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Fermion masses, leptogenesis, and supersymmetric SO(10) unification

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Current neutrino oscillation data indicate the existence of two large lepton mixing angles, while Kobayashi-Maskawa matrix elements are all small. Here we show how supersymmetric SO(10) with extra chiral singlets can easily reconcile large lepton mixing angles with small quark mixing angles within the framework of the successful Fritzsch ansatz. Moreover, we show how this is fully consistent with the thermal leptogenesis scenario, avoiding the so-called gravitino problem. A sizable asymmetry can be generated at scales as low as possible within the leptogenesis mechanism. We present our results in terms of the leptonic *CP* violation parameter that characterizes neutrino oscillations.

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I. INTRODUCTION

An endemic difficulty in unified gauge models is the understanding of fermion masses. Thanks to the brilliant series of experiments which led to the discovery of neutrino oscillations over the past ten years or so, we now have an improved knowledge of neutrino masses and mixings [1]. The maximal atmospheric mixing angle and the large solar mixing angle both came as a surprise, at odds with naive unification-based expectations that lepton and quark mixings are similar. Indeed, although not mandatory, within unification models a similar structure for quark and lepton mixing is far easier to account for than what is currently established by observation. Several approaches have been considered in the literature to circumvent this problem, involving various types of extensions of the multiplet content [2–6].

A virtue of seesaw models is that they bring the possibility of decoupling the angles in the quark and lepton sectors by having additional Majorana-type terms for the latter [7]. Moreover, seesaw models may account for the observed cosmological baryon excess through the elegant mechanism called leptogenesis [8]. However, in minimal supergravity models, with $m_{3/2} \sim 100$ GeV to 10 TeV the supersymmetric type-I seesaw scenario leads to an overproduction of gravitinos after inflation [9,10] unless the reheat temperature is restricted to be much lower than that required for successful leptogenesis [11]. One possible way out is resonant leptogenesis [12] as indeed suggested in Ref. [13]. Another alternative has been considered in Ref. [14] requires going beyond the minimal seesaw by adding a small R-parity violating term in the superpotential.

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Here we consider an alternative approach to supersymmetric SO(10) unification that involves mainly an extension of the lepton sector and a novel realization of the seesaw mechanism already discussed previously [15,16]. We show how the observed structure of lepton masses and mixings fits together, thanks to the presence of the extra states, with the requirements of a predictive pattern of quark mixing, encoded within the successful Fritzsch ansatz [17]. Moreover, we show explicitly how such extended supersymmetric SO(10) seesaw scheme provides a realistic scenario for thermal leptogenesis, avoiding the so-called gravitino problem. Our approach is phenomenological in spirit. Hence, we do not seek to derive a full-fledged unified theory of flavor incorporating a specific symmetry, such as A_4 [18–22] to the unified gauge group level.

The paper is organized as follows. In Sec. II we discuss the limitations of minimal supersymmetric type-I seesaw in naturally providing a framework for thermal leptogenesis in agreement with bounds on the reheat temperature after inflation, in Sec. III we recall the basic features of the considered model, while ansatzes for the coupling matrices are discussed in Sec. IV. In Sec. V we discuss the calculation of the final baryon number asymmetry from the decay asymmetry of the lightest singlet. There we discuss how we can use results obtained previously on production and washout factors in the seesaw type-I case also in our extended seesaw model. We show that leptogenesis scenario is consistent both with the reheat temperature constraint as well as with fitting the lepton and quark mixing angles within the successful Fritzsch ansatz.

II. MINIMAL SUPERSYMMETRIC SEESAW LEPTOGENESIS

In the most general seesaw model, the Dirac fermion mass terms arise from the couplings of the **16** with the **10** and **126**, while the diagonal entries in the seesaw neutrino mass matrix arise only from the couplings of the **16** with

the **126**. The mass matrix expressed in the basis is ν_L, ν_L^c is given as [23]

$$\mathcal{M}_\nu = \begin{pmatrix} Y_L\langle\Delta_L\rangle & Y\langle\Phi\rangle \\ Y^T\langle\Phi\rangle & Y_R\langle\Delta_R\rangle \end{pmatrix}, \tag{1}$$

where the **16** denotes each chiral matter generation while the **10** and **126** are Higgs-type chiral multiplets. Here Y, Y_L, and Y_R denote the Yukawa couplings of the **10** and **126**, respectively. These are all symmetric in flavor, the symmetry of Y_L, Y_R results from the Pauli principle, while that of Y follows from SO(10).

Minimization of the Higgs scalar potential leads to a vev seesaw relation

$$\langle\Delta_L\rangle\langle\Delta_R\rangle \sim \langle\Phi\rangle^2, \tag{2}$$

consistent with the desired vev hierarchy $\langle\Delta_R\rangle \gg \langle\Phi\rangle \gg \langle\Delta_L\rangle$, where $\langle\Phi\rangle$ is the standard vev and $\langle\Delta_{L,R}\rangle$ are the vacuum expectation values (vevs) giving rise to the Majorana terms.

Before we display our proposed model, it is useful to consider the case of the minimal type-I seesaw [24]. This consists in taking $\langle\Delta_L\rangle \to 0$ in the above equations. Although not generally consistent with SO(10) in the presence of the **126**, this approximation is suitable for our purposes as the extension we will be discussing is tripletless, the **126** being replaced by **16**'s.

The matrix $\mathcal{M}_\nu$ is diagonalized by performing a 6×6 unitary transformation U_ν,

$$\nu_i = \sum_{a=1}^{6} (U_\nu)_{ia} n_a \qquad U_\nu^T \mathcal{M}_\nu U_\nu = \mathrm{diag}(m_i, M_i), \tag{3}$$

whose form is given explicitly as a perturbation series in Ref. [25]. The effective mass matrix of the three light neutrinos is given as

$$m_\nu \approx -Y Y_R^{-1} Y^T \frac{\langle\Phi\rangle^2}{\langle\Delta_R\rangle}. \tag{4}$$

The smallness of light neutrino masses follows dynamically from Eq. (4), since $\langle\Delta_R\rangle$ is large.

In order to reproduce the observed pattern of quark masses and CP violation, we assume the complex matrix Y to have the form

$$Y = \begin{pmatrix} 0 & a & 0 \\ a^* & b & c \\ 0 & c^* & d \end{pmatrix} \tag{5}$$

with a, c complex and b, d real. Notice that the large observed lepton mixing angles can always be reconciled with the small quark mixing angles thanks to the presence of the coupling matrices $Y_{L,R}$ which exist only for neutrinos (in the limit of unbroken D-parity, one has $Y_L = Y_R$). However, in a unified theory where a flavor symmetry is assumed to predict the lepton mixing angles, it becomes a real challenge to account naturally also for the quark mixing angles.

Now we turn to cosmology. One of the attractive features of seesaw models is that they open the possibility of accounting for the observed cosmological matter-antimatter asymmetry in the Universe through the leptogenesis mechanism [8]. This requires the out-of-equilibrium decays of the heavy "right-handed" neutrinos N_i (see Fig. 1) to take place before the electroweak phase transition, and the presence of CP violation in the lepton sector. The tree level and one-loop diagrams for N_i decay that interfere in order to generate a lepton asymmetry of the universe are shown.

These decays generate the asymmetry,

$$\epsilon_i = \frac{\Gamma_{N_i L} - \Gamma_{N_i \bar{L}}}{\Gamma_{N_i L} + \Gamma_{N_i \bar{L}}}. \tag{6}$$

Here we recall the estimate of such asymmetry, as given in Ref. [26]. The relevant amplitudes follow from the initial Lagrangian,

$$\mathcal{L} = \lambda_{ji} \bar{N}_i P_L L_j H + \mathrm{H.c.} \tag{7}$$

Here it is sufficient just to sketch their structure. At the tree level we have

$$\mathcal{M}^0_{NL} = i\lambda^*_{ji}, \qquad \mathcal{M}^0_{N\bar{L}} = i\lambda_{ji}, \tag{8}$$

while for the one-loop it is only necessary to evaluate the vertex contribution,

$$\begin{aligned} \mathcal{M}^1_{NL} &= i^7 f M_k \lambda^*_{jk} \lambda^*_{mk} \lambda_{mi}, \\ \mathcal{M}^1_{N\bar{L}} &= i^7 f M_k \lambda_{jk} \lambda_{mk} \lambda^*_{mi}, \end{aligned} \tag{9}$$

where f is some complex loop function. Now tree level plus one-loop will be proportional to

$$|\mathcal{M}^0_{NL} + \mathcal{M}^1_{NL}|^2 \simeq |\mathcal{M}^0_{NL}|^2 + \mathcal{M}^0_{NL}\mathcal{M}^{1*}_{NL} + \mathcal{M}^{0*}_{NL}\mathcal{M}^1_{NL} \tag{10}$$

so that the asymmetry is

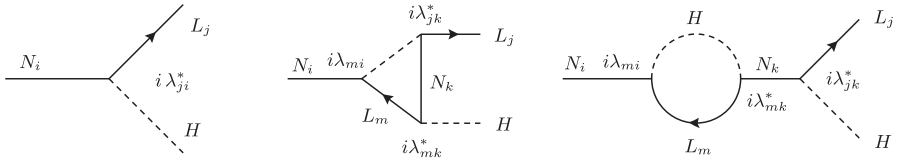

FIG. 1. Diagrams contributing to generate a lepton asymmetry of the universe.

$$\epsilon_i \propto \frac{\mathcal{M}^0_{NL}\mathcal{M}^{1*}_{NL} + \mathcal{M}^{0*}_{NL}\mathcal{M}^1_{NL} - \mathcal{M}^0_{N\bar{L}}\mathcal{M}^{1*}_{N\bar{L}} - \mathcal{M}^{0*}_{N\bar{L}}\mathcal{M}^1_{N\bar{L}}}{|\mathcal{M}^0_{NL}|^2}. \tag{11}$$

Now using Eqs. (8) and (9) we can write

$$\epsilon_i \propto \mathrm{Im}(f)\frac{\sum_{k,j,m}\mathrm{Im}(\lambda^*_{ji}\lambda^*_{mi}\lambda_{mk}\lambda_{jk})M_k}{\sum_j \lambda^*_{ki}\lambda_{ki}}. \tag{12}$$

Finally we get, in the case of hierarchical right-handed neutrinos,

$$\epsilon_1 = -\frac{3}{16\pi}M_1\frac{\sum_{k\neq 1,j,m}\mathrm{Im}(\lambda^*_{j1}\lambda^*_{m1}\lambda_{mk}\lambda_{jk})\frac{1}{M_k}}{\sum_k \lambda^*_{j1}\lambda_{j1}}. \tag{13}$$

Note that in this limit the contribution of the self-energy has the same dependence on the couplings and is already included. The lepton (or B-L) asymmetry thus produced then gets converted, through sphaleron processes, into the baryon asymmetry [8], observed to be $\mathcal{O}(10^{-10})$. In order to provide an acceptable framework for leptogenesis, and taking into account the presence of washout effects, the asymmetry we need is $\epsilon_1 \sim \mathcal{O}(10^{-7})$ or larger, for the same values of parameters that reproduce the observed small neutrino masses, Eq. (4). In order to generate this asymmetry thermally, one requires the Universe to reheat after inflation to a very high temperature [27–30],

$$T_R > 2\times 10^9 \text{ GeV}. \tag{14}$$

Such a large scale leads to an overproduction of cosmological gravitinos. In minimal supergravity models, with $m_{3/2} \sim 100$ GeV to 10 TeV gravitinos are not stable, decaying during or after big bang nucleosynthesis (BBN). Their rate of production can be so large that subsequent gravitino decays completely change the standard BBN scenario. Since the abundance of gravitinos is proportional to T_R, such "gravitino crisis" can only be prevented by requiring a low enough reheat temperature T_R after inflation [9,10]. A recent detailed analysis gives a stringent upper bound

$$T_R \lesssim 10^6 \text{ GeV} \tag{15}$$

when the gravitino has hadronic decay modes [11]. Therefore, thermal leptogenesis seems difficult to reconcile with low energy supersymmetry if gravitino masses lie in the range suggested by the simplest minimal supergravity models. One possible way out is to have resonant leptogenesis [12] as suggested in Ref. [13]. Another alternative considered in Ref. [14] requires going beyond the minimal seesaw by adding a small R-parity violating term in the superpotential. In the following, we discuss quantitatively the alternative suggestion made in [15] in the context of the extended supersymmetric seesaw scheme.

III. THE NEW EXTENDED SEESAW MODEL

For definiteness we work in the context of the supersymmetric SO(10) unified model considered in Refs. [15,16]. The gauge symmetry and D-parity are broken by **45** and **210** multiplets at the unification scale. The B-L symmetry breaks at lower scale thanks to expectation values of the χ fields in the **16**, instead of the more familiar left and right triplets present in **126** which lead to the standard seesaw mechanism. Gauge coupling unification cannot fix the B-L breaking scale, which can be relatively low, as shown in Ref. [16]. The possibility of a low B-L breaking scale also fits with the observed neutrino masses. The relevant Yukawa couplings leading to neutrino masses are

$$-\mathcal{L}_Y = Y_{ij}\nu^c{}_{iL}\nu_{jL}\phi + F_{ij}\nu_{iL}S_j\chi_L + \tilde{F}_{ij}\nu^c{}_{iL}S_j\chi_R + \tfrac{1}{2}M_\Sigma\Sigma\Sigma. \tag{16}$$

Note that a direct Majorana mass term for the singlet fields S_i is forbidden by an additional imposed $U(1)_G$ symmetry and the fact that the only singlet scalar present (σ) is odd under D-parity, while S_iS_j are even under D-parity. For the same reason, σ cannot couple to Σ, but a bare mass is allowed [15]. As we will show, leptogenesis allows this mass to be of the order of TeV. We also introduce a soft term breaking $U(1)_G$, which allows mixing between the scalar components of these fields ΣS_i,

$$-\mathcal{L}_{\not{G}} = \Delta_i\Sigma S_i. \tag{17}$$

This will then give a 10×10 neutrino mass matrix, in the basis (ν_i, Σ, ν^c_{iL}, S_i):

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 & Y\upsilon & F\upsilon_L \\ 0 & M_\Sigma & 0 & \Delta^T \\ Y^T\upsilon & 0 & 0 & \tilde{F}\upsilon_R \\ F^T\upsilon_L & \Delta & \tilde{F}^T\upsilon_R & 0 \end{pmatrix}, \tag{18}$$

where $\upsilon = \langle\phi\rangle$, $\upsilon_L = \langle\chi_L\rangle$, and $\upsilon_R = \langle\chi_R\rangle$ are the vevs for the fields ϕ, χ_L, and χ_R, respectively, and Δ is the $U(1)_G$ breaking entry. Besides the three light neutrinos, this mass matrix will give two heavy states which are dominantly the right-handed neutrino $\nu^c{}_{iL}$ and the singlets S_i, and a lighter state Σ. Notice that we will not impose any specific family symmetry. In Sec. IV B we will, however, consider the case where the Y_{ij} are restricted to follow the Fritzsch texture for the quark sector.

A. Light neutrinos

Using the seesaw diagonalization prescription given in Ref. [25], we obtain the effective left-handed light neutrino mass matrix as [15]

$$m_\nu = -\frac{1}{M_\Sigma}GG^T - [Y(F\tilde{F}^{-1})^T + (F\tilde{F}^{-1})Y^T]\frac{\upsilon\upsilon_L}{\upsilon_R}, \tag{19}$$

where $G \equiv Y(\tilde{F}^{-1})^T \Delta \frac{v}{v_R}$. Since the D-parity breaking scale is much higher than the scale at which the left-right symmetry breaks, and this in turn is higher than the electroweak symmetry breaking scale, one has the "vev-seesaw" relation

$$v_L \propto \frac{v_R v}{M_X}, \tag{20}$$

where M_X is determined by the SO(10) breaking vevs. In the following we will take this proportionality constant equal to one and therefore write Eq. (19) as

$$m_\nu = -\frac{1}{M_\Sigma} GG^T - [Y(F\tilde{F}^{-1})^T + (F\tilde{F}^{-1})Y^T]\frac{v^2}{M_X}. \tag{21}$$

Note that the neutrino masses are suppressed by the unification scale M_X instead of the B-L scale. As noted in [16], we need two pairs of $\mathbf{16}\,\overline{\mathbf{16}}$ in order to boost up the neutrino mass scale and bring in an independent flavor structure beyond that of the charged Dirac couplings, constrained by the quark sector phenomenology.

Now we want to extract as much information from the low energy neutrino data as possible. The matrix m_ν can be diagonalized as

$$U^T m_\nu U = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}), \tag{22}$$

where we use the standard parametrization from Ref. [7] and now adopted by the PDG:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \tag{23}$$

The values of U and $\mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$ are known to some degree from experiment and we want to turn this information into the parameters of the theory, defined by Eq. (16). Of course we have too little experimental information, three angles and two mass differences, to be able to reconstruct in full the three matrices Y, F, $\tilde{F}$, and the vector Δ.

B. Heavy neutrinos

Let us now turn to the discussion of the heavy neutrinos. The following terms in the Lagrangian,

$$\begin{aligned} -\mathcal{L} &= \tfrac{1}{2}(2\tilde{F}_{ij}\nu^c{}_{iL}S_j v_R + M_\Sigma \Sigma\Sigma + 2\Delta_i \Sigma S_i) \\ &= \tfrac{1}{2}(v_R \nu^{cT}{}_L \tilde{F} S + v_R S^T \tilde{F}^T \nu^c{}_L + \Delta^T \Sigma S + \Delta S^T \Sigma \\ &\quad + M_\Sigma \Sigma\Sigma) \\ &= \tfrac{1}{2} N_i M_{Nij} N_j, \end{aligned} \tag{24}$$

lead to a 7×7 mass matrix in the basis $N = (\Sigma, \nu^c_{iL}, S_i)$:

$$M_N = \begin{pmatrix} M_\Sigma & 0 & \Delta^T \\ 0 & 0 & \tilde{F} v_R \\ \Delta & \tilde{F}^T v_R & 0 \end{pmatrix}, \tag{25}$$

which determines the masses of the heavy neutrinos. Now in order to get the states with physical masses we diagonalize this mass matrix in three steps. We start by diagonalizing the matrix $\tilde{F}$. We define

$$U_N^T \tilde{F} U_S v_R = \mathrm{diag}(M_1, M_2, M_3) \equiv M_D, \tag{26}$$

where U_N and U_S are unitary matrices. Then we define new fields

$$\nu'^c = U_N^{-1} \nu^c, \qquad S' = U_S^{-1} S, \tag{27}$$

so that in the basis $N' = (\Sigma, \nu'^c, S')$ we have

$$-\mathcal{L} = \tfrac{1}{2} N'^T M'_N N' \tag{28}$$

with

$$M'_N = R_{NS}^T M_N R_{NS} = \begin{pmatrix} M_\Sigma & 0 & \Delta^T U_S \\ 0 & 0 & M_D \\ U_S^T \Delta & M_D & 0 \end{pmatrix} \tag{29}$$

and

$$R_{NS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & U_N & 0 \\ 0 & 0 & U_S \end{pmatrix}. \tag{30}$$

Now we rotate the fields ν'^c and S' to obtain the almost degenerate states $N_\pm = \frac{1}{\sqrt{2}}(\nu'^c \pm S')$. This is done by defining a new matrix

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{31}$$

that obeys

$$-\mathcal{L} = \tfrac{1}{2} N''^T M''_N N'' \tag{32}$$

with $N' = RN''$, $N'' = (\Sigma, N_-, N_+)$, and

$$M''_N = R^T M'_N R = \begin{pmatrix} M_\Sigma & -\frac{1}{\sqrt{2}}\Delta^T U_S & \frac{1}{\sqrt{2}}\Delta^T U_S \\ -\frac{1}{\sqrt{2}} U_S^T \Delta & -M_D & 0 \\ \frac{1}{\sqrt{2}} U_S^T \Delta & 0 & M_D \end{pmatrix}. \tag{33}$$

We now have to perform the final rotation in order to get the physical states for these heavy neutrinos. As we have seen in the discussion of the light neutrinos, the amount of G-violation must be small, therefore Eq. (33) can be approximately diagonalized using the techniques of

Ref. [25]. We obtain

$$-\mathcal{L} = \tfrac{1}{2} N'''^T M_N''' N''' \tag{34}$$

with $N''' = (\Sigma', N'_-, N'_+)$, $N'' = U_{RH} N'''$, where

$$M_N''' = U_{RH}^T M_N'' U_{RH} = \begin{pmatrix} M_\Sigma & 0 & 0 \\ 0 & -M_D & 0 \\ 0 & 0 & M_D \end{pmatrix} \tag{35}$$

and

$$U_{RH} = \begin{pmatrix} 1 & \frac{1}{\sqrt{2}} \Delta^T (U_S^{-1}) M_D^{-1} & \frac{1}{\sqrt{2}} \Delta^T (U_S^{-1}) M_D^{-1} \\ -\frac{1}{\sqrt{2}} M_D^{-1} U_S^T \Delta & 1 & 0 \\ -\frac{1}{\sqrt{2}} M_D^{-1} U_S^T \Delta & 0 & 1 \end{pmatrix}. \tag{36}$$

If we assume that the eigenvalues of $\tilde{F}$ are hierarchical, we get also a hierarchical spectrum for the heavy states, Σ', N'_-, N'_+, namely, $|M_\Sigma| \ll |M_1| \ll |M_2| \ll |M_3|$. In order to evaluate the asymmetry generated we must evaluate the couplings of the mass eigenstates. For this we notice that

$$N = R_{NS} R U_{RH} N''' \tag{37}$$

and a simple calculation gives

$$R_{NS} R U_{RH} = \begin{pmatrix} 1 & \frac{1}{\sqrt{2}} \Delta^T (U_S^{-1})^T M_D^{-1} & \frac{1}{\sqrt{2}} \Delta^T (U_S^{-1})^T M_D^{-1} \\ -U_N M_D^{-1} U_S^T \Delta & \frac{1}{\sqrt{2}} U_N & \frac{1}{\sqrt{2}} U_N \\ 0 & -\frac{1}{\sqrt{2}} U_S & \frac{1}{\sqrt{2}} U_S \end{pmatrix} \tag{38}$$

which allows us to write

$$\begin{aligned} \Sigma &= \Sigma' + \frac{1}{\sqrt{2}} \Delta^T (U_S^{-1})^T M_D^{-1} N'_- + \frac{1}{\sqrt{2}} \Delta^T (U_S^{-1})^T M_D^{-1} N'_+ \\ \nu^c &= -(\tilde{F}^{-1})^T \frac{\Delta}{v_R} \Sigma' + \frac{1}{\sqrt{2}} U_N N'_- + \frac{1}{\sqrt{2}} U_N N'_+ \\ S &= -\frac{1}{\sqrt{2}} U_S N'_- + \frac{1}{\sqrt{2}} U_S N'_-, \end{aligned} \tag{39}$$

where we have used

$$(\tilde{F}^{-1})^T = U_N M_D^{-1} U_S^T v_R. \tag{40}$$

With this we can rewrite the relevant part of the Lagrangian of Eq. (16) in terms of the eigenstates (we drop the primes from now on),

$$\mathcal{L}_Y = Y_{\Sigma i} L_i H \Sigma + Y_{\pm ij} N_{\pm i} L_j H + \text{H.c.}, \tag{41}$$

where

$$\begin{aligned} Y_\Sigma &= \alpha_H Y (\tilde{F}^{-1})^T \frac{\Delta}{v_R}, \\ Y_{\pm ij} &= -\left(\frac{1}{\sqrt{2}} \alpha_H (U_N^T Y)_{ij} \pm \alpha_\chi^H (F U_S)_{ji} \right), \end{aligned} \tag{42}$$

where α_H denotes the projection of the relevant light minimal supersymmetric extension of the standard model (MSSM) Higgs doublet$h$ into the directions of the defining Higgs doublets living in $H \in 10_H$, and $\alpha_{\chi^k}^H$ are the projections of the light MSSM-like Higgs doublet onto the defining Higgs doublets in the $\overline{16}_H^k$ and M_1 is the mass of the $N_{\pm 1}$.

C. Calculation of the asymmetry

We now discuss the issue of leptogenesis in this model. It can occur only after the local $(B\text{-}L) \subset \mathrm{SO}(10)$ symmetry is broken. This will take place through the out-of-equilibrium decay of the singlet fermion Σ. The total width of Σ is given by (treating Y_Σ as a column vector)

$$\Gamma_\Sigma \sim \frac{1}{8\pi} Y_\Sigma^\dagger Y_\Sigma M_\Sigma, \tag{43}$$

where Y_Σ is given in Eq. (42).

The asymmetry coming from the diagrams of Fig. 2 involves the sum over k which reduces to the sum over the lightest pair of the (almost degenerate) $N_\pm \equiv \frac{1}{\sqrt{2}}(\nu^c \pm S)$ states with masses $\pm M_1$. A comparison of the diagrams in Fig. 1 and 2 gives the following dictionary:

$$\begin{aligned} \lambda_{j1} &\to Y_{\Sigma j} \qquad \lambda_{m1} \to Y_{\Sigma m} \\ \lambda_{mk} &\to Y_{\pm m} \qquad \lambda_{jk} \to Y_{\pm j}. \end{aligned} \tag{44}$$

We have then for the numerator

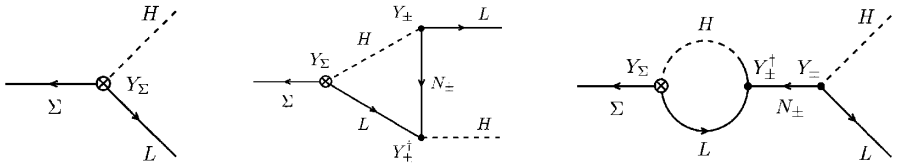

FIG. 2. Tree level and one-loop diagrams for the decay of Σ that interferes to generate a lepton asymmetry of the universe.

$$\sum_{j,m,\pm} Y^*_{\Sigma j} Y^*_{\Sigma m} Y_{\pm m} Y_{\pm j} \frac{1}{M_\pm} = \frac{1}{M_1} \sum_{j,m} Y^*_{\Sigma j} Y^*_{\Sigma m} \times (Y_{+m} Y_{+j} - Y_{-m} Y_{-j}) = \frac{4}{M_1} \sum_{j,m} Y^*_{\Sigma j} Y^*_{\Sigma m} A_m B_j, \tag{45}$$

where we have defined $Y_{\pm m} = A_m \pm B_m$. Comparing with Eq. (42) we get

$$A_m = -\frac{1}{\sqrt{2}} \alpha_H (Y^T U_N)_{m1}, \qquad B_m = -\frac{1}{\sqrt{2}} \alpha^H_\chi (F U_S)_{m1}. \tag{46}$$

Putting everything together we finally obtain

$$\epsilon_\Sigma = -\frac{3}{8\pi} \frac{M_\Sigma}{M_1} \frac{\mathrm{Im}[(Y^\dagger_\Sigma F_k U_S \alpha^H_{\chi^k})_1 (Y^\dagger_\Sigma Y^T U_N \alpha_H)_1]}{Y^\dagger_\Sigma Y_\Sigma}. \tag{47}$$

It is important to note that, in contrast to the asymmetry ϵ_1 in the minimal seesaw discussed in the previous section, ϵ_Σ is not constrained by the light neutrino masses. This can essentially be understood by the fact that the neutrino mass, see Eq. (19) and (20), is suppressed by the small ratio v_L/v_R, whereas in the calculation of ϵ_Σ, the small quantity Δ/v_R appears quadratically in the numerator and the denominator, and thus cancels. ϵ_Σ can therefore be much larger than in the minimal seesaw case, independent of the light neutrino masses. Consequently, there is also no *lower bound on* M_Σ from the asymmetry parameter ϵ_Σ.

IV. ANSATZES FOR THE COUPLING MATRICES

We now estimate the resulting *CP* asymmetry needed for leptogenesis making use of the current values of the neutrino oscillation parameters given in [1]. A simple ansatz is to assume that it comes just from the Dirac phase of the three-neutrino lepton mixing matrix assuming the unitary approximation. In this approximation the asymmetry is proportional to the unique *CP* invariant parameter that can be probed in neutrino oscillations. The maximum value of the asymmetry that can be achieved is obtained by varying randomly all the other model parameters and the results are displayed at each point of the plane ($\sin\theta_{13}$, $\sin\delta$).

A. Generic case: Nonhierarchical Y_{ij}

We start by considering a nonhierarchical ansatz for the coupling matrices. In order to reduce the unknowns, we assume that both F_{ij} and $\tilde{F}_{ij}$ matrices are proportional to the standard SO(10) Dirac Yukawa coupling matrix Y_{ij}, which is symmetric, namely,

$$F_{ij} = f Y_{ij}, \quad \tilde{F}_{ij} = \tilde{f} Y_{ij}, \quad Y_{ij} = Y_{ji}. \tag{48}$$

With this choice Eq. (21) can be written as

$$m_\nu = m^{(a)}_\nu + m^{(b)}_\nu, \tag{49}$$

where

$$m^{(a)}_\nu = -\frac{v^2}{\tilde{f}^2 v_R^2 M_\Sigma} \Delta_i \Delta_j \simeq -3 \times 10^{10} \frac{1}{\tilde{f}^2} \frac{1}{(\frac{M_\Sigma}{10^3\ \mathrm{GeV}})} \left(\frac{\Delta_i}{v_R}\right)\left(\frac{\Delta_j}{v_R}\right)(\mathrm{eV}) \tag{50}$$

$$m^{(b)}_\nu = -\frac{f}{\tilde{f}} \frac{2v^2}{M_X} Y_{ij} \simeq -3 \times 10^{-3} \frac{f}{\tilde{f}} Y_{ij}(\mathrm{eV}).$$

Clearly, the projective nature of $m^{(a)}_\nu$ will not explain the neutrino data, therefore we need the contribution of $m^{(b)}_\nu$. The atmospheric scale can easily be reproduced with $Y \simeq \mathcal{O}(1)$, $f/\tilde{f} \simeq 0.1$, and $\frac{\Delta}{v_R} \simeq 10^{-7}$, for $M_\Sigma = 1$ TeV.

With the assumptions of Eq. (48) we can solve for the Yukawa matrix in terms of the experimentally observed neutrino oscillation parameters via

$$Y_{ij} = -\frac{M_X}{2v^2} \frac{\tilde{f}}{f} \left[(U^T)^{-1} m^{\mathrm{exp}}_\nu U^{-1} + \Delta_i \Delta_j \frac{v^2}{v_R^2 M_\Sigma} \frac{1}{\tilde{f}^2} \right] \tag{51}$$

which also involves other parameters of the model.

With this ansatz and the choice $f = 1$, $\tilde{f} = 0.1$ we take the lepton mixing parameters in the range allowed by the experiment. At 3σ the latest neutrino oscillation data give [1]

$$\sin^2\theta_{12} \in [0.24, 0.40], \qquad \sin^2\theta_{23} \in [0.34, 0.68], \qquad \sin^3\theta_{13} < 0.04. \tag{52}$$

In addition, we take the other model parameters in the following ranges:

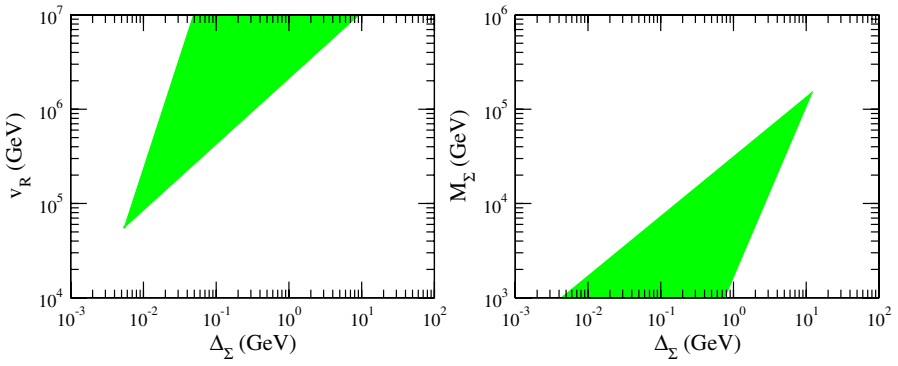

FIG. 3 (color online). v_R, M_Σ, and Δ_Σ values that follow from our first *ansatz*, Eq. (49), for $\tilde{f} = 0.1$, when the parameters are varied as in Eq. (53), see text.

$$M_\Sigma \in [10^3, 10^7]\ \text{GeV}, \qquad \Delta_i \in [10^{-3}, 10^3]\ \text{GeV}, \qquad v_R \in [10^3, 10^7]\ \text{GeV}. \tag{53}$$

With these values we can see the resulting v_R, M_Σ, and Δ_Σ values that follow from our ansatz in Fig. 3. They show that it is possible to fit the neutrino data with this simple type of ansatz. The resulting CP asymmetry produced is given in Fig. 4. Here we have calculated the maximum value of the asymmetry that can be achieved at each point of the ($\sin\theta_{13}$, $\sin\delta$) plane, varying randomly all the other parameters. Clearly the sizable values of the asymmetry can be obtained, especially for large $\sin\theta_{13}$ and $\sin\delta$, as expected, so that the necessary CP asymmetry needed for leptogenesis can be achieved.

One sees that large values of the asymmetry, compatible with the experimental data, can be achieved in the full parameter space, even for very small $\sin\theta_{13}$ and $\sin\delta$ values. However, the ansatz is manifestly inconsistent with the successful Fritzsch texture for the quark masses.

B. Fritzsch case: Hierarchical Y_{ij}

It is natural to ask whether our SO(10) model described by the Lagrangian of Eq. (16) can provide thermal leptogenesis while reconciling the successful Fritzsch texture for the quark masses with the observed structure of lepton masses and mixings that follow from neutrino oscillation experiments. To this end we now assume that the Y_{ij} Yukawa's involved in neutrino mass generation are also restricted by the Fritzsch ansatz for the quark couplings, given in Eq. (5), with a, c complex and b, d real. Aware of the fact that the phases will be necessary in computing the final asymmetry, we will, for the moment take the ansatz parameters all real, as

$$a = \sqrt{\frac{m_u m_c}{v^2}}, \qquad b = \frac{m_c}{v}, \qquad c = \sqrt{\frac{m_c m_t}{v^2}}, \qquad d = \frac{m_t}{v}. \tag{54}$$

These values imply a strong hierarchy among the Yukawa couplings. With this choice, let us now look at the neutrino mass matrix. It is clear from Eq. (21) that this hierarchy must be corrected by a suitable hierarchy in the $\tilde{F}$ coupling matrix. In fact Eq. (21) can be "solved" for $\tilde{F}$ as

$$\tilde{F} = X^{-1}F, \tag{55}$$

where X is obtained from

$$YX^T + XY^T = -\frac{M_X}{v^2}\left[(U^T)^{-1} m_\nu^{\exp} U^{-1} + (Y\tilde{\Delta})(Y\tilde{\Delta})^T \frac{v^2}{v_R^2 M_\Sigma}\right] \equiv Z, \tag{56}$$

where $\tilde{\Delta} \equiv \tilde{F}^{-1}\Delta$, $\tilde{F}$ being, in general, an arbitrary non-symmetric matrix. Now Z is expressed as a combination of neutrino data and additional parameters, for which we take random values.

We have performed a random study of this Fritzsch ansatz assuming as our ansatz that $\tilde{F} = X^{-1}F$ with $F = F^T$ and taking random values of order one in various ways. We have found that in this case, for example, for v_R in the range $[10^7, 10^8]$ GeV, M_Σ of few TeV, and $\Delta_\Sigma \in [10^{-2}, 10^4]$ GeV, one indeed obtains a viable solution. One finds that some of the entries of $\tilde{F}$ are small (of order

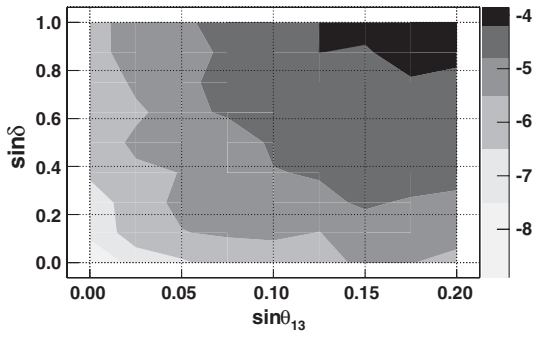

FIG. 4. Contour levels of the logarithm of the maximum value of the asymmetry ϵ_Σ obtained as a function of $\sin\theta_{13}$ and $\sin\delta$ for the nonhierarchical ansatz of Sec. IVA. The other neutrino mixing parameters are taken from the latest neutrino oscillation data given in [1].

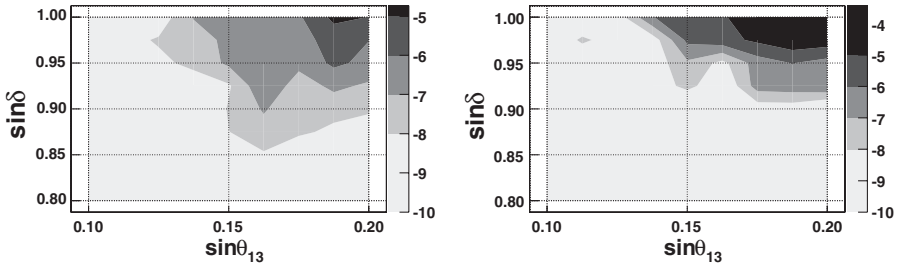

FIG. 5. Contour levels of the maximum asymmetry obtained as a function of $\sin\theta_{13}$ and $\sin\delta$ using the Fritzsch texture with $\tilde{F} = X^{-1}F$, $F = I$ (left panel), or F symmetric (right panel).

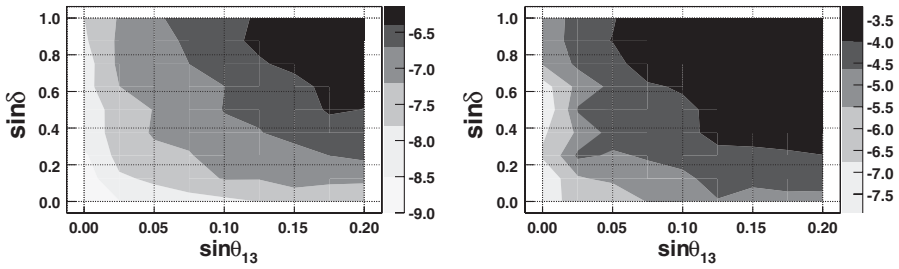

FIG. 6. Contours of the maximum value of the asymmetry obtained using the Fritzsch texture. Left panel: $\tilde{F} = \tilde{f}Y$, F symmetric. Right panel: $\tilde{F}$ diagonal and $F = X\tilde{F}$.

10^{-3}) in order to compensate for the corresponding smallness of Y in Eq. (5).

We have also explicitly calculated the value of the asymmetry ϵ_Σ for this ansatz as explained above. The results are shown in Fig. 5 for the case of the hierarchical ansatz with the current neutrino oscillation parameters from [1] and the Dirac phase is $\delta \in [0, \pi]$.

One sees that, in this case, values of the asymmetry of order 10^{-6}–10^{-7} can only be obtained for large values of $\sin\theta_{13}$ and $\sin\delta$. Therefore, this scenario can be potentially probed by the future neutrino oscillation measurements.

Larger values of the CP asymmetry compatible with current neutrino oscillation measurements can be found for other ansatze of the current model using the Fritzsch texture. For instance, if we consider $\tilde{F} = \tilde{f}Y$ with F symmetric, or $\tilde{F}$ diagonal and $F = X\tilde{F}$, one can obtain very large values of the asymmetry even for very small values of $\sin\theta_{13}$ and $\sin\delta$, as illustrated in Fig. 6.

V. ASYMMETRY WASHOUT, SINGLET PRODUCTION, AND THE SPHALERON CONSTRAINT

Generating a large enough asymmetry in the decay of the lightest singlet is a necessary, but not a sufficient, condition for successful leptogenesis. Additional conditions are required, before one can conclude that any given model can generate the required baryon asymmetry.

The first condition to be satisfied is the out-of-equilibrium decay of the heavy singlet, which is nothing but the basic Sakharov condition [31]. This requires that the decay rate is smaller than the expansion rate of the universe, i.e. $\Gamma_\Sigma \ll \Gamma_{\text{Hubble}}$ at the decay epoch. For large Γ_Σ, say, of order of the expansion rate, part of the asymmetry produced in the decays will be washed out by inverse scattering processes violating lepton number. This constraint will thus put an upper limit on the couplings of Σ to the leptons (and Higgses) for any given mass M_Σ. Second, we need to produce a sufficient number of singlets in the early universe. Singlets could be either produced by their couplings to the thermal bath or through the decay of some heavier particle, which was present in the universe at earlier times. Obviously, a sufficiently strong direct thermal production requires a minimum value for the couplings of Σ, whereas the second option does not. The third constraint comes from the fact that the Σ decays have to take place at a time early enough that the SM sphalerons are still in equilibrium, otherwise we will produce a lepton number, but no nonzero baryon number. This constraint puts a lower limit on Γ_Σ, $\Gamma_\Sigma \geq \Gamma_{\text{Sphaleron}}$, independent of the production mechanism of Σ.

To accurately calculate the first two of the above conditions, in principle, one needs to set up a network of Boltzmann equations, which in general can only be solved numerically [32]. However, under certain simplifying assumptions, one can derive approximate analytical solutions which reproduce the full, numerical calculations quite well. Several analytical approximations have been proposed in the literature, see for example [32–34]. In our earlier paper [15] we have used the following approximate

form for the washout factor:

$$\kappa'(z) = \frac{1}{1 + 10z}, \tag{57}$$

where $z \equiv \frac{\Gamma}{\Gamma_{\rm Hubble}}$, with $\Gamma_{\rm Hubble}$ being the expansion rate of the universe [32]. Reference [34] has numerically solved the Boltzmann system for the case of the simpler type-I seesaw. In this case the decay width of the lightest right-handed neutrino is proportional to $\Gamma \sim (Y^\dagger Y)_{11}$, and the authors of [34] define the "effective mass" parameter $\tilde{m}_1 = (Y^\dagger Y)_{11} v^2/M_1$, where M_1 is the mass of the lightest right-handed neutrino and v the SM vev. Thermal equilibrium for the right-handed neutrino is reached for an "equilibrium mass" $\tilde{m}_* = 1.08 \times 10^{-3}$ eV, for which by definition $z = 1$. The full numerical calculation is then very well approximated by

$$\kappa(z) \simeq 0.24(x_- e^{-x_-} + x_+ e^{-x_+}) \tag{58}$$

with

$$x_\pm = \left(\frac{\tilde{m}_*}{\tilde{m}_\pm}\right)^{\mp 1-\alpha}, \tag{59}$$

and the numerical values

$$\tilde{m}_+ = 8.3 \times 10^{-4} \text{ eV}, \qquad \tilde{m}_- = 3.5 \times 10^{-4} \text{ eV}, \qquad \alpha = 0.1. \tag{60}$$

We can make use of the results of [34] with the straightforward replacement,

$$x_\pm = \left(\frac{\tilde{m}_* z}{\tilde{m}_\pm}\right)^{\mp 1-\alpha}. \tag{61}$$

Our more complicated setup of Boltzmann equations then is solved approximately by the fit, Eq. (58). Note, that this implicitly assumes [35] that the heavier singlets are sufficiently decoupled so as to not contribute significantly to the washout.

As seen in Fig. 7, Eqs. (57) and (58) lead to very similar results for $z > 1$, The two forms differ, however, for $z < 1$. This can be traced to the fact that Eq. (58) also accounts for the suppressed production in the weak coupling regime, while Eq. (57) does not. For definiteness, in the plots shown below, we have used Eq. (58).

We have also used the constraint imposed by the condition that the decay of the singlets must happen while the sphalerons are still active. We estimate the sphaleron time using

$$\Gamma_{\rm Sphaleron} = \Gamma_{\rm Hubble}\Big|_{T=\rm EWPT} = \sqrt{4\pi^3 g_*/45}\left(\frac{T^2}{M_{\rm Pl}}\right)\Big|_{T=\rm EWPT}, \tag{62}$$

with EWPT denoting the energy at which the electroweak phase transition occurs, g_* is the effective number of degrees of freedom, and $M_{\rm Pl}$ is the reduced Planck mass. We cut all points with $\Gamma_\Sigma \leq \Gamma_{\rm Sphaleron}$. This is, of course, a rough approximation to the real, dynamical situation. However, we believe it to be sufficiently accurate for deriving order of magnitude constraints on the model parameters. Finally, when converting the produced lepton asymmetry to the baryon asymmetry, we have to take into account an efficiency factor for the sphalerons. This factor has been calculated in [36] to be

$$\eta_B = \frac{8n_F + 4n_H}{22n_F + 13n_H}\,\eta_L, \tag{63}$$

where $n_F(n_H)$ is the number of families (Higgses). Numerically this factor is $\sim 1/3$.

Figure 8 shows as an example the resulting η_B as a function of Δ for a numerical scan using the ansatz shown

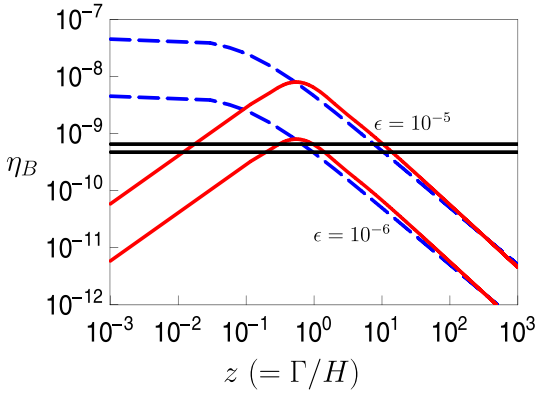

FIG. 7 (color online). Calculated range for η_B as a function of z, for the different fits to the washout factor. The dashed (full) lines show the approximate analytical solutions to the Boltzmann equations, see Eqs. (57) and (58). Results are shown for 2 typical values of $\epsilon = 10^{-5}, 10^{-6}$. The horizontal black band is the currently allowed η_B range [40].

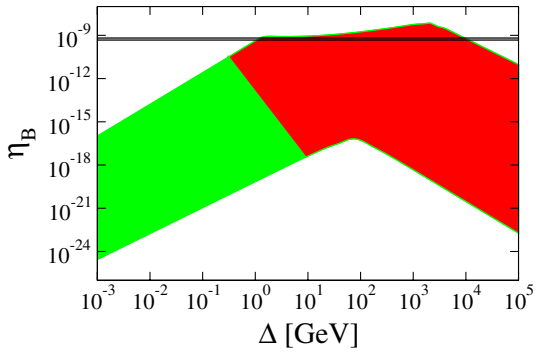

FIG. 8 (color online). Calculated range for η_B as a function of Δ, for the ansatz in Fig. 5, to the left. The horizontal band is the allowed range of η_B. The full shaded region corresponds to the allowed range without the sphaleron constraint, while the superimposed darker/red area indicates the allowed range of parameters once the sphaleron constraint is included. For a discussion see text.

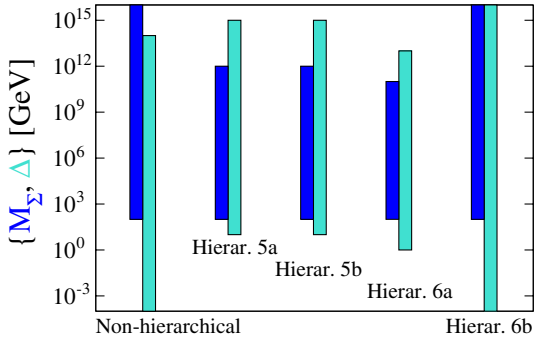

FIG. 9 (color online). Calculated ranges for Δ and M_Σ for the different texture ansätze discussed in the text. This ranges have been obtained with the largest possible scan for the parameters, for example, M_Σ in the full interval $[10^2, 10^{16}]$ GeV.

in Fig. 5, to the left. The light/dark (green/red) area is the calculated range for η_B without/with the sphaleron constraint. One sees that the different constraints discussed above conspire to choose a rather well-defined allowed range for the parameter Δ in this case. A large enough baryon asymmetry can be obtained roughly for $\Delta = [1, 10^4]$ GeV, for the range of the other parameters as given in Eq. (53). It is amusing to note that the requirement of producing a sufficient number of singlets and the sphaleron constraint leads to rather similar lower cuts on Δ [37].

We have repeated this exercise for all the different ansätze discussed above. The resulting allowed ranges for the parameters Δ and M_Σ are shown in Fig. 9. As shown in this figure, the random nonhierarchical ansatz and the hierarchical ansatz shown in the right panel of Fig. 6 lead to the widest allowed ranges for Δ and M_Σ.

Before we close this section, let us briefly illustrate possible values for the relevant parameters. These are Δ, M_Σ the masses M_1, M_2, M_3 of the three isosinglet neutrinos and the resulting baryon asymmetry η_B. Some examples are given in Table I for each of the scenarios we have discussed. As one can see, the acceptable baryon asymmetry may arise for rather low scales, in contrast to the (simpler) seesaw type-I schemes. Note however that the values given in the table are by no means unique, and are also not meant to be "representative" of the classes. The generation of the baryon asymmetry is far easier in this context than in the traditional seesaw model. The presence of the new singlets allows us to have, in addition, acceptable textures for the quark and lepton mixing angles.

VI. CONCLUSIONS

We have reviewed the argument that in minimal type-I SO(10) seesaw one cannot easily reconcile the thermal leptogenesis scenario with the successful Fritzsch texture for the quarks and an acceptable pattern of lepton masses and mixings that follow from neutrino oscillation experiments. This is due to the fact that the large seesaw scale needed to account for small neutrino masses leads to an overproduction of cosmological gravitinos, which destroys the standard predictions of big bang nucleosynthesis (BBN). Barring the very special case of resonant leptogenesis, one must go beyond the minimal type-I seesaw mechanism.

In this paper we have studied in some detail an extended seesaw scenario as a natural way to overcome this limitation. The proposed extension has the added virtue of providing a natural setting for reconciling the observed structure of neutrino mixing angles with the strong hierarchy among quark masses and the smallness of the quark mixings. While this can always be accommodated in a "generic" unified theory with arbitrary multiplet content, it becomes a real challenge for unified predictive models of flavor.

We have provided a quantitative study of fermion masses and (an approximative calculation of) leptogenesis

TABLE I. Some example spectra for the different ansätze considered.

Δ [GeV]	M_Σ [GeV]	M_1 [GeV]	M_2 [GeV]	M_3 [GeV]	$\eta_B[10^{-10}]$
		Nonhierarchical			
8.3×10^{-2}	1.2×10^3	4.1×10^4	5.0×10^4	2.6×10^5	6.3
9.8	2.3×10^4	9.3×10^5	1.2×10^6	6.1×10^6	5.0
		Hierarchical 5a			
3.5×10^3	1.2×10^3	3.6×10^4	6.2×10^4	6.7×10^6	4.9
2.9×10^3	1.1×10^3	4.6×10^4	6.8×10^4	5.7×10^6	5.8
		Hierarchical 5b			
1.8×10^3	6.6×10^2	2.1×10^4	6.4×10^4	3.8×10^6	5.2
8.3×10^2	2.5×10^2	6.1×10^3	6.2×10^4	9.8×10^6	4.8
		Hierarchical 6a			
2.9×10^2	2.9×10^3	1.6×10^5	3.6×10^9	6.0×10^{11}	5.1
3.3×10^3	8.3×10^2	2.5×10^5	1.3×10^{11}	1.4×10^{13}	5.8
		Hierarchical 6b			
1.0×10^1	1.8×10^3	1.9×10^5	1.1×10^6	2.0×10^6	5.4
3.4×10^1	2.8×10^3	1.5×10^6	1.5×10^6	2.5×10^6	5.9

in the context of supersymmetric SO(10) unification. Our approach was phenomenological in that we have not assumed a specific flavor symmetry. We have shown how thermal leptogenesis can occur at relatively low scale through the decay of a new singlet, thereby avoiding the gravitino crisis. Washout of the asymmetry is effectively suppressed by the absence of direct couplings of the singlet to leptons. For illustration we have shown how one can accommodate current oscillation neutrino data and the required value for the asymmetry for successful leptogenesis even if the only source of CP violation is the Dirac phase δ in the low energy neutrino mixing matrix. Finally, we note that we have not taken into account flavor effects [38,39] in the calculation of the asymmetry. However, we believe that the absence of a lower bound on M_Σ in our model is independent on whether flavor effects are taken into account or not.

Using the Fritzsch texture, we have found that some ansatze lead to acceptable values of the asymmetry of order 10^{-6}–10^{-7} only for large values of $\sin\theta_{13}$ and $\sin\delta$. Therefore, these scenarios can be potentially probed by the future neutrino oscillation measurements, as the required value of the CP invariant is nearly maximal. In contrast, we have also presented alternative Fritzsch-type ansatze leading to substantially larger values of the CP asymmetry, even for very small values of $\sin\theta_{13}$ and $\sin\delta$. We have also discussed, how to approximately treat the conversion of the decay asymmetry to the baryon asymmetry, without resorting to a full numerical solution of the Boltzmann equations. To this end, we made use of some approximation formulas derived previously for seesaw type-I and discussed how they can be adapted to cover also our more complicated case.

In summary, our extended seesaw scenario provides a way of reconciling the lepton and quark mixing angles with thermal leptogenesis in a unified scenario. While this by itself does not constitute a complete theory of fermion masses and leptogenesis, at least it provides a useful first step towards an ultimate unified theory incorporating flavor.

ACKNOWLEDGMENTS

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