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Pentaquark in the flux tube model

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We propose a model for pentaquarks in an excited state in the flux tube picture. The pentaquark is assumed to be composed of two diquarks and an antiquark connected by a color flux tube with a junction. If the pentaquark is rotating rapidly, it is polarized into two clusters: one is a diquark and the other is an antiquark plus another diquark. Excited energy of this quasilinear system is calculated with the use of the WKB approximation. It is predicted that there exist quasistable excited pentaquarks: 1690 MeV($3/2^+$), 2000 MeV($5/2^-$), 2250 MeV($7/2^+$) etc., which decay mainly through three-body modes.

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I. INTRODUCTION

In 2003, Nakano *et al.* reported the experimental evidence for the baryon states constructed from four quarks and an antiquark at the LEPS facility [1]. They observed a narrow peak at a mass 1540 MeV. On the other hand, Diakonov had predicted the pentaquark with a mass of about 1530 MeV and a width of 15 MeV or less in 1997 [2]. This coincidence revolutionized the field of baryon spectroscopy. Afterwards, however, the existence of the pentaquark has not been confirmed by further experiments with higher precision.

The fundamental theory of hadron physics is quantum chromodynamics (QCD). According to QCD, it is believed that only hadrons without color exist in nature, which is called "color confinement." This hypothesis explains the fact that the usual hadrons are constructed from a quark and an antiquark (mesons) or three quarks (baryons). But it does not prohibit the existence of a pentaquark composed of four quarks and one antiquark, because such a hadron can be in the color singlet state. This means that the pentaquark may exist in the real world. Its "nonexistence" seems to indicate that it may decay into usual hadrons very quickly even if it is produced.

There have been many theoretical studies on the pentaquark up to now [3]. Most of them discuss the ground state of a stable pentaquark. In this paper we will discuss the possible existence of the pentaquark from a different point of view: We explore *excited* states of the pentaquark. Let us remind the reader of the cluster model in nuclear physics. It is well known that there exist many states with molecular-like structure, which leads to the cluster model [4]. These states are mostly excited states discovered near the threshold energies for alpha decay. If we follow this lesson, it is probable that a pentaquark may exist as excited states. In this paper we study the pentaquark in the rotational excited state, which is well known as the Regge trajectory.

It is the main purpose of this paper to discuss the possible existence of the pentaquark state by using the flux tube (hadron string) model. There are theoretical and phenomenological indications for the probable importance of diquark correlations in hadronic physics [5–7]. We take a semiclassical model of color flux tubes (the flux tube model), taking into account the diquark correlation. We assume that a pentaquark is composed of two diquarks and an s-quark which are connected with a junction by the color flux. When this system rotates rapidly, it may be deformed as a quasilinear shape and polarize into two clusters: One is a diquark and the other an s-quark plus diquark (triquark) [8]. This picture is just the same as that of baryons, which are made up of a quark and a diquark connected by the elongated flux tube. Using this semiclassical model, we try to calculate the excited energies with the use of the WKB approximation. At the same time its decay mode and stability of the excited state will be studied.

In the next section, a string structure of the pentaquark is discussed from the phenomenological point of view. In Sec. III, we calculate the excited energies and the decay widths with the use of the WKB approximation. Their numerical calculation is done in the final section, and discussions on their possible existence are given.

II. A STRING MODEL FOR THE PENTAQUARK

To begin with we present a flux tube model [9–17] for the pentaquark. We assume that a pentaquark is described by two diquarks and an $\bar{s}$-quark which are connected with a junction as shown in Fig. 1(a).

First let us consider the optimized position of the junction when three ends of the flux tube are fixed. As the energy of the string is proportional to the total length, the position of the junction is determined by the minimization of the quantity

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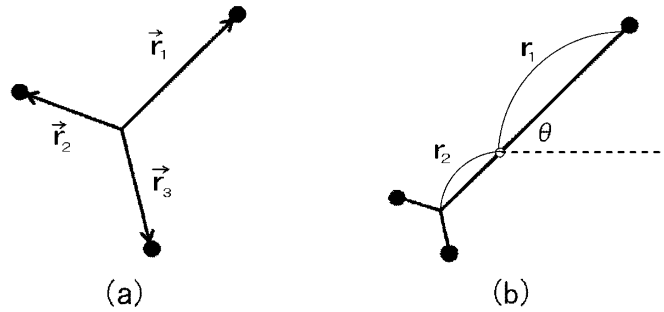

FIG. 1. The schematic pictures of the pentaquark: (a) that in the ground state and (b) that in the rotational excited state.

$$F(x, y) = \sum_{i=1}^{3} \sqrt{(x - x_i)^2 + (y - y_i)^2}, \tag{1}$$

where (x_i, y_i) $(i = 1, 2, 3)$ and (x, y) denote the Cartesian coordinates of the three ends and that of the junction, respectively. Consequently we have

$$\nabla F = \hat{\mathbf{r}}_1 + \hat{\mathbf{r}}_2 + \hat{\mathbf{r}}_3 = 0, \tag{2}$$

with the use of the definition $\hat{\mathbf{r}} \equiv \mathbf{r}/r$. From these equations, we get

$$\hat{\mathbf{r}}_{\mathbf{i}} \cdot \hat{\mathbf{r}}_{\mathbf{j}} = -\tfrac{1}{2} + \tfrac{3}{2}\delta_{i,j}. \tag{3}$$

Therefore each angle in Fig. 1 is the same value, $\theta = 2\pi/3$. This point of the junction is called the Steiner point of a triangle.

Now we consider rotational excited states of this three-body system (the ground state will be discussed in the Appendix). Suppose that this system rotates rapidly in the x-y plane. Then the pentaquark is elongated to a one-dimensional shape and polarized into two clusters whose masses are denoted by M_1 and M_2. The circumstances are shown in Fig. 1(b). This quasilinear shape has the largest moment of inertia, and the excited energy is lowered. There are two cases for the two clusters: a diquark and an $\bar{s}$ + diquark or an $\bar{s}$ and two diquarks. The Lagrangian of our system is written as

$$L = -\sum_{i=1}^{2} (M_i \sqrt{1 - v_i^2}) - V(r_1, r_2, \omega), \tag{4}$$

where r_1 or r_2 is now a length between each end and the center of mass (origin), and ω denotes the angular velocity around the axis perpendicular to the (x,y) plane. The velocity of each end is written as $v_i = \sqrt{\dot{r}_i^2 + \omega^2 r_i^2}$. The second term, which we call potential energy, is given by

$$V(r_1, r_2, \omega) = \sum_{i=1}^{2} \int_0^{r_i} a\sqrt{1 - r^2\omega^2} dr - \frac{e}{r}. \tag{5}$$

The first term denotes the energy of the elongated flux tube with the string tension a. The second one is the Coulomb energy between the two ends with the coupling constant e.

In order to go over to the Hamiltonian formalism, we define the canonical momentum of each variable by

$$p_i = \frac{\partial L}{\partial \dot{r}_i} = \frac{M_i \dot{r}_i}{\sqrt{1 - v_i^2}} \qquad (i = 1, 2), \tag{6}$$

$$l = \frac{\partial L}{\partial \dot{\theta}} = \sum_i \left(\frac{M_i r_i^2 \omega}{\sqrt{1 - v_i^2}} + \int_0^{r_i} \frac{a r^2 \omega}{\sqrt{1 - r^2\omega^2}} dr \right). \tag{7}$$

These quantities p_i and l denote the radial momentum of each cluster and the total (orbital) angular momentum, respectively. Here we impose a condition for the coordinate system that the center of mass of our system is at rest:

$$p_1 - p_2 = 0, \tag{8}$$

$$\frac{\partial L}{\partial r_1} - \frac{\partial L}{\partial r_2} = 0. \tag{9}$$

The second condition comes from $\dot{p}_1 - \dot{p}_2 = 0$ and describes equilibrium of the forces in the rotating coordinates system. These constraints reduce the number of degrees of freedom of our system. In fact, under the variation δr_i, the variation of the Lagrangian is written as

$$\delta L = \sum_i \left(\frac{\partial L}{\partial \dot{r}_i} \delta \dot{r}_i + \frac{\partial L}{\partial r_i} \delta r_i \right) = p_1 \delta(\dot{r}_1 + \dot{r}_2) + \frac{\partial L}{\partial r_1} \delta(r_1 + r_2). \tag{10}$$

If we define the length of the string by $r = r_1 + r_2$, our system is described only by r and θ: $L(r, \dot{r}, \theta)$. Thus we get the Hamiltonian as follows:

$$H = \dot{r}p + \omega l - L = \sum_i \left(\frac{M_i}{\sqrt{1 - v_i^2}} + \int_0^{r_i} \frac{a}{\sqrt{1 - r^2\omega^2}} dr \right) - \frac{e}{r}, \tag{11}$$

where we define the radial momentum conjugate to r by $p \equiv \partial L/\partial \dot{r} = p_1 = p_2$. The first term represents the kinetic energy of each cluster. The second and third ones correspond to the string and the Coulomb energies, respectively.

Finally we must express the Hamiltonian (11) in terms of the canonical variables. Substituting Eq. (6) into Eq. (11), we obtain

$$H(r, p, l) = \sum_i \left(\varepsilon_i + \frac{a}{\omega} \sin^{-1}(\omega r_i) \right) - \frac{e}{r}, \tag{12}$$

where the energy of each end is written as

$$\varepsilon_i = \sqrt{\frac{p^2 + M_i^2}{1 - \omega^2 r_i^2}} \qquad (i = 1, 2). \tag{13}$$

The angular velocity ω is represented by l through the other equation (7):

$$l = \omega \sum_i \left(r_i^2 \varepsilon_i + \frac{a}{2\omega^3} \sin^{-1}(\omega r_i) - \frac{a r_i}{2\omega^2} \right). \tag{14}$$

The ratio of r_1 and r_2 is determined by the condition on the center of mass (9),

$$\omega r_1 \varepsilon_1 - \frac{a}{\omega}\sqrt{1 - \omega^2 r_1^2} = \omega r_2 \varepsilon_2 - \frac{a}{\omega}\sqrt{1 - \omega^2 r_2^2}. \tag{15}$$

If these equations (13) and (14) are substituted into Eq. (12), we would have the Hamiltonian $H(r, p, l)$. However, its function is not expressed explicitly. We must be satisfied to have the implicit representation for $H(r, p, l)$ at this stage.

III. MASS AND DECAY WIDTH OF THE PENTAQUARK

In this section, we calculate the mass spectrum and the decay widths of the pentaquark by using the WKB method [18–20]. The Schrödinger equation of our system is given by

$$H(r, \hat{p}, \hat{l})\Psi(r, \theta) = E\Psi(r, \theta), \tag{16}$$

where Ψ is the wave function of the excited state with the excited energy E. The $\hat{p}$ and $\hat{l}$ denote the operators of the radial and angular momenta, respectively. The Hamiltonian is defined by Eqs. (12) and (14) implicitly. If our wave function has an eigenvalue m of the angular momentum, it is written as

$$\Psi(r, \theta) = \frac{1}{\sqrt{2\pi}} e^{im\theta/\hbar} \phi(r), \tag{17}$$

where the angular momentum is quantized as $m = 0, \hbar, 2\hbar, 3\hbar, \cdots$. Substituting this equation into Eq. (16), we get

$$H(r, \hat{p}, m)\phi(r) = E\phi(r). \tag{18}$$

The operator of the radial momentum in the three-dimensional space is written as

$$\hat{p}^{\,2} = -\hbar^2 \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} = \left(-i\hbar\left(\frac{\partial}{\partial r} + 1/r \right) \right)^2. \tag{19}$$

With the use of the change of variable $\phi(r) = \chi(r)/r$, the Schrödinger equation (18) is rewritten as a one-dimensional type,

$$H\left(r, -i\hbar\frac{\partial}{\partial r}, m \right)\chi(r) = E\chi(r). \tag{20}$$

Now the radial equation is solved by the use of the WKB approximation. It is suitable for this case because our Hamiltonian cannot be given by explicit expression. If the wave function is denoted by $\chi = A(r)\exp(iS(r)/\hbar)$, Eq. (20) is written as

$$H\left(r, \frac{\partial S}{\partial r} - i\hbar\frac{\partial}{\partial r}, m \right)A(r) = EA(r). \tag{21}$$

Here H in the left-hand side is expanded as a power series of $\hbar$, and we take up to the first order of $\hbar$. Thus we get

$$H(r, p(r), m) = E, \tag{22}$$

where the momentum $p(r)$ is defined by $\partial S/\partial r$, and

$$\frac{1}{2}\left(\frac{\partial H}{\partial p}\left(-i\hbar\frac{\partial}{\partial r} \right) + \left(-i\hbar\frac{\partial}{\partial r} \right)\frac{\partial H}{\partial p} \right)A = 0. \tag{23}$$

Here the operator in the left-hand side is symmetrized so as to be Hermitian. These equations determine the real unknown functions $A(r)$ and $S(r)$. The boundary condition for $S(r)$ leads to the Bohr-Sommerfeld relation:

$$\int_a^b p(r)dr = \begin{cases} (n + \frac{1}{2})\pi\hbar & (m > 0), \\ (n + \frac{3}{4})\pi\hbar & (m = 0), \end{cases} \tag{24}$$

where a and b are the turning points $[p(a) = p(b) = 0]$ and n means the quantum number of radial motion. We should be careful in the case of $m = 0$. The boundary condition at $a = 0$ is $\chi(r = 0) = 0$ so that the numerical factor $3/4$ in the right-hand side appears. We can rewrite Eq. (23) as

$$\frac{\partial}{\partial r}\left(\frac{\partial H}{\partial p} A^2 \right) = 0 \to A(r)^2 = \frac{C}{(\frac{\partial H}{\partial p(r)})}. \tag{25}$$

The integration constant C is determined by the normalization condition of the wave function,

$$\int_0^\infty \phi(r)^2 r^2 dr = 1 \to \int_0^\infty A(r)^2 dr = 1. \tag{26}$$

In the above discussion, it is an open problem how to determine the momentum $p(r)$. To this end, we take up the two constants of motion, Eqs. (12) and (14), and the condition on the center of mass (15):

$$\begin{cases} E = \sum_i (\varepsilon_i + \frac{a}{\omega}\sin^{-1}(\omega r_i)) - \frac{e}{r}, \\ m = \omega \sum_i (r_i^2 \varepsilon_i + \frac{a}{2\omega^3}\sin^{-1}(\omega r_i) - \frac{a r_i}{2\omega^2}\sqrt{1 - \omega^2 r_i^2}), \\ \omega r_1 \varepsilon_1 - \frac{a}{\omega}\sqrt{1 - \omega^2 r_1^2} = \omega r_2 \varepsilon_2 - \frac{a}{\omega}\sqrt{1 - \omega^2 r_2^2}. \end{cases} \tag{27}$$

The first and second equations mean the energy and angular momenta, respectively. Under the constants E, m and the parameter $r = r_1 + r_2$, these simultaneous equations determine $p(r)$ and $\omega(r)$. If these solutions are substituted into the Bohr-Sommerfeld condition (24), we have the energy spectrum $E_{n,m}$. This task will be carried out numerically in the next section.

Next let us consider the decay process of our system. We consider two decay processes: (a) $\bar{q}q$ pair production and (b) doubling of the string as shown in Fig. 2. The former is the decay through pair production of $\bar{q}q$ inside the flux tube in Fig. 2(a), which is known as the Schwinger mechanism [21–24]. The latter is the decay process due to doubling of

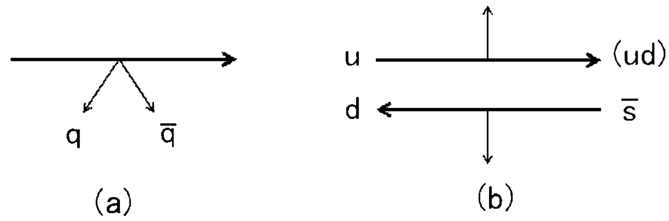

FIG. 2. The schematic pictures for the decay process: (a) quark-antiquark pair production and (b) doubling of the string.

the flux tube as shown in Fig. 2(b). This is a two-body decay mode: $\Theta \to N + K$. Here the pentaquark is denoted by Θ. On the other hand, the former is a three-body one such as $\Theta \to N + K + \pi$.

In this section we discuss the former process, (a), and the other one, (b), will be considered in the next section. The calculation of process (a) is carried out by using the Schwinger formula. If the probability of the pair production in a unit space-time volume is given by w, the probability of the decay (pair production) in the unit time is written as

$$\Gamma = \int_0^\infty wV(r)|\phi(r)|^2 r^2 dr, \qquad (28)$$

where $V(r)$ denotes the volume of the string with its length r in the laboratory coordinates system. It is given by

$$V(r) = \sum_i \int_0^{r_i} \sigma\sqrt{1 - r^2\omega^2}dr = \sum_i \frac{\sigma r_i}{2}\left(\sqrt{1 - r_i^2\omega^2} + \frac{1}{r_i\omega}\sin^{-1} r_i\omega\right). \qquad (29)$$

In the right-hand side, σ denotes the area of the tube in the rest frame of the string and the factor $\sqrt{1-\omega^2 r^2}$ represents its Lorenz contraction. The angular velocity ω is given by the relation (24). Substituting Eqs. (25) and (26) into Eq. (28), we obtain the decay width,

$$\Gamma = \left(\int_0^\infty wV(r)\left(\frac{\partial H}{\partial p}\right)^{-1} dr\right)\left(\int_0^\infty \left(\frac{\partial H}{\partial p}\right)^{-1} dr\right)^{-1}. \qquad (30)$$

The factor $\partial H/\partial p$ on the right-hand side can be rewritten as follows: With the use of the canonical equation $\dot{r} = \partial H/\partial p$ and the relation $\varepsilon_1 \dot{r}_1 = \varepsilon_2 \dot{r}_2 = p$, we get

$$\frac{\partial H}{\partial p} = \frac{dr}{dt} = \frac{p}{\varepsilon_1} + \frac{p}{\varepsilon_2} = \sum_i \frac{p\sqrt{1 - r_i^2\omega^2}}{\sqrt{p^2 + M_i^2}}. \qquad (31)$$

Thus $V(r)$ and $\partial H/\partial r$ are represented by $p(r)$ and $\omega(r)$, which are calculated by the simultaneous equations (24). Hence the decay width can be calculated by using Eqs. (26)–(28).

IV. NUMERICAL RESULTS

Now we are at a stage to calculate the energy spectrum and the decay widths of the pentaquark. The following parameters are present in our model: (a) the string tension a, the constant of the Coulomb potential e, (b) the strange quark mass m_s, the diquark mass m_d, and (c) the probability of the pair production $w\sigma$ inside the color flux tube. Parameters (a) have been studied in connection with Regge phenomenology. Recently many studies have been done by means of lattice QCD [25–27]. From these studies, we take $a = 0.15\ \text{GeV}^2$ and $e = 0.4$. As for the quark mass (b), m_s is taken to be the current mass, 150 MeV. On the other hand, the diquark mass is determined so as to reproduce the nucleon mass, assuming the baryon is made up of a quark and a diquark: $m_d = 330$ MeV. For parameter (c), $w\sigma$ is determined so as to reproduce the decay width of the rho-meson (770) instead of calculating the formula of the Schwinger mechanism: $w\sigma = 0.1\ \text{GeV}^2$.

We have calculated the Regge trajectories with the use of the above parameters. It is assumed that the mass of the light quark ($q = u$ or d) at the end of the string is the current mass, 5 MeV. The structures of the rho-meson and nucleon are assumed to be $s - \bar{s}$ and $q -$ diquark, respectively. The results are shown in Fig. 3. The closed circles and open triangles denote theoretical and experimental values, respectively [28]. It is seen that our present model is valid.

Now let us discuss the Regge trajectory of the pentaquark. Before carrying out computation, it is worthwhile to mention the stability of the triquark cluster in our pentaquark. Suppose that the triquark with mass m_t is made with an s-quark and a diquark. Then it may be possible that a Kaon take off from the tricluster: $\Theta \to N + K$. It is, however, hardly impossible from the energy conservation: $m_t = m_s + m_d = 485\ \text{MeV} < m_K$. As a result, our model for the pentaquark approximately has stability. We have calculated the energy spectrum of the pentaquark for $l > 0$, $n = 0$. The result is shown in Fig. 4. The total angular

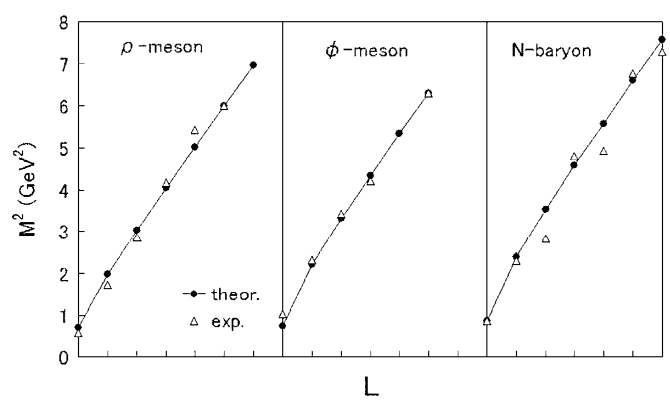

FIG. 3. Regge trajectories of the rho-meson, phi-meson, and nucleon: The closed circles and open triangles denote theoretical and experimental values, respectively.

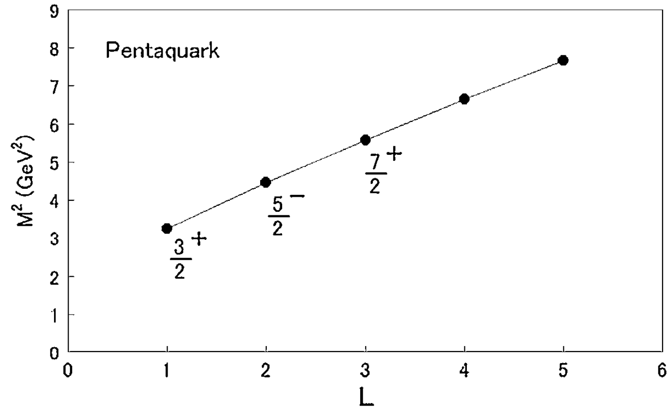

FIG. 4. Regge trajectory of the pentaquark: The theoretical values are denoted by closed circles together with the spin parity.

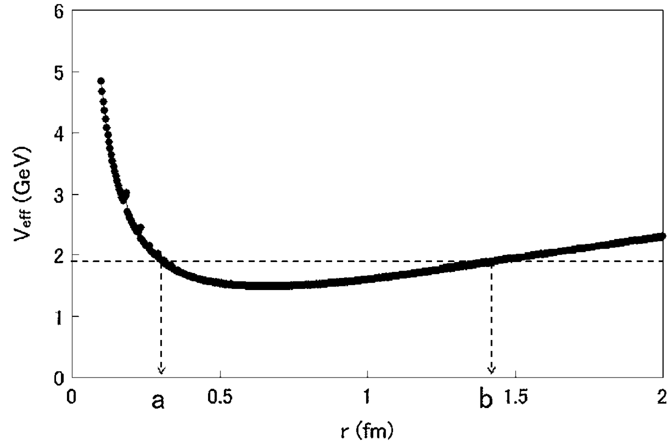

FIG. 5. The relative wave function of the pentaquark: r represents the length of the string. The turning points are denoted by a and b.

momentum J is set to be $J = l + 1/2$, because the spin orbit force is weak in the flux tube model as shown in Ref. [7]. We have assumed that the pentaquark is made up of a diquark and a triquark (s-quark + diquark). Of course, another structure composed of an s-quark and two diquarks is also possible. We have calculated the energy spectrum with this structure and found that their energies are a little larger than those of Fig. 4.

Next we consider the decay process of the pentaquark. There are two decay modes, as is presented in the previous section. First we calculate the decay width through the quark pair creation, that is, Eq. (30). The results are shown in Table I together with their masses. It is seen that their decay widths are the same order of magnitude as those of the usual excited hadrons.

Second we consider the decay mode due to the doubling of the string shown in Fig. 3 (b). The physical properties of these strings are the same as that of the $\bar{q} - q$ string because they have the same quantum number of color. In other words, the doubling needs twice the energy of the string. If the average length of the string is denoted by $\langle r \rangle$, the energy is about $a\langle r \rangle$. In the rightmost column of Table I, we have written the average distance $\langle r \rangle$. An example of the wave function $\psi(r)$ ($n = 0$) is shown in Fig. 5. As the energy $a\langle r \rangle$ is about 1 GeV, it is concluded that the decay process by the doubling mechanism is negligible in our case. In particular, the process becomes hard as the angular momentum increases.

TABLE I. Masses and decay widths of the pentaquarks. The average lengths of the string are also shown in the last column.

Spin (parity)	Mass (GeV)	Decay width (GeV)	Average length (fm)
3/2(+)	1.80	0.215	0.85
5/2(−)	2.11	0.278	1.11
7/2(+)	2.36	0.327	1.33
9/2(−)	2.58	0.356	1.47
11/2(+)	2.77	0.387	1.61

Finally we comment on the ground state of our system: $n = 0$ and $l = 0$. In this case, an s-quark and two diquarks must be treated on an equal footing for the lack of the centrifugal force. The detail of this discussion is done in the Appendix. The energy of the ground state becomes about $E = 1.78$ GeV without the Coulomb term. This means that it would be larger than the threshold energy for $N + K$ so that it is very hard to create such a state. Even if it is produced accidentally, it would decay very quickly.

In conclusion, we investigate the possibility of the existence of the pentaquark with the use of the flux tube model. It is predicted that there exist quasistable excited pentaquarks: 1690 MeV($3/2^+$), 2000 MeV($5/2^-$), 2250 MeV($7/2^+$)$\cdots$. We expect that such states will be found in the future.

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APPENDIX: GROUND STATE OF THE PENTAQUARK

In this appendix, we discuss the ground state ($1/2^-$) of the pentaquark. The dynamical system in Fig. 1(a) has a junction connected with three ends. Their masses, m_i ($i = 1, 2, 3$), are $m_1 = m_2 > m_3$ in this case [17]. The lengths of three legs are represented by r_1, r_2, and r_3, but their intersection angles are equal to $\pi/3$ as shown in Sec. II. Therefore, rotation of the whole system is described by the angle θ. Then the Lagrangian of this system is given by

$$L = -\sum_{i=1}^{3}(m_i\sqrt{1 - \dot{r}_i^2 - r_i^2\omega^2} + V(r_i, \omega)). \qquad \text{(A1)}$$

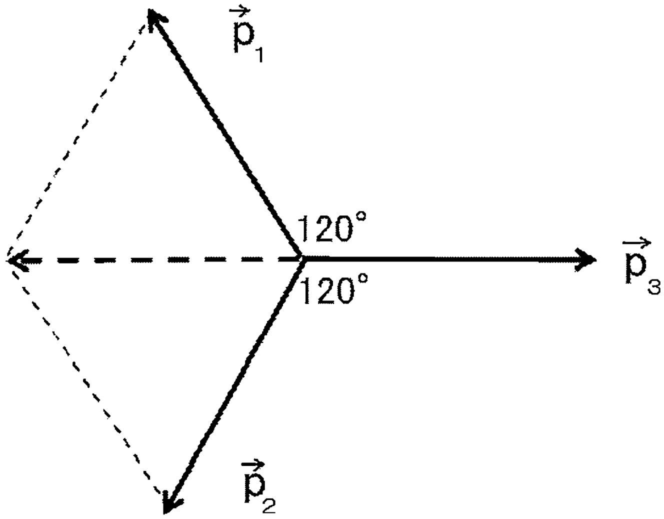

FIG. 6. The schematic picture of the pentaquark in the ground state: The momenta of three ends are represented by $\vec{p}_1$, $\vec{p}_2$, and $\vec{p}_3$.

We define canonical momenta conjugate to r_i by

$$p_i = \frac{\partial L}{\partial \dot{r}_i} = \frac{m_i \dot{r}_i}{\sqrt{1 - v_i^2}} \qquad (i = 1, 2, 3). \tag{A2}$$

Let us impose the condition that the center of mass is rest: The total momentum of this system vanishes. The configuration of our system is drawn in Fig. 6 so that the above condition leads to the following relations:

$$p_1 = p_2 = p_3 \quad \text{and} \quad \frac{\partial L}{\partial r_1} = \frac{\partial L}{\partial r_2} = \frac{\partial L}{\partial r_3}. \tag{A3}$$

The variation of L is rewritten as

$$\delta L = p_1 \delta(\dot{r}_1 + \dot{r}_2 + \dot{r}_3) + \frac{\partial L}{\partial r_1} \delta(r_1 + r_2 + r_3), \tag{A4}$$

so that the Lagrangian depends on only the total length of the string: $r \equiv r_1 + r_2 + r_3$. Then the Hamiltonian is written as

$$H(r, p) = \dot{r} p - L = \sum_{i=1}^{3} \sqrt{p^2 + m_i^2} + ar + V_c(r). \tag{A5}$$

The last term on the right-hand side is the Coulomb energy, which has a complicated form. We can calculate the energy of this system by using the WKB method in the same way as was done in Sec. III. The energy of the ground state is obtained as 1.87 GeV. Although the Coulomb term would lower the energy, the estimated energy is much larger than the threshold energy for $N + K$. Moreover, the minimum of the length of the string is zero so that this ground state would decay very quickly through process (b) in Sec. III.

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