QCD, the coefficients of its EWCL can be estimated by just scaling-up corresponding coefficients in QCD chiral Lagrangian for pseudoscalar mesons [20]. We call this formulation the Gasser-Leutwyler's prescription which naively is only applicable for those QCD-like models. So for QCD-like models, we have two prescriptions which enable us to compare them with each other to check the correctness and set reliability of our formulation. Beyond the traditional one-doublet technicolor model, we choose the topcolor-assisted technicolor model as the first real practice model to perform our computations. The reason we use it is that this model is not QCD-like and does not seriously contradict with experimental data as in the case of one-doublet technicolor model. The dynamics of this model responsible for electroweak symmetry breaking is similar to that in the one-doublet technicolor model. To our knowledge that various electroweak observables computed in the literature for this model all involve fermions of the theory, since we have dropped out fermion contributions and only focus on the pure bosonic part of the theory, this work has no intersection with those existing calculations in the literature. We will find for the pure bosonic part of EWCL that the coefficients for the topcolor-assisted technicolor model can be divided into three parts: the direct TC2 interaction part, the TC1 and TC2 induced effective Z' particle contribution part, and the ordinary quarks contribution part. The first two parts are computed in this paper and we show that direct TC2 interaction part is the same as that in the one-doublet technicolor model, while TC1 and TC2 induced effective Z' particle contributions is at least proportional to p^2 order parameter β_1 in EWCL. Typical features of the topcolor-assisted technicolor model are that it only allows positive T and U parameters and the T parameter varies in the range $0 \sim 1/(25\alpha)$, the upper bound of T parameter will decrease as long as the Z' mass become large. The S parameter can be either positive or negative depending on whether the Z' mass is large or small. The Z' mass is bounded above and the upper bound depends on the value of the T parameter. We obtain the values for all the coefficients of the electroweak chiral Lagrangian up to the order of p^4.

This paper is organized as follows. Section II is the basis of the work in which we discuss the one-doublet technicolor model. We first review the Gasser-Leutwyler's prescription, then build up our dynamical computation prescription. We show how to consistently set in the dynamical computation equation (the Schwinger-Dyson equation) into our formulation. We make comparison between two prescriptions to check validity of the results from our dynamical computation prescription. Section III is the main part of this work in which we apply our formulation developed in the one-doublet technicolor model to the topcolor-assisted technicolor model. We perform dynamical calculations on technicolor interactions and then integrate out colorons and Z' to compute EWCL coefficients. Since this is the first time to systematically perform dynamical computations on the strongly coupled models, we emphasize the technical side more than physics analysis and display the computation procedure in a little bit more detail. Section IV is the conclusion. In the appendix, we list some requisite formulas.

II. DERIVATION OF EWCL FROM THE ONE-DOUBLET TECHNICOLOR MODEL

Consider the one-doublet technicolor (TC) model proposed by Weinberg and Susskind independently [16–19]. The techniquarks are assigned to $(SU(N)_{\rm TC}, SU(3)_C, SU(2)_L, U(1)_Y)$ as $\psi_L \sim (N, 1, 2, 0)$, $U_R = (1/2 + \tau^3/2)\psi_R \sim (N, 1, 1, 1/2)$, $D_R = (1/2 - \tau^3/2)\psi_R \sim (N, 1, 1, -1/2)$. With these assignments, the techniquarks have electric charges defined by $Q = T_3 + Y$, of $+1/2$ for U and $-1/2$ for D. It can be shown below, by dynamical analysis through the Schwinger-Dyson equation, that the $SU(N)_{\rm TC}$ interaction induces the techniquark condensate $\langle\bar{\psi}\psi\rangle \neq 0$, which will trigger the electroweak symmetry breaking $SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$. Neglecting ordinary fermions and gluons, we focus on the action of the techniquark, technicolor-gauge-boson, and electroweak-gauge-boson sector, i.e. the electroweak symmetry breaking sector (SBS) of this model,

$$S_{\rm SBS} = \int d^4x \bigg[-\frac{1}{4}F^{\alpha}_{\mu\nu}F^{\alpha,\mu\nu} - \frac{1}{4}W^{a}_{\mu\nu}W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ - g_{\rm TC}t^{\alpha}G\!\!\!/^{\alpha} - g_2\frac{\tau^a}{2}W\!\!\!\!/^{a}P_L - g_1\frac{\tau^3}{2}B\!\!\!/P_R)\psi \bigg], \quad (1)$$

where $g_{\rm TC}$, g_2, and g_1 (G^α_μ, W^a_μ, and B_μ) are the coupling constants (gauge fields) of $SU(N)_{\rm TC}$, $SU(2)_L$, and $U(1)_Y$ with technicolor index α ($=1,2,\ldots,N^2-1$) and weak index a ($=1,2,3$) respectively; and $F^\alpha_{\mu\nu}$, $W^a_{\mu\nu}$, and $B_{\mu\nu}$ are the corresponding field strength tensors; t^α ($\alpha=1,2,\ldots,N^2-1$) are the generators for the fundamental representation of $SU(N)_{\rm TC}$, while τ^a ($a=1,2,3$) are Pauli matrices; and the left and right chirality projection operators $P_{\substack{L\\R}}=(1\mp\gamma_5)/2$.

To derive low energy effective EWCL from the one-doublet TC model, we need to integrate out the technigluons and techniquarks above the electroweak scale which can be formulated as

$$\int\mathcal{D}G^\alpha_\mu\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{iS_{\rm SBS}[G^\alpha_\mu,W^a_\mu,B_\mu,\bar{\psi},\psi]}=\int\mathcal{D}\mu(U)e^{iS_{\rm eff}[U,W^a_\mu,B_\mu]},\qquad(2)$$

where $U(x)$ is a dimensionless unitary unimodular matrix field in EWCL, and $\mathcal{D}\mu(U)$ denotes the corresponding functional integration measure.

As mentioned in the previous section, there are two different approaches: one is the Gasser-Leutwyler's prescription, the other is the dynamical computation prescription. The second approach we developed in this paper is relatively easily generalized to more complicated theories. We will compare the results obtained in both approaches.

A. The Gasser-Leutwyler's prescription

As we will see, it is easy to relate QCD-like models to chiral Lagrangian using the Gasser-Leutwyler's prescription. To begin with, we substitute (1) into the left-hand side of Eq. (2). The resulting path integral with technicolor interaction is analogous to QCD, then we can use the technique developed by Gasser and Leutwyler relating it with the path integral of chiral Lagrangian for Goldstone bosons induced from SBS [20],

$$\begin{aligned}&\int\mathcal{D}G^\alpha_\mu\mathcal{D}\bar{\psi}\mathcal{D}\psi\exp\Big\{i\int d^4x\Big[-\frac{1}{4}F^\alpha_{\mu\nu}F^{\alpha,\mu\nu}\\&\quad+\bar{\psi}(i\not\partial-g_{\rm TC}t^\alpha\not{G}^\alpha-g_2\frac{\tau^a}{2}\not{W}^aP_L-g_1\frac{\tau^3}{2}\not{B}P_R)\psi\Big]\Big\}\\&\quad\times\Big\{\int\mathcal{D}\bar{\psi}\mathcal{D}\psi\exp\Big\{i\int d^4x\bar{\psi}\Big[i\not\partial-g_2\frac{\tau^a}{2}\not{W}^aP_L\\&\quad-g_1\frac{\tau^3}{2}\not{B}P_R\Big]\psi\Big\}\Big\}^{-1}=\int\mathcal{D}\mu(\tilde{U})e^{iS_{\rm TC\text{-}induced\ eff}[\tilde{U},W,B]},\end{aligned}\qquad(3)$$

in which the denominator of the left-hand side of the above equation is introduced to ensure the technicolor-induced chiral effective action $S_{\rm TC\text{-}induced\ eff}[\tilde{U},W,B]$ is normalized as zero when we switch off technicolor interactions by setting $g_{\rm TC}=0$. $S_{\rm TC\text{-}induced\ eff}[\tilde{U},W,B]$ can be written as

$$\begin{aligned}S_{\rm TC\text{-}induced\ eff}[\tilde{U},W,B]=\int d^4x\Big[&\frac{(F_0^{\rm 1D})^2}{4}{\rm tr}[(\nabla^\mu\tilde{U}^\dagger)(\nabla_\mu\tilde{U})]+L_1^{\rm 1D}[{\rm tr}(\nabla^\mu\tilde{U}^\dagger\nabla_\mu\tilde{U})]^2+L_2^{\rm 1D}{\rm tr}[\nabla_\mu\tilde{U}^\dagger\nabla_\nu\tilde{U}]{\rm tr}[\nabla^\mu\tilde{U}^\dagger\nabla^\nu\tilde{U}]\\&+L_3^{\rm 1D}{\rm tr}[(\nabla^\mu\tilde{U}^\dagger\nabla_\mu\tilde{U})^2]-iL_9^{\rm 1D}{\rm tr}[F^R_{\mu\nu}\nabla^\mu\tilde{U}\nabla^\nu\tilde{U}^\dagger+F^L_{\mu\nu}\nabla^\mu\tilde{U}^\dagger\nabla^\nu\tilde{U}]\\&+L_{10}^{\rm 1D}{\rm tr}[\tilde{U}^\dagger F^R_{\mu\nu}\tilde{U}F^{L,\mu\nu}]+H_1^{\rm 1D}{\rm tr}[F^R_{\mu\nu}F^{R,\mu\nu}+F^L_{\mu\nu}F^{L,\mu\nu}]\Big]+O(p^6),\end{aligned}\qquad(4)$$

where the coefficients $F_0^{\rm 1D}$, $L_1^{\rm 1D}$, $L_2^{\rm 1D}$, $L_3^{\rm 1D}$, $L_{10}^{\rm 1D}$, $H_1^{\rm 1D}$ arise from $SU(N)_{\rm TC}$ dynamics at the scale of 250 GeV, and

$$\begin{aligned}\nabla_\mu\tilde{U}&\equiv\partial_\mu\tilde{U}-ir_\mu\tilde{U}+i\tilde{U}l_\mu,\\\nabla_\mu\tilde{U}^\dagger&=-\tilde{U}^\dagger(\nabla_\mu\tilde{U})\tilde{U}^\dagger,\\F^R_{\mu\nu}&\equiv i[\partial_\mu-ir_\mu,\partial_\nu-ir_\nu]\\F^L_{\mu\nu}&\equiv i[\partial_\mu-il_\mu,\partial_\nu-il_\nu],\\r_\mu&\equiv-g_1\frac{\tau^3}{2}B_\mu\\l_\mu&\equiv-g_2\frac{\tau^a}{2}W^a_\mu.\end{aligned}\qquad(5)$$

Note that conventional $\tilde{U}$ field in Eq. (4) given in the second paper of [20] is a 3×3 unitary matrix. However, for the $SU(2)_L\times SU(2)_R$ EWCL we considered in this paper, $\tilde{U}$ is a 2×2 unitary matrix, and thus the $L_1^{\rm 1D}$ term and the $L_3^{\rm 1D}$ term in the present situation are linearly related,

$$\begin{aligned}L_3^{\rm 1D}{\rm tr}[(\nabla^\mu\tilde{U}^\dagger\nabla_\mu\tilde{U})^2]&=L_3^{\rm 1D}{\rm tr}\{[\tilde{U}^\dagger(\nabla^\mu\tilde{U})\tilde{U}^\dagger(\nabla^\mu\tilde{U})]^2\}\\&=\frac{L_3^{\rm 1D}}{2}\{{\rm tr}[\tilde{U}^\dagger(\nabla^\mu\tilde{U})\tilde{U}^\dagger(\nabla^\mu\tilde{U})]\}^2\end{aligned}\qquad(6)$$

Comparing the covariant derivative for $\tilde{U}$ given in (5) and covariant derivative given in Ref. [2], we find we must recognize $\tilde{U}^\dagger=U$, $\nabla_\mu\tilde{U}^\dagger=D_\mu U$, $F^R_{\mu\nu}=-g_1\frac{\tau^3}{2}B_{\mu\nu}$, and $F^L_{\mu\nu}=-g_2\frac{\tau^a}{2}W^a_{\mu\nu}$. Substituting them back into Eq. (4), we obtain

$$S_{\text{TC-induced eff}}[U,W,B]=\int d^4x\bigg[-\frac{(F_0^{1D})^2}{4}\operatorname{tr}(X_\mu X^\mu)+\bigg(L_1^{1D}+\frac{L_3^{1D}}{2}\bigg)[\operatorname{tr}(X_\mu X^\mu)]^2+L_2^{1D}[\operatorname{tr}(X_\mu X_\nu)]^2 -i\frac{L_9^{1D}}{2}g_1B_{\mu\nu}\operatorname{tr}(\tau^3X^\mu X^\nu)-iL_9^{1D}\operatorname{tr}(\bar{W}_{\mu\nu}X^\mu X^\nu)+\frac{L_{10}^{1D}}{2}g_1B_{\mu\nu}\operatorname{tr}(\tau^3\bar{W}^{\mu\nu}) +\frac{H_1^{1D}}{2}g_1^2B_{\mu\nu}B^{\mu\nu}+H_1^{1D}\operatorname{tr}(\bar{W}_{\mu\nu}\bar{W}^{\mu\nu})\bigg]. \tag{7}$$

where

$$X_\mu\equiv U^\dagger(D_\mu U)\qquad \bar{W}_{\mu\nu}\equiv U^\dagger g_2W_{\mu\nu}U, \tag{8}$$

We have reformulated the EWCL in terms of X_μ and τ^3 instead of V_μ and T in Ref. [2], the corresponding relations are given in Appendix A. Comparing (7) with the standard EWCL given in Ref. [2], we find

$$f^2=(F_0^{1D})^2,\qquad \beta_1=0,\qquad \alpha_1=L_{10}^{1D},\qquad \alpha_2=\alpha_3=-\frac{L_9^{1D}}{2},\qquad \alpha_4=L_2^{1D},\qquad \alpha_5=L_1^{1D}+\frac{L_3^{1D}}{2},\qquad \alpha_i=0\quad(i=6,7,\ldots,14). \tag{9}$$

Note that in (7), the terms with coefficient H_1^{1D} do not affect the resulting α_i coefficients. Unlike the original case of QCD, H_1^{1D} now is a finite constant. The divergences are from the term $\operatorname{Tr}\log(i\not\partial-g_2\frac{\tau^a}{2}\not W^aP_L-g_1\frac{\tau^3}{2}\not BP_R)$ in (3) which will only contribute $\operatorname{tr}(W_{\mu\nu}W^{\mu\nu})$ and $B_\mu B^\mu$ terms with divergent coefficients due to gauge invariance. These divergent coefficients in combination with H_1^{1D} will cause wave function renormalization corrections for W_μ^a and B_μ fields which further lead to redefinitions of W_μ^a and B_μ fields and their gauge couplings g_2 and g_1. This renormalization procedure will have no effect on our EWCL, since all electroweak gauge fields appearing in EWCL are as form of $g_2W_\mu^a$ and g_1B_μ which are renormalization invariant quantities. Because of this consideration, in the rest of this paper, we just left the wave function corrections to electroweak gauge fields W_μ^a and B_μ in the theory and skip the corresponding renormalization procedure.

B. The dynamical computation prescription

Now we develop a dynamical computation program. This program will be applied to a more complicated model in the next section.

We first review the derivation process given in Ref. [15] and start with introducing a local 2×2 operator $O(x)$ as $O(x)\equiv\operatorname{tr}_{lc}[\psi_L(x)\bar{\psi}_R(x)]$ with tr_{lc} is the trace with respect to Lorentz and technicolor indices. The transformation of $O(x)$ under $SU(2)_L\times U(1)_Y$ is $O(x)\to V_L(x)O(x)V_R^\dagger(x)$. Then we decompose $O(x)$ as $O(x)=\xi_L^\dagger(x)\sigma(x)\xi_R(x)$ with the $\sigma(x)$ represented by a Hermitian matrix that describes the modular degree of freedom; while $\xi_L(x)$ and $\xi_R(x)$ represented by unitary matrices describe the phase degree of freedom of $SU(2)_L$ and $U(1)_Y$, respectively. Their transformation under $SU(2)_L\times U(1)_Y$ are $\sigma(x)\to h(x)\sigma(x)h^\dagger(x)$, $\xi_L(x)\to h(x)\xi_L(x)V_L^\dagger(x)$, and $\xi_R(x)\to h(x)\xi_R(x)V_R^\dagger(x)$ where $h(x)=e^{i\theta_h(x)\tau^3/2}$ belongs to an induced hidden local $U(1)$ symmetry group. Now we define a new field $U(x)$ as $U(x)\equiv\xi_L^\dagger(x)\xi_R(x)$, which is the nonlinear realization of the Goldstone boson fields in EWCL. Subtracting out the $\sigma(x)$ field, we find that the present decomposition results in a constraint $\xi_L(x)O(x)\xi_R^\dagger(x)-\xi_R(x)O^\dagger(x)\xi_L^\dagger(x)=0$, the functional expression of it is

$$\int\mathcal{D}\mu(U)\mathcal{F}[O]\delta(\xi_LO\xi_R^\dagger-\xi_RO^\dagger\xi_L^\dagger)=\text{const}, \tag{10}$$

where $\mathcal{D}\mu(U)$ is an effective invariant integration measure; $\mathcal{F}[O]$ only depends on O, and it compensates the integration to make it a constant. It is easy to show that $\mathcal{F}[O]$ is invariant under $SU(2)_L\times U(1)_Y$ transformations. Substituting Eq. (10) into the left-hand side of Eq. (2), we have

$$\int\mathcal{D}G_\mu^\alpha\mathcal{D}\bar{\psi}\mathcal{D}\psi\exp(iS_{\text{SBS}}[G_\mu^\alpha,W_\mu^a,B_\mu,\bar{\psi},\psi]) =\int\mathcal{D}\mu(U)\exp(iS_{\text{eff}}[U,W_\mu^a,B_\mu]), \tag{11}$$

where

$$S_{\text{eff}}[U,W_\mu^a,B_\mu]=\int d^4x\bigg(-\frac{1}{4}W_{\mu\nu}^aW^{a,\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}\bigg)-i\log\int\mathcal{D}G_\mu^\alpha\mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{F}[O]\delta(\xi_LO\xi_R^\dagger-\xi_RO^\dagger\xi_L^\dagger) \times\exp\bigg\{i\int d^4x\bigg[-\frac{1}{4}F_{\mu\nu}^\alpha F^{\alpha,\mu\nu}+\bar{\psi}(i\not\partial-g_{\text{TC}}t^\alpha\not G^\alpha-g_2\frac{\tau^a}{2}\not W^aP_L-g_1\frac{\tau^3}{2}\not BP_R)\psi\bigg]\bigg\} \tag{12}$$

To match the correct normalization, we introduce in the argument of the logarithm function the normalization factor $\int\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{i\int d^4x\bar{\psi}(i\not\partial-g_2\frac{\tau^a}{2}\not W^aP_L-g_1\frac{\tau^3}{2}\not BP_R)\psi}=\exp\operatorname{Tr}\log(i\not\partial-g_2\frac{\tau^a}{2}\not W^aP_L-g_1\frac{\tau^3}{2}\not BP_R)$ and then take a special $SU(2)_L\times U(1)_Y$ rotation, as $V_L(x)=\xi_L(x)$ and $V_R(x)=\xi_R(x)$, on both numerator and denominator of the normalization factor

$$S_{\rm eff}[U,W^a_\mu,B_\mu]=\int d^4x\Big(-\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}\Big)-i\,{\rm Tr}\log\Big(i\partial\!\!\!/-g_2\frac{\tau^a}{2}W\!\!\!\!/^aP_L-g_1\frac{\tau^3}{2}B\!\!\!/P_R\Big)$$
$$-i\log\Big\{\int\mathcal{D}G^\alpha_\mu\mathcal{D}\bar\psi_\xi\mathcal{D}\psi_\xi\mathcal{F}[O_\xi]\delta(O_\xi-O^\dagger_\xi)$$
$$\times\exp\Big\{i\int d^4x\Big[-\frac{1}{4}F^\alpha_{\mu\nu}F^{\alpha,\mu\nu}+\bar\psi_\xi(i\partial\!\!\!/-g_{\rm TC}t^\alpha G\!\!\!/^\alpha-g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi P_L-g_1\frac{\tau^3}{2}B\!\!\!/_\xi P_R)\psi_\xi\Big]\Big\}$$
$$\times\Big\{\int\mathcal{D}\bar\psi_\xi\mathcal{D}\psi_\xi\exp\Big[i\int d^4x\bar\psi_\xi(i\partial\!\!\!/-g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi P_L-g_1\frac{\tau^3}{2}B\!\!\!/_\xi P_R)\psi_\xi\Big]\Big\}^{-1}\Big\},\qquad(13)$$

where rotated fields are denoted as follows

$$\begin{aligned}\psi_\xi&=P_L\xi_L(x)\psi_L(x)+P_R\xi_R(x)\psi_R(x),\\ O_\xi(x)&\equiv\xi_L(x)O(x)\xi^\dagger_R(x),\\ g_2\frac{\tau^a}{2}W^a_{\xi,\mu}(x)&\equiv\xi_L(x)\Big[g_2\frac{\tau^a}{2}W^a_\mu(x)-i\partial_\mu\Big]\xi^\dagger_L(x),\\ g_1\frac{\tau^3}{2}B_{\xi,\mu}(x)&\equiv\xi_R(x)\Big[g_1\frac{\tau^3}{2}B_\mu(x)-i\partial_\mu\Big]\xi^\dagger_R(x).\end{aligned}\qquad(14)$$

In (13), the possible anomalies caused by this special chiral rotation are canceled between the numerator and the denominator. Thus Eq. (13) can be written as

$$S_{\rm eff}[U,W^a_\mu,B_\mu]=\int d^4x\Big(-\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}\Big)+S_{\rm anom}[U,W^a_\mu,B_\mu]+S_{\rm norm}[U,W^a_\mu,B_\mu],\qquad(15a)$$

where

$$S_{\rm norm}[U,W^a_\mu,B_\mu]=-i\log\int\mathcal{D}G^\alpha_\mu\mathcal{D}\bar\psi_\xi\mathcal{D}\psi_\xi\mathcal{F}[O_\xi]\times\delta(O_\xi-O^\dagger_\xi)\times\exp\Big\{i\int d^4x\Big[-\frac{1}{4}F^\alpha_{\mu\nu}F^{\alpha,\mu\nu}+\bar\psi_\xi(i\partial\!\!\!/-g_{\rm TC}t^\alpha G\!\!\!/^\alpha-g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi P_L-g_1\frac{\tau^3}{2}B\!\!\!/_\xi P_R)\psi_\xi\Big]\Big\}.\qquad(15b)$$

and

$$iS_{\rm anom}[U,W^a_\mu,B_\mu]={\rm Tr}\log\Big(i\partial\!\!\!/-g_2\frac{\tau^a}{2}W\!\!\!\!/^aP_L-g_1\frac{\tau^3}{2}B\!\!\!/P_R\Big)-{\rm Tr}\log\Big(i\partial\!\!\!/-g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi P_L-g_1\frac{\tau^3}{2}B\!\!\!/_\xi P_R\Big),\qquad(15c)$$

It is worthwhile to mention that the transformations of the rotated fields under $SU(2)_L\times U(1)_Y$ are $\psi_\xi(x)\to h(x)\psi_\xi(x)$, $O_\xi(x)\to h(x)O_\xi(x)h^\dagger(x)$, where $h(x)$ defined previously describes a hidden local $U(1)$ symmetry. Thus, the chiral symmetry $SU(2)_L\times U(1)_Y$ covariance of the unrotated fields has been transferred totally to the hidden symmetry $U(1)$ covariance of the rotated fields. We can further find the combination of electroweak gauge fields $g_2\frac{\tau^a}{2}W^a_{\xi,\mu}(x)-g_1\frac{\tau^3}{2}B_{\xi,\mu}(x)\to h(x)[g_2\frac{\tau^a}{2}W^a_{\xi,\mu}(x)-g_1\frac{\tau^3}{2}B_{\xi,\mu}(x)]h^\dagger(x)$ transforms covariantly, while an alternative combination $g_2\frac{\tau^a}{2}W^a_{\xi,\mu}(x)+g_1\frac{\tau^3}{2}B_{\xi,\mu}(x)\to h(x)\times[g_2\frac{\tau^a}{2}W^a_{\xi,\mu}(x)+g_1\frac{\tau^3}{2}B_{\xi,\mu}(x)-2i\partial_\mu]h^\dagger(x)$ transforms as the "gauge field" of the hidden local $U(1)$ symmetry.

With the technique used in Ref. [11], technigluon fields in Eq. (15b) can be formally integrated out with the help of a full n-point Green's function of the G^α_μ-field $G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}$, thus Eq. (15b) after integration becomes

$$e^{iS_{\rm norm}[U,W^a_\mu,B_\mu]}=\int\mathcal{D}\bar\psi_\xi\mathcal{D}\psi_\xi\mathcal{F}[O_\xi]\delta(O_\xi-O^\dagger_\xi)\exp\Big[i\int d^4x\bar\psi_\xi\Big(i\partial\!\!\!/-g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi P_L-g_1\frac{\tau^3}{2}B\!\!\!/_\xi P_R\Big)\psi_\xi+\sum_{n=2}^\infty\int d^4x_1\ldots d^4x_n\frac{(-ig_{\rm TC})^n}{n!}G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x_1,\ldots,x_n)J^{\mu_1}_{\xi,\alpha_1}(x_1)\ldots J^{\mu_n}_{\xi,\alpha_n}(x_n)\Big].\qquad(16)$$

where effective sources $J^\mu_{\xi,\alpha}(x)$ are identified as $J^\mu_{\xi,\alpha}(x)\equiv\bar\psi_\xi(x)t^\alpha\gamma^\mu\psi_\xi(x)$.

1. Schwinger-Dyson equation for techniquark propagator

To show that the technicolor interaction indeed induces the condensate $\langle\bar\psi\psi\rangle\neq0$ which triggers the electroweak symmetry breaking, we investigate the behavior of the techniquark propagator $S^{\sigma\rho}(x,x')\equiv\langle\psi^\sigma_\xi(x)\bar\psi^\rho_\xi(x')\rangle$ in the following. *Neglecting the factor* $\mathcal{F}[O_\xi]\delta(O_\xi-O^\dagger_\xi)$ in Eq. (16), the total functional derivative of the integrand with respect to

$\bar{\psi}^{\sigma}_{\xi}(x)$ is zero, (here and henceforth the suffixes σ and ρ are short notations for Lorentz spinor, techniflavor, and technicolor indices) i.e.,

$$0 = \int \mathcal{D}\bar{\psi}_\xi \mathcal{D}\psi_\xi \frac{\delta}{\delta\bar{\psi}^{\sigma}_{\xi}(x)} \exp\bigg[\int d^4x(\bar{\psi}_\xi I + \bar{I}\psi_\xi) + i\int d^4x\bar{\psi}_\xi\bigg(i\partial\!\!\!/ - g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi P_L - g_1\frac{\tau^3}{2}B\!\!\!/_\xi P_R\bigg)\psi_\xi + \sum_{n=2}^{\infty}\int d^4x_1\ldots d^4x_n\frac{(-ig_{\rm TC})^n}{n!}G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x_1,\ldots,x_n)J^{\mu_1}_{\xi,\alpha_1}(x_1)\ldots J^{\mu_n}_{\xi,\alpha_n}(x_n)\bigg], \tag{17}$$

where $I(x)$ and $\bar{I}(x)$ are the external sources for, respectively, $\bar{\psi}_\xi(x)$ and $\psi_\xi(x)$. It leads to

$$0 = \bigg\langle\bigg\langle I_\sigma(x) + i\bigg[i\partial\!\!\!/_x - g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi(x)P_L - g_1\frac{\tau^3}{2}B\!\!\!/_\xi(x)P_R\bigg]_{\sigma\tau}\psi^\tau_\xi(x) + \sum_{n=2}^{\infty}\int d^4x_2\ldots d^4x_n\frac{(-ig_{\rm TC})^n}{(n-1)!}G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x,x_2,\ldots,x_n)(t^{\alpha_1}\gamma^{\mu_1})_{\sigma\tau}\psi^\tau_\xi(x)J^{\mu_2}_{\xi,\alpha_2}(x_2)\ldots J^{\mu_n}_{\xi,\alpha_n}(x_n)\bigg\rangle\bigg\rangle_I, \tag{18}$$

where we have defined the notation $\langle\langle\cdots\rangle\rangle_I$ in this section by

$$\langle\langle\mathcal{O}(x)\rangle\rangle_I \equiv \int \mathcal{D}\bar{\psi}_\xi \mathcal{D}\psi_\xi \mathcal{O}(x)\exp\bigg[\int d^4x(\bar{\psi}_\xi I + \bar{I}\psi_\xi) + i\int d^4x\bar{\psi}_\xi\bigg(i\partial\!\!\!/ - g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi P_L - g_1\frac{\tau^3}{2}B\!\!\!/_\xi P_R\bigg)\psi_\xi + \sum_{n=2}^{\infty}\int d^4x_1\ldots d^4x_n\frac{(-ig_{\rm TC})^n}{n!}G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x_1,\ldots,x_n)J^{\mu_1}_{\xi,\alpha_1}(x_1)\ldots J^{\mu_n}_{\xi,\alpha_n}(x_n)\bigg]. \tag{19}$$

Taking the functional derivative of Eq. (18) with respect to $I_\rho(y)$, and subsequently setting $I = \bar{I} = 0$, we obtain

$$0 = \delta_{\sigma\rho}\delta(x-y) + i\bigg[i\partial\!\!\!/_x - g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi(x)P_L - g_1\frac{\tau^3}{2}B\!\!\!/_\xi(x)P_R\bigg]_{\sigma\tau}\langle\psi^\tau_\xi(x)\bar{\psi}^\rho_\xi(y)\rangle - \sum_{n=2}^{\infty}\int d^4x_2\ldots d^4x_n\frac{(-ig_{\rm TC})^n}{(n-1)!}G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x,x_2,\ldots,x_n)\langle\bar{\psi}^\rho_\xi(y)(t^{\alpha_1}\gamma^{\mu_1})_{\sigma\tau}\psi^\tau_\xi(x)J^{\mu_2}_{\xi,\alpha_2}(x_2)\ldots J^{\mu_n}_{\xi,\alpha_n}(x_n)\rangle, \tag{20}$$

where we have defined vacuum expectation value (VEV) $\langle\cdots\rangle$ by $\langle\mathcal{O}(x)\rangle \equiv \langle\langle\mathcal{O}(x)\rangle\rangle_I/\langle\langle 1\rangle\rangle_I|_{I=\bar{I}=0}$. If we *neglect higher-point Green's functions*, and further taking the following *factorization approximation*,

$$\langle\bar{\psi}^\rho_\xi(y)\psi^\tau_\xi(x)\bar{\psi}^\gamma_\xi(x_2)\psi^\delta_\xi(x_2)\rangle \approx \langle\bar{\psi}^\rho_\xi(y)\psi^\tau_\xi(x)\rangle\langle\bar{\psi}^\gamma_\xi(x_2)\psi^\delta_\xi(x_2)\rangle - \langle\bar{\psi}^\rho_\xi(y)\psi^\delta_\xi(x_2)\rangle \times\langle\bar{\psi}^\gamma_\xi(x_2)\psi^\tau_\xi(x)\rangle,$$

we obtain

$$0 = \delta_{\sigma\rho}\delta(x-y) + i\bigg[i\partial\!\!\!/_x - g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi(x)P_L - g_1\frac{\tau^3}{2}B\!\!\!/_\xi(x)P_R\bigg]_{\sigma\tau}\langle\psi^\tau_\xi(x)\bar{\psi}^\rho_\xi(y)\rangle - g^2_{\rm TC}\int d^4x_2G^{\alpha_1\alpha_2}_{\mu_1\mu_2}(x,x_2)(t^{\alpha_1}\gamma^{\mu_1})_{\sigma\tau}(t^{\alpha_2}\gamma^{\mu_2})_{\gamma\delta} \times\langle\bar{\psi}^\rho_\xi(y)\psi^\delta_\xi(x_2)\rangle\langle\bar{\psi}^\gamma_\xi(x_2)\psi^\tau_\xi(x)\rangle, \tag{21}$$

where we have used the property that $\langle\bar{\psi}_\xi(x_2)t^{\alpha_2}\gamma^{\mu_2}\psi_\xi(x_2)\rangle = 0$, which comes from the Lorentz and gauge invariance of a vacuum. We denote the technifermion propagator $S^{\sigma\rho}(x,x') \equiv \langle\psi^\sigma_\xi(x)\bar{\psi}^\rho_\xi(x')\rangle$, multiplying the inverse of the technifermion propagator in both sides of Eq. (21), it then becomes the Schwinger-Dyson equation (SDE) for the techniquark propagator,

$$0 = S^{-1}_{\sigma\rho}(x,y) + i\bigg[i\partial\!\!\!/_x - g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi(x)P_L - g_1\frac{\tau^3}{2}B\!\!\!/_\xi(x)P_R\bigg]_{\sigma\rho} \times\delta(x-y) - g^2_{\rm TC}G^{\alpha_1\alpha_2}_{\mu_1\mu_2}(x,y)[t^{\alpha_1}\gamma^{\mu_1}S(x,y)t^{\alpha_2}\gamma^{\mu_2}]_{\sigma\rho}. \tag{22}$$

By defining techniquark self-energy $i\Sigma$ as

$$i\Sigma_{\sigma\rho}(x,y) \equiv S^{-1}_{\sigma\rho}(x,y) + i\bigg[i\partial\!\!\!/_x - g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi(x)P_L - g_1\frac{\tau^3}{2}B\!\!\!/_\xi(x)P_R\bigg]_{\sigma\rho}\delta(x-y), \tag{23}$$

the SDE (22) can be written as

$$i\Sigma_{\sigma\rho}(x,y) = g^2_{\rm TC}G^{\alpha_1\alpha_2}_{\mu_1\mu_2}(x,y)[t^{\alpha_1}\gamma^{\mu_1}S(x,y)t^{\alpha_2}\gamma^{\mu_2}]_{\sigma\rho}. \tag{24}$$

Moreover, from the fact that the technigluon propagator is diagonal in the adjoint representation space of the technicolor group, i.e., $G^{\alpha\beta}_{\mu\nu}(x,y) = \delta^{\alpha\beta}G_{\mu\nu}(x,y)$, and the techniquark propagator is diagonal in the techniquark representation space of the technicolor group, and also $(t^\alpha t^\alpha)_{ab} = C_2(N)\delta_{ab}$ for the fundamental representation of $SU(N)$, Eq. (24) is diagonal in technicolor indices a, b

and the diagonal part satisfies

$$i\Sigma^{ij}_{\eta\zeta}(x,y) = C_2(N)g^2_{\rm TC}G_{\mu_1\mu_2}(x,y)[\gamma^{\mu_1}S(x,y)\gamma^{\mu_2}]^{ij}_{\eta\zeta}, \tag{25}$$

where $\{i, j\}$, and $\{\eta, \zeta\}$ are, respectively, techniflavor and Lorentz spinor indices; and the Casimir operator $C_2(N) = (N^2 - 1)/(2N)$.

$B_{\xi,\mu} = W^a_{\xi,\mu} = 0$ *Case: The gap equation*

The technigluon propagator in Landau gauge is $G^{\alpha\beta}_{\mu\nu}(x,y) = \delta^{\alpha\beta}\int\frac{d^4p}{(2\pi)^4}e^{-ip(x-y)}G_{\mu\nu}(p)$ with $G_{\mu\nu}(p) = \frac{i}{-p^2[1+\Pi(-p^2)]}(g_{\mu\nu} - p_\mu p_\nu/p^2)$. And the techniquark self-energy and propagator are, respectively,

$$\Sigma^{ij}_{\eta\zeta}(x,y) = \int\frac{d^4p}{(2\pi)^4}e^{-ip(x-y)}\Sigma^{ij}_{\eta\zeta}(-p^2),$$
$$S^{ij}_{\eta\zeta}(x,y) = \int\frac{d^4p}{(2\pi)^4}e^{-ip(x-y)}S^{ij}_{\eta\zeta}(p), \tag{26}$$

with $S^{ij}_{\eta\zeta}(p) = i[1/(\not{p} - \Sigma(-p^2))]^{ij}_{\eta\zeta}$. Substituting the above results into the SDE (25), we have

$$\Sigma^{ij}_{\eta\zeta}(-p^2) = \int\frac{d^4q}{(2\pi)^4}\frac{-C_2(N)g^2_{\rm TC}}{(p-q)^2[1+\Pi(-(p-q)^2)]} \times\left[g_{\mu\nu} - \frac{(p-q)_\mu(p-q)_\nu}{(p-q)^2}\right] \times\left[\gamma^\mu\frac{i}{\not{q} - \Sigma(-q^2)}\gamma^\nu\right]^{ij}_{\eta\zeta} \tag{27}$$

from which we can see that the solution of the techniquark self-energy must be diagonal in techniflavor space, since the integration kernel is independent of techniflavor indices, i.e., $\Sigma^{ij}_{\eta\zeta}(-p^2) = \delta^{ij}\Sigma_{\eta\zeta}(-p^2)$. With the assumption that the techniquark self-energy is proportional to the unit matrix in Lorentz spinor space, we obtain the following two equations

$$i\Sigma(-p^2) = 3C_2(N)\int\frac{d^4q}{4\pi^3}\frac{\alpha_{\rm TC}[-(p-q)^2]}{(p-q)^2}\frac{\Sigma(-q^2)}{q^2 - \Sigma^2(-q^2)},$$
$$0 = \int d^4q\frac{\alpha_{\rm TC}[-(p-q)^2]}{(p-q)^2} \times\left[g_{\mu\nu} - \frac{(p-q)_\mu(p-q)_\nu}{(p-q)^2}\right]\gamma^\mu \times\frac{\not{q}}{q^2 - \Sigma^2(-q^2)}\gamma^\nu, \tag{28}$$

in which we have labeled the integration kernel with the running coupling constant $\alpha_{\rm TC}(-p^2) \equiv g^2_{\rm TC}(-p^2)/(4\pi) = g^2_{\rm TC}/(4\pi[1+\Pi(-p^2)])$. The second equation of (28) is automatically satisfied when we take *the approximation*, $\alpha_{\rm TC}[(p_E - q_E)^2] = \alpha_{\rm TC}(p_E^2)\theta(p_E^2 - q_E^2) + \alpha_{\rm TC}(q_E^2)\theta(q_E^2 - p_E^2)$. The first equation of (28) can be written in Euclidean space as

$$\Sigma(p_E^2) = 3C_2(N)\int\frac{d^4q_E}{4\pi^3}\frac{\alpha_{\rm TC}[(p_E - q_E)^2]}{(p_E - q_E)^2}\frac{\Sigma_(q_E^2)}{q_E^2 + \Sigma^2(q_E^2)}, \tag{29}$$

If there is a nonzero solution for above equation, we will obtain a nonzero techniquark condensate $\langle\bar\psi^k_\xi\psi^j_\xi\rangle$ with k and j techniflavor indices,

$$\langle\bar\psi^k_\xi(x)\psi^j_\xi(x)\rangle = -{\rm tr}_{lc}[S^{jk}(x,x)] = -4N\delta^{jk}\int\frac{d^4p_E}{(2\pi)^4}\frac{\Sigma(p_E^2)}{p_E^2 + \Sigma^2(p_E^2)}, \tag{30}$$

where ${\rm tr}_{lc}$ is the trace with respect to Lorentz, technicolor indices. In obtaining the above result, we have used the property that techniquark self-energy is diagonal in techniflavor space. Thus, nonzero techniquark self-energy can give a nontrivial diagonal condensate $\langle\bar\psi\psi\rangle \neq 0$, which spontaneously breaks $SU(2)_L \times U(1)_Y$ to $U(1)_{\rm em}$.

$B_{\xi,\mu} \neq 0$ and $W^a_{\xi,\mu} \neq 0$ *Case: the Lowest-order Approximation*

In the following we consider the effects of the nonzero electroweak gauge fields $B_{\xi,\mu}$ and $W^a_{\xi,\mu}$. The SDE (25) in terms of $\Sigma(x,y)$ is explicitly

$$\Sigma(x,y) = C_2(N)g^2_{\rm TC}G_{\mu\nu}(x,y)\gamma^\mu \times\left[i\not{\partial}_x - g_2\frac{\tau^a}{2}\not{W}^a_\xi(x)P_L - g_1\frac{\tau^3}{2}\not{B}_\xi(x)P_R - \Sigma\right]^{-1} \times(x,y)\gamma^\nu, \tag{31}$$

where techniflavor and Lorentz spinor indices of the techniquark self-energy are implicitly contained. In this case, the self-energy can no longer be written as the function on the derivatives with respect to spacetime, i.e., $\Sigma(x,y) \neq \Sigma(\partial_x^2)\delta(x-y)$.

Suppose the function $\Sigma(-p^2)$ is a solution of the SDE in the case $B_{\xi,\mu} = W^a_{\xi,\mu} = 0$, that is, it satisfies the equation

$$\Sigma(-p^2) = C_2(N)g^2_{\rm TC}\int\frac{d^4q}{(2\pi)^4}G_{\mu\nu}(q)\gamma^\mu \times\frac{1}{\not{q} + \not{p} - \Sigma[-(q+p)^2]}\gamma^\nu, \tag{32}$$

where in the right-hand side of the equality the legality of the integration variable translation comes from the logarithmical divergence of the fermion self-energy. Replacing the variable p by $p + \Delta$ in Eq. (32) and subsequently integrating over p with the weight $e^{-ip(x-y)}$, we obtain, as long as Δ is commutative with ∂_x and Dirac matrices, Eq. (32) implies

$$\Sigma[(\partial_x - i\Delta)^2]\delta(x-y) = G_{\mu\nu}(x,y)\gamma^\mu \frac{C_2(N)g_{\rm TC}^2}{i\partial\!\!\!/_x + \Delta\!\!\!/ - \Sigma[-(i\partial_x+\Delta)^2]}\gamma^\nu\delta(x-y). \tag{33}$$

Even if Δ is noncommutative with ∂_x and Dirac matrices, the above equation holds as the lowest-order approximation, for the commutator $[\partial\!\!\!/, \Delta]$ includes extra derivative and therefore belongs to higher order of momentum expansion than Δ itself. Now if we take Δ to be $-g_2\frac{\tau^a}{2}W_\xi^a P_L - g_1\frac{\tau^3}{2}B_\xi P_R$, *ignoring the property that it is noncommutative with* ∂_x *and Dirac matrices*, Eq. (33) is just the SDE (31) in the case $B_{\xi,\mu} \neq 0$ and $W^a_{\xi,\mu} \neq 0$. Thus, $\Sigma[(\partial^x_\mu + ig_2\frac{\tau^a}{2}W^a_{\xi,\mu}P_L + ig_1\frac{\tau^3}{2}B_{\xi,\mu}P_R)^2]\delta(x-y)$, which is the hidden-symmetry $U(1)$ covariant, can be regarded as the lowest-order solution of Eq. (31). To further simplify the calculations and keep the hidden-symmetry covariance of the self-energy in the meantime, we can *reduce the covariant derivative inside the self-energy* $\nabla_\mu \equiv \partial_\mu + ig_2\frac{\tau^a}{2}W^a_{\xi,\mu}P_L + ig_1\frac{\tau^3}{2}B_{\xi,\mu}P_R$ *to the form of minimal coupling* by dropping out its axial vector part,

$$\bar{\nabla}_\mu \equiv \partial_\mu + \frac{i}{2}\left[g_2\frac{\tau^a}{2}W^a_{\xi,\mu}(x) + g_1\frac{\tau^3}{2}B_{\xi,\mu}(x)\right], \tag{34}$$

in which, as we mentioned before, $[g_2\frac{\tau^a}{2}W^a_{\xi,\mu}(x) + g_1\frac{\tau^3}{2}B_{\xi,\mu}(x)]/2$ transforms as a gauge field under hidden $U(1)$ symmetry transformations. Thus, if the function $\Sigma(\partial_x^2)\delta(x-y)$ is the self-energy solution of the SDE in the case $B_{\xi,\mu} = W^a_{\xi,\mu} = 0$, we can replace its argument ∂_x by the minimal-coupling covariant derivative $\bar{\nabla}_x$, i.e., $\Sigma(\bar{\nabla}_x^2)\delta(x-y)$, as an approximate solution of the SDE in the case $B_{\xi,\mu} \neq 0$ and $W^a_{\xi,\mu} \neq 0$.

2. Effective action

The exponential terms on the right-hand side of Eq. (16) can be written explicitly as

$$\sum_{n=2}^{\infty}\int d^4x_1\ldots d^4x_n \frac{(-ig_{\rm TC})^n}{n!}G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x_1,\ldots,x_n)J^{\mu_1}_{\xi,\alpha_1}(x_1)\ldots J^{\mu_n}_{\xi,\alpha_n}(x_n) \approx \int d^4x d^4x' \bar{\psi}^\sigma_\xi(x)\Pi_{\sigma\rho}(x,x')\psi^\rho_\xi(x'), \tag{35}$$

where we have taken the approximation of *replacing the summation over* 2*n-fermion interactions with parts of them by their vacuum expectation values*, that is,

$$\Pi_{\sigma\rho}(x,x') = \sum_{n=2}^{\infty}\Pi^{(n)}_{\sigma\rho}(x,x'), \tag{36}$$

$$\Pi^{(n)}_{\sigma\rho}(x,x') = n\times\int d^4x_2\ldots d^4x_{n-1}\frac{(-ig_{\rm TC})^n}{n!}G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x,x_2\ldots,x_{n-1},x') \times\langle(t^{\alpha_1}\gamma^{\mu_1})_{\sigma\sigma_1}\psi^{\sigma_1}_\xi(x)\bar{\psi}_\xi(x_2)t^{\alpha_2}\gamma^{\mu_2}\psi_\xi(x_2)\ldots\bar{\psi}_\xi(x_{n-1})t^{\alpha_{n-1}}\gamma^{\mu_{n-1}}\psi_\xi(x_{n-1})\bar{\psi}^{\rho_n}_\xi(x')(t^{\alpha_n}\gamma^{\mu_n})_{\rho_n\rho}\rangle \tag{37}$$

where the factor n comes from n different choices of unaveraged $\bar{\psi}_\xi\psi_\xi$, and the lowest term of which is

$$\Pi^{(2)}_{\sigma\rho}(x,x') = -g^2_{\rm TC}G^{\alpha_1\alpha_2}_{\mu_1\mu_2}(x,x')[t^{\alpha_1}\gamma^{\mu_1}S(x,y)t^{\alpha_2}\gamma^{\mu_2}]_{\sigma\rho}. \tag{38}$$

Comparing Eq. (38) with Eq. (24), we have

$$i\Pi^{(2)}_{\sigma\rho}(x,x') = \Sigma_{\sigma\rho}(x,x') \approx \Sigma_{\sigma\rho}(\bar{\nabla}^2_x)\delta(x-y). \tag{39}$$

By neglecting the factor $\mathcal{F}[O_\xi]\delta(O_\xi - O^\dagger_\xi)$ in Eq. (16), we have

$$e^{iS_{\rm norm}[U,W^a_\mu,B_\mu]} \approx \int \mathcal{D}\bar{\psi}_\xi\mathcal{D}\psi_\xi\exp\left[i\int d^4x\bar{\psi}_\xi\left(i\partial\!\!\!/ - g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi P_L - g_1\frac{\tau^3}{2}B\!\!\!/_\xi P_R\right)\psi_\xi + \int d^4xd^4x'\bar{\psi}^\sigma_\xi(x)\Pi_{\sigma\rho}(x,x')\psi^\rho_\xi(x')\right]$$
$$\approx {\rm Det}\left[i\partial\!\!\!/ - g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi P_L - g_1\frac{\tau^3}{2}B\!\!\!/_\xi P_R - \Sigma(\bar{\nabla}^2)\right] \tag{40}$$

where in the second equality we have taken a further approximation of *keeping only the lowest order*, i.e. $\Pi^{(2)}_{\sigma\rho}(x,x')$, *of* $\Pi_{\sigma\rho}(x,x')$. With these three approximations, we have

$$S_{\rm norm}[U,W^a_\mu,B_\mu] = -i{\rm Tr}\log\left[i\partial\!\!\!/ - g_2\frac{\tau^a}{2}W\!\!\!\!/^a_\xi P_L - g_1\frac{\tau^3}{2}B\!\!\!/_\xi P_R - \Sigma(\bar{\nabla}^2)\right], \tag{41}$$

As done in [12], we can parametrize the normal part of the effective action as follows

$$\begin{aligned} S_{\rm norm}[U, W_\mu^a, B_\mu] &= -i\mathrm{Tr}\log[i\partial\!\!\!/ + v\!\!\!/ + a\!\!\!/\gamma_5 - \Sigma(\bar{\nabla}^2)] \\ &= \int d^4x\,\mathrm{tr}_f[(F_0^{\rm 1D})^2 a^2 - \mathcal{K}_1^{\rm 1D}(d_\mu a^\mu)^2 \\ &\quad - \mathcal{K}_2^{\rm 1D}(d_\mu a_\nu - d_\nu a_\mu)^2 + \mathcal{K}_3^{\rm 1D}(a^2)^2 \\ &\quad + \mathcal{K}_4^{\rm 1D}(a_\mu a_\nu)^2 - \mathcal{K}_{13}^{\rm 1D} V_{\mu\nu}V^{\mu\nu} \\ &\quad + i\mathcal{K}_{14}^{\rm 1D} a_\mu a_\nu V^{\mu\nu}] + \mathcal{O}(p^6), \end{aligned} \tag{42}$$

where the fields v_μ, a_μ are identified with $v_\mu \equiv -\frac{1}{2}\times(g_2\frac{\tau^a}{2}W^a_{\xi,\mu} + g_1\frac{\tau^3}{2}B_{\xi,\mu})$ and $a_\mu \equiv \frac{1}{2}(g_2\frac{\tau^a}{2}W^a_{\xi,\mu} - g_1\frac{\tau^3}{2}B_{\xi,\mu})$ and $d_\mu a_\nu \equiv \partial_\mu a_\nu - i[v_\mu, a_\nu]$, $V_{\mu\nu} \equiv i[\partial_\mu - iv_\mu, \partial_\nu - iv_\nu]$. $\mathcal{K}_i^{\rm 1D}$ coefficients with superscript 1D to denote the present one-doublet model are functions of techniquark self-energy $\Sigma(p^2)$ and detailed expressions are already written down in (36) of Ref. [12] with the replacement of $N_c \to N$.

For the anomaly part, compare (15c) and (41), we find its U field dependent part can be produced from the normal part by vanishing techniquark self-energy Σ, i.e.

$$\begin{aligned} iS_{\rm anom}[U, W_\mu^a, B_\mu] &= \mathrm{Tr}\log\Big(i\partial\!\!\!/ - g_2\frac{\tau^a}{2}W\!\!\!\!/^a P_L \\ &\quad - g_1\frac{\tau^3}{2}B\!\!\!/ P_R\Big) \\ &\quad - iS_{\rm norm}[U, W_\mu^a, B_\mu]|_{\Sigma=0} \end{aligned} \tag{43}$$

Notice that the U field independent part of the pure gauge field part is irrelevant to EWCL. Combines with (42), the above relation implies

$$\begin{aligned} iS_{\rm anom}[U, W_\mu^a, B_\mu] &= \mathrm{Tr}\log\Big(i\partial\!\!\!/ - g_2\frac{\tau^a}{2}W\!\!\!\!/^a P_L - g_1\frac{\tau^3}{2}B\!\!\!/ P_R\Big) \\ &\quad + i\int d^4x\,\mathrm{tr}_f[-\mathcal{K}_1^{\rm 1D,(anom)}(d_\mu a^\mu)^2 - \mathcal{K}_2^{\rm 1D,(anom)}(d_\mu a_\nu - d_\nu a_\mu)^2 + \mathcal{K}_3^{\rm 1D,(anom)}(a^2)^2 \\ &\quad + \mathcal{K}_4^{\rm 1D,(anom)}(a_\mu a_\nu)^2 - \mathcal{K}_{13}^{\rm 1D,(anom)}V_{\mu\nu}V^{\mu\nu} + i\mathcal{K}_{14}^{\rm 1D,(anom)}a_\mu a_\nu V^{\mu\nu}] + \mathcal{O}(p^6), \end{aligned} \tag{44}$$

with

$$\mathcal{K}_i^{\rm 1D,(anom)} = -\mathcal{K}_i^{\rm 1D}|_{\Sigma=0} \qquad i = 1, 2, 3, 4, 13, 14 \tag{45}$$

where we have used the result that $F_0^{\rm 1D}|_{\Sigma=0} = 0$. Combine the normal and the anomaly part contribution together, with the help of (15), we finally find

$$\begin{aligned} S_{\rm eff}[U, W_\mu^a, B_\mu] &= -\frac{1}{4}\int d^4x(W^a_{\mu\nu}W^{a,\mu\nu} + B_{\mu\nu}B^{\mu\nu}) + \mathrm{Tr}\log\Big(i\partial\!\!\!/ - g_2\frac{\tau^a}{2}W\!\!\!\!/^a P_L - g_1\frac{\tau^3}{2}B\!\!\!/ P_R\Big) \\ &\quad + i\int d^4x\,\mathrm{tr}_f[(F_0^{\rm 1D})^2a^2 - \mathcal{K}_1^{{\rm 1D},\Sigma\neq0}(d_\mu a^\mu)^2 - \mathcal{K}_2^{{\rm 1D},\Sigma\neq0}(d_\mu a_\nu - d_\nu a_\mu)^2 + \mathcal{K}_3^{{\rm 1D},\Sigma\neq0}(a^2)^2 \\ &\quad + \mathcal{K}_4^{{\rm 1D},\Sigma\neq0}(a_\mu a_\nu)^2 - \mathcal{K}_{13}^{{\rm 1D},\Sigma\neq0}V_{\mu\nu}V^{\mu\nu} + i\mathcal{K}_{14}^{{\rm 1D},\Sigma\neq0}a_\mu a_\nu V^{\mu\nu}] + \mathcal{O}(p^6), \end{aligned} \tag{46}$$

with $\mathcal{K}_i^{{\rm 1D},\Sigma\neq0}$ be Σ dependent part of $\mathcal{K}_i$

$$\mathcal{K}_i^{{\rm 1D},\Sigma\neq0} \equiv \mathcal{K}_i^{\rm 1D} - \mathcal{K}_i^{\rm 1D}|_{\Sigma=0} \qquad i = 1, 2, 3, 4, 13, 14 \tag{47}$$

After some algebra, the terms in Eq. (46) can be reexpressed in terms of X_μ and $\bar{W}_{\mu\nu}$ which are just standard EWCL given in Ref. [2] with coefficients

$$\begin{aligned} &f^2 = (F_0^{\rm 1D})^2, \qquad \beta_1 = 0, \qquad \alpha_1 = \frac{\mathcal{K}_2^{{\rm 1D},\Sigma\neq0} - \mathcal{K}_{13}^{{\rm 1D},\Sigma\neq0}}{2}, \qquad \alpha_2 = \alpha_3 = -\frac{\mathcal{K}_{13}^{{\rm 1D},\Sigma\neq0}}{4} + \frac{\mathcal{K}_{14}^{{\rm 1D},\Sigma\neq0}}{16}, \\ &\alpha_4 = \frac{\mathcal{K}_4^{{\rm 1D},\Sigma\neq0} + 2\mathcal{K}_{13}^{{\rm 1D},\Sigma\neq0} - \mathcal{K}_{14}^{{\rm 1D},\Sigma\neq0}}{16}, \qquad \alpha_5 = \frac{\mathcal{K}_3^{{\rm 1D},\Sigma\neq0} - \mathcal{K}_4^{{\rm 1D},\Sigma\neq0} - 4\mathcal{K}_{13}^{{\rm 1D},\Sigma\neq0} + 2\mathcal{K}_{14}^{\Sigma\neq0}}{32}. \end{aligned} \tag{48}$$

With formulas of $\mathcal{K}_i$ coefficients depending on techniquark self-energy $\Sigma(p^2)$ given in Ref. [12], we can substitute the solution of SD Eq. (29) for $\Sigma(p^2)$ into them and then obtain numerical results for those nonzero α_i coefficients. In Table I, we list down the numerical calculation results for different kinds of dynamics.

To obtain the above numerical result, we have solved the Schwinger-Dyson equation (29) with the following running coupling which was used as model A in Ref. [12]

TABLE I. The obtained nonzero values of the $O(p^4)$ coefficients α_1, $\alpha_2 = \alpha_3$, α_4, α_5 for the one-doublet technicolor model with the conventional strong interaction QCD theory values given in Ref. [12] for model A and experimental values for comparison. $\Lambda_{\rm TC}$ is in TeV and $\Lambda_{\rm QCD} = 484$ MeV. They are determined by $f = 250$ GeV and $f_\pi = 93$ MeV, respectively. The coefficients are in units of 10^{-3}. QCD values are taken by using relation (9).

N	$\Lambda_{\rm TC}$	α_1	$\alpha_2 = \alpha_3$	α_4	α_5
3	1.34	−6.90	−2.43	2.02	−2.69
4	1.15	−9.26	−3.28	2.87	−3.69
5	1.03	−11.6	−4.11	3.60	−4.62
6	0.94	−13.9	−4.93	4.32	−5.54
QCD	Theor	−7.06	−2.54	2.20	−2.81
QCD	Expt	-6.0 ± 0.7	-2.7 ± 0.4	1.7 ± 0.7	-1.3 ± 1.5

$$\alpha_{\rm TC}(p^2) = \begin{cases} 7\frac{12\pi}{(11N-2N_f)} & \ln\frac{p^2}{\Lambda_{\rm TC}^2} \le -2; \\ \frac{12\pi\{7-\frac{4}{5}[2+\ln\frac{p^2}{\Lambda_{\rm TC}^2}]^2\}}{(11N-2N_f)} & -2 \le \ln\frac{p^2}{\Lambda_{\rm TC}^2} \le 0.5; \\ \frac{1}{\ln\frac{p^2}{\Lambda_{\rm TC}^2}}\frac{12\pi}{(11N-2N_f)} & 0.5 \le \ln\frac{p^2}{\Lambda_{\rm TC}^2}, \end{cases} \quad (49)$$

in which the fermion number is taken to be $N_f = 2$ corresponding to the present one-doublet techniquark. Although there is a dimensional parameter $\Lambda_{\rm TC}$ appearing in $\alpha_{\rm TC}(p^2)$, except dimensional coefficient $F_0^{\rm 1D}$, all dimensionless result coefficients α_i, $i = 1, 2, 3, 4, 5$ are independent of this parameter. This can be seen as follows. If we scale up $\Lambda_{\rm TC}$ as $\lambda\Lambda_{\rm TC}$, $\alpha_{\rm TC}(p)$ defined above satisfies $\alpha_{\rm TC}(p^2)|_{\lambda\Lambda_{\rm TC}} = \alpha_{\rm TC}(\lambda^{-2}p^2)|_{\Lambda_{\rm TC}}$ which, by (29), results in a scaling-up techniquark self-energy $\Sigma(p^2)|_{\lambda\Lambda_{\rm TC}} = \lambda\Sigma(\lambda^{-2}p^2)|_{\Lambda_{\rm TC}}$, since an alternative expression of (29) is

$$\lambda\Sigma(\lambda^{-2}p_E^2) = 3C_2(N)\int\frac{d^4q_E}{4\pi^3}\frac{\alpha_{\rm TC}[\lambda^{-2}(p_E-q_E)^2]}{(p_E-q_E)^2} \times\frac{\lambda\Sigma(\lambda^{-2}q_E^2)}{q_E^2+\lambda^2\Sigma^2(\lambda^{-2}q_E^2)}. \quad (50)$$

Further from (36) of Ref. [12], we find coefficients $\mathcal{K}_i^{\Sigma\neq 0}$, $i = 1, 2, 3, 4, 13, 14$ are invariant and F_0 is changed to λF_0 under exchanging $\Sigma(p^2) \to \lambda\Sigma(\lambda^{-2}p^2)$ if we take cutoff in the formulas $\Lambda \to \infty$. Because of this invariance for $\mathcal{K}_i^{\Sigma\neq 0}$, $i = 1, 2, 3, 4, 13, 14$, from (48), we can see then α_i, $i = 1, 2, 3, 4, 5$ are independent of $\Lambda_{\rm TC}$ and $F_0^{\rm 1D}$ scales the same as $\Lambda_{\rm TC}$. It is this scale dependence for F_0 which makes ordinary QCD contribution small to electroweak symmetry breaking and leads the necessity for new strong interactions at a higher energy scale. The scale relations above are the result of present rough approximations; they will simplify our future computations very much in the next section.

From Table I, we see that within the errors of our approximation, the numerical results exhibit the scaling-up behavior among different N.

C. Comparison and discussion on two prescriptions

Compare results from Gasser-Leutwyler's prescription and dynamical computation prescription, (9) and (48), we find results are the same as long as we identify

$$\begin{aligned} H_1^{\rm 1D} &= -\frac{\mathcal{K}_2^{\rm 1D,\Sigma\neq 0}+\mathcal{K}_{13}^{\rm 1D,\Sigma\neq 0}}{4} \\ L_{10}^{\rm 1D} &= \frac{\mathcal{K}_2^{\rm 1D,\Sigma\neq 0}-\mathcal{K}_{13}^{\rm 1D,\Sigma\neq 0}}{2} \\ L_9^{\rm 1D} &= \frac{\mathcal{K}_{13}^{\rm 1D,\Sigma\neq 0}}{2}-\frac{\mathcal{K}_{14}^{\rm 1D,\Sigma\neq 0}}{8} \\ L_2^{\rm 1D} &= \frac{\mathcal{K}_4^{\rm 1D,\Sigma\neq 0}+2\mathcal{K}_{13}^{\rm 1D,\Sigma\neq 0}-\mathcal{K}_{14}^{\rm 1D,\Sigma\neq 0}}{16} \\ L_1^{\rm 1D}+\frac{L_3^{\rm 1D}}{2} &= \frac{\mathcal{K}_3^{\rm 1D,\Sigma\neq 0}-\mathcal{K}_4^{\rm 1D,\Sigma\neq 0}-4\mathcal{K}_{13}^{\rm 1D,\Sigma\neq 0}+2\mathcal{K}_{14}^{\rm 1D,\Sigma\neq 0}}{32} \end{aligned} \quad (51)$$

which is just the result (25) obtained in Ref. [12]. This shows that the two prescriptions are equivalent in results. The merit of Gasser-Leutwyler's prescription is its simplicity and the ability to express resulting coefficients of EWCL in terms of those in Gasser-Leutwyler chiral Lagrangian for pseudoscalar mesons in ordinary strong interaction, but we can only apply this prescription to so-called QCD-like theories for which the technicolor interaction must be vectorlike. On the other hand, the dynamical computation prescription, much more complex but touches the dynamics details, does not limit us in the type of detailed interactions. This has a very strong potential to be applied to more complicated theories, such as chiral-like technicolor models. Since we involve detailed dynamical computation in this prescription, not like Gasser-Leutwyler's prescription where the coefficients are expressed in terms of strong interaction experiment fixed values, we can give detailed theoretical computation results for all coefficients and it further allows us to test possible effects on the coefficients from variations of the dynamics.

The first property qualitatively drew out from (15c) and (41) for their trace operation is that all coefficients are proportional to N. This is the well-known scaling-up result for the one-doublet technicolor model, i.e., present coefficients can be obtained by (9) but identify the L_i with the corresponding Gasser-Leutwyler chiral Lagrangian for pseudoscalar mesons by a scaling-up factor N/N_c with $N_c = 3$ as the number of color for ordinary strong interactions. In fact, it was this direct correspondence that lead to the death of the one-doublet technicolor model, since negative experiment value for $L_{10}^{\rm 1D}$ results in large positive

$S = -16\pi\alpha_1 = -16\pi L_{10}^{1D}$ parameter which contradicts with present electroweak precision measurement data.

The second property quantitatively drawn out from (15c) and (41) is that except for the overall N factor in front of all coefficients, the remaining part of the coefficients depends on dynamics, so exactly speaking, they are not precisely equivalent to their strong interaction partners. For the one-doublet technicolor model, when $N \neq N_c$, not only will we have an overall scaling-up factor N/N_c, but we will also have different techniquark self-energy Σ due to the difference in the running coupling constant (49) appeared in SDE (29). $N_f = 2$ also causes some differences for techniquark self-energy Σ due to a different choice of flavor number in running coupling constant (49). But, since the estimations over the values in the original strong interaction already suffers large errors either in experiment or theories, this difference caused by dynamics hides in the existing uncertainties.

III. DERIVATION OF EWCL FROM A TOPCOLOR-ASSISTED TECHNICOLOR MODEL

There are several options in topcolor-assisted technicolor model building: (1) TC breaks both the EW interactions and the TopC interactions; (2) TC breaks EW, and something else breaks TopC; (3) TC breaks only TopC and something else drives EWSB (e.g., a fourth generation condensate driven by TopC). For definiteness, we will focus on a skeletal model in category (1) in the following.

Consider a schematic TC2 model proposed by C. T. Hill [21]. The technicolor group is chosen to be $G_{\rm TC} = SU(3)_{\rm TC1} \times SU(3)_{\rm TC2}$. The gauge charge assignments of techniquarks in $G_{\rm TC} \times SU(3)_1 \times SU(3)_2 \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2}$ are shown in Table II.

The action of the symmetry breaking sector then is

$$S_{\rm SBS} = \int d^4x(\mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm techniquark}), \qquad (52)$$

with different parts of the Lagrangian given by

TABLE II. Gauge charge assignments of techniquarks for a schematic topcolor-assisted technicolor model. Ordinary quarks and additional fields (such as leptons) required for anomaly cancellation are not shown. The techniquark condensate $\langle\bar{Q}Q\rangle$ breaks $SU(3)_1 \times SU(3)_2 \times U(1)_{Y_1} \times U(1)_{Y_2} \to SU(3) \times U(1)_Y$, while $\langle\bar{T}T\rangle$ breaks $SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$.

field	$SU(3)_{\rm TC1}$	$SU(3)_{\rm TC2}$	$SU(3)_1$	$SU(3)_2$	$SU(2)_L$	$U(1)_{Y_1}$	$U(1)_{Y_2}$
Q_L	3	1	3	1	1	$\frac{1}{2}$	0
Q_R	3	1	1	3	1	0	$\frac{1}{2}$
T_L	1	3	1	1	2	0	$\frac{1}{6}$
T_R	1	3	1	1	1	0	$(\frac{2}{3}, -\frac{1}{3})$

$$\mathcal{L}_{\rm gauge} = -\tfrac{1}{4}F^{\alpha}_{1\mu\nu}F^{\alpha\mu\nu}_1 - \tfrac{1}{4}F^{\alpha}_{2\mu\nu}F^{\alpha\mu\nu}_2 - \tfrac{1}{4}A^{A}_{1\mu\nu}A^{A\mu\nu}_1 - \tfrac{1}{4}A^{A}_{2\mu\nu}A^{A\mu\nu}_2 - \tfrac{1}{4}W^{a}_{\mu\nu}W^{a\mu\nu} - \tfrac{1}{4}B_{1\mu\nu}B^{\mu\nu}_1 - \tfrac{1}{4}B_{2\mu\nu}B^{\mu\nu}_2, \qquad (53)$$

$$\begin{aligned}\mathcal{L}_{\rm techniquark} = \bar{Q}\Big(& i\partial\!\!\!/ - g_{31}r_1^{\alpha}G\!\!\!/_1^{\alpha} - h_1\frac{\lambda^A}{2}A\!\!\!/_1^A P_L - h_2\frac{\lambda^A}{2}A\!\!\!/_2^A P_R - q_1\frac{1}{2}B\!\!\!/_1 P_L - q_2\frac{1}{2}B\!\!\!/_2 P_R\Big)Q \\ & + \bar{T}\Big[i\partial\!\!\!/ - g_{32}r_2^{\alpha}G\!\!\!/_2^{\alpha} - g_2\frac{\tau^a}{2}W\!\!\!/ P_L - q_2\frac{1}{6}B\!\!\!/_2 P_L - q_2\Big(\frac{1}{6}+\frac{\tau^3}{2}\Big)B\!\!\!/_2 P_R\Big]T, \qquad (54)\end{aligned}$$

where g_{31}, g_{32}, h_1, h_2, g_2, q_1, and q_2 are the coupling constants of, respectively, $SU(3)_{\rm TC1}$, $SU(3)_{\rm TC2}$, $SU(3)_1$, $SU(3)_2$, $SU(2)_L$, $U(1)_{Y_1}$, and $U(1)_{Y_2}$; and the corresponding gauge fields (field strength tensors) are denoted by $G^{\alpha}_{1\mu}$, $G^{\alpha}_{2\mu}$, $A^{A}_{1\mu}$, $A^{A}_{2\mu}$, W^{a}_{μ}, $B_{1\mu}$, and $B_{2\mu}$ ($F^{\alpha}_{1\mu\nu}$, $F^{\alpha}_{2\mu\nu}$, $A^{A}_{1\mu\nu}$, $A^{A}_{2\mu\nu}$, $W^{a}_{\mu\nu}$, $B_{1\mu\nu}$, and $B_{2\mu\nu}$) with the superscripts α and A running from 1 to 8 and a from 1 to 3; r_1^{α} and r_2^{α} ($\alpha = 1, \ldots, 8$) are the generators of, respectively, $SU(3)_{\rm TC1}$ and $SU(3)_{\rm TC2}$, while λ^A ($A = 1, \ldots, 8$) and τ^a ($a = 1, 2, 3$) are, respectively, Gell-Mann and Pauli matrices. We do not consider the ordinary quarks in this work for the following considerations, as we mentioned in the Introduction that this paper will only involve the discussion for the bosonic part of EWCL and the matter part of EWCL will be discussed in the future. The matter part of EWCL mainly deals with effective interactions among ordinary fermions which certainly include ordinary quarks. Ignoring discussion of these effective interactions, only concentrating on their contribution to bosonic part EWCL coefficients is not self-consistent and efficient. Furthermore, one special feature of the topcolor-assisted technicolor model is its arrangements on the interactions among ordinary quarks, especially for top and bottom quark mass splitting. The top pions resulted from top quark condensation through topcolor interactions; therefore, dealing with quark interactions is a separate important issue which needs special care. Previous formal derivations from underlying gauge theory to the low energy chiral Lagrangian for QCD and electroweak theory, do not include the matter part of the chiral Lagrangian. Further initial computation shows that we need some special techniques to handle top-bottom splitting which are beyond those techniques developed in this paper. To simplify the computations and reduce the lengthy formulas, we will not discuss ordinary quarks in this paper and would rather focus our attention on this issue in future works.

The strategy to derive the EWCL from the schematic topcolor-assisted technicolor model can be formulated as

$$\begin{aligned}\exp(iS_{\rm EW}[W^a_\mu, B_\mu]) &= \int \mathcal{D}\bar{Q}\mathcal{D}Q\mathcal{D}\bar{T}\mathcal{D}T\mathcal{D}G^\alpha_{1\mu}\mathcal{D}G^\alpha_{2\mu}\mathcal{D}B^A_\mu\mathcal{D}Z'_\mu \\ &\quad\times \exp[iS_{\rm SBS}[G^\alpha_{1\mu}, G^\alpha_{2\mu}, A^A_{1\mu}, A^A_{2\mu}, W^a_\mu, B_{1\mu}, B_{2\mu}, \bar{Q}, Q, \bar{T}, T]]_{A^A_\mu=0} \\ &= \mathcal{N}[W^a_\mu, B_\mu]\int \mathcal{D}\mu(U)\exp(iS_{\rm eff}[U, W^a_\mu, B_\mu]), \end{aligned} \tag{55}$$

where $A^A_\mu(x)$ is the ordinary gluon field, $U(x)$ is a dimensionless unitary unimodular matrix field in the electroweak chiral Lagrangian, and $\mathcal{D}\mu(U)$ denotes a normalized functional integration measure on U. The normalization factor $\mathcal{N}[W^a_\mu, B_\mu]$ is determined through the requirement that when the gauge coupling g_{32} is switched off $S_{\rm eff}[U, W^a_\mu, B_\mu]$ vanishes, this leads to the electroweak gauge fields W^a_μ, B_μ dependent part of $\mathcal{N}[W^a_\mu, B_\mu]$ is

$$\mathcal{N}[W^a_\mu, B_\mu] = \int \mathcal{D}\bar{Q}\mathcal{D}Q\mathcal{D}\bar{T}\mathcal{D}T\mathcal{D}G^\alpha_{1\mu}\mathcal{D}B^A_\mu\mathcal{D}Z'_\mu \exp[iS_{\rm SBS}[G^\alpha_{1\mu}, 0, A^A_{1\mu}, A^A_{2\mu}, W^a_\mu, B_{1\mu}, B_{2\mu}, \bar{Q}, Q, \bar{T}, T]]_{A^A_\mu=0} \tag{56}$$

Since there are different interactions in the present model, in the following several subsections we discuss them and their contributions to EWCL separately.

A. Topcolor symmetry breaking: The contribution of $SU(3)_{\rm TC1}$

It can be shown below, by analysis with the help of SDE, that the $SU(3)_{\rm TC1}$ interaction induces the techniquark condensate $\langle\bar{Q}Q\rangle \neq 0$, which will trigger the topcolor symmetry breaking $SU(3)_1 \times SU(3)_2 \times U(1)_{Y_1} \times U(1)_{Y_2} \to SU(3)_c \times U(1)_Y$ at the scale $\Lambda = 1$ TeV. This typically leaves a degenerate, massive color octet of "colorons," B^A_μ, and a singlet heavy Z'_μ in the coset space $[SU(3)_1 \times SU(3)_2 \times U(1)_{Y_1} \times U(1)_{Y_2}]/[SU(3)_c \times U(1)_Y]$. The gluon A^A_μ and coloron B^A_μ (the SM $U(1)_Y$ field B_μ and the $U(1)'$ field Z'_μ) are defined by orthogonal rotations with mixing angle θ (θ'):

$$\begin{pmatrix} A^A_{1\mu} & A^A_{2\mu}\end{pmatrix} = \begin{pmatrix} B^A_\mu & A^A_\mu\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}, \tag{57a}$$

$$\begin{pmatrix} B_{1\mu} & B_{2\mu}\end{pmatrix} = \begin{pmatrix} Z'_\mu & B_\mu\end{pmatrix}\begin{pmatrix}\cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta'\end{pmatrix}, \tag{57b}$$

which leads to

$$-h_1\frac{\lambda^A}{2}\not{A}^A_1 P_L - h_2\frac{\lambda^A}{2}\not{A}^A_2 P_R \tag{58a}$$

$$= -g_3\frac{\lambda^A}{2}\not{A}^A - g_3(\cot\theta P_L - \tan\theta P_R)\frac{\lambda^A}{2}\not{B}^A, \qquad -q_1\frac{1}{2}\not{B}_1 P_L - q_2\frac{1}{2}\not{B}_2 P_R = -g_1\frac{1}{2}\not{B} - g_1(\cot\theta' P_L - \tan\theta' P_R)\frac{1}{2}\not{Z}', \tag{58b}$$

with

$$g_3 \equiv h_1\sin\theta = h_2\cos\theta \qquad g_1 \equiv q_1\sin\theta' = q_2\cos\theta'. \tag{59}$$

As a first step, we formally integrate out the $SU(3)_{\rm TC1}$ technigluons $G^\alpha_{1\mu}$ in Eq. (55) by introducing full n-point Green's function of the $G^\alpha_{1\mu}$-field $G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}$

$$\begin{aligned}\exp(iS_{\rm EW}[W^a_\mu, B_\mu]) &= \exp\Big[i\int d^4x\Big(-\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu}\Big)\Big]\int \mathcal{D}\bar{T}\mathcal{D}T\mathcal{D}G^\alpha_{2\mu}\mathcal{D}B^A_\mu\mathcal{D}Z'_\mu \\ &\quad\times\exp\Big[i\int d^4x\Big(-\frac{1}{4}F^\alpha_{2\mu\nu}F^{\alpha\mu\nu}_2 - \frac{1}{4}A^A_{1\mu\nu}A^{A\mu\nu}_1 - \frac{1}{4}A^A_{2\mu\nu}A^{A\mu\nu}_2 - \frac{1}{4}B_{1\mu\nu}B^{\mu\nu}_1 - \frac{1}{4}B_{2\mu\nu}B^{\mu\nu}_2\Big) \\ &\quad + iS_{\rm TC1}[A^A_\mu, B^A_\mu, B_{1\mu}, B_{2\mu}] \\ &\quad + i\int d^4x\bar{T}\Big[i\not{\partial} - g_{32}r^\alpha_2\not{G}^\alpha_2 - g_2\frac{\tau^a}{2}\not{W}^a P_L - q_2\frac{1}{6}\not{B}_2 P_L - q_2\Big(\frac{1}{6}+\frac{\tau^3}{2}\Big)\not{B}_2 P_R\Big]T\Big]_{A^A_\mu=0},\end{aligned} \tag{60}$$

where

$$\exp(iS_{\mathrm{TC1}}[A_\mu^A, B_\mu^A, B_{1\mu}, B_{2\mu}]) \equiv \int \mathcal{D}\bar{Q}\mathcal{D}Q \exp\Big[i\int d^4x\bar{Q}\Big(i\partial\!\!\!/ - h_1\frac{\lambda^A}{2}A\!\!\!/_1^A P_L - h_2\frac{\lambda^A}{2}A\!\!\!/_2^A P_R - q_1\frac{1}{2}B\!\!\!/_1 P_L - q_2\frac{1}{2}B\!\!\!/_2 P_R\Big)Q + \sum_{n=2}^{\infty}\int d^4x_1\ldots d^4x_n\frac{(-ig_{31})^n}{n!}G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x_1,\ldots,x_n)J^{\mu_1}_{1\alpha_1}(x_1)\ldots J^{\mu_n}_{1\alpha_n}(x_n)\Big]. \tag{61}$$

$J^\mu_{1\alpha}(x) \equiv \bar{Q}(x)r_1^\alpha\gamma^\mu Q(x)$ is the effective source.

Since the total functional derivative of the integrand in Eq. (61) with respect to $\bar{Q}^\sigma(x)$ is zero, (here and henceforth the suffixes σ and ρ are short notations for Lorentz spinor, techniflavor, and technicolor indices) i.e.,

$$0 = \int \mathcal{D}\bar{Q}\mathcal{D}Q\frac{\delta}{\delta\bar{Q}^\sigma(x)}\exp\Big[\int d^4x(\bar{Q}I + \bar{I}Q) + i\int d^4x\bar{Q}\Big(i\partial\!\!\!/ - h_1\frac{\lambda^A}{2}A\!\!\!/_1^A P_L - h_2\frac{\lambda^A}{2}A\!\!\!/_2^A P_R - q_1\frac{1}{2}B\!\!\!/_1 P_L - q_2\frac{1}{2}B\!\!\!/_2 P_R\Big)Q + \sum_{n=2}^{\infty}\int d^4x_1\ldots d^4x_n\frac{(-ig_{31})^n}{n!}G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x_1,\ldots,x_n)J^{\mu_1}_{1\alpha_1}(x_1)\ldots J^{\mu_n}_{1\alpha_n}(x_n)\Big], \tag{62}$$

where $I(x)$ and $\bar{I}(x)$ are the external sources for, respectively, $\bar{Q}(x)$ and $Q(x)$, then we continue the similar procedure from (18) to (21). By *neglecting higher-point Green's functions* and *taking factorization approximation*, we obtain

$$0 = \delta_{\sigma\rho}\delta(x-y) + i\Big[i\partial\!\!\!/_x - h_1\frac{\lambda^A}{2}A\!\!\!/_1^A(x)P_L - h_2\frac{\lambda^A}{2}A\!\!\!/_2^A(x)P_R - q_1\frac{1}{2}B\!\!\!/_1(x)P_L - q_2\frac{1}{2}B\!\!\!/_2(x)P_R\Big]_{\sigma\tau}\langle Q^\tau(x)\bar{Q}^\rho(y)\rangle - g_{31}^2\int d^4x_2 G^{\alpha_1\alpha_2}_{\mu_1\mu_2}(x,x_2)(r_1^{\alpha_1}\gamma^{\mu_1})_{\sigma\tau}(r_1^{\alpha_2}\gamma^{\mu_2})_{\gamma\delta}\langle\bar{Q}^\rho(y)Q^\delta(x_2)\rangle\langle\bar{Q}^\gamma(x_2)Q^\tau(x)\rangle, \tag{63}$$

where $\langle\mathcal{O}(x)\rangle \equiv \langle\langle\mathcal{O}(x)\rangle\rangle_I/\langle\langle 1\rangle\rangle_I|_{I=\bar{I}=0}$ and we have defined the notation $\langle\langle\cdots\rangle\rangle_I$ in this section by

$$\langle\langle\mathcal{O}(x)\rangle\rangle_I \equiv \int \mathcal{D}\bar{Q}\mathcal{D}Q\mathcal{O}(x)\exp\Big[\int d^4x(\bar{Q}I + \bar{I}Q) + i\int d^4x\bar{Q}\Big(i\partial\!\!\!/ - h_1\frac{\lambda^A}{2}A\!\!\!/_1^A P_L - h_2\frac{\lambda^A}{2}A\!\!\!/_2^A P_R - q_1\frac{1}{2}B\!\!\!/_1 P_L - q_2\frac{1}{2}B\!\!\!/_2 P_R\Big)Q + \sum_{n=2}^{\infty}\int d^4x_1\ldots d^4x_n\frac{(-ig_{31})^n}{n!}G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x_1,\ldots,x_n)J^{\mu_1}_{1\alpha_1}(x_1)\ldots J^{\mu_n}_{1\alpha_n}(x_n)\Big]. \tag{64}$$

Denote the technifermion propagator $S^{\sigma\rho}(x,x') \equiv \langle Q^\sigma(x)\bar{Q}^\rho(x')\rangle$, Eq. (63) can be written as SDE for the techniquark propagator,

$$0 = S^{-1}_{\sigma\rho}(x,y) + i\Big[i\partial\!\!\!/_x - h_1\frac{\lambda^A}{2}A\!\!\!/_1^A(x)P_L - h_2\frac{\lambda^A}{2}A\!\!\!/_2^A(x)P_R - q_1\frac{1}{2}B\!\!\!/_1(x)P_L - q_2\frac{1}{2}B\!\!\!/_2(x)P_R\Big]_{\sigma\rho}\delta(x-y) - g_{31}^2 G^{\alpha_1\alpha_2}_{\mu_1\mu_2}(x,y)[r_1^{\alpha_1}\gamma^{\mu_1}S(x,y)r_1^{\alpha_2}\gamma^{\mu_2}]_{\sigma\rho}. \tag{65}$$

By defining techniquark self-energy Σ as

$$i\Sigma_{\sigma\rho}(x,y) \equiv S^{-1}_{\sigma\rho}(x,y) + i\Big[i\partial\!\!\!/_x - h_1\frac{\lambda^A}{2}A\!\!\!/_1^A(x)P_L - h_2\frac{\lambda^A}{2}A\!\!\!/_2^A(x)P_R - q_1\frac{1}{2}B\!\!\!/_1(x)P_L - q_2\frac{1}{2}B\!\!\!/_2(x)P_R\Big]_{\sigma\rho}\delta(x-y), \tag{66}$$

the SDE (65) can be written as

$$i\Sigma_{\sigma\rho}(x,y) = g_{31}^2 G^{\alpha_1\alpha_2}_{\mu_1\mu_2}(x,y)[r_1^{\alpha_1}\gamma^{\mu_1}S(x,y)r_1^{\alpha_2}\gamma^{\mu_2}]_{\sigma\rho}. \tag{67}$$

Moreover, from the fact that the technigluon propagator is diagonal in the adjoint representation space of $SU(3)_{\mathrm{TC1}}$, i.e., $G^{\alpha\beta}_{\mu\nu}(x,y) = \delta^{\alpha\beta}G_{\mu\nu}(x,y)$, and techniquark propagator $\langle Q\bar{Q}\rangle$ is diagonal in the fundamental representation space of $SU(3)_{\mathrm{TC1}}$, and also $(r_1^\alpha r_1^\alpha)_{ab} = C_2(3)\delta_{ab}$, Eq. (67) is diagonal in indices a, b and the diagonal part becomes

$$i\Sigma^{ij}_{\eta\zeta}(x,y) = C_2(3)g_{31}^2 G_{\mu_1\mu_2}(x,y)[\gamma^{\mu_1}S(x,y)\gamma^{\mu_2}]^{ij}_{\eta\zeta}, \tag{68}$$

where $\{i,j\}$ and $\{\eta,\zeta\}$ are, respectively, techniflavor and Lorentz spinor indices, and the Casimir operator $C_2(3) = (3^2-1)/(2\times 3) = 4/3$.

1. The gap equation

We first consider the case of $A^A_{1\mu} = A^A_{2\mu} = B_{1\mu} = B_{2\mu} = 0$. The $SU(3)_{\mathrm{TC1}}$ technigluon propagator in Landau gauge is

$$G^{\alpha\beta}_{\mu\nu}(x,y) = \int \frac{d^4p}{(2\pi)^4} \frac{\delta^{\alpha\beta} i e^{-ip(x-y)}}{-p^2[1+\Pi(-p^2)]}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right), \tag{69}$$

In the case of $A^A_{1\mu} = A^A_{2\mu} = B_{1\mu} = B_{2\mu} = 0$, the $SU(3)_{\rm TC1}$ techniquark self-energy and propagator are, respectively,

$$\begin{aligned} \Sigma^{ij}_{\eta\zeta}(x,y) &= \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \Sigma^{ij}_{\eta\zeta}(-p^2), \\ S^{ij}_{\eta\zeta}(x,y) &= \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} S^{ij}_{\eta\zeta}(p), \end{aligned} \tag{70}$$

with $S^{ij}_{\eta\zeta}(p) = i\{1/(\not{p} - \Sigma(-p^2))\}^{ij}_{\eta\zeta}$. Substituting the above equations into the SDE (68), we have

$$\begin{aligned} \Sigma^{ij}_{\eta\zeta}(-p^2) = \int \frac{d^4q}{(2\pi)^4} & \frac{-C_2(3) g^2_{31}}{(p-q)^2[1+\Pi(-(p-q)^2)]} \\ & \times \left[g_{\mu\nu} - \frac{(p-q)_\mu (p-q)_\nu}{(p-q)^2} \right] \\ & \times \left[\gamma^\mu \frac{i}{\not{q} - \Sigma(-q^2)} \gamma^\nu \right]^{ij}_{\eta\zeta} \end{aligned} \tag{71}$$

As discussions of the dynamical computation prescription for the one-doublet technicolor model, the above equation will lead $\Sigma^{ij}_{\eta\zeta}(-p^2) = \delta^{ij}\delta_{\eta\zeta}\Sigma_{\rm TC}(p_E^2)$ and in Euclidean space $\Sigma_{\rm TC}(p_E^2)$ satisfies

$$\begin{aligned} \Sigma_{\rm TC1}(p_E^2) = 3C_2(3) \int \frac{d^4q_E}{4\pi^3} & \frac{\alpha_{31}[(p_E - q_E)^2]}{(p_E - q_E)^2} \\ & \times \frac{\Sigma_{\rm TC1}(q_E^2)}{q_E^2 + \Sigma^2_{\rm TC1}(q_E^2)}. \end{aligned} \tag{72}$$

The corresponding techniquark condensate $\langle \bar{Q}^k Q^j \rangle$ with k and j techniflavor indices,

$$\langle \bar{Q}^k(x) Q^j(x) \rangle = -12\delta^{jk} \int \frac{d^4p_E}{(2\pi)^4} \frac{\Sigma_{\rm TC}(p_E^2)}{p_E^2 + \Sigma^2_{\rm TC}(p_E^2)}, \tag{73}$$

Nonzero techniquark self-energy can give a nontrivial diagonal condensate $\langle \bar{Q} Q \rangle \neq 0$, which spontaneously breaks $SU(3)_1 \times SU(3)_2 \times U(1)_{Y_1} \times U(1)_{Y_2} \to SU(3)_c \times U(1)_Y$.

In the following we consider the effects of the nonzero electroweak gauge fields $A^A_{1\mu}, A^A_{2\mu}, B_{1\mu}$, and $B_{2\mu}$. The SDE (68) is explicitly

$$\begin{aligned} \Sigma(x,y) = C_2(3) g^2_{31} G_{\mu\nu}(x,y) \gamma^\mu \Bigg[\Bigg(& i\not{\partial}_x - h_1 \frac{\lambda^A}{2} \not{A}^A_1(x) P_L \\ & - h_2 \frac{\lambda^A}{2} \not{A}^A_2(x) P_R - q_1 \frac{1}{2} \not{B}_1(x) P_L \\ & - q_2 \frac{1}{2} \not{B}_2(x) P_R \Bigg) \delta(x-y) - \Sigma(x,y) \Bigg]^{-1} \gamma^\nu, \end{aligned} \tag{74}$$

where the techniflavor and Lorentz spinor indices of the techniquark self-energy are implicitly contained.

Suppose the function $\Sigma_{\rm TC1}(-p^2)$ is a solution of the SDE in the case $A^A_{1\mu} = A^A_{2\mu} = B_{1\mu} = B_{2\mu} = 0$, that is, it satisfies the equation

$$\begin{aligned} \Sigma_{\rm TC1}(-p^2) = C_2(3) g^2_{31} \int & \frac{d^4q}{(2\pi)^4} G_{\mu\nu}(q^2) \gamma^\mu \\ & \times \frac{1}{\not{q} + \not{p} - \Sigma_{\rm TC1}[-(q+p)^2]} \gamma^\nu, \end{aligned} \tag{75}$$

Replacing the variable p by $p + \Delta$ in Eq. (75) and subsequently integrating over p with the weight $e^{-ip(x-y)}$, we obtain, as long as Δ is commutative with ∂_x and Dirac matrices,

$$\begin{aligned} & \Sigma_{\rm TC1}[(\partial_x - i\Delta)^2]\delta(x-y) \\ & = G_{\mu\nu}(x,y)\gamma^\mu \frac{C_2(3) g^2_{31}}{i\not{\partial}_x + \not{\Delta} - \Sigma_{\rm TC1}[-(i\partial_x + \Delta)^2]} \delta(x-y)\gamma^\nu. \end{aligned} \tag{76}$$

Now if we take Δ to be $-h_1 \frac{\lambda^A}{2} A^A_1 P_L - h_2 \frac{\lambda^A}{2} A^A_2 P_R - q_1 \frac{1}{2} B_1 P_L - q_2 \frac{1}{2} B_2 P_R$, *ignoring the property that it is noncommutative with* ∂_x *and Dirac matrices*, Eq. (76) is just the SDE (74) in the case $A^A_{1\mu} \neq 0$, $A^A_{2\mu} \neq 0$, $B_{1\mu} \neq 0$, and $B_{2\mu} \neq 0$. Thus, $\Sigma_{\rm TC1}[(\partial^x_\mu + ih_1 \frac{\lambda^A}{2} A^A_{1\mu} P_L + ih_2 \frac{\lambda^A}{2} A^A_{2\mu} P_R + iq_1 \frac{1}{2} B_{1\mu} P_L + iq_2 \frac{1}{2} B_{2\mu} P_R)^2]\delta(x-y)$, which is $SU(3)_1 \times SU(3)_2 \times U(1)_{Y_1} \times U(1)_{Y_2}$ covariant, can be regarded as the lowest-order solution of Eq. (74). From Eqs. (58a) and (58b), we can write the covariant derivative of $SU(3)_1 \times SU(3)_2 \times U(1)_{Y_1} \times U(1)_{Y_2}$ as

$$\begin{aligned} \nabla_\mu \equiv & \, \partial_\mu + ih_1 \frac{\lambda^A}{2} A^A_{1\mu} P_L + ih_2 \frac{\lambda^A}{2} A^A_{2\mu} P_R + iq_1 \frac{1}{2} B_{1\mu} P_L \\ & + iq_2 \frac{1}{2} B_{2\mu} P_R \\ = & \, \partial_\mu + ig_3 \frac{\lambda^A}{2} A^A_\mu + ig_1 \frac{1}{2} B_\mu \\ & + ig_3 (\cot\theta P_L - \tan\theta P_R) \frac{\lambda^A}{2} B^A_\mu \\ & + ig_1 (\cot\theta' P_L - \tan\theta') \frac{1}{2} Z'_\mu, \end{aligned} \tag{77}$$

where A^A_μ and B_μ are gauge fields of the unbroken symmetry group $SU(3)_c \times U(1)_Y$. To further simplify the calculations, we can *only keep* $SU(3)_c \times U(1)_Y$ *covariance of the self-energy*, that is, we replace ∇_μ by the covariant derivative of $SU(3)_c \times U(1)_Y$,

$$\bar{\nabla}_\mu \equiv \partial_\mu + ig_3 \frac{\lambda^A}{2} A^A_\mu + ig_1 \frac{1}{2} B_\mu \tag{78}$$

inside the techniquark self-energy. Thus, if the function $\Sigma(\partial_x^2)\delta(x-y)$ is the self-energy solution of the SDE in the case $A^A_{1\mu} = A^A_{2\mu} = B_{1\mu} = B_{2\mu} = 0$, we can replace its argument ∂_x by the $SU(3)_c \times U(1)_Y$ covariant derivative $\bar{\nabla}_x$, i.e., $\Sigma(\bar{\nabla}_x^2)\delta(x-y)$, as an approximate solution for

SDE in the case $A^A_{1\mu} \neq 0$, $A^A_{2\mu} \neq 0$, $B_{1\mu} \neq 0$, and $B_{2\mu} \neq 0$.

Now we are ready to integrate out the techniquarks Q and $\bar{Q}$. The exponential terms on the right-hand side of Eq. (61) can be written explicitly as

$$\sum_{n=2}^{\infty} \int d^4x_1 \ldots d^4x_n \frac{(-ig_{31})^n}{n!} G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x_1,\ldots,x_n) J^{\mu_1}_{1,\alpha_1}(x_1)\ldots J^{\mu_n}_{1,\alpha_n}(x_n) \approx \int d^4x d^4x' \bar{Q}^\sigma(x)\Pi_{\sigma\rho}(x,x')Q^\rho(x'), \tag{79}$$

where in the last equality we have taken the approximation of *replacing the summation over* $2n$*-fermion interactions with parts of them by their vacuum expectation values*, that is,

$$\Pi_{\sigma\rho}(x,x') = \sum_{n=2}^{\infty} \Pi^{(n)}_{\sigma\rho}(x,x'), \tag{80}$$

$$\begin{aligned}\Pi^{(n)}_{\sigma\rho}(x,x') = n \times \int d^4x_2 \ldots d^4x_{n-1} \frac{(-ig_{31})^n}{n!} G^{\alpha_1\ldots\alpha_n}_{\mu_1\ldots\mu_n}(x,x_2\ldots,x_{n-1},x') \\ \times \langle (r_1^{\alpha_1}\gamma^{\mu_1})_{\sigma\sigma_1} Q^{\sigma_1}(x)\bar{Q}(x_2) r_1^{\alpha_2}\gamma^{\mu_2} Q(x_2)\ldots \bar{Q}(x_{n-1}) r_1^{\alpha_{n-1}}\gamma^{\mu_{n-1}} Q(x_{n-1})\bar{Q}^{\rho_n}(x')(r_1^{\alpha_n}\gamma^{\mu_n})_{\rho_n\rho}\rangle,\end{aligned} \tag{81}$$

where the factor n comes from n different choices of unaveraged $\bar{Q}Q$, and the lowest term of which is

$$\begin{aligned}\Pi^{(2)}_{\sigma\rho}(x,x') &= 2\cdot\frac{(-ig_{31})^2}{2!} G^{\alpha_1\alpha_2}_{\mu_1\mu_2}(x,x')\langle (r_1^{\alpha_1}\gamma^{\mu_1})_{\sigma\sigma_1} Q^{\sigma_1}(x)\bar{Q}^{\rho_2}(x')(r_1^{\alpha_2}\gamma^{\mu_2})_{\rho_2\rho}\rangle \\ &= -g_{31}^2 G^{\alpha_1\alpha_2}_{\mu_1\mu_2}(x,x')[r_1^{\alpha_1}\gamma^{\mu_1} S(x,y) r_1^{\alpha_2}\gamma^{\mu_2}]_{\sigma\rho}.\end{aligned} \tag{82}$$

Comparing Eq. (82) with Eq. (67), we have

$$i\Pi^{(2)}_{\sigma\rho}(x,x') = \Sigma_{\sigma\rho}(x,x') \approx \Sigma_{\sigma\rho}(\bar{\nabla}^2_x)\delta(x-y). \tag{83}$$

Substituting Eq. (79) into Eq. (61), we obtain

$$\begin{aligned}\exp(iS_{\rm TC1}[A^A_\mu, B^A_\mu, B_{1\mu}, B_{2\mu}]) \approx {\rm Det}\Big[i\not{\partial} - g_3\frac{\lambda^A}{2}\not{A}^A - g_3(\cot\theta P_L - \tan\theta P_R)\frac{\lambda^A}{2}\not{B}^A - g_1\frac{1}{2}\not{B} \\ - g_1(\cot\theta' P_L - \tan\theta' P_R)\frac{1}{2}\not{Z}' - \Sigma_{\rm TC1}(\bar{\nabla}^2)\Big],\end{aligned} \tag{84}$$

where we have taken further approximation of *keeping only the lowest-order*, i.e. $\Pi^{(2)}_{\sigma\rho}(x,x')$, *of* $\Pi_{\sigma\rho}(x,x')$. With all these approximations, we have

$$\begin{aligned}iS_{\rm TC1}[A^A_\mu, B^A_\mu, B_\mu, Z'_\mu] = {\rm Tr}\log\Big[i\not{\partial} - g_3\frac{\lambda^A}{2}\not{A}^A - g_3(\cot\theta P_L - \tan\theta P_R)\frac{\lambda^A}{2}\not{B}^A - g_1\frac{1}{2}\not{B} \\ - g_1(\cot\theta' P_L - \tan\theta' P_R)\frac{1}{2}\not{Z}' - \Sigma_{\rm TC1}(\bar{\nabla}^2)\Big],\end{aligned} \tag{85}$$

We know the QCD-induced condensate is too weak to give sufficiently large masses of W and Z bosons, and thus it is negligible when we consider the main cause responsible for the electroweak symmetry breaking, which implies that ordinary QCD gluon fields have very little effect on our technicolor and electroweak interactions. Therefore for simplicity, we ignore them by just vanishing gluon field $A^A_\mu = 0$. In the next two subsubsections, we perform low energy expansion and explicitly expand above action up to the order of p^4.

2. Low energy expansion for $iS_{\rm TC1}[0, B^A_\mu, B_\mu, Z'_\mu]$

We have

$$iS_{\rm TC1}[0, B^A_\mu, B_\mu, Z'_\mu] = {\rm Tr}\log[i\not{\partial} + \not{v}_1 + \not{a}_1\gamma_5 - \Sigma_{\rm TC1}(\bar{\nabla}^2)] = i\int d^4x (F_0^{\rm TC1})^2\,{\rm tr}[a_1^2(x)] + S^{(4)}_{\rm TC1}[0, B^A_\mu, B_\mu, Z'_\mu] + O(p^6), \tag{86}$$

where the parameter $F_0^{\rm TC1}$ depends on the techniquark self-energy $\Sigma_{\rm TC1}$. The fields v_μ and a_μ from (85) and (86) are identified with

$$\begin{aligned} v_{1,\mu} &\equiv -\frac{g_3}{2}(\cot\theta - \tan\theta)\frac{\lambda^A}{2}B^A_\mu - \frac{g_1}{2}B_\mu - \frac{g_1}{4}(\cot\theta' - \tan\theta')Z'_\mu \\ a_{1,\mu} &\equiv -\frac{g_3}{2}(\cot\theta + \tan\theta)\frac{\lambda^A}{2}B^A_\mu - \frac{g_1}{4}(\cot\theta' + \tan\theta')Z'_\mu. \end{aligned} \tag{87}$$

Substituting the above equations into Eq. (86), we obtain, at the order of p^2,

$$S^{(2)}_{\mathrm{TC1}}[0, B^A_\mu, B_\mu, Z'_\mu] = \frac{(F_0^{\mathrm{TC1}})^2}{16}\int d^4x[2g_3^2(\cot\theta + \tan\theta)^2 B^A_\mu B^{A,,\mu} + 3g_1^2(\cot\theta' + \tan\theta')^2 Z'^2]. \tag{88}$$

Now we come to consider the p^4 order effective action. It can be divided into two parts

$$S^{(4)}_{\mathrm{TC1}}[0, B^A_\mu, B_\mu, Z'_\mu] = S^{(4d)}_{\mathrm{TC1}}[0, B^A_\mu, B_\mu, Z'_\mu] + S^{(4c)}_{\mathrm{TC1}}[0, B^A_\mu, B_\mu, Z'_\mu] \tag{89}$$

with

$$\begin{aligned} iS^{(4d)}_{\mathrm{TC1}}[0, B^A_\mu, B_\mu, Z'_\mu] &= \mathrm{Tr}\log[i\not\partial + \not v_1 + \not a_1\gamma_5] \\ iS^{(4c)}_{\mathrm{TC1}}[0, B^A_\mu, B_\mu, Z'_\mu] &= \mathrm{Tr}\log[i\not\partial + \not v_1 + \not a_1\gamma_5 - \Sigma_{\mathrm{TC1}}(\bar\nabla^2)] - \mathrm{Tr}\log[i\not\partial + \not v_1 + \not a_1\gamma_5] \end{aligned}$$

$S^{(4d)}_{\mathrm{TC1}}[0, B^A_\mu, B_\mu, Z'_\mu]$ is the divergent part of the action, which can be calculated by the following standard formula

$$i\mathrm{Tr}\log[i\not\partial + \not l P_L + \not r P_R] = -\frac{1}{2}\mathcal{K}\int d^4x\, \mathrm{tr}[r^{\mu\nu}r_{\mu\nu} + l^{\mu\nu}l_{\mu\nu}] \tag{90}$$

$$r_{\mu\nu} = \partial_\mu r_\nu - \partial_\nu r_\mu - i(r_\mu r_\nu - r_\nu r_\mu) \qquad l_{\mu\nu} = \partial_\mu l_\nu - \partial_\nu l_\mu - i(l_\mu l_\nu - l_\nu l_\mu) \qquad \mathcal{K} = -\frac{1}{48\pi^2}\left(\log\frac{\kappa^2}{\Lambda^2} + \gamma\right) \tag{91}$$

with $\mathcal{K}$ a divergent constant dependent on the ratio between ultraviolet cutoff Λ and infrared cutoff κ of the theory. We identify

$$\begin{aligned} r_\mu &= v_{1,\mu} + a_{1,\mu} = -g_3\cot\theta\frac{\lambda^A}{2}B^A_\mu - g_1\frac{1}{2}B_\mu - \frac{g_1}{2}\cot\theta' Z'_\mu \\ l_\mu &= v_{1,\mu} - a_{1,\mu} = g_3\tan\theta\frac{\lambda^A}{2}B^A_\mu - g_1\frac{1}{2}B_\mu + \frac{g_1}{2}\tan\theta' Z'_\mu \end{aligned}$$

With these preparations,

$$\begin{aligned} S^{(4d)}_{\mathrm{TC1}}[0, B^A_\mu, B_\mu, Z'_{c,\mu}] = -\frac{1}{2}\mathcal{K}\int d^4x\Big[&\frac{g_3^2}{2}(\cot^2\theta B^A_{r,\mu\nu}B^{A,\mu\nu}_r + \tan^2\theta B^A_{l,\mu\nu}B^{A,\mu\nu}_l) + \frac{3g_1^2}{2}B_{\mu\nu}B^{\mu\nu} \\ &+ \frac{3g_1^2}{4}(\cot^2\theta' + \tan^2\theta')Z'_{\mu\nu}Z'^{,\mu\nu} + \frac{3g_1^2}{2}(\cot\theta' - \tan\theta')B_{\mu\nu}Z'^{,\mu\nu}\Big] \end{aligned}$$

with

$$\begin{aligned} B^A_{r,\mu\nu} &= \partial_\mu B^A_\nu - \partial_\nu B^A_\mu - g_3\cot\theta f^{ABC}B^B_\mu B^C_\nu \qquad B^A_{l,\mu\nu} = \partial_\mu B^A_\nu - \partial_\nu B^A_\mu + g_3\tan\theta f^{ABC}B^B_\mu B^C_\nu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \qquad Z'_{\mu\nu} = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu \end{aligned} \tag{92}$$

$S^{(4c)}_{\mathrm{TC1}}[0, B^A_\mu, B_\mu, Z'_\mu]$ is the convergent part of the action, which can be calculated by the following standard formula

$$S^{(4c)}_{\rm TC1}[0,B^A_\mu,B_\mu,Z'_\mu]=\int d^4x\,{\rm tr}[-\mathcal{K}^{\rm TC1,\Sigma\neq0}_1(d_\mu a^\mu_1)^2-\mathcal{K}^{\rm TC1,\Sigma\neq0}_2(d_\mu a_{1,\nu}-d_\nu a_{1,\mu})^2+\mathcal{K}^{\rm TC1,\Sigma\neq0}_3(a_1^2)^2+\mathcal{K}^{\rm TC1,\Sigma\neq0}_4(a_{1,\mu}a_{1,\nu})^2-\mathcal{K}^{\rm TC1,\Sigma\neq0}_{13}V_{1,\mu\nu}V_1^{\mu\nu}+i\mathcal{K}^{\rm TC1,\Sigma\neq0}_{14}V_{1,\mu\nu}a^\mu_1a^\nu_1],\tag{93}$$

with $V_{1,\mu\nu}\equiv\partial_\mu v_{1,\nu}-\partial_\nu v_{1,\mu}-i(v_{1,\mu}v_{1,\nu}-v_{1,\nu}v_{1,\mu})$ and $d_\mu a_{1,\nu}\equiv\partial_\mu a_{1,\nu}-i(v_{1,\mu}a_{1,\nu}-a_{1,\nu}v_{1,\mu})$. $S^{(4c)}_{\rm TC1}[0,B^A_\mu,B_\mu,Z'_\mu]$ can be further divided into four parts

$$S^{(4c)}_{\rm TC1}[0,B^A_\mu,B_\mu,Z'_\mu]=S^{(4c,B^A)}_{\rm TC1}[B^A]+S^{(4c,B)}_{\rm TC1}[B]+S^{(4c,Z')}_{\rm TC1}[Z']+S^{(4c,B^AZ')}_{\rm TC1}[B^A,Z']+S^{(4c,BZ')}_{\rm TC1}[B,Z'].\tag{94}$$

The detail form of $S^{(4c,B^A)}_{\rm TC1}[B^A]$, $S^{(4c,B)}_{\rm TC1}[B]$, $S^{(4c,Z')}_{\rm TC1}[Z']$, $S^{(4c,B^AZ')}_{\rm TC1}[B^A,Z']$ and $S^{(4c,BZ')}_{\rm TC1}[B,Z']$ is given in (A5)–(A7) respectively. Since TC1 interaction is $SU(3)$ gauge interaction which is same as QCD interaction and the quark number N_f are all equal to three,[1] as we discussed before, due to scale invariance we have $\mathcal{K}^{\rm TC1,\Sigma\neq0}_i$, $i=2,3,4,13,14$ are equal to those of QCD values within our approximations

$$\mathcal{K}^{\rm TC1,\Sigma\neq0}_i=\mathcal{K}^{\Sigma\neq0}_i\qquad i=2,3,4,13,14\tag{95}$$

Using the relation given in Ref. [12],

$$\begin{aligned}H_1&=-\tfrac14(\mathcal{K}^{\Sigma\neq0}_2+\mathcal{K}^{\Sigma\neq0}_{13})\\ L_{10}&=\tfrac12(\mathcal{K}^{\Sigma\neq0}_2-\mathcal{K}^{\Sigma\neq0}_{13})\\ L_9&=\tfrac18(4\mathcal{K}^{\Sigma\neq0}_{13}-\mathcal{K}^{\Sigma\neq0}_{14})\\ L_1&=\tfrac1{32}(\mathcal{K}^{\Sigma\neq0}_4+2\mathcal{K}^{\Sigma\neq0}_{13}-\mathcal{K}^{\Sigma\neq0}_{14})\\ L_3&=\tfrac1{16}(\mathcal{K}^{\Sigma\neq0}_3-2\mathcal{K}^{\Sigma\neq0}_4-6\mathcal{K}^{\Sigma\neq0}_{13}+3\mathcal{K}^{\Sigma\neq0}_{14})\end{aligned}\tag{96}$$

TABLE III. The obtained nonzero values of the $O(p^4)$ coefficients H_1, L_{10}, L_9, L_2, L_1, L_3 for the topcolor-assisted technicolor model. The $F_0^{\rm TC1}$ and $\Lambda_{\rm TC1}$ are in units of TeV and coefficients are in units of 10^{-3}.

$F_0^{\rm TC1}$	$\Lambda_{\rm TC1}$	H_1	L_{10}	L_9	L_2	L_1	L_3
1	5.21	43.0	−7.04	5.06	2.19	1.10	−7.81

In Table III, we list down original QCD calculation results given in Ref. [12], the value for H_1 in the original paper is a divergent constant therefore we have not given its value. Now the divergent part is already extracted out by $S^{(4d)}_{\rm TC1}[0,B^A_\mu,B_\mu,Z'_\mu]$, H_1 here is a convergent quantity which can be obtained in the original formula for H_1 by subtracting out its divergent part caused by terms with $\Sigma=0$.

We finally obtain

$$\begin{aligned}\mathcal{K}^{\rm TC1,\Sigma\neq0}_2&=L_{10}-2H_1\\ \mathcal{K}^{\rm TC1,\Sigma\neq0}_3&=64L_1+16L_3+8L_9+2L_{10}+4H_1\\ \mathcal{K}^{\rm TC1,\Sigma\neq0}_4&=32L_1-8L_9-2L_{10}-4H_1\\ \mathcal{K}^{\rm TC1,\Sigma\neq0}_{13}&=-L_{10}-2H_1\\ \mathcal{K}^{\rm TC1,\Sigma\neq0}_{14}&=-4L_{10}-8L_9-8H_1.\end{aligned}\tag{97}$$

B. Electroweak symmetry breaking: The contribution of $SU(3)_{\rm TC2}$

Likewise, it is easy to check that the $SU(3)_{\rm TC2}$ interaction does induce the techniquark condensate $\langle\bar TT\rangle\neq0$, which triggers the electroweak symmetry breaking $SU(2)_L\times U(1)_Y\to U(1)_{\rm EM}$. Integrating out the $SU(3)_{\rm TC2}$ technigluons $G^\alpha_{2\mu}$ and the techniquarks T and $\bar T$, Eq. (60) can be written as

$$\exp(iS_{\rm EW}[W^a_\mu,B_\mu])=\exp\Big[i\int d^4x\Big(-\frac14W^a_{\mu\nu}W^{a\mu\nu}\Big)\Big]\int\mathcal{D}B^A_\mu\mathcal{D}Z'_\mu\exp\Big[i\int d^4x\Big(-\frac14A^A_{1\mu\nu}A^{A\mu\nu}_1-\frac14A^A_{2\mu\nu}A^{A\mu\nu}_2-\frac14B_{1\mu\nu}B^{\mu\nu}_1-\frac14B_{2\mu\nu}B^{\mu\nu}_2\Big)+iS_{\rm TC1}[A^A_\mu,B^A_\mu,B_\mu,Z'_\mu]+iS_{\rm TC2}[W^a_\mu,B_{2\mu}]\Big],\tag{98}$$

where $S_{\rm TC1}[A^A_\mu,B^A_\mu,B_\mu,Z'_\mu]$ has been given in Eq. (85) for its general form and expanded up to the order of p^2 in (88) and p^4 in (89), and $S_{\rm TC2}[W^a_\mu,B_{2\mu}]$ is given by

$$\exp(iS_{\rm TC2}[W^a_\mu,B_{2\mu}])=\int\mathcal{D}\bar T\mathcal{D}T\mathcal{D}G^\alpha_{2\mu}\exp\Big[i\int d^4x\Big(-\frac14F^\alpha_{2\mu\nu}F^{\alpha\mu\nu}_2+\bar T[i\not\partial-g_{32}r_2^\alpha\not G^\alpha_2+\not l_2P_L+\not r_2P_R]T\Big)\Big]\tag{99}$$

[1] This is in fact an approximation in which we have ignored possible effects on the running of the TC1 gauge coupling constant from ordinary color gauge fields and coloron fields.

with $l_{2,\mu} \equiv -g_2\frac{\tau^a}{2}W^a_\mu - q_2\frac{1}{6}B_{2\mu}$ and $r_{2,\mu} \equiv -q_2(\frac{1}{6}+\frac{\tau^3}{2})B_{2\mu}$.

By means of the Gasser-Leutwyler's prescription presented in Sec. II, the functional integration (99) is related to the QCD-type chiral Lagrangian by

$$\int \mathcal{D}\bar{T}\mathcal{D}T\mathcal{D}G^\alpha_{2\mu}\exp\bigg[i\int d^4x\bigg[-\frac{1}{4}F^\alpha_{2\mu\nu}F^{\alpha\mu\nu}_2 + \bar{T}(i\not{\partial} - g_{32}r^\alpha_2\not{G}^\alpha_2 + \not{l}_2P_L + \not{r}_2P_R)T\bigg]\bigg] \times\bigg\{\int \mathcal{D}\bar{T}\mathcal{D}Te^{i\int d^4x\bar{T}[i\not{\partial}+\not{l}_2P_L+\not{r}_2P_R]T}\bigg\}^{-1} = \int \mathcal{D}\mu(\tilde{U})e^{iS_{\mathrm{TC2-induced\ eff}}[\tilde{U},l_{2,\mu},r_{2,\mu}]}, \tag{100}$$

with the $SU(3)_{\mathrm{TC2}}$-induced chiral effective action

$$\begin{aligned} S_{\mathrm{TC2-induced\ eff}}[\tilde{U},l_{2,\mu},r_{2,\mu}] = \int d^4x\bigg[&\frac{(F_0^{\mathrm{TC2}})^2}{4}\mathrm{tr}[(\nabla^\mu\tilde{U}^\dagger)(\nabla_\mu\tilde{U})] + L_1^{\mathrm{TC2}}[\mathrm{tr}(\nabla^\mu\tilde{U}^\dagger\nabla_\mu\tilde{U})]^2 \\ &+ L_2^{\mathrm{TC2}}\mathrm{tr}[\nabla_\mu\tilde{U}^\dagger\nabla_\nu\tilde{U}]\mathrm{tr}[\nabla^\mu\tilde{U}^\dagger\nabla^\nu\tilde{U}] + L_3^{\mathrm{TC2}}\mathrm{tr}[(\nabla^\mu\tilde{U}^\dagger\nabla_\mu\tilde{U})^2] \\ &- iL_9^{\mathrm{TC2}}\mathrm{tr}[F^R_{\mu\nu}\nabla^\mu\tilde{U}\nabla^\nu\tilde{U}^\dagger + F^L_{\mu\nu}\nabla^\mu\tilde{U}^\dagger\nabla^\nu\tilde{U}] + L_{10}^{\mathrm{TC2}}\mathrm{tr}[\tilde{U}^\dagger F^R_{\mu\nu}\tilde{U}F^{L,\mu\nu}] \\ &+ H_1^{\mathrm{TC2}}\mathrm{tr}[F^R_{\mu\nu}F^{R,\mu\nu} + F^L_{\mu\nu}F^{L,\mu\nu}]\bigg], \end{aligned} \tag{101}$$

where

$$\nabla_\mu\tilde{U} \equiv \partial_\mu\tilde{U} - ir_{2,\mu}\tilde{U} + i\tilde{U}l_{2,\mu}, \qquad \nabla_\mu\tilde{U}^\dagger = -\tilde{U}^\dagger(\nabla_\mu\tilde{U})\tilde{U}^\dagger,$$
$$F^R_{\mu\nu} \equiv i[\partial_\mu - ir_{2,\mu}, \partial_\nu - ir_{2,\nu}], \qquad F^L_{\mu\nu} \equiv i[\partial_\mu - il_{2,\mu}, \partial_\nu - il_{2,\nu}]. \tag{102}$$

The coefficients F_0^{TC2}, L_1^{TC2}, L_2^{TC2}, L_3^{TC2}, L_{10}^{TC2}, H_1^{TC2} arise from $SU(3)_{\mathrm{TC2}}$ dynamics. These coefficients relate to the $\mathcal{K}_i^{\mathrm{TC2}}$ coefficients as those that appeared in the one-doublet technicolor model as

$$H_1^{\mathrm{TC2}} = -\frac{\mathcal{K}_2^{\mathrm{TC2},\Sigma\neq0} + \mathcal{K}_{13}^{\mathrm{TC2},\Sigma\neq0}}{4} \qquad L_{10}^{\mathrm{TC2}} = \frac{\mathcal{K}_2^{\mathrm{TC2},\Sigma\neq0} - \mathcal{K}_{13}^{\mathrm{TC2},\Sigma\neq0}}{2} \qquad L_9^{\mathrm{TC2}} = \frac{\mathcal{K}_{13}^{\mathrm{TC2},\Sigma\neq0}}{2} - \frac{\mathcal{K}_{14}^{\mathrm{TC2},\Sigma\neq0}}{8}$$
$$L_2^{\mathrm{TC2}} = \frac{\mathcal{K}_4^{\mathrm{TC2},\Sigma\neq0} + 2\mathcal{K}_{13}^{\mathrm{TC2},\Sigma\neq0} - \mathcal{K}_{14}^{\mathrm{TC2},\Sigma\neq0}}{16} \qquad L_1^{\mathrm{TC2}} + \frac{L_3^{\mathrm{TC2}}}{2} = \frac{\mathcal{K}_3^{\mathrm{TC2},\Sigma\neq0} - \mathcal{K}_4^{\mathrm{TC2},\Sigma\neq0} - 4\mathcal{K}_{13}^{\mathrm{TC2},\Sigma\neq0} + 2\mathcal{K}_{14}^{\mathrm{TC2},\Sigma\neq0}}{32} \tag{103}$$

$\mathcal{K}_i^{\mathrm{TC2}}$ coefficients with superscript TC2 denote the present TC2 interaction. They are functions of technifermion T self-energy $\Sigma_{\mathrm{TC2}}(p^2)$ and detailed expressions are already written down in (36) of Ref. [12] with the replacement of $N_c \to 3$ and subtracting out their $\Sigma_{\mathrm{TC2}}(p^2) = 0$ parts. TC2 interactions among techiquark doublet T is $SU(3)$, which is the same as the one-doublet technicolor model discussed before. The only difference is that the Λ_{TC} in the original one-doublet technicolor model must be replaced with Λ_{TC2} now. But as we discussed before, $\mathcal{K}_i^{\mathrm{TC2}}$, $i = 1, 2, 3, 4, 13, 14$ are independent of Λ_{TC2}, therefore our $\mathcal{K}_i^{\mathrm{TC2}}$, $i = 1, 2, 3, 4, 13, 14$ are the same as those obtained in the one-doublet technicolor model. This result presents L_i^{TC2}, $i = 1, 2, 3, 9, 10$ and H_1^{TC2} coefficients as the same as those in the one-doublet technicolor model. In Table IV we list down the numerical calculation results in which the method is already mentioned in a previous section and, except for the result for H_1^{TC2}, all others are already used in Table I.

Similar to the one-doublet case for $\tilde{U}$ is a 2×2 unitary matrix, and thus the L_1^{TC2} term and the L_3^{TC2} term are linearly related,

$$L_3^{\mathrm{TC2}}\mathrm{tr}[(\nabla^\mu\tilde{U}^\dagger\nabla_\mu\tilde{U})^2] = \frac{L_3^{\mathrm{TC2}}}{2}\{\mathrm{tr}[\tilde{U}^\dagger(\nabla^\mu\tilde{U})\tilde{U}^\dagger(\nabla_\mu\tilde{U})]\}^2. \tag{104}$$

Comparing Eqs. (102) with the standard covariant derivative given in Ref. [2], we need to recognize

TABLE IV. The obtained nonzero values of the $O(p^4)$ coefficients $H_1^{\mathrm{TC2}} = H_1^{\mathrm{1D}}$, $L_{10}^{\mathrm{TC2}} = L_{10}^{\mathrm{1D}}$, $L_9^{\mathrm{TC2}} = L_9^{\mathrm{1D}}$, $L_2^{\mathrm{TC2}} = L_2^{\mathrm{1D}}$, $L_1^{\mathrm{TC2}} = L_1^{\mathrm{1D}}$, $L_3^{\mathrm{TC2}} = L_3^{\mathrm{1D}}$ for the topcolor-assisted technicolor model. The F_0^{TC2} and Λ_{TC2} are in units of TeV and coefficients are in units of 10^{-3}.

F_0^{TC2}	Λ_{TC2}	H_1^{TC2}	L_{10}^{TC2}	L_9^{TC2}	L_2^{TC2}	L_1^{TC2}	L_3^{TC2}
0.25	1.34	43.0	−6.90	4.87	2.02	1.01	−7.40

$$\tilde{U}^\dagger = U, \qquad \nabla_\mu \tilde{U}^\dagger = \tilde{D}_\mu U \equiv \partial_\mu U + ig_2 \frac{\tau^a}{2} W^a_\mu U - U i q_2 \frac{\tau^3}{2} B_{2\mu}. \tag{105}$$

And $F^R_{\mu\nu} = -q_2(\frac{1}{6} + \frac{\tau^3}{2})B_{2\mu\nu}$, $F^L_{\mu\nu} = -g_2 \frac{\tau^a}{2} W^a_{\mu\nu} - q_2 \frac{1}{6} B_{2\mu\nu}$ with $B_{2\mu\nu} \equiv \partial_\mu B_{2\nu} - \partial_\nu B_{2\mu}$ is the $U(1)_{Y_2}$ gauge field strength tensor.

Substituting the above equations back into Eq. (101), we obtain

$$\begin{aligned} S_{\mathrm{TC2-induced\ eff}}[U, W, B_2] = \int d^4x \Big[&-\frac{(F_0^{\mathrm{TC2}})^2}{4} \mathrm{tr}(\tilde{X}_\mu \tilde{X}^\mu) + \Big(L_1^{\mathrm{1D}} + \frac{L_3^{\mathrm{1D}}}{2}\Big)[\mathrm{tr}(\tilde{X}_\mu \tilde{X}^\mu)]^2 + L_2^{\mathrm{1D}}[\mathrm{tr}(\tilde{X}_\mu \tilde{X}_\nu)]^2 \\ &- i\frac{L_9^{\mathrm{1D}}}{2} q_2 B_{2\mu\nu} \mathrm{tr}(\tau^3 \tilde{X}^\mu \tilde{X}^\nu) - i L_9^{\mathrm{1D}} \mathrm{tr}(\bar{W}_{\mu\nu} \tilde{X}^\mu \tilde{X}^\nu) + \frac{L_{10}^{\mathrm{1D}}}{2} q_2 B_{2\mu\nu} \mathrm{tr}(\tau^3 \bar{W}^{\mu\nu}) \\ &+ \frac{1}{18}(L_{10}^{\mathrm{1D}} + 11 H_1^{\mathrm{1D}}) q_2^2 B_{2\mu\nu} B_2^{\mu\nu} + H_1^{\mathrm{1D}} \mathrm{tr}(\bar{W}_{\mu\nu} \bar{W}^{\mu\nu}) \Big], \end{aligned} \tag{106}$$

where $\tilde{X}_\mu$ is defined by

$$\tilde{X}_\mu \equiv U^\dagger (\tilde{D}_\mu U). \tag{107}$$

With (107) and from Eqs. (105), (57b), and (59), we obtain

$$\tilde{X}_\mu = X_\mu + i g_1 \tan\theta' Z'_\mu \frac{\tau^3}{2}. \tag{108}$$

Substituting Eq. (108) into Eq. (106), we obtain, at the order of p^2,

$$S^{(2)}_{\mathrm{TC2-induced\ eff}}[U, W^a, B\cos\theta' - Z'\sin\theta'] = \frac{(F_0^{\mathrm{TC2}})^2}{4} \int d^4x \Big[-\mathrm{tr}(X_\mu X^\mu) - i g_1 \tan\theta' Z'_\mu \mathrm{tr}(\tau^3 X^\mu) + \frac{g_1^2}{2} \tan^2\theta' Z'^2 \Big]. \tag{109}$$

Similarly detailed algebra gives

$$\begin{aligned} &i S^{(4)}_{\mathrm{TC2-induced\ eff}}[U, W^a, B\cos\theta' - Z'\sin\theta'] \\ &= i \int d^4x \Big[\Big(L_1^{\mathrm{1D}} + \frac{L_3^{\mathrm{1D}}}{2}\Big)[\mathrm{tr}(X_\mu X^\mu)]^2 + L_2^{\mathrm{1D}}[\mathrm{tr}(X_\mu X_\nu)]^2 - i L_9^{\mathrm{1D}} \mathrm{tr}(\bar{W}_{\mu\nu} X^\mu X^\nu) - i \frac{L_9^{\mathrm{1D}}}{2} g_1 B_{\mu\nu} \mathrm{tr}(\tau^3 X^\mu X^\nu) \\ &+ \frac{L_{10}^{\mathrm{1D}}}{2} g_1 B_{\mu\nu} \mathrm{tr}(\tau^3 \bar{W}^{\mu\nu}) + \frac{1}{18}(L_{10}^{\mathrm{1D}} + 11 H_1^{\mathrm{1D}}) g_1^2 B_{\mu\nu} B^{\mu\nu} + H_1^{\mathrm{1D}} \mathrm{tr}(\bar{W}_{\mu\nu} \bar{W}^{\mu\nu}) - \Big(L_1^{\mathrm{1D}} + \frac{L_3^{\mathrm{1D}}}{2}\Big) g_1^2 \tan^2\theta' Z'^{,2} (\mathrm{tr} X^2) \\ &+ \Big(L_1^{\mathrm{1D}} + L_2^{\mathrm{1D}} + \frac{L_3^{\mathrm{1D}}}{2}\Big)\Big[\frac{1}{4} g_1^4 \tan^4\theta' Z'^{,4} - i g_1^3 \tan^3\theta' Z'^{,2} Z'^{,\mu} \mathrm{tr}(X_\mu \tau^3)\Big] - \frac{1}{2} L_2^{\mathrm{1D}} g_1^2 \tan^2\theta' [Z'^2 \mathrm{tr}(X_\nu \tau^3) \mathrm{tr}(X^\nu \tau^3) \\ &+ Z'^\mu Z'^\nu \mathrm{tr}(X_\mu \tau^3) \mathrm{tr}(X_\nu \tau^3)] - \Big(L_1^{\mathrm{1D}} + \frac{L_3^{\mathrm{1D}}}{2}\Big) g_1^2 \tan^2\theta' Z'^{,\mu} Z'^{,\nu} \mathrm{tr}(X_\mu \tau^3)(X_\nu \tau^3) - L_2^{\mathrm{1D}} g_1^2 \tan^2\theta' Z'^{,\mu} Z'^{,\nu} \mathrm{tr}(X_\mu X_\nu) \\ &+ 2i\Big(L_1^{\mathrm{1D}} + \frac{L_3^{\mathrm{1D}}}{2}\Big) g_1 \tan\theta' Z'_\mu \mathrm{tr}(X^\mu \tau^3)(\mathrm{tr} X^2) + 2i L_2^{\mathrm{1D}} g_1 \tan\theta' Z'_\mu \mathrm{tr}(X_\nu \tau^3) \mathrm{tr}(X^\mu X^\nu) + i \frac{L_9^{\mathrm{1D}}}{2} g_1 \tan\theta' Z'_{\mu\nu} \mathrm{tr}(\tau^3 X^\mu X^\nu) \\ &+ \frac{1}{2} g_1 L_9^{\mathrm{1D}} \tan\theta' [\mathrm{tr}(\bar{W}^{\mu\nu} X_\mu \tau^3) Z'_\nu + \mathrm{tr}(\bar{W}^{\mu\nu} \tau^3 X_\nu) Z'_\mu] - \frac{L_{10}^{\mathrm{1D}}}{2} g_1 \tan\theta' Z'_{\mu\nu} \mathrm{tr}(\tau^3 \bar{W}^{\mu\nu}) \\ &+ \frac{1}{18}(L_{10}^{\mathrm{1D}} + 11 H_1^{\mathrm{1D}})[g_1^2 \tan^2\theta' Z'_{\mu\nu} Z'^{,\mu\nu} - 2 g_1^2 \tan\theta' Z'_{\mu\nu} B^{\mu\nu}] \Big]. \end{aligned} \tag{110}$$

Thus, from Eqs. (99)–(107) we obtain

$$\begin{aligned} i S_{\mathrm{TC2}}[W^a_\mu, B_{2\mu}] = &\mathrm{Tr} \ln \Big[i\partial\!\!\!/ - g_2 \frac{\tau^a}{2} W\!\!\!\!/^a P_L - q_2 \frac{1}{6} B\!\!\!/_2 P_L - q_2\Big(\frac{1}{6} + \frac{\tau^3}{2}\Big) B\!\!\!/_2 P_R \Big] \\ &+ \log \int \mathcal{D}\mu(U) \exp(i S^{(2)}_{\mathrm{TC2-induced\ eff}}[U, W, B_2] + i S^{(4)}_{\mathrm{TC2-induced\ eff}}[U, W, B_2]). \end{aligned} \tag{111}$$

We still have left to compute $\mathrm{Tr} \ln[i\partial\!\!\!/ - g_2 \frac{\tau^a}{2} W\!\!\!\!/^a P_L - q_2 \frac{1}{6} B\!\!\!/_2 P_L - q_2(\frac{1}{6} + \frac{\tau^3}{2}) B\!\!\!/_2 P_R]$, which is at least of the order of p^4. We can write it as

$$\begin{aligned} &\mathrm{Tr}\,\ln\left[i\not{\partial} - g_2\frac{\tau^a}{2}\not{W}^a P_L - q_2\frac{1}{6}\not{B}_2 P_L - q_2\left(\frac{1}{6}+\frac{\tau^3}{2}\right)\not{B}_2 P_R\right] = \mathrm{Tr}\log[i\not{\partial} + \not{l}_2 P_L + \not{r}_2 P_R] \\ &l_2^\mu = -g_2\frac{\tau^a}{2}W^{a,\mu} - q_2\frac{1}{6}B_2^\mu = -g_2\frac{\tau^a}{2}W^{a,\mu} - \frac{1}{6}(g_1 B^\mu - g_1\tan\theta' Z'^{,\mu}) \\ &r_2^\mu = -q_2\left(\frac{1}{6}+\frac{\tau^3}{2}\right)B_2^\mu = -\left(\frac{1}{6}+\frac{\tau^3}{2}\right)(g_1 B^\mu - g_1\tan\theta' Z'^{,\mu}) \end{aligned} \tag{112}$$

Then with the help of (90), computation gives

$$\begin{aligned} &i\mathrm{Tr}\,\ln\left[i\not{\partial} - g_2\frac{\tau^a}{2}\not{W}^a P_L - q_2\frac{1}{6}\not{B}_2 P_L - q_2\left(\frac{1}{6}+\frac{\tau^3}{2}\right)\not{B}_2 P_R\right] \\ &= -\frac{1}{2}\mathcal{K}\int d^4x\left[\frac{11}{18}g_1^2(B_{\mu\nu}B^{\mu\nu} + \tan^2\theta' Z'_{\mu\nu}Z'^{,\mu\nu} - 2\tan\theta' Z'_{\mu\nu}B^{\mu\nu}) + \frac{1}{2}g_2^2 W^a_{\mu\nu}W^{a,\mu\nu}\right] \end{aligned} \tag{113}$$

Substituting Eq. (111) into Eq. (98) and then comparing it with the last line of Eq. (55), we have

$$\begin{aligned} \mathcal{N}[W^a_\mu, B_\mu]\exp(iS_{\mathrm{eff}}[U, W^a_\mu, B_\mu]) = {}& \exp\left[i\int d^4x\left(-\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu}\right)\right]\int \mathcal{D}B^A_\mu \mathcal{D}Z'_\mu \\ &\times\exp\left[i\int d^4x\left(-\frac{1}{4}A^A_{1\mu\nu}A^{A\mu\nu}_1 - \frac{1}{4}A^A_{2\mu\nu}A^{A\mu\nu}_2 - \frac{1}{4}B_{1\mu\nu}B_1^{\mu\nu} - \frac{1}{4}B_{2\mu\nu}B_2^{\mu\nu}\right)\right. \\ &+ \mathrm{Tr}\,\ln\left[i\not{\partial} - g_2\frac{\tau^a}{2}\not{W}^a P_L - q_2\frac{1}{6}\not{B}_2 P_L - q_2\left(\frac{1}{6}+\frac{\tau^3}{2}\right)\not{B}_2 P_R\right] \\ &\left.+ iS_{\mathrm{TC1}}[A^A_\mu, B^A_\mu, B_\mu, Z'_\mu] + iS_{\mathrm{TC2-induced\ eff}}[U, W, B_2]\right]_{A^A_\mu = 0}, \end{aligned} \tag{114}$$

where we have put the $SU(3)_c$ gluon fields $A^A_\mu = 0$ on the right-hand side, for the QCD effects are small here. The normalization factor from its definition (56) can be calculated similarly to a previous procedure. The only difference is that we switch off the TC2 interaction by taking $g_{32} = 0$, This will result in $S_{\text{TC2-induced eff}}[U, W, B_2]$ vanishing, which then leads to ignoring the term $iS_{\text{TC2-induced eff}}[U, W, B_2]$ in the above expression, and we get the expression for $\mathcal{N}[W^a_\mu, B_\mu]$.

C. Integrating out of colorons

Now, as shown in Eqs. (114), the next work is to integrate out the $SU(3)_c$ octet of colorons, B^A_μ. From Eqs. (57a) and (59), it is straightforward to get

$$A^A_{1\mu\nu} = A^A_{\mu\nu}\sin\theta + B^A_{1,\mu\nu}\cos\theta, \tag{115a}$$

$$A^A_{2\mu\nu} = A^A_{\mu\nu}\cos\theta - B^A_{2,\mu\nu}\sin\theta, \tag{115b}$$

where

$$\begin{aligned} B^A_{1,\mu\nu} &\equiv \partial_\mu B^A_\nu - \partial_\nu B^A_\mu + g_3 f^{ABC}(\cot\theta B^B_\mu B^C_\nu + B^B_\mu A^C_\nu + A^B_\mu B^C_\nu) \\ B^A_{2,\mu\nu} &\equiv \partial_\mu B^A_\nu - \partial_\nu B^A_\mu - g_3 f^{ABC}(\tan\theta B^B_\mu B^C_\nu + B^B_\mu A^C_\nu + A^B_\mu B^C_\nu) \end{aligned}$$

then Eqs. (114) become

$$\begin{aligned}\mathcal{N}[W^a_\mu, B_\mu]\exp(iS_{\rm eff}[U, W^a_\mu, B_\mu]) = \exp\Big[i\int d^4x\Big(-\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu}\Big)\Big]\int\mathcal{D}B^A_\mu\mathcal{D}Z'_\mu \\ \times\exp\Big[iS_{\rm TC1}[0, B^A_\mu, B_\mu, Z'_\mu] + i\int d^4x\Big(-\frac{1}{4}B^A_{1\mu\nu}B^{A\mu\nu}_1\cos^2\theta - \frac{1}{4}B^A_{2\mu\nu}B^{A\mu\nu}_2\sin^2\theta \\ -\frac{1}{4}B_{1\mu\nu}B^{\mu\nu}_1 - \frac{1}{4}B_{2\mu\nu}B^{\mu\nu}_2\Big) \\ + \mathrm{Tr}\ln\Big[i\not\partial - g_2\frac{\tau^a}{2}\not W^aP_L - q_2\frac{1}{6}\not B_2P_L - q_2\Big(\frac{1}{6}+\frac{\tau^3}{2}\Big)\not B_2P_R\Big] \\ + iS_{\rm TC2-induced\ eff}[U, W, B_2]\Big],\end{aligned} \tag{116}$$

Ignoring term $iS_{\rm TC2-induced\ eff}[U, W, B_2]$ in the above expression, we get an expression for $\mathcal{N}[W^a_\mu, B_\mu]$. In the above result, if we denote the coloron involved part as $\int\mathcal{D}B^A_\mu e^{iS_{\rm coloron}[B^A,Z']}$, then

$$\begin{aligned}S_{\rm coloron}[B^A, Z'] = S^{4c,B^A}_{\rm TC1}[B^A] + S^{4c,B^AZ'}_{\rm TC1}[B^A, Z'] + \int d^4x\Big(-\frac{1}{4}B^A_{1\mu\nu}B^{A\mu\nu}_1\cos^2\theta - \frac{1}{4}B^A_{2\mu\nu}B^{A\mu\nu}_2\sin^2\theta \\ + \frac{(F^{\rm TC1}_0)^2}{8}g_3^2(\cot\theta + \tan\theta)^2B^A_\mu B^{A,\mu} - \frac{g_3^2}{4}\mathcal{K}(\cot^2\theta B^A_{r,\mu\nu}B^{A,\mu\nu}_r + \tan^2\theta B^A_{l,\mu\nu}B^{A,\mu\nu}_l)\Big)\end{aligned} \tag{117}$$

$$= S^0_{\rm coloron}[B^A, Z'] + S^{\rm int}_{\rm coloron}[B^A, Z'] \tag{118}$$

with $S^0_{\rm coloron}[B^A, Z']$ linear and quadratic in coloron fields and $S^{\rm int}_{\rm coloron}[B^A, Z']$ cubic and quartic in coloron fields. The detailed form of them is given in (A8) and (A9). Now coloron fields are not correctly normalized, since the coefficient in front of the kinetic term is not standard $-1/4$. We now introduce normalized fields $B^A_{R,\mu}$ as

$$B^A_\mu = \frac{1}{c}B^A_{R,\mu} \tag{119}$$

$$\begin{aligned}c^2 = 1 + g_3^2\Big[\frac{1}{2}\mathcal{K}^{\rm TC1,\Sigma\neq0}_2(\cot\theta + \tan\theta)^2 \\ + \frac{1}{2}\mathcal{K}^{\rm TC1,\Sigma\neq0}_{13}(\cot\theta - \tan\theta)^2 + \mathcal{K}(\cot^2\theta + \tan^2\theta)\Big]\end{aligned} \tag{120}$$

With them, $S^0_{\rm coloron}[B^A, Z']$ in terms of normalized coloron fields become

$$S^0_{\rm coloron}[B^A, Z'] = \int d^4x\frac{1}{2}B^A_{R,\mu}(x)D^{-1,\mu\nu}_B(Z')B^A_{R,\nu}(x) \tag{121}$$

with

$$\begin{aligned}D^{-1,\mu\nu}_B(Z') = D^{-1,\mu\nu}_{B0} + \Delta^{\mu\nu}(Z') \\ D^{-1,\mu\nu}_{B0} = g^{\mu\nu}(\partial^2 + M^2_{\rm coloron}) - (1+\lambda_B)\partial^\mu\partial^\nu\end{aligned} \tag{122}$$

$$\begin{aligned}\Delta^{\mu\nu}(Z') = \Big[g^{\mu\nu}\Big(\frac{1}{2}\mathcal{K}^{\rm TC1,\Sigma\neq0}_3 + \mathcal{K}^{\rm TC1,\Sigma\neq0}_4\Big)Z'_{\nu'}Z'^{,\nu'} \\ + \Big(2\mathcal{K}^{\rm TC1,\Sigma\neq0}_3 + \frac{5}{4}\mathcal{K}^{\rm TC1,\Sigma\neq0}_4\Big)Z'^{,\mu}Z'^{,\nu}\Big] \\ \times\frac{g_1^2g_3^4(\cot\theta + \tan\theta)^2(\cot\theta' + \tan\theta')^2}{32c^2}\end{aligned} \tag{123}$$

$$M_{\rm coloron} = \frac{1}{2}g_3\frac{\cot\theta + \tan\theta}{c}F^{\rm TC1}_0 = \frac{g_3F^{\rm TC1}_0}{2c\sin\theta\cos\theta} \tag{124}$$

$$\lambda_B = -\frac{1}{4}g_3^2\frac{(\cot\theta + \tan\theta)^2}{c^2}\mathcal{K}^{\rm TC1,\Sigma\neq0}_1. \tag{125}$$

Here we recover the estimation for coloron mass $M_{\rm coloron}\sim g_3\Lambda/(\sin\theta\cos\theta)$ given in Ref. [19] if we identify $\Lambda = F^{\rm TC1}_0/(2c)$. We now denote the resulting action after integration over colorons as

$$\int\mathcal{D}B^A_\mu e^{iS_{\rm coloron}[B^A,Z']} = e^{i\bar{S}_{\rm coloron}[Z']} \tag{126}$$

$\bar{S}_{\rm coloron}[Z']$ are all vacuum diagrams with propagator $D^{\mu\nu}_B(Z')$ and vertices determined by $S^{\rm int}_{\rm coloron}[B^A, Z']$. The loop expansion result is

$$\begin{aligned}i\bar{S}_{\rm coloron}[Z'] = -\tfrac{1}{2}\mathrm{Tr}\log D^{-1}_B(Z') \\ + \text{two and more loop contributions}\end{aligned} \tag{127}$$

The first term in the right-hand side of the above equation is a one-loop result. If we further perform low energy expansion for it and drop out total derivative terms, we find the contributions from the one-loop term is quartically divergent up to the order of p^4, which will vanish if we take dimensional regularization. Then up to the order of 1-loop precision, colorons make no contributions.

D. Integrating out of Z'

Equations. (57b) and (59) imply

$$B_{1\mu\nu} = B_{\mu\nu}\sin\theta' + (\partial_\mu Z'_\nu - \partial_\nu Z'_\mu)\cos\theta', \tag{128a}$$

$$B_{2\mu\nu} = B_{\mu\nu}\cos\theta' - (\partial_\mu Z'_\nu - \partial_\nu Z'_\mu)\sin\theta'. \tag{128b}$$

Substituting $B_{1\mu} = B_\mu \sin\theta' + Z'_\mu \cos\theta'$, $B_{2\mu} = B_\mu \cos\theta' - Z'_\mu \sin\theta'$ and Eq. (128) into the right-hand side of Eq. (116), combined with (126), we get

$$\begin{aligned}
&\mathcal{N}[W^a_\mu, B_\mu]\exp(iS_{\rm eff}[U, W^a_\mu, B_\mu])\\
&= \exp\left[i\int d^4x\left(-\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu}\right)\right]\int \mathcal{D}Z'_\mu \exp\left[i\int d^4x\left(-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}(\partial_\mu Z'_\nu - \partial_\nu Z'_\mu)^2\right.\right.\\
&\quad \left.+ \frac{3g_1^2(F_0^{\rm TC1})^2}{16}(\cot\theta' + \tan\theta')^2 Z'^2 - \frac{1}{2}\mathcal{K}\left[\frac{3g_1^2}{2}B_{\mu\nu}B^{\mu\nu} + \frac{3g_1^2}{4}(\cot^2\theta' + \tan^2\theta')Z'_{\mu\nu}Z'^{,\mu\nu} + \frac{3g_1^2}{2}(\cot\theta' - \tan\theta')B_{\mu\nu}Z'^{,\mu\nu}\right]\right)\\
&\quad + i\bar{S}_{\rm coloron}[Z'] + iS^{(4c,B)}_{\rm TC1}[B] + iS^{(4c,Z')}_{\rm TC1}[Z'] + iS^{(4c,BZ')}_{\rm TC1}[B,Z'] + {\rm Tr}\ln\left[i\not\partial - g_2\frac{\tau^a}{2}\not W^a P_L - q_2\frac{1}{6}\not B_2 P_L - q_2\left(\frac{1}{6} + \frac{\tau^3}{2}\right)\not B_2 P_R\right]\\
&\quad \left.+ iS_{\text{TC2-induced eff}}[U, W, B_2]\right],
\end{aligned} \tag{129}$$

Ignoring term $iS_{\text{TC2-induced eff}}[U, W, B_2]$ in the above expression, we get an expression for $\mathcal{N}[W^a_\mu, B_\mu]$. In the above result, if we denote the Z' involved part as $\int \mathcal{D}Z'_\mu e^{iS_{Z'}[Z',U,W^a,B]}$, then we will find that Z' field in $S_{Z'}[Z', U, W^a, B]$ is not correctly normalized since the coefficient in front of kinetic term is not standard $-1/4$. We now introduce normalized fields $Z'_{R,\mu}$ as

$$Z'_\mu = \frac{1}{c'}Z'_{R,\mu} \tag{130}$$

$$\begin{aligned}
c'^{,2} = 1 &+ \mathcal{K}\frac{3g_1^2}{2}(\cot^2\theta' + \tan^2\theta')\\
&+ \frac{3g_1^2}{4}[\mathcal{K}_2^{\text{TC1},\Sigma\neq 0}(\cot\theta' + \tan\theta')^2\\
&+ \mathcal{K}_{13}^{\text{TC1},\Sigma\neq 0}(\cot\theta' - \tan\theta')^2]\\
&- \frac{11}{9}g_1^2\mathcal{K}\tan^2\theta' - \frac{2}{9}(L_{10}^{\rm 1D} + 11H_1^{\rm 1D})g_1^2\tan^2\theta'
\end{aligned} \tag{131}$$

then

$$\begin{aligned}
S_{Z'}[Z', U, W^a, B] = \int d^4x\Big[&\frac{1}{2}Z'_{R,\mu}(x)D_Z^{-1,\mu\nu}Z'_{R,\nu}(x)\\
&+ Z_R'^{,\mu}J_{Z,\mu} + Z_R'^2 Z'_{R,\mu}J^\mu_{3Z}\\
&+ g_{4Z}\frac{g_1^4}{c'^4}Z_R'^{,4}\Big]
\end{aligned} \tag{132}$$

with

$$D_Z^{-1,\mu\nu} = g^{\mu\nu}(\partial^2 + M^2_{Z'}) - (1 + \lambda_Z)\partial^\mu\partial^\nu + \Delta_Z^{\mu\nu}(X) \tag{133}$$

$$M^2_{Z'} = \frac{3g_1^2(F_0^{\rm TC1})^2}{8c'^2}(\cot\theta' + \tan\theta')^2 + \frac{g_1^2(F_0^{\rm TC2})^2}{4c'^2}\tan^2\theta' \tag{134}$$

$$\lambda_Z = -\mathcal{K}_1^{\text{TC1},\Sigma\neq 0}\frac{3g_1^2}{8c'^2}(\cot\theta' + \tan\theta')^2 \tag{135}$$

$$\begin{aligned}
\Delta_Z^{\mu\nu}(X) = -g_1^2\frac{\tan^2\theta'}{c'^2}\{&(2L_1^{\rm 1D} + L_3^{\rm 1D})g^{\mu\nu}({\rm tr}X^2)\\
&+ (2L_1^{\rm 1D} + L_3^{\rm 1D})\,{\rm tr}(X^\mu\tau^3)\,{\rm tr}(X^\nu\tau 3)\\
&+ 2L_2^{\rm 1D}\,{\rm tr}(X^\mu X^\nu) + L_2^{\rm 1D}[g^{\mu\nu}\,{\rm tr}(X_{\nu'}\tau^3)\,{\rm tr}(X^{\nu'}\tau^3)\\
&+ {\rm tr}(X^\mu\tau^3)\,{\rm tr}(X^\nu\tau^3)]\}
\end{aligned} \tag{136}$$

and

$$J^\mu_Z = J^\mu_{Z0} + \frac{g_1^2\gamma}{c'}\partial^\nu B_{\mu\nu} + \tilde{J}^\mu_Z \tag{137}$$

$$J^\mu_{Z0} = -ig_1\frac{(F_0^{\rm TC2})^2}{4c'}\tan\theta'\,{\rm tr}(\tau^3 X^\mu) \tag{138}$$

$$\gamma = -(3\mathcal{K} + \mathcal{K}_{13}^{\mathrm{TC1},\Sigma\neq 0})\frac{1}{2}(\cot\theta' - \tan\theta') - (11\mathcal{K} + 2L_{10}^{\mathrm{1D}} + 22H_1^{\mathrm{1D}})\frac{1}{9}\tan\theta' \quad (139)$$

$$\begin{aligned}\tilde{J}_{Z,\mu} &= \frac{2i}{c'}\left(L_1^{\mathrm{1D}} + \frac{L_3^{\mathrm{1D}}}{2}\right)g_1\tan\theta'\,\mathrm{tr}(X_\mu\tau^3)(\mathrm{tr}X^2) \\ &+ \frac{2i}{c'}L_2^{\mathrm{1D}}g_1\tan\theta'\,\mathrm{tr}(X^\nu\tau^3)(\mathrm{tr}X_\mu X_\nu) \\ &+ \frac{1}{2c'}g_1L_9^{\mathrm{1D}}\tan\theta'\,\mathrm{tr}[(\bar{W}_{\mu\nu}\tau^3 - \tau^3\bar{W}_{\mu\nu})X^\nu] \\ &+ \frac{i}{2c'}L_9^{\mathrm{1D}}g_1\tan\theta'\partial^\nu[\mathrm{tr}\tau^3(X_\mu X_\nu - X_\nu X_\mu)] \\ &- \frac{1}{c'}L_{10}^{\mathrm{1D}}g_1\tan\theta'\partial^\nu\,\mathrm{tr}(\tau^3\bar{W}_{\mu\nu}) \end{aligned} \quad (140)$$

$$g_{4Z} = (\mathcal{K}_3^{\mathrm{TC1},\Sigma\neq 0} + \mathcal{K}_4^{\mathrm{TC1},\Sigma\neq 0})\frac{3}{256}(\cot\theta' + \tan\theta')^4 + \left(L_1^{\mathrm{1D}} + L_2^{\mathrm{1D}} + \frac{L_3^{\mathrm{1D}}}{2}\right)\frac{1}{4}\tan^4\theta' \quad (141)$$

$$J_{3Z}^\mu = -\frac{i}{c'^3}\left(L_1^{\mathrm{1D}} + L_2^{\mathrm{1D}} + \frac{L_3^{\mathrm{1D}}}{2}\right)g_1^3\tan^3\theta'\,\mathrm{tr}(X^\mu\tau^3). \quad (142)$$

We denote the resulting action after the integration over Z' as

$$\int \mathcal{D}Z'_\mu e^{iS_{Z'}[Z',U,W^a,B]} = e^{i\bar{S}_{Z'}[U,W^a,B]} \quad (143)$$

We can use loop expansion to calculate the above integration

$$\bar{S}_{Z'}[U, W^a, B] = S_{Z'}[Z'_c, U, W^a, B] + \text{loop terms} \quad (144)$$

where classical field Z'_c satisfies

$$\frac{\partial}{\partial Z'_{c,\mu}(x)}[S_{Z'}[Z'_c, U, W^a, B] + \text{loop terms}] = 0. \quad (145)$$

With (132), the solution is

$$Z_c'^\mu(x) = -D_Z^{\mu\nu}J_{Z,\nu}(x) + O(p^3) + \text{loop terms} \quad (146)$$

then

$$\bar{S}_{Z'}[U, W^a, B] = \int d^4x\Big[-\frac{1}{2}J_{Z,\mu}D_Z^{\mu\nu}J_{Z,\nu} - J_{3Z,\mu'}(D_Z^{\mu'\nu'}J_{Z,\nu'})(D_Z^{\mu\nu}J_{Z,\nu})^2 + g_{4Z}\frac{g_1^4}{c'^4}(D_Z^{\mu\nu}J_{Z,\nu})^4\Big] + \text{loop terms} \quad (147)$$

where

$$D_Z^{-1,\mu\nu}D_{Z,\nu\lambda} = D_Z^{\mu\nu}D_{Z,\nu\lambda}^{-1} = g_\lambda^\mu. \quad (148)$$

It is not difficult to show that if we are accurate up to the order of p^4, then the order p solution for Z'_c is enough. All contributions from the order p^3 Z'_c at least belong to the order of p^6.

With these results, (129) becomes

$$\begin{aligned}&\mathcal{N}[W_\mu^a, B_\mu]\exp(iS_{\mathrm{eff}}[U, W_\mu^a, B_\mu]) \\ &= \exp\Big[i\bar{S}_{Z'}[U, W^a, B] + iS_{\mathrm{TC1}}^{(4c,B)}[B] + i\int d^4x\Big[-\frac{1}{4}W_{\mu\nu}^aW^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \mathcal{K}\frac{3g_1^2}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ \frac{1}{2}\mathcal{K}\left(\frac{11}{18}g_1^2B_{\mu\nu}B^{\mu\nu} + \frac{1}{2}g_2^2W_{\mu\nu}^aW^{a,\mu\nu}\right) - \frac{(F_0^{\mathrm{TC2}})^2}{4}\mathrm{tr}(X_\mu X^\mu) + \left(L_1^{\mathrm{1D}} + \frac{L_3^{\mathrm{1D}}}{2}\right)[\mathrm{tr}(X_\mu X^\mu)]^2 + L_2^{\mathrm{1D}}[\mathrm{tr}(X_\mu X_\nu)]^2 \\ &- iL_9^{\mathrm{1D}}\,\mathrm{tr}(\bar{W}_{\mu\nu}X^\mu X^\nu) - i\frac{L_9^{\mathrm{1D}}}{2}g_1B_{\mu\nu}\,\mathrm{tr}(\tau^3X^\mu X^\nu) + \frac{L_{10}^{\mathrm{1D}}}{2}g_1B_{\mu\nu}\,\mathrm{tr}(\tau^3\bar{W}^{\mu\nu}) + \frac{1}{18}(L_{10}^{\mathrm{1D}} + 11H_1^{\mathrm{1D}})g_1^2B_{\mu\nu}B^{\mu\nu} \\ &+ H_1^{\mathrm{1D}}\,\mathrm{tr}(\bar{W}_{\mu\nu}\bar{W}^{\mu\nu})\Big]\Big]. \end{aligned} \quad (149)$$

Ignoring the term with coefficients F_0^{TC2}, L_i^{1D}, and H_1^{1D} in the above expression, we get an expression for $\mathcal{N}[W_\mu^a, B_\mu]$. With it we finally obtain $S_{\mathrm{eff}}[U, W_\mu^a, B_\mu]$

$$\begin{aligned}S_{\mathrm{eff}}[U, W_\mu^a, B_\mu] = \int d^4x\Big[&-\frac{(F_0^{\mathrm{TC2}})^2}{4}\mathrm{tr}(X_\mu X^\mu) + \left(L_1^{\mathrm{1D}} + \frac{L_3^{\mathrm{1D}}}{2}\right)[\mathrm{tr}(X_\mu X^\mu)]^2 + L_2^{\mathrm{1D}}[\mathrm{tr}(X_\mu X_\nu)]^2 - iL_9^{\mathrm{1D}}\,\mathrm{tr}(\bar{W}_{\mu\nu}X^\mu X^\nu) \\ &- i\frac{L_9^{\mathrm{1D}}}{2}g_1B_{\mu\nu}\,\mathrm{tr}(\tau^3X^\mu X^\nu) + \frac{L_{10}^{\mathrm{1D}}}{2}g_1B_{\mu\nu}\,\mathrm{tr}(\tau^3\bar{W}^{\mu\nu}) + \frac{1}{18}(L_{10}^{\mathrm{1D}} + 11H_1^{\mathrm{1D}})g_1^2B_{\mu\nu}B^{\mu\nu} \\ &+ H_1^{\mathrm{1D}}\,\mathrm{tr}(\bar{W}_{\mu\nu}\bar{W}^{\mu\nu})\Big] + \Delta S_{\mathrm{eff}}[U, W_\mu^a, B_\mu] \end{aligned} \quad (150)$$

i.e. our result EWCL is equal to the standard one-doublet technicolor model result plus contributions from Z'. We denote this Z' contribution part $\Delta S_{\rm eff}[U, W_\mu^a, B_\mu]$,

$$\Delta S_{\rm eff}[U, W_\mu^a, B_\mu] = \bar{S}_{Z'}[U, W^a, B] - \bar{S}_{Z'}[U, W^a, B]|_{F_0^{\rm TC2}=0, L_i^{\rm 1D}=H_1^{\rm 1D}=0}. \tag{151}$$

Correspondingly, EWCL coefficients for the topcolor-assisted technicolor model can also be divided into two parts

$$f^2 = (F_0^{\rm TC2})^2 \qquad \beta_1 = \Delta\beta_1$$
$$\alpha_i = \alpha_i|_{\rm one\ doulet} + \Delta\alpha_i \qquad i = 1, 2, \ldots, 14 \tag{152}$$

in which $\alpha_i|_{\rm one\ doulet}$ $i = 1, 2, \ldots, 14$ are coefficients from the one-doublet technicolor model. Their values are given in (9) and Table I. $\Delta\beta_1$ and $\Delta\alpha_i$ $i = 1, 2, \ldots, 14$ are contributions from Z' and ordinary quarks. Since we do not consider ordinary quarks in this work, in the following we calculate Z' contributions.

With the help of (133), (137), and (147),

$$\Delta S_{\rm eff}[U, W_\mu^a, B_\mu] = \int d^4x\bigg[-\frac{1}{2}J_{Z0,\mu}D_Z^{\mu\nu}J_{Z0,\nu} - \frac{1}{M_{Z'}^2}J_{Z0,\mu}\bigg(\tilde{J}_Z^\mu + \frac{g_1^2\gamma}{c'}\partial_\nu B^{\mu\nu}\bigg) - \frac{1}{M_{Z'}^6}J_{3Z,\mu}J_{Z0}^\mu J_{Z0}^2 + \frac{g_{4Z}g_1^4}{c'^4M_{Z'}^8}J_{Z0}^4\bigg]. \tag{153}$$

With the help of the following algebra relations,

$$\partial_\mu\,{\rm tr}[\tau^3X^\mu] = 0$$
$${\rm tr}[\tau^3(\partial_\mu X_\nu - \partial_\nu X_\mu)] = -2\,{\rm tr}(\tau^3X_\mu X_\nu) + i\,{\rm tr}(\tau^3\bar{W}_{\mu\nu}) - ig_1B_{\mu\nu}\,{\rm tr}(\tau^3X_\mu X_\nu)\,{\rm tr}(\tau^3X^\mu X^\nu)$$
$$= [{\rm tr}(X_\mu X_\nu)]^2 - [{\rm tr}(X_\mu X^\mu)]^2 - {\rm tr}(X_\mu X_\nu)\,{\rm tr}(\tau^3X^\mu)\,{\rm tr}(\tau^3X^\nu) + {\rm tr}(X_\mu X^\mu)[{\rm tr}(\tau^3X_\nu)]^2\,{\rm tr}(TA)$$
$$\times\,{\rm tr}(TBC) + {\rm tr}(TB)\,{\rm tr}(TCA) + {\rm tr}(TC)\,{\rm tr}(TAB) = 2\,{\rm tr}(ABC) \tag{154}$$

where ${\rm tr}A = {\rm tr}B = {\rm tr}C = 0$ and $T^2 = 1$. We can show that (153) leads to the form of standard EWCL, further combined with (133) and (138) and Table I, we can read out p^2 coefficient

$$\beta_1 = \frac{g_1^2(F_0^{\rm TC2})^2}{8c'^2M_{Z'}^2}\tan^2\theta' = \frac{(F_0^{\rm TC2})^2}{3(F_0^{\rm TC1})^2(\cot^2\theta' + 1)^2 + 2(F_0^{\rm TC2})^2} \tag{155}$$

which implies a positive and bounded above β_1. With the fact that $f = F_0^{\rm TC2} = 250$ GeV and original model requirement $F_0^{\rm TC1} = 1$ TeV, we find

$$2\beta_1 = \frac{1}{24(\cot^2\theta' + 1)^2 + 1}. \tag{156}$$

Combine with $\alpha T = 2\beta_1$ given in Ref. [2], we obtain the result that the topcolor-assisted technicolor model produces a positive and bounded above T parameter. The upper limit of β_1 is $1/50$, which corresponds to the upper limit of T parameter $1/(25\alpha) \sim 5.1$. Note this upper limit does not mean parameter T can take this maximum value. Instead we will see later that there exist some more stringent upper bounds for T which depend on the Z' mass. From (155), we know the β_1 coefficient is uniquely determined by parameter θ', therefore instead of using θ' as the input parameter of the theory, we can further use β_1 or $T = 2\beta_1/\alpha$ as the input parameter of the theory.

The p^4 order coefficients can be read out from derived EWCL (150) and (153); we list down the results as follows:

TABLE V. The symmetry breaking sector of the electroweak chiral Lagrangian.

	Formulation I	Formulation II
$\mathcal{L}^{(2)}$	$-\frac{1}{4}f^2\,{\rm tr}(V_\mu V^\mu)$	$-\frac{1}{4}f^2\,{\rm tr}(X_\mu X^\mu)$
$\mathcal{L}^{(2)\prime}$	$\frac{1}{4}\beta_1 f^2[{\rm tr}(TV_\mu)]^2$	$\frac{1}{4}\beta_1 f^2[{\rm tr}(\tau^3X_\mu)]^2$
$\frac{\mathcal{L}_1}{\alpha_1}$	$\frac{1}{2}g_2g_1B_{\mu\nu}\,{\rm tr}(TW^{\mu\nu})$	$\frac{1}{2}g_1B_{\mu\nu}\,{\rm tr}(\tau^3\bar{W}^{\mu\nu})$
$\frac{\mathcal{L}_2}{\alpha_2}$	$\frac{1}{2}ig_1B_{\mu\nu}\,{\rm tr}(T[V^\mu, V^\nu])$	$ig_1B_{\mu\nu}\,{\rm tr}(\tau^3X^\mu X^\nu)$
$\frac{\mathcal{L}_3}{\alpha_3}$	$ig_2\,{\rm tr}(W_{\mu\nu}[V^\mu, V^\nu])$	$2i\,{\rm tr}(\bar{W}_{\mu\nu}X^\mu X^\nu)$
$\frac{\mathcal{L}_4}{\alpha_4}$	$[{\rm tr}(V_\mu V_\nu)]^2$	$[{\rm tr}(X_\mu X_\nu)]^2$
$\frac{\mathcal{L}_5}{\alpha_5}$	$[{\rm tr}(V_\mu V^\mu)]^2$	$[{\rm tr}(X_\mu X^\mu)]^2$
$\frac{\mathcal{L}_6}{\alpha_6}$	${\rm tr}(V_\mu V_\nu)\,{\rm tr}(TV^\mu)\,{\rm tr}(TV^\nu)$	${\rm tr}(X_\mu X_\nu)\,{\rm tr}(\tau^3X^\mu)\,{\rm tr}(\tau^3X^\nu)$
$\frac{\mathcal{L}_7}{\alpha_7}$	${\rm tr}(V_\mu V^\mu)\,{\rm tr}(TV_\nu)\,{\rm tr}(TV^\nu)$	${\rm tr}(X_\mu X^\mu)\,{\rm tr}(\tau^3X_\nu)\,{\rm tr}(\tau^3X^\nu)$
$\frac{\mathcal{L}_8}{\alpha_8}$	$\frac{1}{4}g_2^2[{\rm tr}(TW_{\mu\nu})]^2$	$\frac{1}{4}[{\rm tr}(\tau^3\bar{W}_{\mu\nu})]^2$
$\frac{\mathcal{L}_9}{\alpha_9}$	$\frac{i}{2}g_2\,{\rm tr}(TW_{\mu\nu})\,{\rm tr}(T[V^\mu, V^\nu])$	$i\,{\rm tr}(\tau^3\bar{W}_{\mu\nu})\,{\rm tr}(\tau^3X^\mu X^\nu)$
$\frac{\mathcal{L}_{10}}{\alpha_{10}}$	$\frac{1}{2}[{\rm tr}(TV_\mu)\,{\rm tr}(TV_\nu)]^2$	$\frac{1}{2}[{\rm tr}(\tau^3X_\mu)\,{\rm tr}(\tau^3X_\nu)]^2$
$\frac{\mathcal{L}_{11}}{\alpha_{11}}$	$g_2\epsilon^{\mu\nu\rho\lambda}\,{\rm tr}(TV_\mu)\,{\rm tr}(V_\nu W_{\rho\lambda})$	$\epsilon^{\mu\nu\rho\lambda}\,{\rm tr}(\tau^3X_\mu)\,{\rm tr}(X_\nu\bar{W}_{\rho\lambda})$
$\frac{\mathcal{L}_{12}}{\alpha_{12}}$	$g_2\,{\rm tr}(TV_\mu)\,{\rm tr}(V_\nu W^{\mu\nu})$	${\rm tr}(\tau^3X_\mu)\,{\rm tr}(X_\nu\bar{W}^{\mu\nu})$
$\frac{\mathcal{L}_{13}}{\alpha_{13}}$	$g_2g_1\epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}\,{\rm tr}(TW_{\rho\sigma})$	$\epsilon^{\mu\nu\rho\sigma}g_1B_{\mu\nu}\,{\rm tr}(\tau^3\bar{W}_{\rho\sigma})$
$\frac{\mathcal{L}_{14}}{\alpha_{14}}$	$g_2^2\epsilon^{\mu\nu\rho\sigma}\,{\rm tr}(TW_{\mu\nu})\,{\rm tr}(TW_{\rho\sigma})$	$\epsilon^{\mu\nu\rho\sigma}\,{\rm tr}(\tau^3\bar{W}_{\mu\nu})\,{\rm tr}(\tau^3\bar{W}_{\rho\sigma})$

$$\alpha_1 = (1-2\beta_1)L_{10}^{1D} + \frac{(F_0^{TC2})^2}{2M_{Z'}^2}\beta_1 - 2\gamma\beta_1\cot\theta'$$
$$\alpha_2 = -\frac{1}{2}(1-2\beta_1)L_9^{1D} + \frac{(F_0^{TC2})^2}{2M_{Z'}^2}\beta_1 - 2\gamma\beta_1\cot\theta'$$
$$\alpha_3 = -\frac{1}{2}(1-2\beta_1)L_9^{1D}$$
$$\alpha_4 = L_2^{1D} + \frac{(F_0^{TC2})^2}{2M_{Z'}^2}\beta_1 + 2\beta_1 L_9^{1D}$$
$$\alpha_5 = L_1^{1D} + \frac{L_3^{1D}}{2} - \frac{(F_0^{TC2})^2}{2M_{Z'}^2}\beta_1 - 2\beta_1 L_9^{1D}$$
$$\alpha_6 = -\frac{(F_0^{TC2})^2}{2M_{Z'}^2}\beta_1 + 4\beta_1^2 L_2^{1D} - 4\beta_1\Big(L_2^{1D} + \frac{L_9^{1D}}{2}\Big)$$
$$\alpha_7 = \beta_1\frac{(F_0^{TC2})^2}{2M_{Z'}^2} + 2(\beta_1^2-\beta_1)(2L_1^{1D}+L_3^{1D}) + 2\beta_1 L_9^{1D}$$
$$\alpha_8 = -\beta_1\frac{(F_0^{TC2})^2}{2M_{Z'}^2} + 4\beta_1 L_{10}^{1D}$$
$$\alpha_9 = -\beta_1\frac{(F_0^{TC2})^2}{2M_{Z'}^2} + 2\beta_1(-L_9^{1D} + L_{10}^{1D})$$
$$\alpha_{10} = (4\beta_1^2 - 8\beta_1^3)(2L_1^{1D} + 2L_2^{1D} + L_3^{1D}) + 32\beta_1^4 g_{4Z}\cot^4\theta'$$
$$\alpha_{11} = \alpha_{12} = \alpha_{13} = \alpha_{14} = 0 \qquad (157)$$

Several features of this result are:

(1) Except for part of the one-doublet technicolor model result, all corrections from the Z' particle are at least proportional to β_1, which vanish if the mixing disappears by $\theta' = 0$.

(2) Since $L_{10}^{1D} < 0$, (157) then tells us α_8 is negative and then $U = -16\pi\alpha_8$ is always positive in this model.

(3) Except for α_1, α_2, and α_{10}, all other coefficients are determined by the one-doublet technicolor model coefficients given in Table IV and two other parameter β_1 and $F_0^{TC2}/M_{Z'}$.

(4) α_{10} further depends on parameter g_{4Z} which from (141) further depends on $\mathcal{K}_3^{TC1,\Sigma\neq 0} + \mathcal{K}_4^{TC1,\Sigma\neq 0}$ which is already given by (97) and Table III.

(5) α_1 and α_2 depend on γ which from (139) further relies on an extra parameter $\mathcal{K}$. We can combine (131) and (155) together to fix $\mathcal{K}$,

$$\frac{(F_0^{TC2})^2}{8\beta_1 M_{Z'}^2}\tan^2\theta' = \frac{1}{g_1^2} + \mathcal{K}\Big(\frac{3}{2}\cot^2\theta' + \frac{5}{18}\tan^2\theta'\Big) + \frac{3}{4}[\mathcal{K}_2^{TC1,\Sigma\neq 0}(\cot\theta' + \tan\theta')^2 + \mathcal{K}_{13}^{TC1,\Sigma\neq 0}(\cot\theta' - \tan\theta')^2] - \frac{2}{9}(L_{10}^{1D} + 11H_1^{1D})\tan^2\theta'.$$

Once $\mathcal{K}$ is fixed, with help of (91), we can determine the ratio of infrared cutoff κ and ultraviolet cutoff Λ, in Fig. 1, we draw the κ/Λ as function of T and $M_{Z'}$, we find our calculations do produce very large hierarchy and we further find that not all of the T and $M_{Z'}$ region is available if we consider the natural criteria $\Lambda > \kappa$. This criteria leads to the constraint that as long as the Z' mass becomes large, the allowed range for the T parameter becomes smaller and smaller, approaching zero. For example, $T < 0.37$ for $M_{Z'} = 0.5$ TeV, $T < 0.0223$ for $M_{Z'} = 1$ TeV, $T < 0.004$ for $M_{Z'} = 2$ TeV and $T < 0.002$ for $M_{Z'} = 4$ TeV. In Fig. 2, we draw the Z' mass as function of the T parameter and κ/Λ. The line of $\kappa/\Lambda = 1$ gives the upper bound of the Z' mass. The upper bound of the Z' mass depends on the value of the T parameter: the smaller

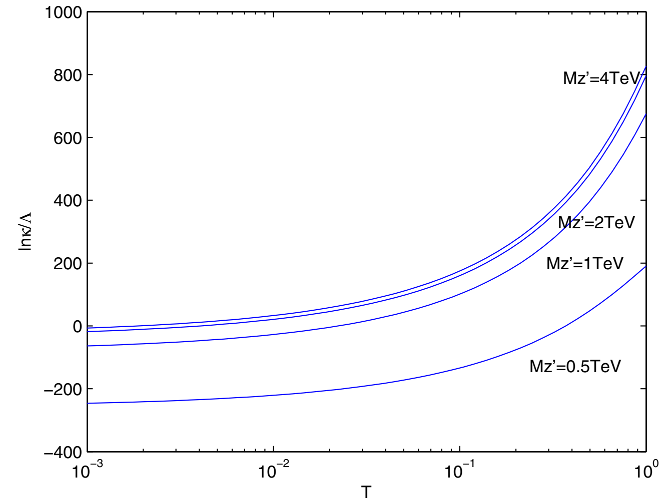

FIG. 1 (color online). The ratio of infrared cutoff and ultraviolet cutoff κ/Λ as a function of the T parameter and Z' mass in units of TeV.

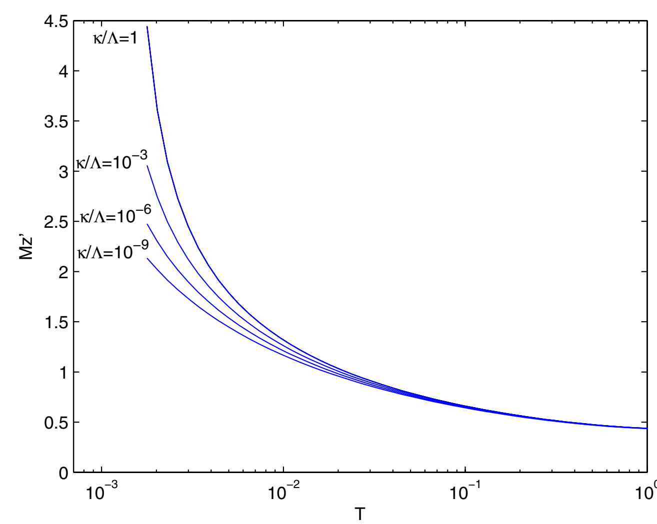

FIG. 2 (color online). Z' mass in units of TeV as a function of the T parameter and κ/Λ.

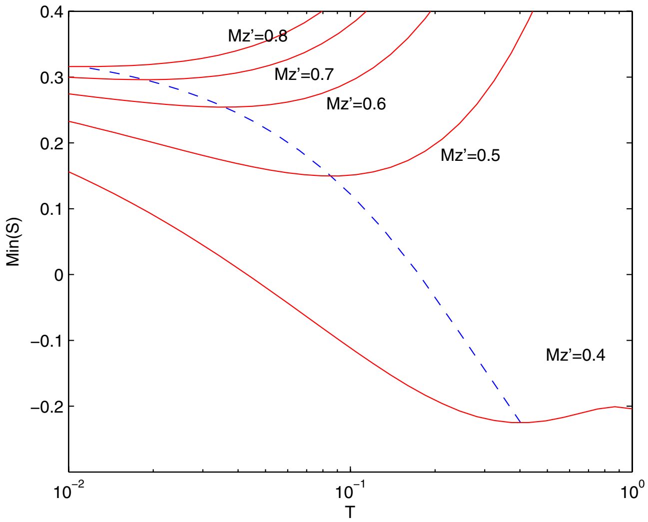

FIG. 3 (color online). The dashed line is the minimal S parameter in the topcolor-assisted technicolor model for different T. The solid lines are the isolines for different choices of the Z' mass in units of TeV.

the T, the larger the upper bound of $M_{Z'}$.

(6) For fixed $M_{Z'}$, there exists a special θ' value which maximizes α_1. The parameter $S = -16\pi\alpha_1$ is of special importance in the search for new physics. In Fig. 3, we draw a graph of minimal S parameter with a different T parameter. We see that if the Z' mass is low enough, say $M_{Z'} < 0.441$ TeV or $T > 0.176$, S will become negative.

Since we already know $F_0^{\rm TC2} = 250$ GeV, all EWCL coefficients then depend on two physical parameters β_1 and $M_{Z'}$. Combined with $\alpha T = 2\beta_1$, we can use the present experimental result for the T parameter to fix β_1. In Fig. 4, we draw a graph for the S in terms of the T parameter. We take three typical Z' masses $M_{Z'} = 0.5,\ 1,\ 2$ TeV for references. We do not draw the corresponding U parameter diagram because it typically is smaller than 10^{-2}. In Figs. 5–8, we draw graphs for all p^4 order nonzero coefficients in terms of the T parameter, where for α_3 and α_{10},

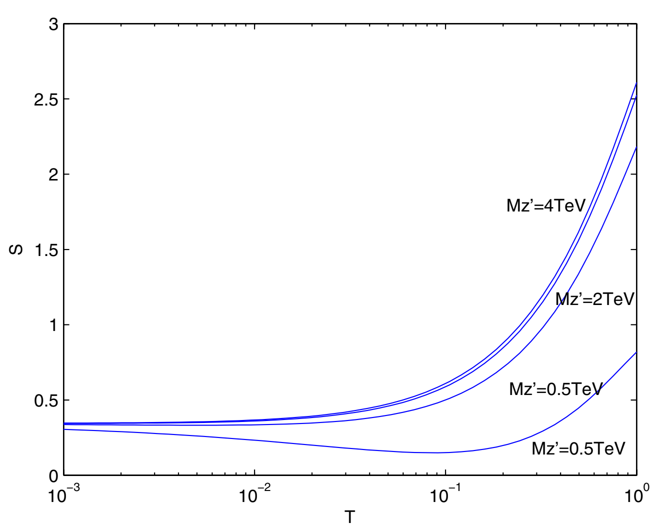

FIG. 4 (color online). The S parameter for the topcolor-assisted technicolor model. $F_0^{\rm TC2} = 250$ GeV, the T parameter and $M_{Z'} = \{0.5, 1, 2, 4\}$ TeV are as input parameters of the model.

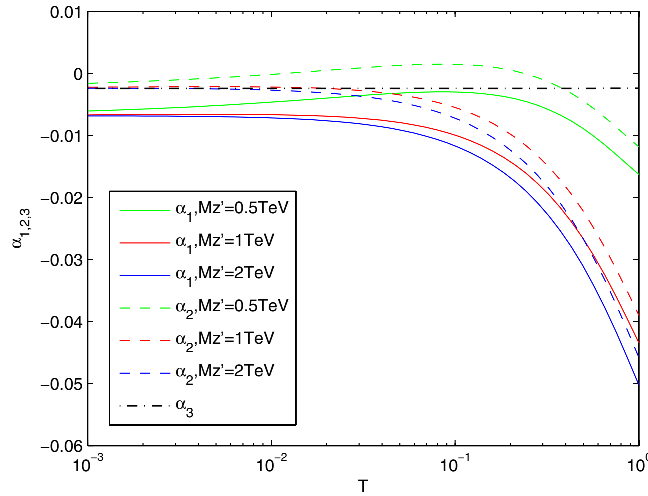

FIG. 5 (color online). Coefficients α_1, α_2, and α_3. $F_0^{\rm TC2} = 250$ GeV, T, and $M_{Z'}$ TeV are as input parameters. Except a horizontal dash-dot line represent α_3 independent of $M_{Z'}$, other curves are divided into three groups, each include a solid (α_1) and a dash (α_2) curve. The upper, middle, and nether group is for the case of $M_{Z'} = 0.5$, 1, 2 TeV, respectively.

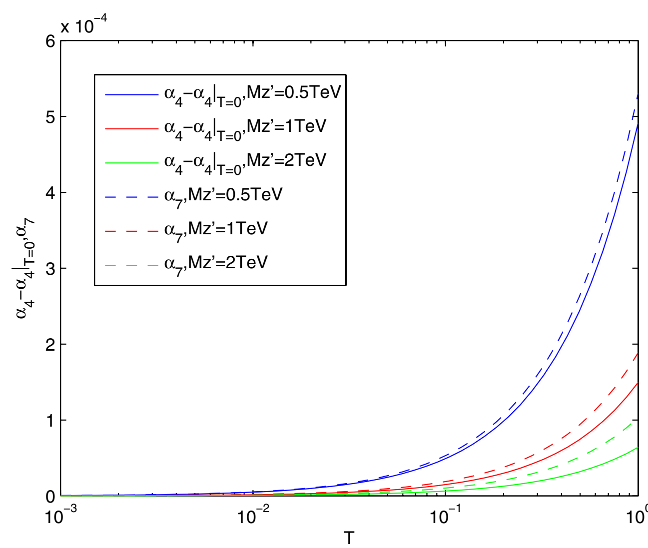

FIG. 6 (color online). Coefficients $\alpha_4 - \alpha_4|_{T=0} = -\alpha_5 + \alpha_5|_{T=0}$ and α_7. In which $\alpha_4|_{T=0} = 0.0020$ and $\alpha_5|_{T=0} = -0.0027$. $F_0^{\rm TC2} = 250$ GeV. Curves are divided into three groups, each include a solid ($\alpha_4 - \alpha_4|_{T=0}$) and a dash ($\alpha_7$) curve. The upper, middle, and nether group is for the case of $M_{Z'} = 0.5$, 1, 2 TeV, respectively.

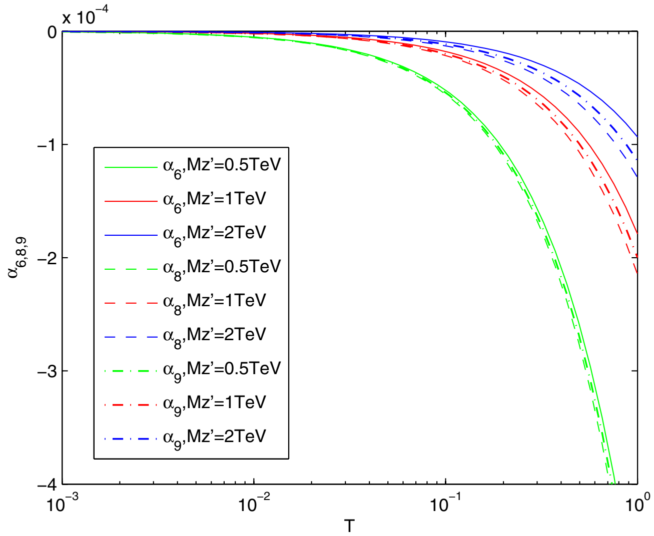

FIG. 7 (color online). Coefficients α_6, α_8, and α_9. $F_0^{\mathrm{TC2}} = 250$ GeV. Curves are divided into three groups, each include a solid (α_6), a dash (α_8), and a dash-dot (α_9) curve. The upper, middle, and nether group is for the case of $M_{Z'} = 2, 1, 0.5$ TeV, respectively.

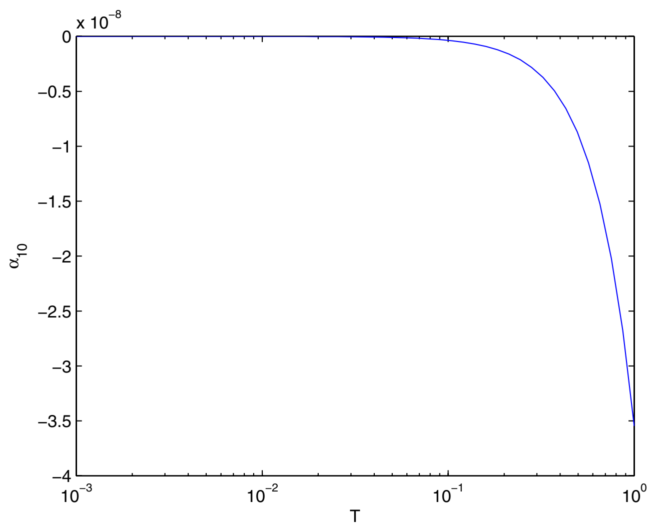

FIG. 8 (color online). Coefficients α_{10}. $F_0^{\mathrm{TC2}} = 250$ GeV.

we only draw one line for each of them since they are independent of the Z' mass. For α_4, we combine the result $\alpha_4|_{T=0} = 0.0020$ and draw its translation $\alpha_4 - \alpha_4|_{T=0}$ with α_7 in the same diagram. For α_5, to save space, we do not draw its diagram separately, since from (157), we find the relation $\alpha_5 - \alpha_5|_{T=0} = -\alpha_4 + \alpha_4|_{T=0}$. Furthermore, with the help of the result $\alpha_5|_{T=0} = -0.0027$, we can obtain its graph from the diagram for $\alpha_4 - \alpha_4|_{T=0}$.

IV. CONCLUSION

In this paper, we have set up a formulation to perform dynamical computation of the bosonic part of EWCL for the one-doublet and topcolor-assisted technicolor models. The one-doublet technicolor model as the earliest and simplest dynamical symmetry breaking model is taken as the trial model to test our formulation. We find our formulation recovers standard scaling-up results. The topcolor-assisted technicolor model is the main model we handle in this paper. We have computed its TC1 dynamics in detail and verify the dynamical symmetry breaking of the theory. The TC1 interaction will induce effective interactions among colorons and Z', which are characterized by a divergent constant $\mathcal{K}$, a dimensional constant F_0^{TC1}, and a series of dimensionless QCD constants L_1, L_3, L_9, L_{10}, H_1. For TC2 dynamics, it will induce effective interaction for Z', electroweak gauge fields and their goldstone bosons. Because of its similarity with QCD, we use Gasser-Leutwyler prescription to describe its low energy effects in terms of the low energy effective Lagrangian with a divergent constant $\mathcal{K}$, dimensional constant F_0^{TC2}, and a series of dimensionless constants L_1^{1D}, L_2^{1D}, L_3^{1D}, L_9^{1D}, L_{10}^{1D}, H_1^{1D} which are the same as those in the one-doublet technicolor model. Due to the similarity between TC2 dynamics and QCD, TC2 interactions make a direct contribution to EWCL coefficients, which is the same as that of the one-doublet technicolor model. Further corrections are from effective interactions among colorons, Z', and ordinary quarks induced by TC1 and TC2 interactions. We have shown that colorons make no contributions to EWCL coefficients within the approximations we have made in this paper, while ordinary quarks are ignored in this paper for future investigations. In fact, for some special EWCL coefficients, such as $S = -16\pi\alpha_1$, $\alpha T = 2\beta_1$, and $U = -16\pi\alpha_8$ parameters, general fermion contributions to them are already calculated [22]. S, T, U, and triple-gauge-vertices from a heavy nondegenerate fermion doublet have been estimated in Refs. [9,23]. One can use these general results to estimate possible contributions to some of the EWCL coefficients. For the topcolor-assisted technicolor model in this paper, the main work is to estimate the effects of the Z' particle. Our computation shows that contributions from the Z' particle are at least proportional to β_1 and then vanish if β_1 is zero. One typical feature of the model is the positivity and bounding above of β_1 parameter which means the T parameter must vary in the range $0 \sim 1/(25\alpha)$ and the positive U parameter. If we consider the natural criteria $\Lambda > \kappa$ which will further constrain the allowed range for the T parameter approaching zero as long as the Z' mass becomes large, for example, $T < 0.37$ for $M_{Z'} = 0.5$ TeV, $T < 0.0223$ for $M_{Z'} = 1$ TeV, and $T < 0.004$ for $M_{Z'} = 2$ TeV. For the S parameter, it can be either positive and negative depending on whether the Z' mass is large or small. As long as $M_{Z'} < 0.441$ TeV or $T > 0.176$, we may find negative S. There exists an upper bound for the mass of Z' which is dependent on the value of the T parameter: the smaller the T, the larger the upper bound of $M_{Z'}$. Except for $U(1)_Y$ coupling

g_1 and coefficients determined in the one-doublet technicolor model and QCD, all EWCL coefficients rely on experimental T parameter and coloron mass $M_{Z'}$. We have taken typical values of $M_{Z'}$ and a varied T parameter to estimate all EWCL coefficients up to the order of p^4. Further works on the matter part of EWCL and computing EWCL coefficients for other dynamical symmetry-breaking new physics models are in progress and will be reported elsewhere.

ACKNOWLEDGMENTS

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APPENDIX: NECESSARY FORMULAS FOR EWCL

In this appendix, we list down the necessary formulas needed in the text. First we describe two equivalent EWCL formalisms used in the literature and our work. EWCL is constructed using a dimensionless unitary unimodular 2×2 matrix field $U(x)$. In Ref. [2], it has been constructed with the building blocks, which are $SU(2)_L$ covariant and $U(1)_Y$ invariant, as

$$T \equiv U\tau^3 U^\dagger, \qquad V_\mu \equiv (D_\mu U)U^\dagger, \qquad g_1 B_{\mu\nu}, \qquad g_2 W_{\mu\nu} \equiv g_2 \frac{\tau^a}{2} W^a_{\mu\nu}. \tag{A1}$$

Alternatively, we reformulate EWCL equivalently with $SU(2)_L$ invariant and $U(1)_Y$ covariant building blocks as

$$\tau^3, \qquad X_\mu \equiv U^\dagger (D_\mu U), \qquad g_1 B_{\mu\nu}, \qquad \bar{W}_{\mu\nu} \equiv U^\dagger g_2 W_{\mu\nu} U, \tag{A2}$$

among which, τ^3 and $g_1 B_{\mu\nu}$ are both $SU(2)_L$ and $U(1)_Y$ invariant, while X_μ and $\bar{W}_{\mu\nu}$ are bilinearly $U(1)_Y$ covariant. This second formulation is largely used throughout this paper. In Table V, we list down the corresponding relations of the two formalisms.

Next we list down the detailed results for convergent part of p^4 order TC1 interaction $S^{(4)}_{\rm TC1}[0, B^A_\mu, B_\mu, Z'_\mu]$ given in (89). It is decomposed into five different parts in (94): $S^{(4c,B^A)}_{\rm TC1}[B^A]$, $S^{(4c,Z')}_{\rm TC1}[Z']$, $S^{(4c,B)}_{\rm TC1}[B]$ are parts only dependent on field B^A, Z', and B, respectively. $S^{(4c,BZ')}_{\rm TC1}[B, Z']$ are part B coupled with Z'. $S^{(4c,B^A Z')}_{\rm TC1}[B^A, Z']$ are part B^A coupled with Z'.

$$\begin{aligned} S^{(4c,B^A)}_{\rm TC1}[B^A] = \int d^4x\bigg[& -\mathcal{K}_1^{\rm TC1,\Sigma\neq 0}\frac{g_3^2}{8}(\cot\theta+\tan\theta)^2(\partial_\mu B^{A,\mu})^2 - \mathcal{K}_2^{\rm TC1,\Sigma\neq 0}\frac{g_3^2}{8}(\cot\theta+\tan\theta)^2 B^A_{a,\mu\nu}B_a^{A,\mu\nu} \\ & + \mathcal{K}_3^{\rm TC1,\Sigma\neq 0}\bigg[\frac{g_3^4}{192}(\cot\theta+\tan\theta)^4(B^A_\mu B^{A,\mu})^2 + \frac{g_3^4}{128}(\cot\theta+\tan\theta)^4(d^{ABC}B^B_\mu B^{C,\mu})^2\bigg] \\ & + \mathcal{K}_4^{\rm TC1,\Sigma\neq 0}\bigg\{\frac{g_3^4}{192}(\cot\theta+\tan\theta)^4(B^A_\mu B^A_\nu)^2 + \frac{g_3^4}{128}(\cot\theta+\tan\theta)^4[(if^{ABC}+d^{ABC})B^B_\mu B^C_\nu]^2\bigg\} \\ & - \mathcal{K}_{13}^{\rm TC1,\Sigma\neq 0}\frac{g_3^2}{8}(\cot\theta-\tan\theta)^2 B^A_{v,\mu\nu}B_v^{A,\mu\nu} + \mathcal{K}_{14}^{\rm TC1,\Sigma\neq 0}\frac{g_3^3}{32}(\cot\theta-\tan\theta)(\cot\theta+\tan\theta)^2 B_v^{A,\mu\nu}f^{ABC}B^B_\mu B^C_\nu\bigg], \end{aligned} \tag{A3}$$

$$\begin{aligned} S^{(4c,Z')}_{\rm TC1} = \int d^4x\bigg[& -\mathcal{K}_1^{\rm TC1,\Sigma\neq 0}\frac{3g_1^2}{16}(\cot\theta'+\tan\theta')^2(\partial_\mu Z'^{,\mu})^2 + (\mathcal{K}_3^{\rm TC1,\Sigma\neq 0}+\mathcal{K}_4^{\rm TC1,\Sigma\neq 0})\frac{3g_1^4}{256}(\cot\theta'+\tan\theta')^4(Z'_\mu Z'^{,\mu})^2 \\ & -\frac{3g_1^2}{16}[\mathcal{K}_2^{\rm TC1,\Sigma\neq 0}(\cot\theta'+\tan\theta')^2 + \mathcal{K}_{13}^{\rm TC1,\Sigma\neq 0}(\cot\theta'-\tan\theta')^2]Z'_{\mu\nu}Z'^{,\mu\nu}\bigg], \end{aligned} \tag{A4}$$

$$S^{(4c,B)}_{\rm TC1}[B] = \int d^4x\bigg[-\mathcal{K}_{13}^{\rm TC1,\Sigma\neq 0}\frac{3g_1^2}{4}B_{\mu\nu}B^{\mu\nu}\bigg], \tag{A5}$$

with $B^A_{v,\mu\nu} = \partial_\mu B^A_\nu - \partial_\nu B^A_\mu - \frac{g_3}{2}(\cot\theta - \tan\theta)f^{ABC}B^B_\mu B^C_\nu$ and

$$S^{(4c,BZ')}_{\rm TC1}[B,Z'] = \int d^4x\bigg[-\mathcal{K}_{13}^{\rm TC1,\Sigma\neq 0}\frac{g_1^2}{4}(\cot\theta' - \tan\theta')B_{\mu\nu}Z'^{,\mu\nu}\bigg], \tag{A6}$$

$$\begin{aligned} S^{(4c,B^AZ')}_{\rm TC1}[B^A,Z'] = \int d^4x\bigg[&\mathcal{K}_3^{\rm TC1,\Sigma\neq 0}\bigg[\frac{g_1^2g_3^2}{64}(\cot\theta+\tan\theta)^2(\cot\theta'+\tan\theta')^2B_\mu^AB^{A,\mu}Z'_\nu Z'^{,\nu} \\ &+\frac{g_1^2g_3^2}{32}(\cot\theta+\tan\theta)^2(\cot\theta'+\tan\theta')^2(B_\mu^AZ'^{,\mu})^2 \\ &+\frac{g_1g_3^3}{32}(\cot\theta+\tan\theta)^3(\cot\theta'+\tan\theta')d^{ABC}B_\mu^BB^{C,\mu}B_\mu^AZ'^{,\mu}\bigg] \\ &+\mathcal{K}_4^{\rm TC1,\Sigma\neq 0}\bigg\{\frac{g_1^2g_3^2}{256}(\cot\theta+\tan\theta)^2(\cot\theta'+\tan\theta')^2(B_\mu^AZ'^{,\mu})^2 \\ &+\frac{g_1^2g_3^2}{64}(\cot\theta+\tan\theta)^2(\cot\theta'+\tan\theta')^2[B_\mu^AB^{A,\mu}Z'_\nu Z'^{,\nu}+(B_\mu^AZ'^{,\mu})^2] \\ &+\frac{g_1g_3^3}{32}(\cot\theta+\tan\theta)^3(\cot\theta'+\tan\theta')d^{ABC}B_\mu^BB_\nu^CB^{A,\mu}Z'^{,\nu}\bigg\}\bigg]. \end{aligned} \tag{A7}$$

Finally, we list down the coloron interaction $S_{\rm coloron}[B^A,Z']$ given by (117) for which in (118) we have decomposed it into two parts. $S^0_{\rm coloron}[B^A,Z']$ is the part linear and quadratic in coloron fields and $S^{\rm int}_{\rm coloron}[B^A,Z']$ is the part cubic and quartic in coloron fields.

$$\begin{aligned} S^0_{\rm coloron}[B^A,Z'] = \int d^4x\bigg[\bigg(&g^{\mu\nu}\bigg\{\frac{(F_0^{\rm TC1})^2}{8}g_3^2(\cot\theta+\tan\theta)^2 \\ &+\bigg[\mathcal{K}_3^{\rm TC1,\Sigma\neq 0}\frac{g_1^2g_3^2}{64}(\cot\theta+\tan\theta)^2(\cot\theta'+\tan\theta')^2 \\ &+\mathcal{K}_4^{\rm TC1,\Sigma\neq 0}\frac{g_1^2g_3^2}{64}(\cot\theta+\tan\theta)^2(\cot\theta'+\tan\theta')^2\bigg]Z'_\lambda Z'^{,\lambda}\bigg\} \\ &+\bigg[\mathcal{K}_3^{\rm TC1,\Sigma\neq 0}\frac{g_1^2g_3^2}{32}(\cot\theta+\tan\theta)^2(\cot\theta'+\tan\theta')^2 \\ &+\mathcal{K}_4^{\rm TC1,\Sigma\neq 0}\frac{5g_1^2g_3^2}{256}(\cot\theta+\tan\theta)^2(\cot\theta'+\tan\theta')^2\bigg]Z'^{,\mu}Z'^{,\nu}\bigg)B_\mu^AB_{A,\nu} \\ &-\mathcal{K}_1^{\rm TC1,\Sigma\neq 0}\frac{g_3^2}{8}(\cot\theta+\tan\theta)^2(\partial_\mu B^{A,\mu})^2+\bigg[-\mathcal{K}_2^{\rm TC1,\Sigma\neq 0}\frac{g_3^2}{8}(\cot\theta+\tan\theta)^2 \\ &-\mathcal{K}_{13}^{\rm TC1,\Sigma\neq 0}\frac{g_3^2}{8}(\cot\theta-\tan\theta)^2-\frac{1}{4}-\frac{g_3^2}{4}\mathcal{K}(\cot^2\theta+\tan^2\theta)\bigg](\partial_\mu B_\nu^A-\partial_\nu B_\mu^A)(\partial^\mu B^{A,\nu}-\partial^\nu B^{A,\mu})\bigg] \end{aligned} \tag{A8}$$

and

$$
\begin{aligned}
S^{\rm int}_{\rm coloron}[B^A, Z'] = \int d^4x \Big[& -\mathcal{K}_2^{\rm TC1,\Sigma\neq 0}\frac{g_3^2}{8}(\cot\theta+\tan\theta)^2[(\partial_\mu B^A_\nu - \partial_\nu B^A_\mu)2g_3(-\cot\theta+\tan\theta)f^{ABC}B^{B,\mu}B^{C,\nu} \\
& + g_3^2(-\cot\theta+\tan\theta)^2 f^{ABC}B^B_\mu B^C_\nu f^{AB'C'}B^{B',\mu}B^{C',\nu}] \\
& + \mathcal{K}_3^{\rm TC1,\Sigma\neq 0}\bigg[\frac{g_3^4}{192}(\cot\theta+\tan\theta)^4(B^A_\mu B^{A,\mu})^2 + \frac{g_3^4}{128}(\cot\theta+\tan\theta)^4(d^{ABC}B^B_\mu B^{C,\mu})^2\bigg] \\
& + \mathcal{K}_4^{\rm TC1,\Sigma\neq 0}\bigg\{\frac{g_3^4}{192}(\cot\theta+\tan\theta)^4(B^A_\mu B^A_\nu)^2 + \frac{g_3^4}{128}(\cot\theta+\tan\theta)^4[(if^{ABC}+d^{ABC})B^B_\mu B^C_\nu]^2\bigg\} \\
& - \mathcal{K}_{13}^{\rm TC1,\Sigma\neq 0}\frac{g_3^2}{8}(\cot\theta-\tan\theta)^2\bigg[(\partial_\mu B^A_\nu - \partial_\nu B^A_\mu)g_3(-\cot\theta+\tan\theta)f^{ABC}B^{B,\mu}B^{C,\nu} \\
& + \frac{1}{4}g_3^2(-\cot\theta+\tan\theta)^2 f^{ABC}B^B_\mu B^C_\nu f^{AB'C'}B^{B',\mu}B^{C',\nu}\bigg] + \mathcal{K}_{14}^{\rm TC1,\Sigma\neq 0}\frac{g_3^3}{32}(\cot\theta-\tan\theta)(\cot\theta+\tan\theta)^2 \\
& \times\bigg[\partial^\mu B^{A,\nu} - \partial^\nu B^{A,\mu} + \frac{g_3}{2}(-\cot\theta+\tan\theta)f^{ABC}B^{B,\mu}B^{C,\nu}\bigg]f^{ABC}B^B_\mu B^C_\nu \\
& + \mathcal{K}_3^{\rm TC1,\Sigma\neq 0}\frac{g_1 g_3^3}{32}(\cot\theta+\tan\theta)^3(\cot\theta'+\tan\theta')d^{ABC}B^B_\mu B^{C,\mu}B^A_\nu Z'^{,\nu} \\
& + \mathcal{K}_4^{\rm TC1,\Sigma\neq 0}\frac{g_1 g_3^3}{32}(\cot\theta+\tan\theta)^3(\cot\theta'+\tan\theta')d^{ABC}B^B_\mu B^C_\nu B^{A,\mu}Z'^{,\nu} \\
& - \frac{1}{4}\cos^2\theta[2g_3(\partial_\mu B^A_\nu - \partial_\nu B^A_\mu)\cot\theta f^{ABC}B^{B,\mu}B^{C,\nu} + g_3^2\cot^2\theta f^{ABC}B^{B,\mu}B^{C,\nu}f^{AB'C'}B^{B'}_\mu B^{C'}_\nu] \\
& - \frac{1}{4}\sin^2\theta[-2g_3(\partial_\mu B^A_\nu - \partial_\nu B^A_\mu)\tan\theta f^{ABC}B^{B,\mu}B^{C,\nu} + g_3^2\tan^2\theta f^{ABC}B^{B,\mu}B^{C,\nu}f^{AB'C'}B^{B'}_\mu B^{C'}_\nu] \\
& - \frac{g_3^2}{4}\mathcal{K}\cot^2\theta[-2g_3(\partial_\mu B^A_\nu - \partial_\nu B^A_\mu)\cot\theta f^{ABC}B^{B,\mu}B^{C,\nu} + g_3^2\cot^2\theta f^{ABC}B^{B,\mu}B^{C,\nu}f^{AB'C'}B^{B'}_\mu B^{C'}_\nu] \\
& - \frac{g_3^2}{4}\mathcal{K}\tan^2\theta[2g_3(\partial_\mu B^A_\nu - \partial_\nu B^A_\mu)\tan\theta f^{ABC}B^{B,\mu}B^{C,\nu} + g_3^2\tan^2\theta f^{ABC}B^{B,\mu}B^{C,\nu}f^{AB'C'}B^{B'}_\mu B^{C'}_\nu]\Big], \qquad \text{(A9)}
\end{aligned}
$$

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