

Inclusive production of J/ψ meson in proton-proton collisions at BNL RHIC

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Inclusive cross sections for J/ψ production in proton-proton collisions were calculated in the k_T -factorization approach for the Relativistic Heavy Ion Collider (RHIC) energy. Several mechanisms were considered, including direct color-singlet mechanism, radiative decays of χ_c mesons, decays of ψ' , open-charm-associated production of J/ψ as well as weak decays of B mesons. Different unintegrated gluon distributions from the literature were used. We find that radiative χ_c decays and direct color-singlet contributions constitute the dominant mechanism of J/ψ production. These processes cannot be consistently treated within a collinear-factorization approach. The results are compared with recent RHIC data. The new precise data at small transverse momenta impose stringent constraints on unintegrated gluon distribution functions (UGDFs). Some UGDFs are inconsistent with the new data. The Kwieciński UGDFs give the best description of the data. In order to verify the mechanism suggested here we propose J/ψ -jet correlation measurement and an independent measurement of χ_c meson production in $\pi^+\pi^-$ and/or K^+K^- decay channels. Finally, we address the issue of J/ψ spin alignment.

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I. INTRODUCTION

For the last decade, the inclusive production of J/ψ mesons was a serious theoretical puzzle challenging our understanding of QCD, parton model, and the bound-state formation dynamics. The roots of the puzzling J/ψ history trace back to the middle 1990s, when the data on J/ψ and Υ hadroproduction cross sections [1–3] revealed a more than one order-of-magnitude discrepancy with theoretical expectations. This fact has induced extensive theoretical activity and led to the introduction of new production mechanisms, known as the color-octet model [4,5] and gluon vector dominance model [6]. Since then, the color-octet model has been believed to give the most likely explanation of the quarkonium production phenomena, although there were also some indications that it was not working well. The situation became even more intriguing after the measurements of J/ψ spin alignment [7,8] were carried out showing inconsistency with the newly accepted theory.

At the same time, it has been shown that the incorporation of the usual color-singlet production scheme with the k_T -factorization approach can provide a reasonable and consistent picture of the phenomenon under study in its entirety. Within the latter approach, a good description of data on the production of J/ψ , χ_c , and Υ mesons both at the Tevatron [9,10] and HERA [11] has been achieved, and even a solution to the J/ψ spin-alignment problem has been guessed [9,12]. The issue of the quarkonium production mechanism continues to be under intense debate.

Recently, the PHENIX Collaboration at the Brookhaven National Laboratory (BNL) Relativistic Heavy Ion Collider (RHIC) has measured inclusive J/ψ production in elementary proton-proton collisions [13]. While for the RHIC community the elementary pp cross section is only the baseline for the nuclear case, we wish to demonstrate that the elementary data by itself constitute very valuable information about QCD dynamics in the region of intermediate $x \simeq 10^{-2}$ – 10^{-1} . In our paper we present a detailed analysis of RHIC data based on the k_T -factorization approach and a large variety of unintegrated gluon distribution functions (UGDFs). We show that the new precise data at small J/ψ transverse momenta impose stringent constraints on UGDFs and, consequently, stimulate better understanding of the underlying gluon dynamics.

The outline of the paper is the following. In Sec. II we describe the production mechanisms employed in our analysis and discuss the different parametrizations of UGDFs. In Sec. III we compare our theoretical predictions with experimental results and derive new predictions on the quantities which as yet have not been measured but could serve as an important cross-check of our understanding of the reaction mechanism. Our findings and recommendations for the forthcoming experiments are summarized in Sec. IV.

II. FORMALISM**A. Different mechanisms of J/ψ production**

In this paper, we take into account a number of different mechanisms leading to the appearance of J/ψ mesons in the final state (of course, they are not thought to be all of

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equal importance). The considered mechanisms are the following.

Direct color-singlet J/ψ production via gluon-gluon fusion

$$g + g \rightarrow J/\psi + g; \quad (1)$$

direct production of ψ' meson

$$g + g \rightarrow \psi' + g \quad (2)$$

and its subsequent decay $\psi' \rightarrow J/\psi + X$; production of P -wave charmonium states χ_{cJ} ($J = 0, 1, 2$)

$$g + g \rightarrow \chi_{cJ} \quad (3)$$

followed by their radiative decays $\chi_{cJ} \rightarrow J/\psi + \gamma$; production of b quarks and antiquarks

$$g + g \rightarrow b + \bar{b} \quad (4)$$

followed by their fragmentation into B mesons and subsequent weak decays $B \rightarrow J/\psi + X$; and production of J/ψ mesons in association with unbound charmed quarks

$$g + g \rightarrow J/\psi + c + \bar{c}. \quad (5)$$

Examples of the relevant Feynman diagrams for all the mentioned processes are shown in Fig. 1. Every subprocess is accompanied by the emission of gluon jets, as is shown in Fig. 2.

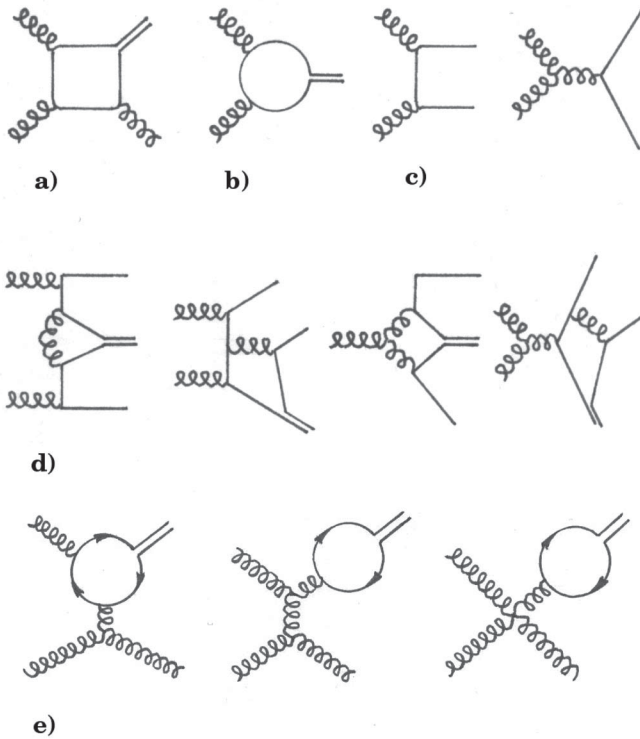


FIG. 1. Processes included in our approach: (a) direct color-singlet production, (b) production of χ_c mesons, (c) open bottom quark production, (d) open-charm-associated production, (e) - color-octet production.

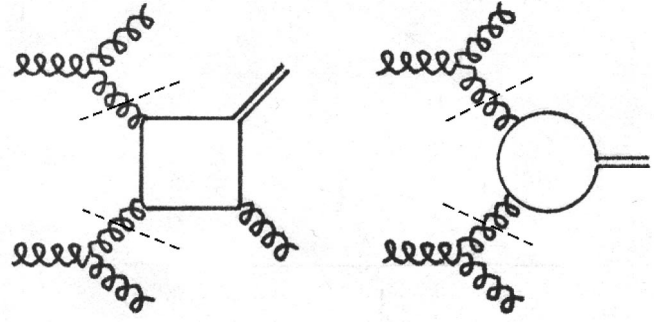


FIG. 2. Application of UGDs to inclusive production of J/ψ (left) and χ_c (right). The upper and the lower parts of these diagrams are included in the k_t evolution of gluon densities. The emitted gluons can realize in the final state hadronic jets.

In general, there could also exist color-octet contributions. The latter cannot be calculated from the first principles and are usually estimated from fits to existing data. It has been shown already that, within the k_t -factorization approach, these contributions are consistent with zero both at the Tevatron [9] and HERA [11]. In view of the uncertainties coming from other contributions we find it not useful to include color-octet contributions in the present analysis.

A few words are in order to describe the formation of $c\bar{c}$ bound states. First of all, it should be noted that the amplitudes of the subprocesses (1)–(3) and (5) contain projection operators $J(S, L)$, which guarantee the proper quantum numbers of the $c\bar{c}$ state under consideration. These operators read for the different spin and orbital angular momentum states [14,15]:

$$J(^1S_0) \equiv J(S = 0, L = 0) = \gamma_5(\not{p}_c + m_c)/m_\psi^{1/2}, \quad (6)$$

$$J(^3S_1) \equiv J(S = 1, L = 0) = \not{\epsilon}(S_z)(\not{p}_c + m_c)/m_\psi^{1/2}, \quad (7)$$

$$J(^3P_J) \equiv J(S = 1, L = 1) \\ = (\not{p}_{\bar{c}} - m_c)\not{\epsilon}(S_z)(\not{p}_c + m_c)/m_\psi^{3/2}, \quad (8)$$

where m_ψ is the mass of the specifically considered $c\bar{c}$ state and $m_c = m_\psi/2$ the mass of the charmed quark (always set equal to 1/2 of the meson mass, as is required by the nonrelativistic bound-state model).

States with various projections of the spin momentum onto the z axis are represented by the polarization vector $\epsilon(S_z)$.

The probability for the two quarks to form a meson depends on the bound-state wave function $\Psi(q)$. In the nonrelativistic approximation which we are using here, the relative momentum q of the quarks in the bound state is treated as a small quantity. So, it is useful to represent the quark momenta as $p_c = p_\psi/2 + q$, $p_{\bar{c}} = p_\psi/2 - q$. Then, we multiply the matrix elements by $\Psi(q)$ and perform

integration with respect to q . The integration is performed after expanding the integrand around $q = 0$:

$$\mathcal{M}(q) = \mathcal{M}|_{q=0} + (\partial\mathcal{M}/\partial q^\alpha)|_{q=0}q^\alpha + \dots \quad (9)$$

Since the expressions for $\mathcal{M}|_{q=0}$, $(\partial\mathcal{M}/\partial q^\alpha)|_{q=0}$, etc. are no longer dependent on q , they may be factored outside the integral sign. A term-by-term integration of this series then yields [15]

$$\int \frac{d^3q}{(2\pi)^3} \Psi(q) = \frac{1}{\sqrt{4\pi}} \mathcal{R}(x=0), \quad (10)$$

$$\int \frac{d^3q}{(2\pi)^3} q^\alpha \Psi(q) = -i\epsilon^\alpha(L_z) \frac{\sqrt{3}}{\sqrt{4\pi}} \mathcal{R}'(x=0), \quad (11)$$

etc., where $\mathcal{R}(x)$ is the radial wave function in the coordinate representation [the Fourier transform of $\Psi(q)$]. The first term contributes only to S -waves, but vanishes for P -waves because $\mathcal{R}_P(0) = 0$. On the contrary, the second term contributes only to P -waves, but vanishes for S -waves because $\mathcal{R}'_S(0) = 0$. States with various projections of the orbital angular momentum onto the z axis are represented by the polarization vector $\epsilon(L_z)$. The numerical values of the wave functions are either known from the leptonic decay widths (for J/ψ and ψ' mesons) or can be taken from potential models (for χ_{cJ} mesons). Including radiative corrections changes the values of the wave functions by a factor of 2 (the NLO result compared to the LO result), and so, one can also expect a large effect from higher order corrections. This leads to a sizeable theoretical uncertainty, which, on the other hand, can only affect the absolute normalization but not the shape of the p_t spectrum.

When calculating the spin average of the matrix elements squared, we adopt the k_t -factorization prescription [16] for the off-shell gluon spin density matrix:

$$\overline{\epsilon^\mu \epsilon^{*\nu}} = k_t^\mu k_t^\nu / |k_t|^2, \quad (12)$$

where k_t is the component of the gluon momentum perpendicular to the beam axis, and the bar stands for the averaging over the gluon spin. In the collinear limit, when $k_t \rightarrow 0$, this expression converges to the ordinary $\overline{\epsilon_g^\mu \epsilon_g^{*\nu}} = -\frac{1}{2}g^{\mu\nu}$. In all other respects, the evaluation of the diagrams is straightforward and follows the standard QCD Feynman rules. This has been done using the algebraic manipulation systems FORM [17] and REDUCE [18].

For the direct production mechanism (1) the fully differential cross section reads

$$\begin{aligned} d\sigma(pp \rightarrow \psi X) &= \frac{\pi\alpha_s^3 |\mathcal{R}(0)|^2}{\hat{s}^2} \frac{1}{4} \sum_{\text{spins}} \frac{1}{64} \sum_{\text{colors}} |\mathcal{M}(gg \rightarrow \psi g)|^2 \\ &\times \mathcal{F}_g(x_1, k_{1T}^2, \mu^2) \\ &\times \mathcal{F}_g(x_2, k_{2T}^2, \mu^2) dk_{1T}^2 dk_{2T}^2 dp_{\psi T}^2 dy_3 dy_\psi \\ &\times \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \frac{d\phi_\psi}{2\pi}, \end{aligned} \quad (13)$$

where ϕ_1 and ϕ_2 are the azimuthal angles of the initial gluons, and y_ψ and ϕ_ψ the rapidity and the azimuthal angle of J/ψ particle. The explicit expressions for the parton level matrix elements $|\mathcal{M}(gg \rightarrow \psi g)|^2$ are presented in Ref. [9].

The phase space physical boundary is determined by the inequality [19]

$$G(\hat{s}, \hat{t}, k_3^2, k_1^2, k_2^2, m_\psi^2) \leq 0, \quad (14)$$

with k_1 , k_2 , and k_3 being the initial and final gluon momenta, $\hat{s} = (k_1 + k_2)^2$, $\hat{t} = (k_1 - p_\psi)^2$, and G is the standard kinematic function [19]. The initial gluon momentum fractions x_1 and x_2 appearing in the unintegrated gluon distribution functions $\mathcal{F}_g(x_i, k_{iT}^2, \mu^2)$ are calculated from the energy-momentum conservation laws in the light cone projections:

$$\begin{aligned} (k_1 + k_2)_{E+p_{\parallel}} &= x_1\sqrt{s} = m_{\psi T} \exp(y_\psi) + |k_{3T}| \exp(y_3), \\ (k_1 + k_2)_{E-p_{\parallel}} &= x_2\sqrt{s} \\ &= m_{\psi T} \exp(-y_\psi) + |k_{3T}| \exp(-y_3), \end{aligned} \quad (15)$$

where $m_{\psi T} = (m_\psi^2 + |p_{\psi T}|^2)^{1/2}$.

The production scheme of ψ' meson (2) is identical to that of J/ψ , and only the numerical value of the wave function $|\mathcal{R}(0)|^2$ is different. In both cases, the values of the wave functions were extracted from the known leptonic decay widths [20] using the formula $|\mathcal{R}(0)|^2 = \Gamma_{ee} m_\psi^2 / (4\alpha^2 e_c^2 [1 - 16\alpha_s / (3\pi)])$ and were set equal to $|\mathcal{R}_{J/\psi}(0)|^2 = 0.8 \text{ GeV}^3$ for J/ψ meson, and $|\mathcal{R}_{\psi'}(0)|^2 = 0.4 \text{ GeV}^3$ for ψ' meson. To calculate the feed-down to J/ψ states, the ψ' production cross section has to be multiplied by the branching fraction $\text{Br}(\psi' \rightarrow J/\psi X) = 56\%$ [20].

For the production of χ_{cJ} mesons via the subprocess (3) we have

$$\begin{aligned} d\sigma(pp \rightarrow \chi_{cJ} X) &= \frac{12\pi^2 \alpha_s^2 |\mathcal{R}'(0)|^2}{x_1 x_2 s \lambda^{1/2}(\hat{s}, k_1^2, k_2^2)} \frac{1}{4} \sum_{\text{spins}} \\ &\times \frac{1}{64} \sum_{\text{colors}} |\mathcal{M}'(gg \rightarrow \chi_{cJ})_{q=0}|^2 \times \mathcal{F}_g(x_1, k_{1T}^2, \mu^2) \\ &\times \mathcal{F}_g(x_2, k_{2T}^2, \mu^2) dk_{1T}^2 dk_{2T}^2 dy_\chi \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi}. \end{aligned} \quad (16)$$

The squares of the matrix elements, as being too lengthy, are not presented there but the full fortran code is available on request. The numerical value of the wave function is taken from the potential model [21]: $|\mathcal{R}'_\chi(0)|^2 = 0.075 \text{ GeV}^5$. The decay branchings to J/ψ meson are known to be [20] $\text{Br}(\chi_{cJ} \rightarrow J/\psi\gamma) = 0.013, 0.35, \text{ and } 0.20$ for $J = 0, 1, \text{ and } 2$, respectively. Here, the off-shell gluon flux factor is defined as $F = 2\lambda^{1/2}(\hat{s}, k_1^2, k_2^2)$, according to the general definition given by Eq. (2.3) in Ref. [19]. For all other subprocesses one can use the approximations $\lambda^{1/2}(\hat{s}, k_1^2, k_2^2) \simeq \hat{s} \simeq x_1 x_2 s$, but they are not suitable for the present case because the invariant mass of the final state is small and the difference between $\hat{s} \equiv m_\chi^2$ and $x_1 x_2 s \equiv m_{\chi,t}^2 = m_\chi^2 + p_t^2$ can make a pronounced effect on the p_T spectrum. The numerical accuracy of the above definition was tested in a toy calculation regarding the leptonic production of χ_{cJ} mesons via photon-photon fusion: $e + e \rightarrow e' + e' + \chi_c$. We have compared the exact $\mathcal{O}(\alpha^4)$ result with a number of calculations based on equivalent photon approximation and using different definitions of the effective photon flux (such as $F = 2\hat{s}$, $F = 2x_1 x_2 s$, etc.). We find that the “ $\lambda^{1/2}$ ” definition is in the best agreement with exact calculation.

For the production of beauty quarks in (4) we have

$$\begin{aligned} d\sigma(pp \rightarrow b\bar{b}X) &= \frac{4\pi\alpha_s^2}{\hat{s}^2} \frac{1}{4} \sum_{\text{spins}} \frac{1}{64} \sum_{\text{colors}} |\mathcal{M}(gg \rightarrow b\bar{b})|^2 \\ &\times \mathcal{F}_g(x_1, k_{1r}^2, \mu^2) \mathcal{F}_g(x_2, k_{2r}^2, \mu^2) \\ &\times dk_{1r}^2 dk_{2r}^2 dp_{bT}^2 dy_b dy_{\bar{b}} \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \frac{d\phi_b}{2\pi}. \end{aligned} \quad (17)$$

The explicit expressions for the parton level matrix elements $|\mathcal{M}(gg \rightarrow b\bar{b})|^2$ can be found elsewhere [22]. In calculations the b -quark mass was set to $m_b = 4.5 \text{ GeV}$. Further on, the produced b -quarks undergo fragmentation into B -mesons according to the Peterson fragmentation function [23] with $\epsilon = 0.006$. The outgoing B -mesons undergo then a decay according to the three body decay mode $B \rightarrow J/\psi + K + \pi$, to which the net effective branching fraction [20] was attributed: $\text{Br}(b \rightarrow J/\psi X) = 1.16\%$ (respectively, $\text{Br}(b \rightarrow \psi' X) = 0.48\%$). This decay mode was taken as a typical representative for all B -meson decays. As the decay matrix elements are unknown, the decays were generated according to the phase space. However, the fine details of fragmentation and decay are rather unimportant for our purposes, because b -quarks play only a marginal role at RHIC energies, except large transverse momenta of J/ψ or ψ' . We shall discuss the region of the large transverse momenta somewhat later.

Finally, for the charm-associated production (5) we write

$$\begin{aligned} d\sigma(pp \rightarrow \psi c\bar{c}X) &= \frac{\alpha_s^4}{4\hat{s}^2} |\mathcal{R}(0)|^2 \frac{1}{4} \sum_{\text{spins}} \frac{1}{64} \sum_{\text{colors}} |\mathcal{M}(gg \rightarrow \psi c\bar{c})|^2 \\ &\times \mathcal{F}_g(x_1, k_{1r}^2, \mu^2) \mathcal{F}_g(x_2, k_{2r}^2, \mu^2) \\ &\times dk_{1r}^2 dk_{2r}^2 dp_{\psi T}^2 dp_{cT}^2 dy_\psi dy_c dy_{\bar{c}} \\ &\times \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \frac{d\phi_\psi}{2\pi} \frac{d\phi_c}{2\pi}. \end{aligned} \quad (18)$$

The explicit expressions for the parton level matrix elements $|\mathcal{M}(gg \rightarrow \psi c\bar{c})|^2$ as well as detailed description of the kinematics are presented in Ref. [24].

To close the description of the production mechanisms, we wish to state that we do not consider explicitly color-octet contributions in the present analysis. In fact, we know no data which would clearly manifest the presence of color-octet contributions. On the contrary, the numerical fits of the color-octet matrix elements based on the Tevatron and HERA data are incompatible with each other. Moreover, a conflict between the model predictions and the data on the J/ψ spin alignment indicate that the production of vector quarkonia is certainly not dominated by the color-octet mechanism. Some small contribution is not excluded but cannot be calculated from first principles.

B. Unintegrated gluon distributions

In general, there are no simple relations between unintegrated and integrated parton distributions. Some UGDFs in the literature are obtained based on familiar collinear distributions, some are obtained by solving evolution equations, some are just modeled, or some are even parametrized. A brief review of unintegrated gluon distributions that will be used also here can be found in Ref. [25].

At very low x the unintegrated gluon distributions are believed to fulfil the BFKL (Balitskij-Fadin-Kuraev-Lipatov) equation [26]. Here in the practical applications we shall use a simple parametrization [27] for the numerical solution [28] and use the acronym BFKL. Another distribution closely related to the BFKL approach was constructed by Blümlein [29].

At large energies (small x) one expects in addition saturation effects due to gluon recombinations. A simple parametrization of unintegrated gluon distribution in the proton can be obtained based on the Golec-Biernat–Wüsthoff parametrization of the dipole-nucleon cross section with parameters fitted to the HERA data. The dipole-nucleon cross section can be transformed to corresponding unintegrated gluon distribution. The resulting gluon distribution can be found in [30]. In the following we call it GBW UGDF for brevity. Another parametrization, also based on the idea of gluon saturation, was proposed in [31]. In contrast to the GBW approach [30], where the dipole-nucleon cross section is parametrized, in the Kharzeev-Levin approach it is the unintegrated gluon distribution which is parametrized. More details can be found in Ref. [25].

Another useful parametrization, which describes the HERA data, and therefore is valid for $10^{-4} < x < 10^{-2}$, was constructed by Ivanov and Nikolaev [32]. We refer the reader for details to the original paper.

In some of the approaches one imposes the following relation between the standard collinear distributions and UGDFs:

$$g(x, \mu^2) = \int_0^{\mu^2} f_g(x, \mathbf{k}_t^2, \mu^2) \frac{d\mathbf{k}_t^2}{\mathbf{k}_t^2}. \quad (19)$$

Because of its simplicity, the Gaussian smearing of initial transverse momenta is a good and popular reference for other approaches. It allows one to study phenomenologically the role of transverse momenta in several high-energy processes. We define a simple unintegrated gluon distribution:

$$\mathcal{F}_g^{\text{Gauss}}(x, k_t^2, \mu_F^2) = x g_i^{\text{coll}}(x, \mu_F^2) \cdot f_{\text{Gauss}}(k_t^2), \quad (20)$$

where $g^{\text{coll}}(x, \mu_F^2)$ is a standard collinear (integrated) gluon distribution and $f_{\text{Gauss}}(k_t^2)$ is a Gaussian two-dimensional function:

$$f_{\text{Gauss}}(k_t^2) = \frac{1}{2\pi\sigma_0^2} \exp(-k_t^2/2\sigma_0^2)/\pi. \quad (21)$$

The UGDF defined by Eq. (20) and (21) is normalized such

that

$$\int \mathcal{F}_g^{\text{Gauss}}(x, k_t^2, \mu_F^2) dk_t^2 = x g_i^{\text{coll}}(x, \mu_F^2). \quad (22)$$

At small values of x the unintegrated gluon distribution can be obtained from integrated distribution as [16]

$$\mathcal{F}(x, k_t^2) = \frac{d(xg(x, \mu^2))}{d\mu^2} \Big|_{\mu^2=k_t^2}. \quad (23)$$

This method cannot be directly used at small transverse momenta (small factorization scales) and must be supplemented by a further prescription. One possible prescription is freezing of the gluon distribution at $k_t^2 < \mu_{f_r}^2$, another is a shift of the scale: $\mu^2 \rightarrow \mu^2 + \mu_s^2$. Of course $\mu_{f_r}^2$ and μ_s^2 are bigger than the lowest possible scale for standard collinear distributions. This method cannot be also applied at larger x as here the scaling violation reverses and negative values are obtained.

At intermediate and large x more careful methods must be used. Kwieciński has shown that the evolution equations for unintegrated parton distributions take a particularly simple form in the variable conjugated to the parton transverse momentum. In the impact-parameter space, the Kwieciński equation takes the following simple form [33]:

$$\begin{aligned} \frac{\partial \tilde{f}_{NS}(x, b, \mu^2)}{\partial \mu^2} &= \frac{\alpha_s(\mu^2)}{2\pi\mu^2} \int_0^1 dz P_{qq}(z) \left[\Theta(z-x) J_0((1-z)\mu b) \tilde{f}_{NS}\left(\frac{x}{z}, b, \mu^2\right) - \tilde{f}_{NS}(x, b, \mu^2) \right], \\ \frac{\partial \tilde{f}_S(x, b, \mu^2)}{\partial \mu^2} &= \frac{\alpha_s(\mu^2)}{2\pi\mu^2} \int_0^1 dz \left\{ \Theta(z-x) J_0((1-z)\mu b) \left[P_{qq}(z) \tilde{f}_S\left(\frac{x}{z}, b, \mu^2\right) + P_{qg}(z) \tilde{f}_G\left(\frac{x}{z}, b, \mu^2\right) \right] \right. \\ &\quad \left. - [zP_{qq}(z) + zP_{gq}(z)] \tilde{f}_S(x, b, \mu^2) \right\}, \\ \frac{\partial \tilde{f}_G(x, b, \mu^2)}{\partial \mu^2} &= \frac{\alpha_s(\mu^2)}{2\pi\mu^2} \int_0^1 dz \left\{ \Theta(z-x) J_0((1-z)\mu b) \left[P_{gq}(z) \tilde{f}_S\left(\frac{x}{z}, b, \mu^2\right) + P_{gg}(z) \tilde{f}_G\left(\frac{x}{z}, b, \mu^2\right) \right] \right. \\ &\quad \left. - [zP_{gq}(z) + zP_{gg}(z)] \tilde{f}_G(x, b, \mu^2) \right\}. \end{aligned} \quad (24)$$

We have introduced here the shorthand notation

$$\begin{aligned} \tilde{f}_{NS} &= \tilde{f}_u - \tilde{f}_{\bar{u}}, & \tilde{f}_d - \tilde{f}_{\bar{d}}, \\ \tilde{f}_S &= \tilde{f}_u + \tilde{f}_{\bar{u}} + \tilde{f}_d + \tilde{f}_{\bar{d}} + \tilde{f}_s + \tilde{f}_{\bar{s}}. \end{aligned} \quad (25)$$

The unintegrated parton distributions in the impact factor representation are related to the familiar collinear distributions as follows:

$$\tilde{f}_k(x, b=0, \mu^2) = \frac{x}{2} p_k(x, \mu^2). \quad (26)$$

On the other hand, the transverse momentum dependent UPDFs are related to the integrated parton distributions as

$$x p_k(x, \mu^2) = \int_0^\infty dk_t^2 f_k(x, k_t^2, \mu^2). \quad (27)$$

The two possible representations are interrelated via Fourier-Bessel transform

$$\begin{aligned} f_k(x, k_t^2, \mu^2) &= \int_0^\infty db b J_0(k_t b) \tilde{f}_k(x, b, \mu^2), \\ \tilde{f}_k(x, b, \mu^2) &= \int_0^\infty dk_t k_t J_0(k_t b) f_k(x, k_t^2, \mu^2). \end{aligned} \quad (28)$$

The index k above numerates either gluons ($k=0$), quarks ($k>0$), or antiquarks ($k<0$).

The perturbative solutions $\tilde{f}_k^{\text{pert}}(x, b, \mu_F^2)$ do not include nonperturbative effects such as, for instance, intrinsic transverse momenta of partons in colliding hadrons. One

of the reasons is e.g. internal motion of constituents of the proton. In order to include such effects we modify the perturbative solution $\tilde{f}_g^{\text{pert}}(x, b, \mu_F^2)$ and write the modified parton distributions $\tilde{f}_g(x, b, \mu_F^2)$ in the simple factorized form

$$\tilde{f}_g(x, b, \mu_F^2) = \tilde{f}_g^{\text{pert}}(x, b, \mu_F^2) \cdot F_g^{np}(b). \quad (29)$$

In the present study we shall use the following functional form for the nonperturbative form factor

$$F_k^{np}(b) = F^{np}(b) = \exp\left(-\frac{b^2}{4b_0^2}\right). \quad (30)$$

In Eq. (30) b_0 is the only free parameter.

While physically $f_k(x, k_t^2, \mu^2)$ should be positive, there is no obvious reason for such a limitation for $\tilde{f}_k(x, b, \mu^2)$.

In the following we use leading-order parton distributions from Ref. [34] as the initial condition for QCD evolution. The set of integro-differential equations in b -space was solved by the method based on the discretization made with the help of the Chebyshev polynomials (see [33]). Then the unintegrated parton distributions were put on a grid in x , b , and μ^2 and the grid was used in practical applications for Chebyshev interpolation.

For the calculation of the direct J/ψ production here the parton distributions in momentum space are more useful. This calculation requires a time-consuming multidimensional integration. An explicit calculation of the Kwieciński UPDFs via Fourier transform for that needed in the main calculation values of $(x_1, k_{1,t}^2)$ and $(x_2, k_{2,t}^2)$ (see next section) is not possible. Therefore auxiliary grids of the momentum-representation UPDFs are prepared before the actual calculation of the cross sections. These grids are then used via a two-dimensional interpolation in the spaces $(x_1, k_{1,t}^2)$ and $(x_2, k_{2,t}^2)$ associated with each of the two incoming partons.

The Kwieciński unintegrated parton distributions were used recently in applications to $c\bar{c}$ photoproduction [35], $c\bar{c}$ correlations in nucleon-nucleon collisions [25], production of gauge bosons [36], production of a standard model Higgs boson [37], inclusive production of pions [38], and production of direct photons [39]. Good agreement with experimental data was obtained in the case when the data existed.

In the approach of Ref. [29], based on leading-order perturbative solution of the BFKL equation, the unintegrated gluon density $\mathcal{F}_g(x, k_t^2, \mu^2)$ is calculated as a convolution of the ordinary (collinear) gluon density $g(x, \mu^2)$ with universal weight factors:

$$\mathcal{F}_g(x, k_t^2, \mu^2) = \int_x^1 \mathcal{G}(\eta, k_t^2, \mu^2) \frac{x}{\eta} g\left(\frac{x}{\eta}, \mu^2\right) d\eta. \quad (31)$$

$$\mathcal{G}(\eta, k_t^2, \mu^2) = \frac{\bar{\alpha}_s}{\eta k_t^2} J_0(2\sqrt{\bar{\alpha}_s \ln(1/\eta) \ln(\mu^2/k_t^2)}), \quad k_t^2 < \mu^2, \quad (32)$$

$$\mathcal{G}(\eta, k_t^2, \mu^2) = \frac{\bar{\alpha}_s}{\eta k_t^2} I_0(2\sqrt{\bar{\alpha}_s \ln(1/\eta) \ln(k_t^2/\mu^2)}), \quad k_t^2 > \mu^2, \quad (33)$$

where J_0 and I_0 stand for Bessel functions (of real and imaginary arguments, respectively), and $\bar{\alpha}_s = \alpha_s/3\pi$. The leading-order GRV set [40] was used in our calculations as the input collinear density. Here the value of α_s and the scale μ^2 are parameters of the model. The resulting unintegrated gluon distributions depend on them rather strongly. Sometimes for brevity we shall denote the distribution from Ref. [29] by JB.

III. RESULTS

Now we shall compare contributions of different processes discussed in the previous section. Here a Monte Carlo method based on the VEGAS routine [41] is used to allow an easy comparison of processes with a different number of particles in the final state. In Fig. 3 we show the contribution of the different mechanisms discussed above to the rapidity distributions of the J/ψ meson for the RHIC energy. This calculation is based on so-called derivative UGDFs, i.e. the ones obtained by differentiating the standard collinear distributions (see the previous section).

In Fig. 4 we show corresponding contributions to the transverse momentum distribution of the J/ψ meson. In this exploratory calculation the cross section is integrated over the full range of rapidity. We obtain a rather surprising result that the sequential production of J/ψ mesons via radiative decays of χ_c mesons is comparable to or even dominates over the direct color-singlet contribution almost in the whole phase space. The reason can be seen in the fact that the production of χ_{cJ} states refers to much lower

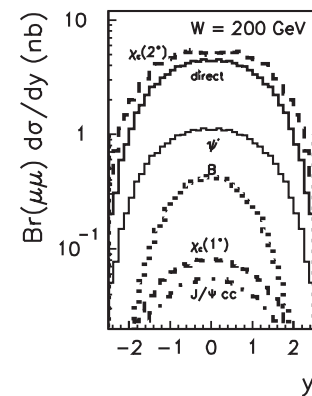


FIG. 3. Contributions of different mechanisms for the production of J/ψ in $d\sigma/dy$ distributions. In this calculation we have used simple “derivative UGDF.”

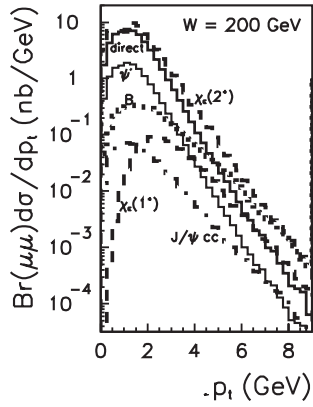


FIG. 4. Contributions of different mechanisms for the production of J/ψ in $d\sigma/dp_t$ distributions. In this calculation we have used simple “derivative UGDF.” The cross section is integrated over the full range of rapidity.

values of the final state invariant mass, $m_\chi^2 \ll (p_\psi + p_g)^2$, giving emphasis to the small x region, where the gluon distributions are growing up. This property becomes even more pronounced as the “direct” matrix element (1) vanishes when the emitted final gluon is soft. Our conclusion on the relative size of the direct and indirect contributions is compatible with the preliminary estimates obtained by the CDF Collaboration [3].

We wish to note now some difficulties of the standard collinear approach for the χ_c mesons. The leading-order contribution coming from the subprocess (3) shows unphysical δ -like p_T spectrum. The usual excuse that the particles produced at zero p_T disappear in the beam pipe and remain invisible does not work, because the decay products do have nonzero p_T and can be detected. At the same time, introducing the next-to-leading contributions (i.e., the processes with extra gluons in the final state) causes a problem of infrared divergences, which need artificial tricks to regularize them.

It is well known that a large fraction of the ψ' mesons decays into channels with J/ψ (BR = 0.56 [20]). This contribution was not considered in the literature and requires a separate discussion. The inclusive cross section for ψ' can be calculated in exactly the same way (color-singlet model) as the cross section for direct J/ψ meson production. The decays of $\psi' \rightarrow J/\psi + X$ change the kinematics only slightly. Finally the ψ' contribution constitutes about 25% of the direct (color-singlet) production.

Also the B -meson decay mechanism gives a sizeable contribution at large transverse momenta.

Summarizing, at the RHIC energy the dominant production mechanisms are radiative decays of $\chi_c(2^+)$ and the direct color-singlet mechanism. In the following we shall concentrate exclusively on these two dominant mechanisms.

Let us start with the color-singlet mechanism. In Fig. 5 we present distributions in rapidity of J/ψ produced by the

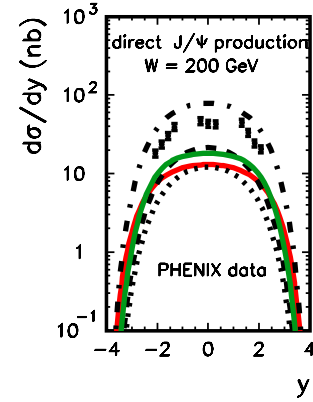


FIG. 5 (color online). Direct color-singlet contribution to rapidity distribution of J/ψ for different models of UGDFs. The black lower solid (red) curve corresponds to the Kwiciński UGDF, the dashed line to the Kharzeev-Levin UGDF, the dotted line to the BFKL UGDF, the dashed-dotted line to the Ivanov-Nikolaev UGDF, and the gray upper solid curve to the Blümlein UGDF. The ψ' contribution is not included here. The new PHENIX data are shown as full circles.

direct color-singlet mechanism for different UGDFs. The distribution obtained with Ivanov-Nikolaev glue exceeds the experimental PHENIX data [13], while the other theoretical distributions are smaller than experimental data. This is rather natural as contributions of other mechanisms are not included. The corresponding distributions in transverse momentum are shown in Fig. 6 for two different intervals in rapidity. Very similar distributions are obtained for mid- and intermediate rapidity intervals. The result with Ivanov-Nikolaev UGDF exceeds the experimental data in the region of small transverse momenta. This is

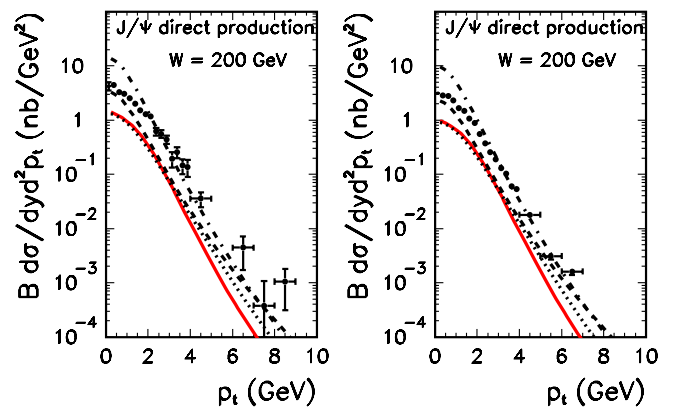


FIG. 6 (color online). Direct color-singlet contribution to transverse momentum distribution of J/ψ for different models of UGDFs for different intervals in rapidity: (a) $-0.35 < y < 0.35$ (left panel), (b) $1.2 < |y| < 2.2$ (right panel). The meaning of the curves is the same as in Fig. 5. The ψ' contribution is not included here. The new PHENIX data are shown as full circles.

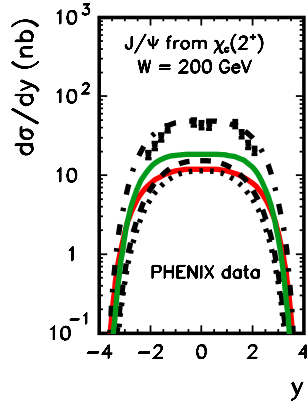


FIG. 7 (color online). χ_c -decay contribution to rapidity distribution of J/ψ for different models of UGDFs. The meaning of the curves is the same as in Fig. 5. The new PHENIX data are shown as full circles.

probably due to an extra nonperturbative contribution at small gluon transverse momenta [32].

Now we shall show results obtained with different UGDFs for radiative decays of $\chi_c(2^+)$. The rapidity distribution of corresponding J/ψ is shown in Fig. 7. Different UGDFs give a similar result. The distributions obtained with Ivanov-Nikolaev UGDF is slightly higher than those obtained with other distributions. In Fig. 8 we show distributions in transverse momentum of radiatively produced J/ψ . The differences in the results for different UGDFs are up to a factor 2 or even larger. Again Ivanov-Nikolaev UGDF gives the highest cross section for small transverse momenta. The Blümlein UGDF shown intentionally for the large value of $\alpha_s = 0.6$ (solid gray, green online) gives completely the wrong shape. The shape in this case depends strongly on the value of α_s . It would be much better for smaller values of α_s .

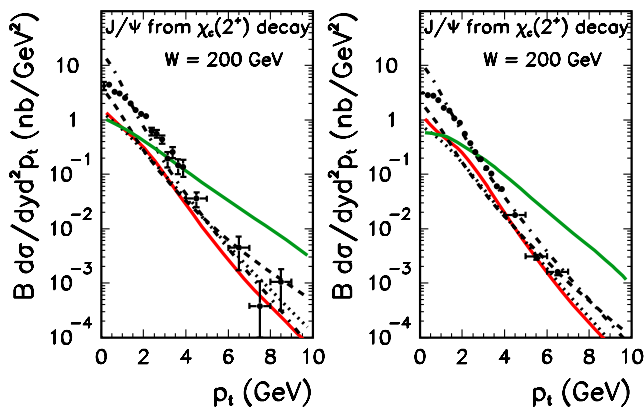


FIG. 8 (color online). χ_c -decay contribution to transverse momentum distribution of J/ψ for different models of UGDFs for different intervals in rapidity: (a) $-0.35 < y < 0.35$ (left panel), (b) $1.2 < |y| < 2.2$ (right panel). The meaning of the curves is the same as in Fig. 5. The new PHENIX data are shown as full circles.

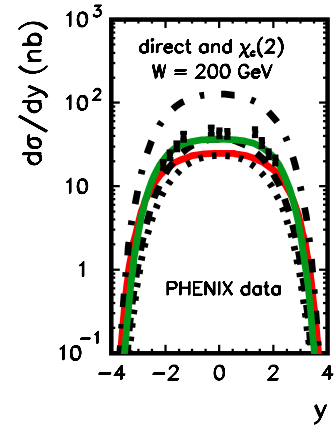


FIG. 9 (color online). Direct color-singlet and $\chi_c(2^+)$ contributions to rapidity distribution of J/ψ for different models of UGDFs. The meaning of the curves is the same as in Fig. 5. The new PHENIX data are shown as full circles.

Finally we would like to show how the sum of the two dominant contributions [direct color-singlet and radiative $\chi_c(2^+)$ decay] compares with the experimental data from RHIC. The distribution in rapidity is shown in Fig. 9 and distributions in transverse momentum in Fig. 10. The theoretical cross sections obtained with the Kwieciński, BFKL, and Kharzeev-Levin UGDFs stay slightly below the experimental data. This seems to be consistent with the fact that the smaller contributions discussed in Figs. 3 and 4 are not included here. They are expected to produce contributions of the order of 20%–30% (see Figs. 3 and 4).

At the RHIC energy $W = 200$ GeV the longitudinal momentum fractions of the order $x \sim 10^{-2}$ – 10^{-1} come into play. This is the place where application of many UGDFs may be questionable. Let us concentrate now on Kwieciński parton distributions, which are constructed for

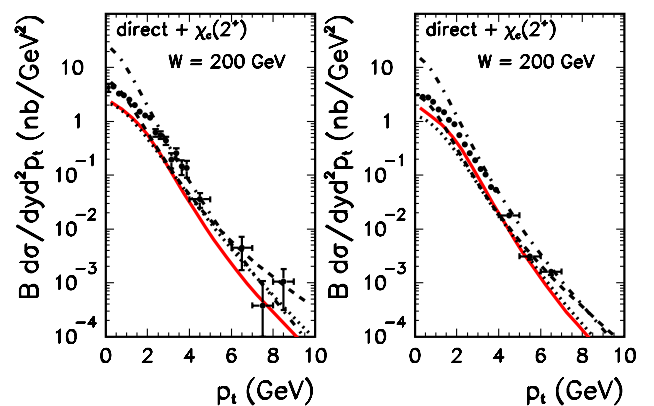


FIG. 10 (color online). Direct and χ_c -decay contributions to transverse momentum distribution of J/ψ for different models of UGDFs for different intervals in rapidity: (a) $-0.35 < y < 0.35$ (left panel), (b) $1.2 < |y| < 2.2$ (right panel). The meaning of the curves is the same as in Fig. 5. The new PHENIX data are shown as full circles.

the region of x under discussion. In the left panel of Fig. 11 we show the invariant cross section for the direct component as a function of J/ψ transverse momentum p_t for mid rapidity range $-0.35 < y < 0.35$. We show results for different factorization scales: $\mu^2 = 10 \text{ GeV}^2$ (solid line) and $\mu^2 = 100 \text{ GeV}^2$ (dashed line). In the right panel of Fig. 11 we show a similar result for J/ψ coming from the decays of the $\chi_c(2^+)$. Here the result depends more strongly on the choice of the scale. The solid line here corresponds to the running factorization scale: $\mu^2 = m_i^2 = m_{\chi_c(2^+)}^2 + p_t^2$.

In Fig. 12 we compare the sum of both processes calculated with the running factorization scale with the PHENIX experimental data. The calculation underestimates the data at small transverse momenta. This is most probably due to the omission of other components, especially the ψ' -decay component.

Let us concentrate now on the region of large transverse momenta of J/ψ . In Fig. 13 we show the contribution of J/ψ from decays of the B and \bar{B} mesons. The cross section for the $b\bar{b}$ is obtained with the Kwieciński UGDF (fixed factorization and renormalization scales, $\mu^2 = 4m_b^2$) within the k_f -factorization approach. The details of the calculation can be found in Ref. [25]. In the present illustrative calculation we neglect hadronization, i.e. we assume that the distribution of B (\bar{B}) mesons is the same as the distribution of b (\bar{b}) quarks. This seems justified for heavy quark to heavy meson transitions. There are several decay channels with final state J/ψ . The inclusive branching ratio is known experimentally $\text{BR} = 1.09\%$ [20]. However, the momentum distribution of J/ψ in the B meson center-of-mass system was not yet measured [42]. Here, in order to demonstrate the dependence on the details of the decay, we consider three academic models of the

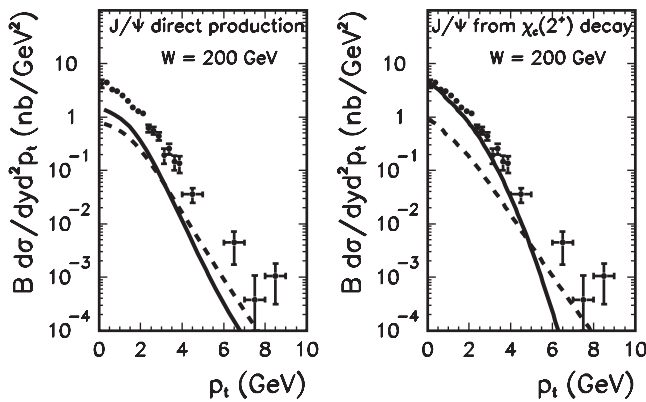


FIG. 11. Factorization scale dependence of the transverse momentum distribution for Kwieciński UGDF. The mid rapidity range $-0.35 < y < 0.35$ was taken as an example. The left panel is for direct production and the right panel for the $\chi_c(2^+)$ decay mechanism. The solid and dashed curves are for $\mu^2 = 10 \text{ GeV}^2$ and for $\mu^2 = 100 \text{ GeV}^2$, respectively. In this calculation $b_0 = 1 \text{ GeV}^{-1}$.

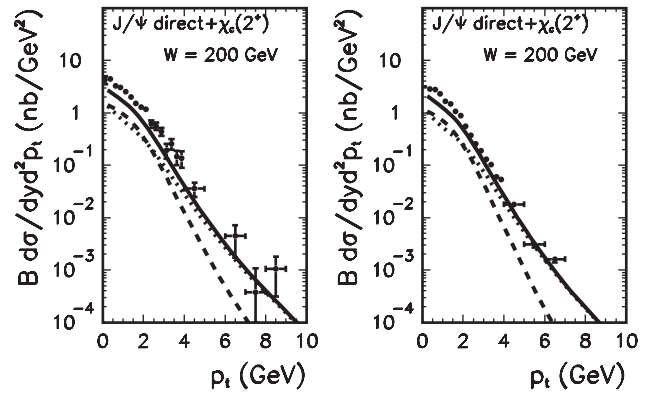


FIG. 12. Invariant cross section for the Kwieciński UGDF with running scale. The left panel is for central rapidity range ($-0.35 < y < 0.35$) and the right panel for intermediate rapidity range ($1.2 < y < 2.2$). The direct contribution is denoted by the dashed line, the $\chi_c(2^+)$ -decay contribution by the dotted line and the sum of both by the solid line.

decays: (a) uniform distribution in p^* (momentum of J/ψ in the meson rest frame) in the interval $(0, p_{\text{max}})$ —dashed line, (b) uniform distribution inside the sphere with radius p_{max} —dotted line, (c) distribution on the sphere with radius p_{max} —dashed-dotted line. Here p_{max} is the momentum obtained assuming a two-body decay: $B \rightarrow J/\psi X$. We assume the effective mass of the state X to be $m_X = 0.5 \text{ GeV}$. As can be seen from the figure, the B decays become an important ingredient at larger transverse momenta. There is a relatively mild dependence on the details of the decay. However, these details may become important with better statistics, when J/ψ with $p_t > 10 \text{ GeV}$ will be measured. The present estimate of the B -decay contribution may be an underestimation because of the two following reasons: (a) it is based on the leading-order approach,

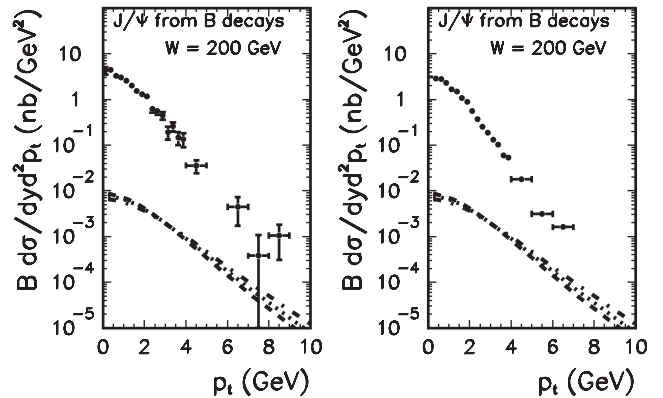


FIG. 13. Invariant cross section for J/ψ from decays of the B mesons as a function of p_t for midrapidity and intermediate rapidity intervals. Kwieciński UGDFs are used with factorization scale $\mu^2 = 4m_b^2$. Different decay models are described in the text.

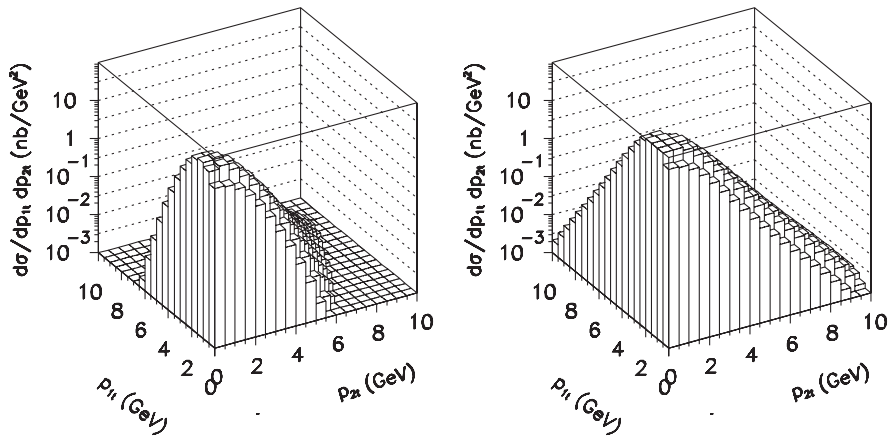


FIG. 14. Factorization scale dependence of the $p_{J/\psi,t} \times p_{g,t}$ distribution for the Kwieciński UGDF. The left panel is for $\mu^2 = 10 \text{ GeV}^2$ and the right panel is for $\mu^2 = 100 \text{ GeV}^2$.

(b) choice of the renormalization scale (see above) [43]. Therefore, at presently measured maximal transverse momenta of J/ψ $p_t \sim 8 \text{ GeV}$ the B -decay contribution at the level of 20%–30% is not excluded.

Let us concentrate now on correlations between produced J/ψ and associated gluon(s). In Fig. 14 we present a two-dimensional distribution in transverse momentum of J/ψ (p_{1t}) and transverse momentum of the associated (the

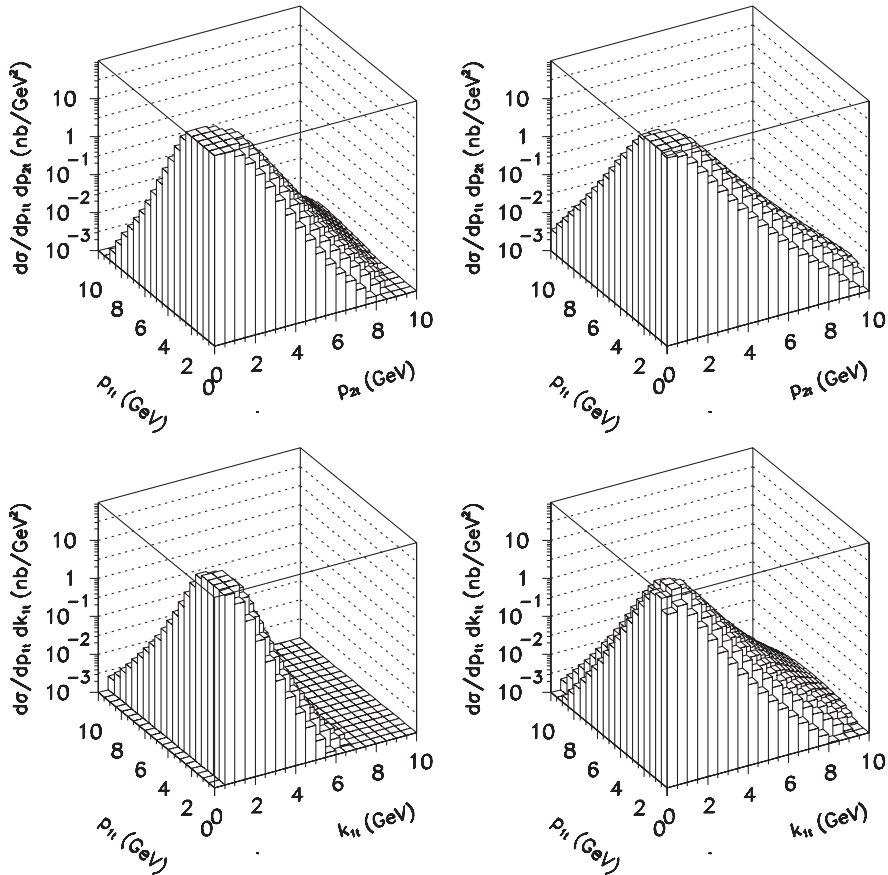


FIG. 15. Two-dimensional distribution of the J/ψ and gluon transverse momenta. The left-top panel is for Kwieciński UGDF (running scale) and matrix element gluon, the right-top panel is for the BFKL UGDF and matrix element gluon, the left-bottom panel is for the Kwieciński UGDF (running scale) and “last from the ladder” gluon and the right-bottom panel is for the BFKL UGDF and last from the ladder gluon.

gluon related with the matrix element gluon (p_{2i}) for two different scales of the Kwieciński UGDF. The bigger the scale is, the bigger is the spread of the cross section in the (p_{1i}, p_{2i}) space. This can be understood by the fact that the bigger scales mean more gluonic emissions which statistically means the bigger spread. This figure is rather of academic value as in practice there are also gluons emitted in the process of the ladder-type emissions. Strictly speaking, the latter have to be described using a full gluon evolution generator. On the other hand, the relevant effects can also be estimated in an approximate way, as follows. On the average, the gluon transverse momentum increases from the proton line towards the hard interaction block (although there is no strict ordering in the transverse momentum in the BFKL equation). So, it is most likely, that the last gluon in the parton ladder has the largest k_i value. As a rough approximation, one can neglect the transverse momenta of all the other emitted gluons [note that the evolution is in the $\log(k_i)$ space rather than k_i space] and use the conservation law in the last splitting vertex to set the k'_i of the emitted gluon opposite to the k_i of the gluon entering the partonic matrix element: $\vec{k}'_i \simeq -\vec{k}_i$. The latter is known from the unintegrated gluon distribution. This trick gives an estimate for the transverse momentum of the final state gluon jet.

In Fig. 15 we show distributions of the cross section on the plane $p_t(J/\psi) \times p_t$ (matrix element gluon or last gluon in the ladder) for the Kwieciński UGDF with running scale (left part) and BFKL UGDF (right part). Comparing these distributions we conclude that the gluons from the ladder (LFL—last from the ladder) contribute to lower transverse momenta than those associated with the matrix element $g + g \rightarrow J/\psi + g$ (ME) for the Kwieciński UGDF, where at $p_t(\text{gluon}) > 5$ GeV the matrix element gluons dominate over the ladder gluons. For the BFKL gluons the situation is much more complicated as here the distribution for ME gluons and LFL gluons are similar.

In Fig. 16 we show an average value of transverse momentum of the matrix element gluon (dashed line) and of the last gluon from the ladder (solid line) as a function of J/ψ transverse momentum. These average values have completely different dependence on $p_t(J/\psi)$. While the average value of the LFL gluon transverse momentum is only weakly dependent on $p_t(J/\psi)$, the average value of the ME gluon transverse momentum grows monotonically with $p_t(J/\psi)$. At low J/ψ transverse momenta $\langle p_t(\text{LFL}) \rangle \sim \langle p_t(\text{ME}) \rangle$. At higher J/ψ transverse momenta $\langle p_t(\text{LFL}) \rangle < \langle p_t(\text{ME}) \rangle$. For the Kwieciński distribution this happens at smaller transverse momentum than for the BFKL UGDF.

Our calculations presented up to now show that the production of J/ψ through radiative decays of χ_c mesons is one of two dominant mechanisms. It would be worthwhile verifying this theoretical prediction experimentally. This would require measuring the χ_c mesons indepen-

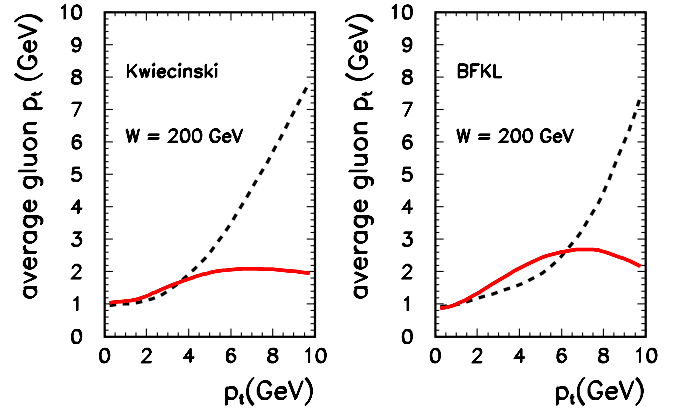


FIG. 16 (color online). Average transverse momenta of the ME (dashed line) and LFL (solid line) gluons as a function of the J/ψ transverse momentum for the Kwieciński UGDF with running scale (left panel) and the BFKL UGDF (right panel) at the RHIC energy $W = 200$ GeV.

dently. In Fig. 17 we show the distributions in rapidity for $\chi_c(0^+)$, $\chi_c(1^+)$, and $\chi_c(2^+)$. These results were obtained with the Kwieciński UGDF, which seems to be the most reliable for the RHIC energy range.

For completeness in Fig. 18 we show the corresponding distributions in transverse momentum. In this calculation $-1 < y < 1$. We wish to point out that the cross sections show no singularity at small transverse momentum. This contrasts with the collinear-factorization predictions, which are either unphysical (δ -like) or divergent (if based on a $2 \rightarrow 2$ subprocess $g + g \rightarrow \chi_c + g$). There is also a significant difference in shape between the transverse momentum distribution for $\chi_c(1^+)$ meson and those for $\chi_c(0^+)$ and $\chi_c(2^+)$ mesons. This property emerges from the Landau-Yang theorem which prohibits the coupling of vector states to massless photons (just because of quantum numbers incompatible with Bose statistics). The production of $\chi_c(1^+)$ states at small p_T is strongly suppressed

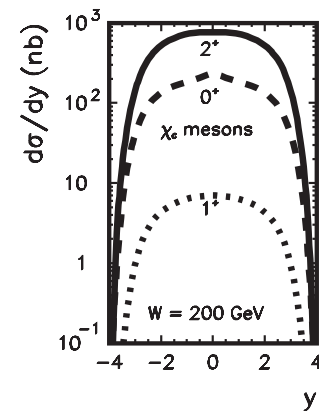


FIG. 17. Rapidity distribution of $\chi_c(0^+)$ (dashed line), $\chi_c(1^+)$ (dotted line) and $\chi_c(2^+)$ (solid line) for the RHIC energy obtained with the Kwieciński UGDF [$b_0 = 1 \text{ GeV}^{-1}$, $\mu^2 = p_T^2(\chi_c)$].

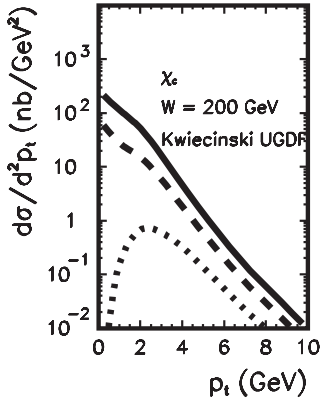


FIG. 18. Transverse momentum distribution of $\chi_c(0^+)$ (dashed line), $\chi_c(1^+)$ (dotted line) and $\chi_c(2^+)$ (solid line) for the RHIC energy obtained with the Kwieciński UGDF [$b_0 = 1 \text{ GeV}^{-1}$, $\mu^2 = p_t^2(\chi_c)$].

because the initial gluons are almost on shell. The suppression goes away at higher p_T , as the off-shellness of the initial gluons becomes larger. These features are discussed in detail in Ref. [44].

In contrast to the transverse momentum distribution of J/ψ from the color-singlet mechanism, the distributions of χ_c mesons (and consequently the distribution of J/ψ from radiative decays) strongly depend on the model of UGDF. In particular, in the limiting case of vanishing initial gluon transverse momenta: $d\sigma/d^2p_t \propto \delta^2(\vec{p}_t)$. For illustrating the effect quantitatively in Fig. 19 we present transverse momentum distributions of $\chi_c(2^+)$ for the Gaussian UGDF with different values of the smearing parameter $\sigma_0 = 0.5, 1, 2 \text{ GeV}$. The example clearly demonstrates that a measurement of transverse momentum distribution of χ_c mesons would open a new and unique possibility to test model unintegrated gluon distributions.

In principle the χ_c mesons [mainly $\chi_c(1^+)$ and $\chi_c(2^+)$] can be identified via photon- J/ψ decay channel. At RHIC the χ_c production mechanism could be also identified

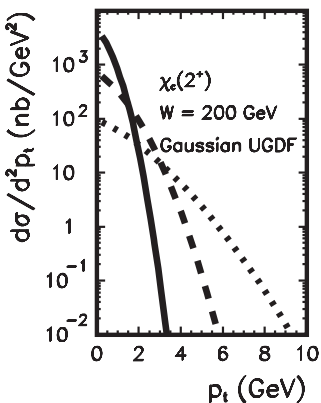


FIG. 19. Transverse momentum distribution of $\chi_c(2^+)$ for the RHIC energy obtained with the Gaussian UGDF and different values of $\sigma_0 = 0.5, 1.0, 2.0 \text{ GeV}$.

using the $\pi^+\pi^-$ and K^+K^- final channels. The corresponding branching ratios are [20] $\text{BR}(\chi_c(0^+) \rightarrow \pi^+\pi^-) = 7.2 \pm 0.6 \times 10^{-3}$, $\text{BR}(\chi_c(0^+) \rightarrow K^+K^-) = 5.4 \pm 0.6 \times 10^{-3}$, $\text{BR}(\chi_c(2^+) \rightarrow \pi^+\pi^-) = 2.14 \pm 0.25 \times 10^{-3}$, $\text{BR}(\chi_c(2^+) \rightarrow K^+K^-) = 7.7 \pm 1.4 \times 10^{-4}$.

Now we are coming to the issue of J/ψ spin alignment, which was, and still is, under intense debates in the literature. We want to stress once again that measuring the polarization of quarkonium states produced at high energies may serve as a crucial test discriminating the different concepts of parton dynamics.

The polarization state of a vector meson is characterized by the spin-alignment parameter α which is defined as a function of any kinematic variable as

$$\alpha(\mathcal{P}) = (d\sigma/d\mathcal{P} - 3d\sigma_L/d\mathcal{P}) / (d\sigma/d\mathcal{P} + d\sigma_L/d\mathcal{P}), \quad (34)$$

where σ is the reaction cross section, \mathcal{P} is a selected kinematical variable, and σ_L is the part of the cross section corresponding to mesons with longitudinal polarization (zero helicity state). The limiting values $\alpha = 1$ and $\alpha = -1$ refer to the totally transverse and totally longitudinal polarizations. Here we consider only the behavior of α as a function of the J/ψ transverse momentum: $\mathcal{P} \equiv |\mathbf{p}_T|$. The experimental definition of α is based on measuring the angular distributions of the decay leptons

$$d\Gamma(J/\psi \rightarrow \mu^+\mu^-)/d\cos\theta \sim 1 + \alpha\cos^2\theta, \quad (35)$$

where θ is the polar angle of the final state muon measured in the decaying meson rest frame.

The results of our calculations for the kinematic conditions of RHIC are displayed in Fig. 20. In order to show the theoretical uncertainty band connected with the choice of UGDF, we use two different parametrizations, which are known to show the largest difference with each other, namely, the ones proposed in Refs. [16] (called “derivative” for brevity) and the one from Ref. [29].

The upper panel in Fig. 20 shows the behavior of the spin-alignment parameter α for J/ψ mesons produced in the direct subprocess (1). The increase in the fraction of longitudinally polarized mesons comes from the increasing virtuality (and longitudinal polarization) of the initial gluons. These predictions shown here are also valid for ψ' mesons.

As far as the contribution from P -waves is concerned, nothing is known on the polarization properties of their decays. If we assume that the quark spin is conserved in radiative transitions, and the emission of a photon only changes the quark orbital momentum (as it is known to be true in the electric dipole transitions in atomic physics, $\Delta S = 0, \Delta L = \pm 1$), then the predictions on α appear to be similar to those made for the direct channel (see lower panel in Fig. 20, dotted curves). If, on the contrary, we assume that the transition $\chi_c \rightarrow J/\psi + \gamma$ leads to a complete depolarization, then we arrive at a more moderate

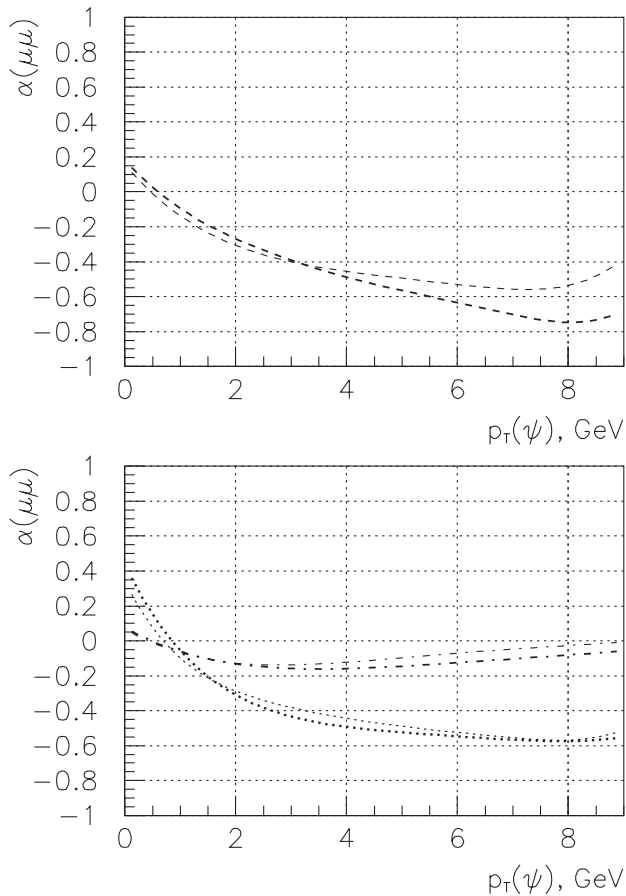


FIG. 20. Predictions for the spin-alignment parameter α for J/ψ and $W = 200$ GeV. Thick lines correspond to the Blümlein parametrization [29] and the thin lines correspond to the derivative UGDF parametrization and the GRV collinear distribution. The top panel is for direct contribution only. The bottom panel includes the feed-down from χ_c decays taken into account. The dotted lines are for the quark spin conservation hypothesis, and the dashed-dotted lines are for the full depolarization hypothesis.

behavior of the parameter α (dashed-dotted curves in Fig. 20). The overall polarization remains slightly longitudinal ($\alpha \simeq -0.1$) in the whole range of p_T due to the “direct” contribution. A comparison between the data on J/ψ and ψ' polarization at the Tevatron [8] seems to give support to the depolarization hypothesis. The difference between the J/ψ and ψ' polarization data can be naturally explained by the presence of the depolarizing contribution in the case of J/ψ and the absence of this contribution in the case of ψ' .

IV. DISCUSSION AND CONCLUSIONS

We have considered different mechanisms contributing to the inclusive production of J/ψ mesons in pp collisions

at RHIC kinematics. The outcome of our study is the following.

We have inspected the hierarchy of contributions and found that the dominant contribution to the cross section comes from radiative decay of χ_c mesons, mainly from a $\chi_c(2^{++})$ state. The second most important mechanism is the direct color-singlet production. The sequential process through the intermediate ψ' turned out to be nonnegligible and constitutes about a quarter of the direct color-singlet contribution. To our knowledge, these processes were not included in previous calculations in the literature on the subject.

As a by-product, we have demonstrated the advantage of the k_t -factorization approach in calculating the χ_c spectra: the latter can hardly be calculated in a consistent way in the collinear scheme. In order to verify the production mechanism suggested in our analysis, we have proposed an independent measurement of inclusive χ_c cross sections in the $\pi^+\pi^-$ and K^+K^- decay channels.

We have applied our approach to describe the data on inclusive J/ψ production recently collected by the PHENIX Collaboration at the BNL. Both rapidity and transverse momentum distributions have been discussed. The new precise data at small J/ψ transverse momenta appeared to show very strong analyzing power, imposing stringent constraints on unintegrated gluon distributions. The best description of the data is obtained with the UGDF proposed by Kwiciński.

Another piece of important information on the underlying gluon dynamics can be extracted from studying kinematic correlations between J/ψ mesons and co-produced gluon jets. In this paper we have presented our predictions for the two dominant contributing mechanisms.

Finally, we have presented our predictions on the J/ψ spin alignment. The latter can serve as an important test discriminating two different concepts of the parton model.

In the present paper we have discussed mechanisms of J/ψ production in elementary collisions. We believe that our findings here may be also useful for nuclear collisions, where J/ψ suppression was originally suggested as a useful indication of the presence of the quark-gluon plasma.

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