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ν induced threshold production of two pions and $N^*(1440)$ electroweak form factors

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We study the threshold production of two pions induced by neutrinos in nucleon targets. The contribution of nucleon, pion, and contact terms are calculated using a chiral Lagrangian. The contribution of the Roper resonance, neglected in earlier studies, has also been taken into account. The numerical results for the cross sections are presented and compared with the available experimental data. It has been found that in the two-pion channels with $\pi^+\pi^-$ and $\pi^0\pi^0$ in the final state, the contribution of the $N^*(1440)$ is quite important and could be used to determine the $N^*(1440)$ electroweak transition form factors if experimental data with better statistics become available in the future.

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I. INTRODUCTION

The important ongoing experimental effort addressing the questions of neutrino oscillations is bringing out, as a fortunate by-product, much information on the structure of hadrons and nuclei. Apart from the intrinsic interest of the knowledge of axial form factors, structure functions, or the strange quark content of the nucleon, a proper and precise understanding of various processes induced by neutrino interactions is required in the experimental analysis of background substraction, ν-flux determination, and particle identification in the neutrino oscillation experiments.

In the study of the neutrino-nucleus interaction, we can distinguish several energy regions. At low energies we have quasielastic scattering (QE) in which a nucleon is knocked out of the target nucleus. There has been a strong effort both experimentally and theoretically to measure and describe this process [1–9]. The basic ingredient is the relatively well-known neutrino-nucleon elastic interaction, although nuclear effects like Fermi motion, Pauli blocking, or long range RPA correlations are also needed. These nuclear effects have been found to strongly modify the cross section and also to distort angular and energy distributions of the final particles.

At intermediate energies, above 0.5 GeV, one pion production becomes relevant. The knowledge of the elementary process $\nu + N \rightarrow \ell + N' + \pi$ is not so well established. One of the reasons is the scarcity of data [10,11]. Most of the theoretical models assume the dominance of $\Delta(1232)$ resonance mechanisms [12–18] but others also include background terms [19–21]. The major uncertainties of these models appear in the $N\Delta(1232)$ transition axial form factors that are fitted, with some theoretical ansatz to the available data. It is hoped that new data on single pion production from neutrino experiments at K2K [22] and MiniBooNE [23,24] could help to determine these form factors.

In nuclei, the description of pion production requires a realistic model to account for the final state interaction (FSI) of the pion. This is usually implemented in Monte Carlo codes. However, that is not enough. It has been shown in several works that also the production mechanisms are modified in the medium. Furthermore, in some particular cases, like the coherent pion production, a quantum treatment of FSI is necessary. In any case, there is clear progress in our understanding of this reaction and some recent [25] and coming data [26] may already put strong constraints on the Δ form factors [27].

Above these energies, but still below the DIS region, new inelastic channels are open and several baryonic resonances beyond the $\Delta(1232)$ can be excited [18,28]. Recently, there has been an important progress in the determination of their vector form factors with the advent of high quality electromagnetic data [29–31]. Our knowledge of the axial form factors is, in general, poorer due to the scarcity of experimental information [18].

The first of these resonances is the $N^*(1440)$, for which the weak excitation can allow to study the axial sector. Although its main decay channel is to $N\pi$, its contribution to the one pion production cross section has been found to be negligible [14] because of the strong dominance of Δ mechanisms. However, the situation can be different for the production of two pions. This channel starts at invariant masses of the hadronic sector just below the Δ. However, the Δ does not couple to two pions in s-wave and thus it is not very relevant at these energies, where only slow pions are produced. On the other hand, the Roper resonance $N^*(1440)$ has a sizable decay into a scalar pion pair and it is very wide so that its contribution could be large. Indeed, Roper excitation mechanisms are known to play a major role in other two-pion production reactions close to threshold, like $\pi N \rightarrow \pi\pi N$ [32–34] or $NN \rightarrow \pi\pi NN$ [35,36].

In the weak interaction sector, there exist very few attempts to study the two-pion production induced by neutrinos and antineutrinos. The older experiments done at CERN [37–40], in the regime of high energy have studied two and three pion production to investigate the diffractive production of meson resonances like ρ and η. The later experiments done at ANL [41,42] and BNL [43] at lower energies have investigated the two-pion production processes, specially in the threshold region, in order to test the predictions of the chiral symmetry of the strong interaction Lagrangian. Such studies were theoretically proposed by Biswas *et al.* [44] and Adjei *et al.* [45,46]. Biswas *et al.* used PCAC and current algebra methods to calculate the threshold production of two pions. On the other hand, the work of Adjei *et al.* made specific predictions for the threshold production of two pions in a restricted kinematic region using an effective Lagrangian incorporating chiral symmetry and its breaking, governed by a free parameter (ξ). Imposing these kinematical restrictions, the experimental data of ANL and BNL were analyzed and compared with Adjei *et al.* results. However, the model did not include any resonance production and the contribution of the $N^*(1440)$ should be taken into account. Furthermore we use an expansion of the chiral Lagrangian that includes terms up to $\mathcal{O}(1/f_\pi^3)$, while Adjei *et al.* kept only terms up to $\mathcal{O}(1/f_\pi^2)$. Now that the pattern of chiral symmetry breaking is well known and the background terms contribution to the threshold production of two pions is fully determined, the process could be used to study the electroweak transition form factors of the Roper resonance.

In this paper, we will study the $\nu_l N \to l^- \pi\pi N$ channel close to threshold. Apart from Roper resonance contribution, many other background mechanisms that only involve nucleons and pions appear and are described by an effective Lagrangian. We will use the lowest order chiral perturbation theory Lagrangian to derive the needed axial and vector currents.

In Sec. II, we present the formalism and the Lagrangians used in our model. We also give the expressions of the Roper form factors. In Sec. III, we present our results and compare them with the available data. Finally, the appendix gives the detailed formulas of the contributions of the background mechanisms for all channels.

II. MODEL FOR ν INDUCED TWO-PION PRODUCTION

A. Kinematics

We will focus on the neutrino-pion production reaction off the nucleon driven by charged currents,

$$\nu_l(k) + N(p) \to l^-(k') + N(p') + \pi(k_{\pi_1}) + \pi(k_{\pi_2}) \quad (1)$$

though the generalization of the obtained expressions to antineutrino-induced reactions is straightforward. The unpolarized differential cross section, with respect to the outgoing lepton kinematical variables, is given in the laboratory (LAB) frame by

$$\frac{d\sigma_{\nu_l l}}{d\Omega(\hat{k}')dE'} = \frac{G^2}{4\pi^2}\frac{|\vec{k}'|}{|\vec{k}|}L_{\mu\sigma}(W^{\mu\sigma}_{\mathrm{CC}2\pi}) \quad (2)$$

with $\vec{k}$ and $\vec{k}'$ the LAB lepton momenta, $E' = (\vec{k}'^2 + m_l^2)^{1/2}$ and m_l the energy and the mass of the outgoing lepton, $G = 1.1664 \times 10^{-11}\ \mathrm{MeV}^{-2}$, the Fermi constant. We take $\epsilon_{0123} = +1$ and the metric $g^{\mu\nu} = (+, -, -, -)$, thus the leptonic tensor is given by

$$\begin{aligned} L_{\mu\sigma} &= (L_s)_{\mu\sigma} \pm i(L_a)_{\mu\sigma} \\ &= k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma} k\cdot k' \pm i\epsilon_{\mu\sigma\alpha\beta}k'^\alpha k^\beta \end{aligned} \quad (3)$$

where the $+(-)$ sign corresponds to neutrino (antineutrino) induced processes.

The hadronic tensor reads as

$$\begin{aligned} W^{\mu\sigma}_{\mathrm{CC}2\pi} = \overline{\sum}_{\mathrm{spins}} \int \frac{d^3p'}{(2\pi)^3}\frac{M}{E'_N}\frac{d^3k_{\pi_1}}{(2\pi)^3}\frac{1}{2E_{\pi_1}}\frac{d^3k_{\pi_2}}{(2\pi)^3}\frac{1}{2E_{\pi_2}} \\ \times (2\pi)^3\delta^4(p' + k_{\pi_1} + k_{\pi_2} - q - p) \\ \times \langle N'\pi_1\pi_2|j^\mu_{cc+}(0)|N\rangle\langle N'\pi_1\pi_2|j^\sigma_{cc+}(0)|N\rangle^* \end{aligned} \quad (4)$$

with M the nucleon mass, $q = k - k'$ and E'_N the energy of the outgoing nucleon. The bar over the sum of initial and final spins, denotes the average on the initial ones. As for the baryon states, they are normalized so that $\langle\vec{p}|\vec{p}'\rangle = (2\pi)^3\delta^3(\vec{p} - \vec{p}')p_0/m$. By construction, the hadronic tensor can be split in

$$(W^{\mu\sigma}_{\mathrm{CC}2\pi}) = (W^{\mu\sigma}_{\mathrm{CC}2\pi})_s + \mathrm{i}(W^{\mu\sigma}_{\mathrm{CC}2\pi})_a \quad (5)$$

with $(W^{\mu\sigma}_{\mathrm{CC}2\pi})_s$ and $(W^{\mu\sigma}_{\mathrm{CC}2\pi})_a$ real symmetric and antisymmetric parts, respectively.

B. Lagrangians for the nonresonant terms

For the derivation of the hadronic tensor we use the effective Lagrangian of the SU(2) nonlinear σ model. This model was used previously in [21] for the description of the nonresonant contributions to one pion weak production processes off nucleon. We refer the reader to that paper for details. Up to $\mathcal{O}(1/f_\pi^3)$, this SU(2) chiral Lagrangian reads

$$\mathcal{L} = \bar{\Psi}[\mathrm{i}\not{\partial} - M]\Psi + \tfrac{1}{2}\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - \tfrac{1}{2}m_\pi^2\vec{\phi}^2 + \mathcal{L}^\sigma_{\mathrm{int}} \quad (6)$$

$$\mathcal{L}^{\sigma}_{\rm int} = \frac{g_A}{f_\pi}\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}(\partial_\mu\vec{\phi})\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma_\mu\vec{\tau}(\vec{\phi}\times\partial^\mu\vec{\phi})\Psi - \frac{1}{6f_\pi^2}(\vec{\phi}^2\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - (\vec{\phi}\partial_\mu\vec{\phi})(\vec{\phi}\partial^\mu\vec{\phi})) + \frac{m_\pi^2}{24f_\pi^2}(\vec{\phi}^2)^2 - \frac{g_A}{6f_\pi^3}\bar{\Psi}\gamma^\mu\gamma_5\Big[\vec{\phi}^2\frac{\vec{\tau}}{2}\partial_\mu\vec{\phi} - (\vec{\phi}\partial_\mu\vec{\phi})\frac{\vec{\tau}}{2}\vec{\phi}\Big]\Psi, \quad (7)$$

where $\Psi = \binom{p}{n}$ is the nucleon field, $\vec{\phi}$ is the isovector pion field, $\vec{\tau}$ are the Pauli matrices and $f_\pi = 93$ MeV is the pion decay constant. The vector and axial currents generated from the Lagrangian in Eq. (6) are given by [21]

$$\vec{V}^\mu = \underbrace{\vec{\phi}\times\partial^\mu\vec{\phi}}_{\vec{V}_a^\mu} + \underbrace{\bar{\Psi}\gamma^\mu\frac{\vec{\tau}}{2}\Psi}_{\vec{V}_b^\mu} + \underbrace{\frac{g_A}{2f_\pi}\bar{\Psi}\gamma^\mu\gamma_5(\vec{\phi}\times\vec{\tau})\Psi}_{\vec{V}_c^\mu} \overbrace{-\frac{1}{4f_\pi^2}\bar{\Psi}\gamma^\mu[\vec{\tau}\vec{\phi}^2 - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})]\Psi - \frac{\vec{\phi}^2}{3f_\pi^2}(\vec{\phi}\times\partial^\mu\vec{\phi})}^{\vec{V}_d^\mu} + \mathcal{O}\Big(\frac{1}{f_\pi^3}\Big) \quad (8)$$

$$\vec{A}^\mu = \underbrace{f_\pi\partial^\mu\vec{\phi}}_{\vec{A}_a^\mu} + \underbrace{g_A\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi}_{\vec{A}_b^\mu} + \underbrace{\frac{1}{2f_\pi}\bar{\Psi}\gamma^\mu(\vec{\phi}\times\vec{\tau})\Psi}_{\vec{A}_c^\mu} + \overbrace{\frac{2}{3f_\pi}[\vec{\phi}(\vec{\phi}\cdot\partial^\mu\vec{\phi}) - \vec{\phi}^2\partial^\mu\vec{\phi}] - \frac{g_A}{4f_\pi^2}\bar{\Psi}\gamma^\mu\gamma_5[\vec{\tau}\vec{\phi}^2 - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})]\Psi}^{\vec{A}_d^\mu} + \mathcal{O}\Big(\frac{1}{f_\pi^3}\Big) \quad (9)$$

and determine the weak transition vertex where the W-boson is absorbed. These currents are, up to a factor, the hadronic realization of the electroweak quark current j^μ_{cc+} for a system of interacting pions and nucleons. Thus, $\vec{A}^\mu_a$ and $\vec{V}^\mu_a$ account for the W-decay into one and two pions, respectively, while $\vec{V}^\mu_b$ and $\vec{A}^\mu_b$ provide the WNN vector and axial vector couplings. Besides, $\vec{A}^\mu_c$ and $\vec{V}^\mu_c$ lead to contact $WNN\pi$ vertices and finally $\vec{A}^\mu_d$ and $\vec{V}^\mu_d$ either contribute to processes with more than one pion in the final state or provide loop corrections.

The overall normalization can be obtained, for instance, by relating the currents of Eq. (8) and (9) with the phenomenological vector and axial nucleon currents in the $\langle N'\pi|j^\mu_{cc+}(0)|N\rangle$ matrix element

$$\langle p;\vec{p}' = \vec{p}+\vec{q}|j^\alpha_{cc+}(0)|n;\vec{p}\rangle = \cos\theta_C\bar{u}(\vec{p}')(V^\alpha_N(q) - A^\alpha_N(q))u(\vec{p}) = \mathcal{A}^\alpha \quad (10)$$

where the u's are Dirac spinors for the neutron and proton, normalized such that $\bar{u}u = 1$, and vector and axial nucleon currents are given by

$$V^\alpha_N(q) = 2\times\Big(F_1^V(q^2)\gamma^\alpha + \mathrm{i}\mu_V\frac{F_2^V(q^2)}{2M}\sigma^{\alpha\nu}q_\nu\Big), \quad A^\alpha_N(q) = G_A(q^2)\times\Big(\gamma^\alpha\gamma_5 + \frac{\not{q}}{m_\pi^2 - q^2}q^\alpha\gamma_5\Big). \quad (11)$$

We find[1] that $-\sqrt{2}\cos\theta_C([V^\mu]_{+1} - [A^\mu]_{+1})$ provides the W^+-absorption vertex, with the appropriate normalization.

The magnetic part in Eq. (11) is not provided by the nonlinear sigma model, which assumes structureless nucleons. We will improve on that by including the q^2 dependence induced by the form factors in Eq. (11) and adding the magnetic contribution, F_2^V term, to the vector part of the $W^+N\to N$ amplitude.

For the nucleon vector form factors (FF) we use the parametrization of Ref. [47]

$$F_1^N = \frac{G_E^N + \kappa G_M^N}{1+\kappa}, \qquad \mu_N F_2^N = \frac{G_M^N - G_E^N}{1+\kappa},$$
$$G_E^p = \frac{G_M^p}{\mu_p} = \frac{G_M^n}{\mu_n} = -(1+\lambda_n\kappa)\frac{G_E^n}{\mu_n\kappa} = \Big(\frac{1}{1-q^2/M_D^2}\Big)^2 \quad (12)$$

with $\kappa = -q^2/4M^2$, $M_D = 0.843$ GeV, $\mu_p = 2.792\,847$, $\mu_n = -1.913\,043$ and $\lambda_n = 5.6$.

$$F_1^V(q^2) = \tfrac{1}{2}(F_1^p(q^2) - F_1^n(q^2)), \quad \mu_V F_2^V(q^2) = \tfrac{1}{2}(\mu_p F_2^p(q^2) - \mu_n F_2^n(q^2)). \quad (13)$$

The axial form factor is given by [48]

$$G_A(q^2) = \frac{g_A}{(1-q^2/M_A^2)^2}, \qquad g_A = 1.26, \qquad M_A = 1.05\ \text{GeV} \quad (14)$$

Using this current we obtain the 16 Feynman diagrams, depicted in Fig. 1, constructed out of the $W^+N\to N$, $W^+N\to N\pi$, $W^+N\to N\pi\pi$, and the contact $W^+\pi\to\pi$ weak transition vertices [Eqs. (8) and (9)] and the πNN, $\pi\pi NN$, $\pi\pi\pi NN$, $\pi\pi\pi\pi$ couplings [Eqs. (6) and (7)].

[1]The $+1$ spherical component of a vector $\vec{A}$ is defined as $A_{+1} = -(A_x + \mathrm{i}A_y)/\sqrt{2}$.

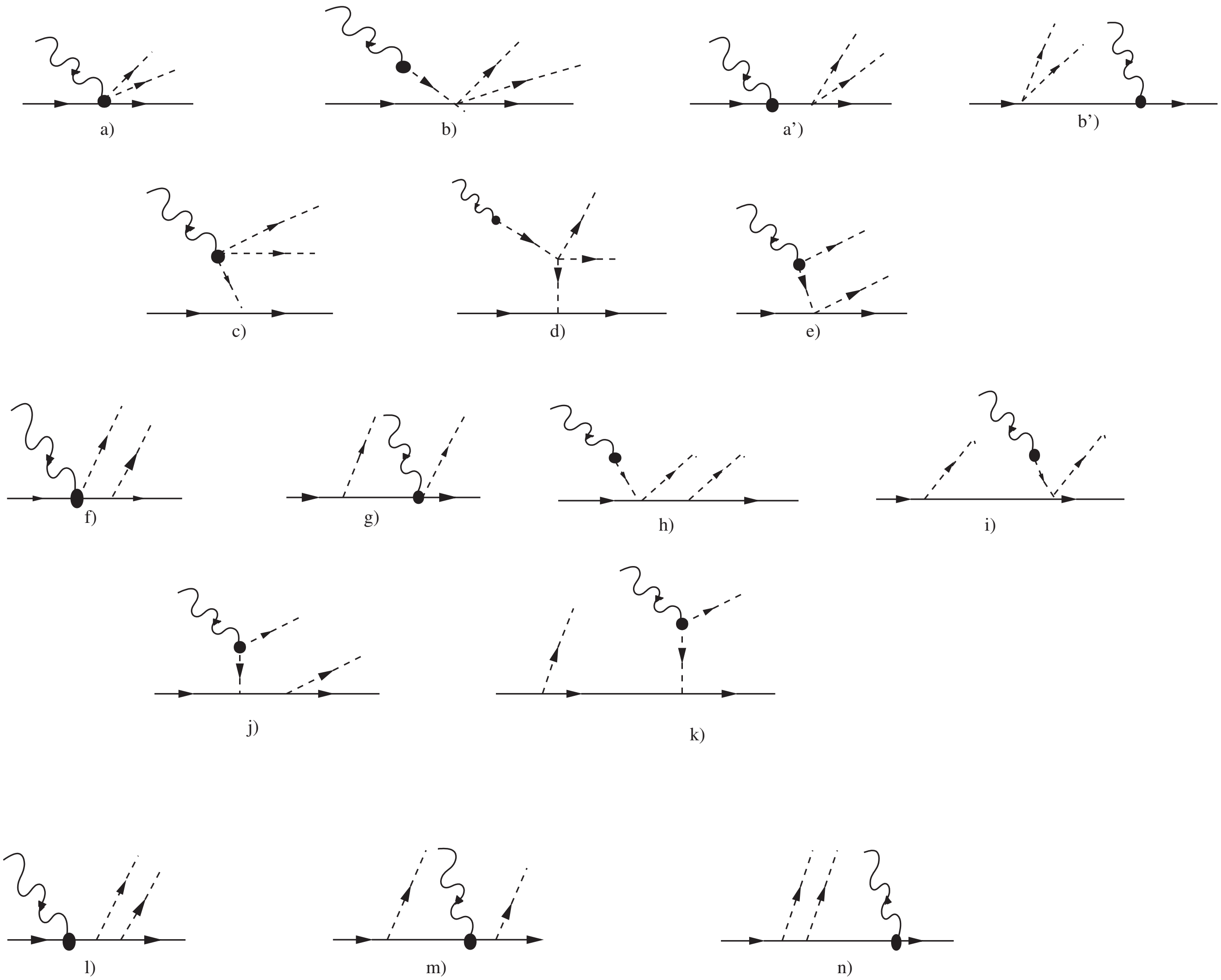

FIG. 1. Nucleon pole, pion pole, and contact terms contributing to 2π production.

Since we have included a q^2 dependence ($F_1^V(q^2)$) on the Dirac part of the vector WNN vertex and to preserve vector current conservation, we include the same form factor in the $V^\mu_{a,b,c,d}$ weak operators.

The expressions for the matrix elements of these terms, for all possible channels, can be found in the appendix.

C. Contribution of the $N^*(1440)$ resonance

The Roper $N^*(1440)$ P_{11} is the lowest lying baryon resonance with an s-wave isoscalar two-pion decay. This suggests the possible relevance of Roper excitation mechanisms in two-pion production processes close to threshold. Indeed, its importance has been clearly established in the $\pi N \to \pi\pi N$ [32,33] and the $NN \to N\pi\pi$ [35,36] reactions where it plays a dominant role for certain isospin channels. However, that is not the case for electromagnetically induced reactions, due to the relatively weak coupling of the Roper to the photons. See, for instance, Ref. [49].

We include in our study the two mechanisms depicted in Fig. 2, which account for the Roper production and its decay into a nucleon and two pions in a s-wave isoscalar state. However, we do not include the contribution of mechanisms in which the Roper decays into $\pi\Delta(1232)$ states, because the two produced pions are in p-wave and thus are not expected to be relevant close to threshold.

1. Roper interaction Lagrangians and currents

The Lagrangian for the s-wave $N^* \to N\pi\pi$ decay can be written as

$$\mathcal{L}_{N^*N\pi\pi} = -c_1^* \frac{m_\pi^2}{f_\pi^2} \bar{\psi}_{N^*} \vec{\phi}^2 \Psi + c_2^* \frac{1}{f_\pi^2} \bar{\psi}_{N^*} (\vec{\tau}\partial_0 \vec{\phi}) \times (\vec{\tau}\partial_0 \vec{\phi}) \Psi + \text{H.c.}, \quad (15)$$

neglecting terms of order $\mathcal{O}(p^2/M^{*2})$ [50]. Here, $\bar{\psi}_{N^*}$ is the Roper field. In Ref. [35] the best agreement with

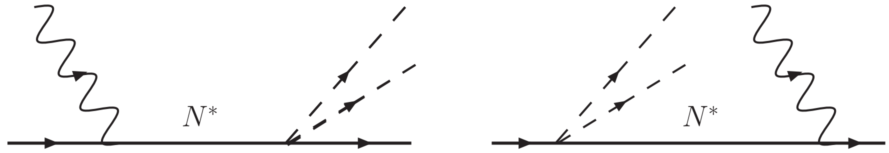

FIG. 2. Direct (left) and crossed (right) Roper excitation contributions to 2π production.

$NN \to \pi\pi NN$ and the $\pi N \to \pi\pi N$ data was obtained using $c_1^* = -7.27\ \mathrm{GeV}^{-1}$, $c_2^* = 0\ \mathrm{GeV}^{-1}$. This result was obtained assuming a branching ratio of 7.5% for the $N(\pi\pi)_{J=0}^{I=0}$ decay mode and a total decay width of the N^*, $\Gamma_{\mathrm{tot}} = 350$ MeV [51].

The other required ingredient is the coupling of the Roper to the charged weak current, that is the vertex $W^+ n \to N^{*+}(1440)$. The matrix elements can be written as

$$\langle N^{*+}; \vec{p}_* = \vec{p} + \vec{q} | j^\alpha_{cc+}(0) | n; \vec{p} \rangle = \cos\theta_C \bar{u}_*(\vec{p}_*) J^\alpha_{cc*} u(\vec{p}), \tag{16}$$

where u_* is the Roper spinor and

$$J^\alpha_{cc*} = \frac{F_1^{V*}(q^2)}{\mu^2}(q^\alpha \not{q} - q^2\gamma^\alpha) + i\frac{F_2^{V*}(q^2)}{\mu}\sigma^{\alpha\nu}q_\nu - G_A\gamma^\alpha\gamma_5 - \frac{G_P}{\mu}q^\alpha\not{q}\gamma_5 - \frac{G_T}{\mu}\sigma^{\alpha\nu}q_\nu\gamma_5 \tag{17}$$

is the most general form compatible with conservation of the vector current and Dirac equation for the nucleon and Roper Dirac spinors. A factor $\mu = M + M_*$, with $M_* = 1440$ MeV the mass of the Roper resonance, is introduced in order to make the vector form factors dimensionless. The G_T term, and unlike the elastic nucleon case where it is zero due to G-parity invariance, does not need to vanish in the present case because the nucleon and the Roper do not belong to the same isospin multiplet. Nevertheless, most analyses neglect its contribution and we shall do so here.

The $N^*N\pi$ coupling is described by the pseudovector Lagrangian

$$\mathcal{L}_{N^*N\pi} = \frac{\tilde{f}}{m_\pi}\bar{\Psi}_{N^*}\gamma^\mu\gamma_5\vec{\tau}\cdot\partial_\mu\vec{\phi}\Psi + \mathrm{H.c.}. \tag{18}$$

The decay width for this process is given by

$$\Gamma_{N^*\to\pi N} = \frac{3}{2\pi}\left(\frac{\tilde{f}}{m_\pi}\right)^2\frac{M}{W}|q_{\mathrm{cm}}|^3, \tag{19}$$

where W is the N^* invariant mass and $|q_{\mathrm{cm}}|$ is the momentum of the outgoing pion in the outgoing πN center of mass frame. Taking for this decay channel a branching ratio of 65% and the total width of the Roper $\Gamma = 350$ MeV [51], we get $\tilde{f} = 0.48$.

2. $N^*(1440)$ *form factors*

Assuming the pseudoscalar coupling G_P is dominated by the pion pole contribution, and imposing partial conservation of the axial current (PCAC) hypothesis we can relate G_P with the axial coupling G_A

$$G_P(q^2) = \frac{\mu}{m_\pi^2 - q^2}G_A(q^2). \tag{20}$$

Furthermore we can relate G_A with the $N^*N\pi$ coupling constant at $q^2 = 0$ using the nondiagonal Goldberger-Treiman relation

$$G_A(0) = 2f_\pi\frac{\tilde{f}}{m_\pi} = 0.63 \tag{21}$$

The q^2 dependence of $G_A(q^2)$ is not constrained by theory so we shall assume for it a dipole form of the type

$$G_A(q^2) = \frac{G_A(0)}{(1 - q^2/M_{A*}^2)^2}, \tag{22}$$

with an axial mass $M_{A*} = 1$ GeV.

The vector-isovector form factors F_1^{V*}/μ^2 and F_2^{V*}/μ can be related to the isovector part of the electromagnetic (EM) form factors, $F_i^{V*} = F_i^{p*} - F_i^{n*}$, that can be determined from photo- and electroproduction experiments. The relevant experimental information about these EM form factors is usually given in terms of helicity amplitudes for the EM current $j^N_{\mathrm{e.m.}}$, defined[2] as [53]

$$A^N_{1/2} = \sqrt{\frac{2\pi\alpha}{k_R}}\langle N^*\uparrow|\sum_{\mathrm{pol}}\epsilon\cdot j_{\mathrm{e.m.}}(0)|N\downarrow\rangle\xi \tag{23}$$

$$S^N_{1/2} = \sqrt{\frac{2\pi\alpha}{k_R}}\frac{|\vec{q}|}{\sqrt{-q^2}}\langle N^*\uparrow|\sum_{\mathrm{pol}}\epsilon\cdot j_{\mathrm{e.m.}}(0)|N\uparrow\rangle\xi, \tag{24}$$

where N stands for proton or neutron, $\alpha = 1/137$, q is the momentum of the virtual photon, $k_R = (W^2 - M^2)/2W$, with W the energy of the Roper in its center of mass, and the polarization vectors are given by

$$\epsilon^\pm = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0), \tag{25}$$

and for a photon of momentum q moving along the positive z-axis

[2]Please note that the MAID group analysis [52] defines $S^N_{1/2}$ with the opposite sign, which we take into account when comparing to their data.

$$\epsilon^0 = \frac{1}{\sqrt{-q^2}}(|\vec{q}|, 0, 0, q^0). \tag{26}$$

The factor ξ in Eqs. (23) and (24) is given by the relative sign between the $NN\pi$ and $N^*N\pi$ couplings [53] which we have taken to be positive (we will discuss this point later).

Finally, the EM $\gamma N \to N^*$ current is written as

$$\langle N^*; \vec{p}_* = \vec{p} + \vec{q} | j^{\alpha}_{\rm e.m.}(0) | N; \vec{p}\rangle = \bar{u}_*(\vec{p}_*)\left[\frac{F_1^{N*}(q^2)}{\mu^2}(q^\alpha \not{q} - q^2\gamma^\alpha) + i\frac{F_2^{N*}(q^2)}{\mu}\sigma^{\alpha\nu}q_\nu\right]u(\vec{p}). \tag{27}$$

Using Eqs. (24)–(27) we obtain the following relations:

$$A^N_{1/2} = |\vec{q}|g(q^2)\left[\frac{F_2^{N*}}{\mu} - \frac{q^2}{W+M}\frac{F_1^{N*}}{\mu^2}\right] \tag{28}$$

$$S^N_{1/2} = \frac{1}{\sqrt{2}}|\vec{q}|^2 g(q^2)\left[\frac{F_1^{N*}}{\mu^2} - \frac{F_2^{N*}}{\mu}\frac{1}{W+M}\right], \tag{29}$$

with

$$g(q^2) = \sqrt{\frac{8\pi\alpha(W+M)W^2}{M(W-M)((W+M)^2 - q^2)}}. \tag{30}$$

Inverting Eqs. (28) and (29) we can obtain the proton FF F_i^{p*} as a function of the experimental helicities. Unfortunately, there is no such information on the $\gamma n \to N^{*0}$ transition so we need some theoretical assumptions to relate the isovector helicity amplitudes with the EM ones. Following the quark models predictions of [54,55] we shall assume $A^n_{1/2} = -2/3A^p_{1/2}$ and $S^n_{1/2} = 0$. For a review of different models see Ref. [56]. Using these relations we can write the neutron FF as a function of the proton ones and express the vector FF in terms of only F_1^{p*} and F_2^{p*} as

$$F_1^{V*} = \frac{F_1^{p*}((M+W)^2 - 5q^2/3) + 2/3F_2^{p*}(M+W)\mu}{(M+W)^2 - q^2} \tag{31}$$

$$F_2^{V*} = \frac{F_2^{p*}(5(M+W)^2 - 3q^2)\mu - 2F_1^{p*}q^2(M+W)}{3((M+W)^2 - q^2)\mu}. \tag{32}$$

We have fitted the proton-Roper EM transition form factors to the experimental results for helicity amplitudes given in [30,31] using a parametrization inspired by Lalakulich *et al.* [18]

$$F_1^{p*}(q^2) = \frac{g_1^p/D_V}{1 - q^2/X_1 M_V^2}, \tag{33}$$

$$F_2^{p*}(q^2) = \frac{g_2^p}{D_V}\left(1 - X_2 \ln\left(1 - \frac{q^2}{1\ \mathrm{GeV}^2}\right)\right) \tag{34}$$

where $D_V = (1 - q^2/M_V^2)^2$ with $M_V = 0.84$ GeV. We have fitted the dimensionless parameters g_1^p, g_2^p, X_1 and X_2 to the available experimental data, and found the following best fit (labeled as FF 1 in the results):

$$g_1^p = -5.7 \pm 0.9, \qquad g_2^p = -0.64 \pm 0.04,$$
$$X_1 = 1.4 \pm 0.5, \qquad X_2 = 2.47 \pm 0.12, \tag{35}$$

with a χ^2/d.o.f $= 5.2$. Our best fit parameters g_1^p and X_1 differ appreciably from those quoted in Ref. [18], in particular g_1^p comes out with opposite sign. This is because the authors of this latter reference did not consider the existing extra minus sign among their definition of $S^N_{1/2}$, which agrees with that of Eq. (24) and used in this work, and the definition used in the MAID work. Indeed, if we do not consider this minus sign, we find values of the parameters in good agreement with those quoted in [18], with minor differences due to the use of different data sets.

The Roper EM data have large error bars and it is possible to accommodate quite different functional forms and values for these FF. Thus, we shall compare other different models for the vector form factors. Firstly, we consider the constituent quark model with gluon, pion, and σ-meson exchange potentials as residual interactions of Meyer *et al.* [57] in set FF 2. In this model the electromagnetic current included, in addition to the one-body current, two-body exchange currents associated with the quark-quark potentials. Here no assumption about the relation between proton and nucleon form factor was assumed. We will also consider the recent parametrizations[3] of Lalakulich *et al.* [18] in the set labeled FF 3. Note that this work employed a pseudoscalar form for the πNN coupling used in deriving the Goldberger-Treiman relation, instead of the pseudovector, Eq. (18). We shall finally use the predictions of the recent MAID analysis [52] in the set labeled FF 4.

From all this information we can now obtain definite expressions for the currents of the direct (N^*d) and crossed (N^*c) diagrams shown in Fig. 2

$$\mathcal{A}^{\mu}_{N*d} = 2g^*\bar{u}(\vec{p}')S_*(p+q)J^{\mu}_{cc*}(q)u(\vec{p}) \tag{36}$$

$$\mathcal{A}^{\mu}_{N*c} = 2g^*\bar{u}(\vec{p}')\tilde{J}^{\mu}_{cc*}(q)S_*(p'-q)u(\vec{p}), \tag{37}$$

with $g^* = c_1^*(m_\pi/f_\pi)^2$. In the crossed diagram it appears $\tilde{J}^{\mu}_{cc*}$, that corresponds to the crossed vertex $W^+N^* \to N$ and it is given by $\tilde{J}^{\mu}_{cc*} = \gamma^0(J^{\mu}_{cc*})^\dagger\gamma^0$. We have here introduced the Roper propagator

$$S_*(p_*) = \frac{\not{p}_* + M_*}{p_*^2 - M_*^2 + i(M_* + W)\Gamma_{\rm tot}(W)/2}. \tag{38}$$

The total W-dependent decay width $\Gamma_{\rm tot}(W)$ includes all the possible decay channels, namely $N\pi$, given by

[3] Notice the opposite sign convention for F_V in [18].

Eq. (19), $\pi\Delta$ and $N(\pi\pi)^{T=0}_{S=0}$. For these two cases we take

$$\Gamma_{\pi\Delta}(W) = 350 \times 0.275|p_\pi(W)|^3/|p_\pi(M_*)|^3 \text{ MeV} \quad (39)$$

and

$$\Gamma_{\pi\pi N} = 350 \times 0.075\text{PhSp}(W)/\text{PhSp}(M_*) \text{ MeV}, \quad (40)$$

where PhSp(W) is the phase space for the three body $\pi\pi N$ decay, and p_π is the pion momentum in the Roper rest frame.

The isospin structure of Eq. (15) determines that the only allowed channels for the two pion s-wave decay of the Roper resonance are $\pi^+\pi^-$ and $\pi^0\pi^0$. The above expressions are given for the $\pi^+\pi^-$ channel, therefore we must include an additional $1/2$ symmetry factor for the $\pi^0\pi^0$ channel.

3. Relative sign between the Roper and nonresonant contributions

Here we give some details on the used scheme to set up the relative signs between the different contributions:

(i) All relative phases in the nonresonant terms are completely fixed by the nonlinear sigma model Lagrangian of Eqs. (6) and (7), the currents deduced from it and shown in Eqs. (8) and (9), and the phenomenological WNN vertex.

(ii) We have assumed the sign of the $N^*N\pi$ coupling to be the same as that of the $NN\pi$ coupling (positive). This choice fixes the global phases in Eqs. (23) and (24) [53] and thus the phase of the vector part of the WNN^* current. Besides the nondiagonal Goldberger-Treiman relation [Eq. (21)] fixes the axial part of the WNN^* current.

(iii) Furthermore, once the relative sign of the $NN\pi$ and $N^*N\pi$ couplings has been set, the study of the reactions $NN \to NN\pi\pi$ and $\pi N \to N\pi\pi$ in Ref. [35] fixes the values for the c_1^* and c_2^* $N^*N\pi\pi$ couplings.

Taking the opposite sign for the $N^*N\pi$ coupling, does not affect the results. This is because the signs of both the WNN^* current (see previous discussion) and the c_1^* and c_2^* couplings[4] would change and therefore the whole resonant contribution would not be affected at all.

III. RESULTS

In this section, we present the results for all the two-pion states produced in the neutrino induced reactions on nucleon targets, i.e. $\nu p \to \mu^- p\pi^+\pi^0$, $\nu p \to \mu^- n\pi^+\pi^+$, $\nu n \to \mu^- p\pi^+\pi^-$, $\nu n \to \mu^- n\pi^+\pi^0$, and $\nu n \to \mu^- p\pi^0\pi^0$.

Our model includes all relevant terms for neutrino induced two-pion production close to threshold. In addition to the contribution of nucleon and pion pole and contact terms described by the chiral Lagrangian the contribution of the $N^*(1440)$ resonance coupling to two s-wave pions is included. The $\Delta(1232)$ and other higher resonances are not considered as their contributions would vanish at threshold.

In Fig. 3, we present the results for the cross section for the process $\nu n \to \mu^- p\pi^+\pi^-$. We show separately the contribution of the background terms coming from the nucleon pole, pion pole and contact terms as well as the contribution of the Roper resonance as calculated by using the various form factors described in Sec. II. The interference between background and the Roper contribution is not shown. We see that the background terms dominate the cross section for neutrino energies $E_\nu > 0.7$ GeV. At lower energies the contribution from the Roper could be larger or smaller than the background depending upon the vector form factors used for the W^+NN^* transition. The parameterization determined by the recent MAID analysis [52] gives the largest Roper contribution to the cross section. The differences in the predictions for the cross sections using the various parametrizations could reach a factor two. The Roper contribution is specially sensitive to $F_2^{V*}(q^2)$ which is negative in contrast to the positive value which one gets in the case of the nucleon. This has interesting implications for the vector-axial vector interference contribution for the cases of ν and $\bar{\nu}$ excitation of the $N^*(1440)$. The comparison of data on ν and $\bar{\nu}$ induced two-pion production in this channel, if available in the future, could be used to study the WNN^* transition and better constrain the corresponding form factors.

In Fig. 4, we compare our results for the same process with experiment. The largest results are obtained with the

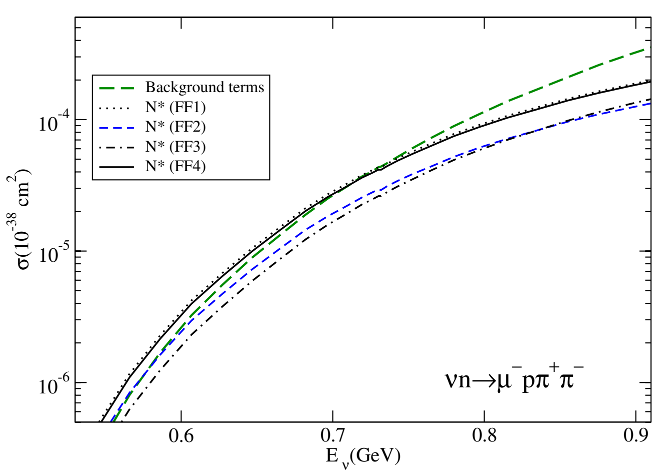

FIG. 3 (color online). Cross section for the $\nu n \to \mu^- p\pi^+\pi^-$ reaction as a function of the neutrino energy. The interference between background and the N^* contribution is not shown. See text for details.

[4]These couplings would swap sign when the sign of the $N^*N\pi$ coupling is changed, since they are fitted to the $NN \to NN\pi\pi$ and $\pi N \to N\pi\pi$ data.

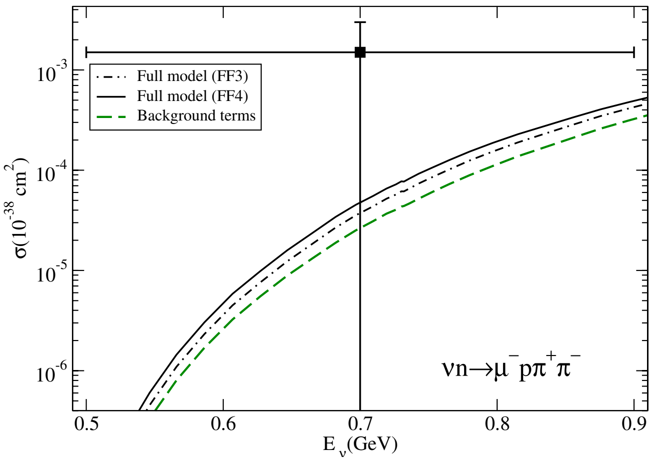

FIG. 4 (color online). Cross section for the $\nu n \to \mu^- p\pi^+\pi^-$ reaction as a function of the neutrino energy. Data from Ref. [43].

set 4, and the smallest from the set 3 of nucleon-Roper transition form factors. The other two sets give cross sections, not shown in the figure, lying between these two limits. Our results with the full model are larger than those obtained using only the background terms as done by Adjei *et al.* [45], however the results are still lower than the central value of the experiment. Experimental data with better statistics is highly desirable in this channel to make a decisive comparison with the predictions of our model.

We expect our model to have a limited region of applicability from threshold to an invariant mass of the $\pi\pi N$ system below 1.4 GeV. For muon neutrinos, that implies a LAB energy under 750 MeV, beyond which some additional contributions from the $\Delta(1232)$ and other higher resonances will become relevant. In order to make a meaningful comparison with the theoretical calculations for threshold two-pion production, Adjei *et al.* [45] suggested a kinematical cut on phase space which was implemented in the experimental analysis made by Day *et al.* [42] and Kitagaki *et al.* [43]. These cuts were defined as

$$q_\pi^2 \leq ((1+\eta/2)m_\pi)^2, \tag{41}$$

$$p\cdot q_\pi \leq (M+(1+\eta)m_\pi)^2 - M^2 - m_\pi^2 \tag{42}$$

$$p'\cdot q_\pi \leq (M+(1+\eta)m_\pi)^2 - M^2 - m_\pi^2, \tag{43}$$

with $q_\pi = (k_{\pi_1} + k_{\pi_2})/2$. Different choices for η, specifically $\eta = 1/4, 2/4, 3/4$, were proposed. As explained in Ref. [45], the first inequality keeps the individual pion momenta close to the average pion momentum, while the last two restrict the phase space to regions of small invariant mass of the three hadrons.

Using these kinematic restrictions on the phase space with $\eta = 3/4$, as used by Refs. [42,43], we present the results for the cross section for the $\nu n \to \mu^- p\pi^+\pi^-$ channel in Fig. 5 and compare with their data. We show

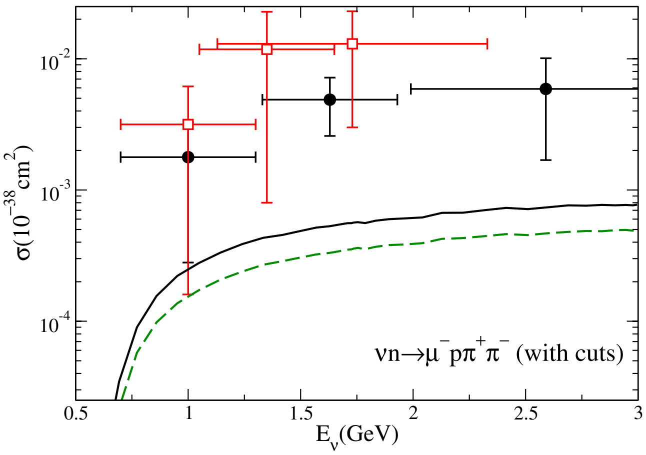

FIG. 5 (color online). Cross section for the $\nu n \to \mu^- p\pi^+\pi^-$ with cuts as explained in the text. Dashed line: Background terms. Solid line: Full model with set 1 of nucleon-Roper transition FF. Data from Ref. [43] (solid circles) and Ref. [42] (open squares).

our results with only background terms and with the full model evaluated using the set 1 of nucleon-Roper transition form factors. Other sets give a similar result in this case. We find that, even in this kinematic region, the theoretical results including the resonance contribution are lower than the experimental.

In Fig. 6, we present the results, with the same cuts, for the total cross section for the channel $\nu p \to \mu^- n\pi^+\pi^+$ and compare with the data of Ref. [42]. For this channel there is no contribution from the $N^*(1440)$ resonance. Our results are in agreement with those of Adjei *et al.* [45] and are consistent, within errors, with the experiment in the higher energy region. However, we underestimate the first point ($E_\nu = 1.25$ GeV) by more than 1 order of magni-

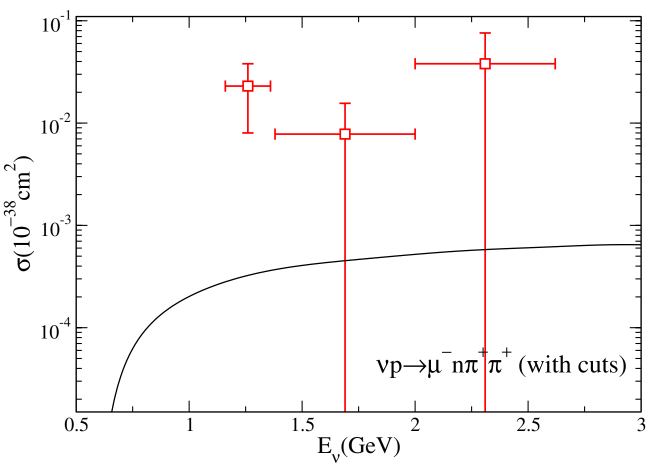

FIG. 6 (color online). Cross section for the $\nu p \to \mu^- n\pi^+\pi^+$ with cuts as explained in the text. Note that there are no contributions from the $N^*(1440)$ resonance to this channel. Data from Ref. [42].

tude. This disagreement may be ascribed to the low statistics of the experiment after the kinematical cuts have been implemented [42], to inadvertent inclusion of some p wave pions while implementing the kinematical cuts in the experimental analysis but also to possible additional reaction mechanisms not included in our model. Note that the kinematics of the plot explore the region just up to 1.4 GeV of final hadron state invariant mass. At this energy we do not expect our threshold production model to account for all the mechanisms of pion production, many of which would be in p-wave and therefore beyond the scope of our work. These mechanisms would include Δ resonance terms similar to those studied in [49] in the context of photo-induced reactions. In any case, the disagreement at $E = 1.25$ GeV underscores the need for better experimental data in this channel around neutrino energy $E = 1$ GeV.

Finally, in Fig. 7, we show our results for the threshold production of two pions in the other channels. Again we only show our results with only background terms and with the full model evaluated using the set 1 of nucleon-Roper transition form factors. The use of different sets gives similar results for the full model calculation. There is no data in the low energy region; the limited data available in these channels at higher energies have not been analyzed in the kinematical region of the threshold production of two pions and thus a comparison with our present results is not possible.

To summarize, we have studied the threshold production of two pions induced by neutrinos on nucleon targets using a chiral Lagrangian to calculate the nucleon pole, pion pole, and contact terms. The contribution from the excitation of the Roper resonance, $N^*(1440)$ has been included. The vector transition form factors are determined from the available data on the helicity amplitudes of its electromagnetic excitation. The axial form factors are obtained using PCAC and the experimental data on $N^*N\pi$ decay. The q^2 dependence of the axial form factor has been assumed to be of the dipole form. It is found that the Roper resonance contributes strongly to the $\nu n \to \mu^- p\pi^+\pi^-$ and $\nu n \to \mu^- p\pi^0\pi^0$ channels. Its contribution is as important as the contribution of the nucleon, pion pole, and contact terms, up to energies of $E_\nu \sim 700$ MeV. The resonance contribution is sensitive to the form factors used for the WNN^* transition, and the experimental data, if available with better statistics for the $\nu n \to \mu^- p\pi^+\pi^-$ and $\nu n \to \mu^- p\pi^0\pi^0$ channels, could be used for the determination of these form factors. For other channels, as well as for these, the theoretical results underestimate the experimental results. Furthermore, physical channels involving Δ production could become relevant at beam energies higher than 750 MeV, where the threshold approximation we have assumed in our model would no longer be valid. Nevertheless, the low statistical significance of data does not provide a conclusive test of our model. Availability of improved data, especially from future experiments on neutrino induced two-pion production in the region of energies $E_\nu < 1$ GeV, will be very useful in the study of the electroweak properties of the $N^*(1440)$ resonance.

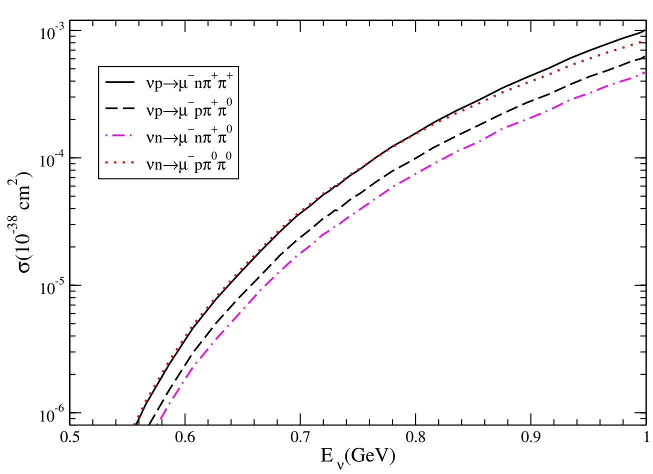

FIG. 7 (color online). Cross sections as a function of the neutrino energy. All calculations correspond to the full model with the FF1 set of nucleon-Roper transition form factors.

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APPENDIX: HADRONIC CURRENT FOR THE DIFFERENT CHARGE CHANNELS

The contributions are labeled following Fig. 1. We give explicit expressions for all channels except for $W^+n \to p\pi^0\pi^0$ which can be obtained via the following isospin relation

$$\begin{aligned}\mathcal{A}(W^+n \to p\pi^0\pi^0) = {}& \mathcal{A}(W^+n \to p\pi^+\pi^-) \\ & - \frac{1}{\sqrt{2}}(\mathcal{A}(W^+p \to p\pi^+\pi^0) \\ & - \mathcal{A}(W^+n \to n\pi^+\pi^0)). \qquad \text{(A1)}\end{aligned}$$

In all these expressions we have used for the nucleon propagator

$$S(p) = \frac{\not{p} + M}{p^2 - M^2 + i\epsilon} \tag{A2}$$

and $V^\mu - A^\mu$ are defined as in Eq. (11).

1. $W_q^+ p_p \to p_{p'} \pi^+_{k_1} \pi^0_{k_2}$

$$\mathcal{A}_a^\mu = \frac{1}{2\sqrt{2} f_\pi^2} \cos\theta_c \bar{u}(\vec{p}\,')(2F_1^V \gamma^\mu - g_A \gamma^\mu \gamma_5) u(\vec{p}) \tag{A3}$$

$$\mathcal{A}_b^\mu = \frac{g_A}{6\sqrt{2} f_\pi^2} \cos\theta_c \frac{q^\mu}{q^2 - m_\pi^2} \bar{u}(\vec{p}\,')(3\not{k}_2 + 2M)\gamma_5 u(\vec{p}) \tag{A4}$$

$$\begin{aligned}\mathcal{A}_c^\mu = {}& \frac{g_A \sqrt{2}}{3 f_\pi^2} \\ & \times \cos\theta_c \frac{q^\mu - 3k_1^\mu}{(q - k_1 - k_2)^2 - m_\pi^2} 2M \bar{u}(\vec{p}\,')\gamma_5 u(\vec{p})\end{aligned} \tag{A5}$$

$$\begin{aligned}\mathcal{A}_d^\mu = {}& \frac{g_A}{3\sqrt{2} f_\pi^2} \cos\theta_c \frac{q^\mu}{q^2 - m_\pi^2} \\ & \times \frac{4qk_1 - 2qk_2 + 2k_1k_2 - q^2}{(q - k_1 - k_2)^2 - m_\pi^2} 2M \bar{u}(\vec{p}\,')\gamma_5 u(\vec{p})\end{aligned} \tag{A6}$$

$$\mathcal{A}_e^\mu = \frac{1}{\sqrt{2} f_\pi^2} \cos\theta_c \frac{2k_2^\mu - q^\mu}{(q - k_2)^2 - m_\pi^2} 2F_1^V \bar{u}(\vec{p}\,') \not{k}_1 u(\vec{p}) \tag{A7}$$

$$\begin{aligned}\mathcal{A}_{b'}^\mu = {}& -\frac{\sqrt{2}}{4 f_\pi^2} \cos\theta_c \bar{u}(\vec{p}\,')(V^\mu - A^\mu) S(p' - q)(\not{k}_1 - \not{k}_2) \\ & \times u(\vec{p})\end{aligned} \tag{A8}$$

$$\begin{aligned}\mathcal{A}_f^\mu = {}& -\frac{g_A}{2\sqrt{2} f_\pi^2} \cos\theta_c \bar{u}(\vec{p}\,')(\not{k}_2 \gamma_5 S(p' + k_2) 2F_1^V g_A \gamma^\mu \gamma_5 \\ & - \not{k}_2 \gamma_5 S(p' + k_2)\gamma^\mu) u(\vec{p})\end{aligned} \tag{A9}$$

$$\begin{aligned}\mathcal{A}_g^\mu = {}& \frac{g_A}{2\sqrt{2} f_\pi^2} \cos\theta_c \{\bar{u}(\vec{p}\,') 2F_1^V g_A \gamma^\mu \gamma_5 (-S(p - k_2)\not{k}_2 \gamma_5 \\ & + 2S(p - k_1)\not{k}_1 \gamma_5) u(\vec{p}) \\ & - \bar{u}(\vec{p}\,')\gamma^\mu (-S(p - k_2)\not{k}_2 \gamma_5 \\ & + 2S(p - k_1)\not{k}_1 \gamma_5) u(\vec{p})\}\end{aligned} \tag{A10}$$

$$\begin{aligned}\mathcal{A}_h^\mu = {}& -\frac{g_A}{4\sqrt{2} f_\pi^2} \cos\theta_c \frac{q^\mu}{q^2 - m_\pi^2} \bar{u}(\vec{p}\,')\not{k}_2 \gamma_5 S(p' + k_2) \\ & \times (\not{q} + \not{k}_1) u(\vec{p})\end{aligned} \tag{A11}$$

$$\begin{aligned}\mathcal{A}_i^\mu = {}& -\frac{g_A}{4\sqrt{2} f_\pi^2} \cos\theta_c \frac{q^\mu}{q^2 - m_\pi^2} \bar{u}(\vec{p}\,')((\not{q} + \not{k}_1) S(p - k_2) \\ & \times \not{k}_2 \gamma_5 - 2(\not{q} + \not{k}_2) S(p - k_1)\not{k}_1 \gamma_5) u(\vec{p})\end{aligned} \tag{A12}$$

$$\begin{aligned}\mathcal{A}_j^\mu = {}& \frac{g_A^2}{2\sqrt{2} f_\pi^2} \cos\theta_c 2F_1^V \frac{q^\mu - 2k_1^\mu}{(q - k_1)^2 - m_\pi^2} \\ & \times \bar{u}(\vec{p}\,')\not{k}_2 \gamma_5 S(p' + k_2)(\not{q} - \not{k}_1)\gamma_5 u(\vec{p})\end{aligned} \tag{A13}$$

$$\begin{aligned}\mathcal{A}_k^\mu = {}& \frac{g_A^2}{2\sqrt{2} f_\pi^2} \cos\theta_c 2F_1^V \bigg(\frac{q^\mu - 2k_1^\mu}{(q - k_1)^2 - m_\pi^2} \bar{u}(\vec{p}\,')(\not{q} - \not{k}_1) \\ & \times \gamma_5 S(p - k_2)\not{k}_2 \gamma_5 u(\vec{p}) - 2\frac{q^\mu - 2k_2^\mu}{(q - k_2)^2 - m_\pi^2} \\ & \times \bar{u}(\vec{p}\,')(\not{q} - \not{k}_2)\gamma_5 S(p - k_1)\not{k}_1 \gamma_5 u(\vec{p}) \bigg)\end{aligned} \tag{A14}$$

$$\begin{aligned}\mathcal{A}_m^\mu = {}& -\frac{g_A^2}{2\sqrt{2} f_\pi^2} \cos\theta_c \bar{u}(\vec{p}\,')\not{k}_2 \gamma_5 S(p' + k_2)(V^\mu - A^\mu) \\ & \times S(p - k_1)\not{k}_1 \gamma_5 u(\vec{p})\end{aligned} \tag{A15}$$

$$\begin{aligned}\mathcal{A}_n^\mu = {}& \frac{g_A^2}{2\sqrt{2} f_\pi^2} \cos\theta_c \bar{u}(\vec{p}\,')(V^\mu - A^\mu) S(p - k_1 - k_2) \\ & \times (\not{k}_2 \gamma_5 S(p - k_1)\not{k}_1 \gamma_5 - \not{k}_1 \gamma_5 S(p - k_2) \\ & \times \not{k}_2 \gamma_5) u(\vec{p})\end{aligned} \tag{A16}$$

2. $W_q^+ p_p \to n_{p'} \pi^+_{k_1} \pi^+_{k_2}$

$$\mathcal{A}_a^\mu = \frac{1}{f_\pi^2} \cos\theta_c \bar{u}(\vec{p}\,')(2F_1^V \gamma^\mu - g_A \gamma^\mu \gamma_5) u(\vec{p}) \tag{A17}$$

$$\mathcal{A}_b^\mu = \frac{g_A}{6 f_\pi^2} \cos\theta_c \frac{q^\mu}{q^2 - m_\pi^2} \bar{u}(\vec{p}\,')(3\not{k}_1 + 3\not{k}_2 + 4M)\gamma_5 u(\vec{p}) \tag{A18}$$

$$\begin{aligned}\mathcal{A}_c^\mu = {}& \frac{2g_A}{3 f_\pi^2} \cos\theta_c \frac{(q^\mu - 3k_1^\mu) + (q^\mu - 3k_2^\mu)}{(q - k_1 - k_2)^2 - m_\pi^2} \\ & \times 2M \bar{u}(\vec{p}\,')\gamma_5 u(\vec{p})\end{aligned} \tag{A19}$$

$$\begin{aligned}\mathcal{A}_d^\mu = {}& \frac{g_A}{3 f_\pi^2} \cos\theta_c \frac{q^\mu}{q^2 - m_\pi^2} \\ & \times \bigg\{ \frac{(4qk_1 - 2qk_2 + 2k_1k_2 - q^2)}{(q - k_1 - k_2)^2 - m_\pi^2} \\ & + \frac{(4qk_2 - 2qk_1 + 2k_1k_2 - q^2)}{(q - k_1 - k_2)^2 - m_\pi^2} \bigg\} 2M \bar{u}(\vec{p}\,')\gamma_5 u(\vec{p})\end{aligned} \tag{A20}$$

$$\mathcal{A}_e^\mu = 2F_1^V \frac{1}{f_\pi^2}\cos\theta_c\left\{\frac{2k_1^\mu - q^\mu}{(q-k_1)^2 - m_\pi^2}\bar{u}(\vec{p}')\not{k}_2 u(\vec{p}) + \frac{2k_2^\mu - q^\mu}{(q-k_2)^2 - m_\pi^2}\bar{u}(\vec{p}')\not{k}_1 u(\vec{p})\right\} \tag{A21}$$

$$\mathcal{A}_f^\mu = -\frac{g_A}{2f_\pi^2}\cos\theta_c\{\bar{u}(\vec{p}')(\not{k}_1\gamma_5 S(p'+k_1) + \not{k}_2\gamma_5 S(p'+k_2))2F_1^V g_A\gamma^\mu\gamma_5 u(\vec{p}) - \bar{u}(\vec{p}')(\not{k}_1\gamma_5 S(p'+k_1) + \not{k}_2\gamma_5 S(p'+k_2)) \times \gamma^\mu u(\vec{p})\} \tag{A22}$$

$$\mathcal{A}_g^\mu = \frac{g_A}{2f_\pi^2}\cos\theta_c\{\bar{u}(\vec{p}')2F_1^V g_A\gamma^\mu\gamma_5(S(p-k_1)\not{k}_1\gamma_5 + S(p-k_2)\not{k}_2\gamma_5)u(\vec{p}) - \bar{u}(\vec{p}')\gamma^\mu(S(p-k_1)\not{k}_1\gamma_5 + S(p-k_2)\not{k}_2\gamma_5)u(\vec{p})\} \tag{A23}$$

$$\mathcal{A}_h^\mu = -\frac{g_A}{4f_\pi^2}\cos\theta_c\frac{q^\mu}{q^2 - m_\pi^2}\bar{u}(\vec{p}')(\not{k}_1\gamma_5 S(p'+k_1)(\not{q}+\not{k}_2) + \not{k}_2\gamma_5 S(p'+k_2)(\not{q}+\not{k}_1))u(\vec{p}) \tag{A24}$$

$$\mathcal{A}_i^\mu = \frac{g_A}{4f_\pi^2}\cos\theta_c\frac{q^\mu}{q^2 - m_\pi^2}\bar{u}(\vec{p}')((\not{q}+\not{k}_1)S(p-k_2)\not{k}_2\gamma_5 + (\not{q}+\not{k}_2)S(p-k_1)\not{k}_1\gamma_5)u(\vec{p}) \tag{A25}$$

$$\mathcal{A}_j^\mu = \frac{g_A^2}{2f_\pi^2}\cos\theta_c 2F_1^V\bigg(\frac{q^\mu - 2k_1^\mu}{(q-k_1)^2 - m_\pi^2} \times \bar{u}(\vec{p}')\not{k}_2\gamma_5 S(p'+k_2)(\not{q}-\not{k}_1)\gamma_5 u(\vec{p}) + \frac{q^\mu - 2k_2^\mu}{(q-k_2)^2 - m_\pi^2}\bar{u}(\vec{p}')\not{k}_1\gamma_5 S(p'+k_1)(\not{q}-\not{k}_2)\gamma_5 \times u(\vec{p})\bigg) \tag{A26}$$

$$\mathcal{A}_k^\mu = -\frac{g_A^2}{2f_\pi^2}\cos\theta_c 2F_1^V\bigg(\frac{q^\mu - 2k_1^\mu}{(q-k_1)^2 - m_\pi^2}\bar{u}(\vec{p}')(\not{q}-\not{k}_1)\gamma_5 \times S(p-k_2)\not{k}_2\gamma_5 u(\vec{p}) + \frac{q^\mu - 2k_2^\mu}{(q-k_2)^2 - m_\pi^2} \times \bar{u}(\vec{p}')(\not{q}-\not{k}_2)\gamma_5 S(p-k_1)\not{k}_1\gamma_5 u(\vec{p})\bigg) \tag{A27}$$

$$\mathcal{A}_m^\mu = -\frac{g_A^2}{2f_\pi^2}\cos\theta_c\bar{u}(\vec{p}')(\not{k}_1\gamma_5 S(p'+k_1)(V^\mu - A^\mu) \times S(p-k_2)\not{k}_2\gamma_5 + \not{k}_2\gamma_5 S(p'+k_2)(V^\mu - A^\mu) \times S(p-k_1)\not{k}_1\gamma_5)u(\vec{p}) \tag{A28}$$

3. $W_q^+ n_p \to p_{p'}\pi_{k_1}^+\pi_{k_2}^-$

$$\mathcal{A}_a^\mu = -\frac{1}{2f_\pi^2}\cos\theta_c\bar{u}(\vec{p}')(2F_1^V\gamma^\mu - g_A\gamma^\mu\gamma_5)u(\vec{p}) \tag{A29}$$

$$\mathcal{A}_b^\mu = -\frac{g_A}{6f_\pi^2}\cos\theta_c\frac{q^\mu}{q^2 - m_\pi^2}\bar{u}(\vec{p}')(3\not{k}_1 + 2M)\gamma_5 u(\vec{p}) \tag{A30}$$

$$\mathcal{A}_c^\mu = -\frac{2g_A}{3f_\pi^2} \times \cos\theta_c\frac{q^\mu - 3k_2^\mu}{(q-k_1-k_2)^2 - m_\pi^2}2M\bar{u}(\vec{p}')\gamma_5 u(\vec{p}) \tag{A31}$$

$$\mathcal{A}_d^\mu = -\frac{g_A}{3f_\pi^2}\cos\theta_c\frac{q^\mu}{q^2 - m_\pi^2} \times \left\{\frac{(4qk_2 - 2qk_1 + 2k_1k_2 - q^2)}{(q-k_1-k_2)^2 - m_\pi^2} + \frac{3m_\pi^2}{(q-k_1-k_2)^2 - m_\pi^2}\right\}2M\bar{u}(\vec{p}')\gamma_5 u(\vec{p}) \tag{A32}$$

$$\mathcal{A}_e^\mu = -\frac{1}{f_\pi^2}\cos\theta_c\frac{2k_1^\mu - q^\mu}{(q-k_1)^2 - m_\pi^2}2F_1^V\bar{u}(\vec{p}')\not{k}_2 u(\vec{p}) \tag{A33}$$

$$\mathcal{A}_{a'}^\mu = \frac{1}{4f_\pi^2}\cos\theta_c\bar{u}(\vec{p}')(\not{k}_1 - \not{k}_2)S(p+q)(V^\mu - A^\mu)u(\vec{p}) \tag{A34}$$

$$\mathcal{A}_{b'}^\mu = -\frac{1}{4f_\pi^2}\cos\theta_c\bar{u}(\vec{p}')(V^\mu - A^\mu)S(p'-q)(\not{k}_1 - \not{k}_2)u(\vec{p}) \tag{A35}$$

$$\mathcal{A}_f^\mu = \frac{g_A}{2f_\pi^2}\cos\theta_c\bar{u}(\vec{p}')(\not{k}_2\gamma_5 S(p'+k_2)2F_1^V g_A\gamma^\mu\gamma_5 - \not{k}_2\gamma_5 S(p'+k_2)\gamma^\mu)u(\vec{p}) \tag{A36}$$

$$\mathcal{A}_g^\mu = -\frac{g_A}{2f_\pi^2}\cos\theta_c\bar{u}(\vec{p}')(2F_1^V g_A\gamma^\mu\gamma_5 S(p-k_2)\not{k}_2\gamma_5 - \gamma^\mu S(p-k_2)\not{k}_2\gamma_5)u(\vec{p}) \tag{A37}$$

$$\mathcal{A}_h^\mu = \frac{g_A}{4f_\pi^2}\cos\theta_c\frac{q^\mu}{q^2 - m_\pi^2}\bar{u}(\vec{p}')\not{k}_2\gamma_5 S(p'+k_2)(\not{q}+\not{k}_1) \times u(\vec{p}) \tag{A38}$$

$$\mathcal{A}_i^\mu = -\frac{g_A}{4f_\pi^2}\cos\theta_c \frac{q^\mu}{q^2 - m_\pi^2}\bar{u}(\vec{p}\,')(\not{q} + \not{k}_1)S(p - k_2) \times \not{k}_2\gamma_5 u(\vec{p}) \quad \text{(A39)}$$

$$\mathcal{A}_j^\mu = -\frac{g_A^2}{2f_\pi^2}\cos\theta_c 2F_1^V \frac{q^\mu - 2k_1^\mu}{(q - k_1)^2 - m_\pi^2}\bar{u}(\vec{p}\,')\not{k}_2\gamma_5 \times S(p' + k_2)(\not{q} - \not{k}_1)\gamma_5 u(\vec{p}) \quad \text{(A40)}$$

$$\mathcal{A}_k^\mu = \frac{g_A^2}{2f_\pi^2}\cos\theta_c 2F_1^V \frac{q^\mu - 2k_1^\mu}{(q - k_1)^2 - m_\pi^2}\bar{u}(\vec{p}\,')(\not{q} - \not{k}_1)\gamma_5 \times S(p - k_2)\not{k}_2\gamma_5 u(\vec{p}) \quad \text{(A41)}$$

$$\mathcal{A}_l^\mu = -\frac{g_A^2}{2f_\pi^2}\cos\theta_c \bar{u}(\vec{p}\,')\not{k}_2\gamma_5 S(p' + k_2)\not{k}_1\gamma_5 \times S(p' + k_1 + k_2)(V^\mu - A^\mu)u(\vec{p}) \quad \text{(A42)}$$

$$\mathcal{A}_n^\mu = -\frac{g_A^2}{2f_\pi^2}\cos\theta_c \bar{u}(\vec{p}\,')(V^\mu - A^\mu)S(p - k_1 - k_2)\not{k}_1\gamma_5 \times S(p - k_2)\not{k}_2\gamma_5 u(\vec{p}) \quad \text{(A43)}$$

4. $W_q^+ n_p \to n_{p'}\pi_{k_1}^+\pi_{k_2}^0$

$$\mathcal{A}_a^\mu = -\frac{1}{2\sqrt{2}f_\pi^2}\cos\theta_c \bar{u}(\vec{p}\,')(2F_1^V\gamma^\mu - g_A\gamma^\mu\gamma_5)u(\vec{p}) \quad \text{(A44)}$$

$$\mathcal{A}_b^\mu = -\frac{g_A}{6\sqrt{2}f_\pi^2}\cos\theta_c \frac{q^\mu}{q^2 - m_\pi^2}\bar{u}(\vec{p}\,')(3\not{k}_2 + 2M)\gamma_5 u(\vec{p}) \quad \text{(A45)}$$

$$\mathcal{A}_c^\mu = -\frac{g_A\sqrt{2}}{3f_\pi^2}\cos\theta_c \frac{q^\mu - 3k_1^\mu}{(q - k_1 - k_2)^2 - m_\pi^2} \times 2M\bar{u}(\vec{p}\,')\gamma_5 u(\vec{p}) \quad \text{(A46)}$$

$$\mathcal{A}_d^\mu = -\frac{g_A}{3\sqrt{2}f_\pi^2}\cos\theta_c \frac{q^\mu}{q^2 - m_\pi^2} \times \frac{4qk_1 - 2qk_2 + 2k_1k_2 - q^2}{(q - k_1 - k_2)^2 - m_\pi^2}2M\bar{u}(\vec{p}\,')\gamma_5 u(\vec{p}) \quad \text{(A47)}$$

$$\mathcal{A}_e^\mu = -\frac{1}{\sqrt{2}f_\pi^2}\cos\theta_c \frac{2k_2^\mu - q^\mu}{(q - k_2)^2 - m_\pi^2}2F_1^V\bar{u}(\vec{p}\,')\not{k}_1 u(\vec{p}) \quad \text{(A48)}$$

$$\mathcal{A}_{a'}^\mu = -\frac{\sqrt{2}}{4f_\pi^2}\cos\theta_c \bar{u}(\vec{p}\,')(\not{k}_1 - \not{k}_2)S(p + q)(V^\mu - A^\mu)u(\vec{p}) \quad \text{(A49)}$$

$$\mathcal{A}_f^\mu = -\frac{g_A}{2\sqrt{2}f_\pi^2}\cos\theta_c\{\bar{u}(\vec{p}\,')(\not{k}_2\gamma_5 S(p' + k_2) - 2\not{k}_1\gamma_5 S(p' + k_1))2F_1^V g_A\gamma^\mu\gamma_5 u(\vec{p}) - \bar{u}(\vec{p}\,') \times (\not{k}_2\gamma_5 S(p' + k_2) - 2\not{k}_1\gamma_5 S(p' + k_1))\gamma^\mu u(\vec{p})\} \quad \text{(A50)}$$

$$\mathcal{A}_g^\mu = -\frac{g_A}{2\sqrt{2}f_\pi^2}\cos\theta_c \bar{u}(\vec{p}\,')(2F_1^V g_A\gamma^\mu\gamma_5 S(p - k_2)\not{k}_2\gamma_5 - \gamma^\mu S(p - k_2)\not{k}_2\gamma_5)u(\vec{p}) \quad \text{(A51)}$$

$$\mathcal{A}_h^\mu = -\frac{g_A}{4\sqrt{2}f_\pi^2}\cos\theta_c \frac{q^\mu}{q^2 - m_\pi^2}\bar{u}(\vec{p}\,')(\not{k}_2\gamma_5 S(p' + k_2) \times (\not{q} + \not{k}_1) - 2\not{k}_1\gamma_5 S(p' + k_1)(\not{q} + \not{k}_2))u(\vec{p}) \quad \text{(A52)}$$

$$\mathcal{A}_i^\mu = -\frac{g_A}{4\sqrt{2}f_\pi^2}\cos\theta_c \frac{q^\mu}{q^2 - m_\pi^2}\bar{u}(\vec{p}\,')(\not{q} + \not{k}_1)S(p - k_2) \times \not{k}_2\gamma_5 u(\vec{p}) \quad \text{(A53)}$$

$$\mathcal{A}_j^\mu = \frac{g_A^2}{2\sqrt{2}f_\pi^2}\cos\theta_c 2F_1^V\bigg(\frac{q^\mu - 2k_1^\mu}{(q - k_1)^2 - m_\pi^2}\bar{u}(\vec{p}\,')\not{k}_2\gamma_5 \times S(p' + k_2)(\not{q} - \not{k}_1)\gamma_5 u(\vec{p}) - 2\frac{q^\mu - 2k_2^\mu}{(q - k_2)^2 - m_\pi^2}\bar{u}(\vec{p}\,')\not{k}_1\gamma_5 S(p' + k_1) \times (\not{q} - \not{k}_2)\gamma_5 u(\vec{p})\bigg) \quad \text{(A54)}$$

$$\mathcal{A}_k^\mu = \frac{g_A^2}{2\sqrt{2}f_\pi^2}\cos\theta_c 2F_1^V \frac{q^\mu - 2k_1^\mu}{(q - k_1)^2 - m_\pi^2}\bar{u}(\vec{p}\,')(\not{q} - \not{k}_1)\gamma_5 \times S(p - k_2)\not{k}_2\gamma_5 u(\vec{p}) \quad \text{(A55)}$$

$$\mathcal{A}_l^\mu = \frac{g_A^2}{2\sqrt{2}f_\pi^2}\cos\theta_c \bar{u}(\vec{p}\,')(\not{k}_2\gamma_5 S(p' + k_2)\not{k}_1\gamma_5 - \not{k}_1\gamma_5 S(p' + k_1)\not{k}_2\gamma_5)S(p' + k_1 + k_2)(V^\mu - A^\mu) \times u(\vec{p}) \quad \text{(A56)}$$

$$\mathcal{A}_m^\mu = \frac{g_A^2}{2\sqrt{2}f_\pi^2}\cos\theta_c \bar{u}(\vec{p}\,')\not{k}_1\gamma_5 S(p' + k_1)(V^\mu - A^\mu) \times S(p - k_2)\not{k}_2\gamma_5 u(\vec{p}) \quad \text{(A57)}$$

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