Ultrahigh energy neutrino scattering

Edmond L. Berger,[1] Martin M. Block,[2] Douglas W. McKay,[3] and Chung-I Tan[4]
[1]*High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[2]*Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA*
[3]*Department of Physics and Astronomy, University of Kansas, Lawrence, Kansas 66045, USA*
[4]*Physics Department, Brown University, Providence, Rhode Island 02912, USA*
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Estimates are made of ultrahigh energy neutrino cross sections based on an extrapolation to very small Bjorken x of the logarithmic Froissart dependence in x shown previously to provide an excellent fit to the measured proton structure function $F_2^p(x, Q^2)$ over a broad range of the virtuality Q^2. Expressions are obtained for both the neutral current and the charged current cross sections. Comparison with an extrapolation based on perturbative QCD shows good agreement for energies where both fit data, but our rates are as much as a factor of 10 smaller for neutrino energies above 10^9 GeV, with important implications for experiments searching for extragalactic neutrinos.

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I. INTRODUCTION

The experimental effort to detect extragalactic, ultra-high-energy (UHE) neutrinos has grown rapidly in the past decade. Optical [1] and radio [2] telescopes and cosmic ray air shower arrays [3] are now searching for evidence of point and diffuse neutrino sources up to and beyond EeV energies. Proposals have been made and others are in preparation [4] for new telescopes or expansions of ones currently deployed, and ambitious satellite-born telescopes have been proposed [5]. The highest energies proposed reach beyond 10^{12} GeV.

Critical to all of this effort are accurate estimates of event rates, based on the extrapolation of measured neutrino deep-inelastic scattering (DIS) cross sections to energies *far* beyond currently available data [6–8]. The estimates are only as reliable as the extrapolations, and determination of fluxes and extraction of signals of new physics at UHE depend on them. Most existing extrapolations are done within the framework of perturbative quantum chromodynamics (pQCD), and they involve extending fitted parton distribution functions (PDFs) into domains in Bjorken x much below those now accessible experimentally, and into domains in which linear pQCD evolution [9] is of questionable applicability. Other physical phenomena are expected to alter the x dependence in this very small x region [10], although a complete analytic solution does not yet exist.

New, alternative methods of extrapolation in x are of significant interest, both theoretically and for phenomenological applications. Imposition of the Froissart [11] unitarity and analyticity constraints on inclusive deep-inelastic cross sections [12] leads to the expectation that the x dependence of the proton structure function $F_2^p(x, Q^2)$ should grow no more rapidly at very small x than $\ln^2(1/x)$. This relatively slow growth may be contrasted with the more rapid inverse power dependence characteristic of PDFs. Excellent fits to data were obtained [12] for $x < 0.1$ with an assumed logarithmic expansion, for a wide range of virtuality Q^2. We explore in this paper the consequences of the Froissart logarithmic form for UHE neutrino phenomena, computing both neutral and charged current cross sections. In doing so, rather than working with parton distribution functions for the decomposition into quark and antiquark contributions, we devise and test a procedure based *directly* on experimental F_2^p data. We obtain excellent agreement with extrapolations based on the CTEQ4-DIS parton densities in the neutrino energy range less than 10^8 GeV. However, we predict an important departure for larger energies, with our neutrino cross sections being about a decade smaller at the highest energies. At the very least, our results suggest that estimates that fall between ours and those obtained from PDF extrapolations be used for guidance in the consideration of new experiments.

II. NEUTRINO-ISOSCALAR NUCLEON CROSS SECTIONS

In the standard parton model the inclusive differential cross section for the charged current (CC) reaction $\nu_\ell + N \to \ell^- + X$ on an isoscalar nucleon $N = (n + p)/2$ and the neutral current (NC) cross section $\nu_\ell + N \to \nu_\ell + X$, where in both cases, $\ell = e, \mu, \tau$, is

$$\frac{d^2\sigma}{dxdy}(E_\nu) = \frac{2G_F^2 m E_\nu}{\pi}\left(\frac{M_V^2}{Q^2 + M_V^2}\right)^2 \times [xq_i(x, Q^2) + x\bar{q}_i(x, Q^2)(1 - y)^2], \quad (1)$$

where $-Q^2$ is the invariant squared momentum transfer between the incoming neutrino and the outgoing muon, m is the proton mass, and G_F is the Fermi coupling constant. The intermediate vector boson mass, M_V, is $M_W = 80.4$ GeV for CC and $M_Z = 91.2$ GeV for NC. Symbols q_i and $\bar{q}_i$, $i =$ CC, NC, are linear combinations of quark and antiquark PDFs. The Bjorken scaling variables, where

$\nu = E_\nu - E_\ell$ is the energy loss in the laboratory frame, are given by

$$x \equiv \frac{Q^2}{2m\nu}, \qquad y \equiv \frac{\nu}{E_\nu}, \qquad 0 \le x, y \le 1. \tag{2}$$

III. CHARGED CURRENT CROSS SECTION

With valence and sea quark distributions denoted by subscripts v and s, respectively, the relevant PDFs in Eq. (1) are

$$q_{\rm CC}(x, Q^2) = \frac{u_v(x, Q^2) + d_v(x, Q^2)}{2} + \frac{u_s(x, Q^2) + d_s(x, Q^2)}{2} + s_s(x, Q^2) + b_s(x, Q^2) \tag{3}$$

and

$$\bar{q}_{\rm CC}(x, Q^2) = \frac{u_s(x, Q^2) + d_s(x, Q^2)}{2} + c_s(x, Q^2) + t_s(x, Q^2), \tag{4}$$

where u, d, c, s, t, and b represent the contributions from the up, down, charm, strange, top, and bottom flavors.

IV. NEUTRAL CURRENT CROSS SECTION

The relevant PDFs in Eq. (1) involve chiral couplings $L_u = 1 - \frac{4}{3}\sin^2\theta_W$, $L_d = -1 + \frac{2}{3}\sin^2\theta_W$, $R_u = -\frac{4}{3}\sin^2\theta_W$, $R_d = \frac{2}{3}\sin^2\theta_W$, where $\sin^2\theta_W = 0.226$ is the weak mixing parameter. For details, see Ref. [7].

V. KINEMATICS

Replacing Q^2 in Eq. (1) by $Q^2 = 2mE_\nu xy$, we obtain an expression in terms of E_ν, x, and y. We choose to integrate first over y. To avoid singularities in the integration, we introduce $Q^2_{\rm min} = 0.01\ {\rm GeV}^2$, such that $Q^2 = 2mE_\nu xy \ge Q^2_{\rm min}$. This defines $x_{\rm min}$, the x-integration minimum, as $x_{\rm min} \equiv Q^2_{\rm min}/(2mE_\nu)$. Thus, for $x_{\rm min} \le x \le 1$, our integration limits for y are $y_{\rm min} = x_{\rm min}/x \le y \le 1$.

The vector boson propagator, $(M_V^2/(Q^2 + M_V^2))^2$, essentially fixes an "effective" x at $x_{\rm eff} \sim M_V^2/(2mE_\nu)$. For $E_\nu = 10^{12}$ GeV, this means we must explore quark distributions having $x_{\rm eff} \sim 5 \times 10^{-9}$, at $Q^2 \sim M_V^2 \sim 10\,000\ {\rm GeV}^2$, both of which involve *enormous* extrapolations from currently available structure function data. At these energies, the propagator also serves to make the calculation insensitive to the choice of $Q^2_{\rm min}$.

VI. ANALYTIC EXPRESSION FOR THE STRUCTURE FUNCTION

In prior work [12], it was shown that an excellent fit to the DIS structure function for $x \le x_P$ is given by

$$F_2^p(x, Q^2) = (1 - x)\left(\frac{F_P}{1 - x_P} + A(Q^2)\ln\left[\frac{x_P}{x}\frac{1 - x}{1 - x_P}\right] + B(Q^2)\ln^2\left[\frac{x_P}{x}\frac{1 - x}{1 - x_P}\right]\right), \tag{5}$$

where

$$A(Q^2) = a_0 + a_1 \ln Q^2 + a_2 \ln^2 Q^2, \quad B(Q^2) = b_0 + b_1 \ln Q^2 + b_2 \ln^2 Q^2. \tag{6}$$

The fitted numerical values of a_j and b_k and their uncertainties may be found in Ref. [12]; $F_P = 0.41$ and $x_P = 0.09$.

The bulk of the neutrino cross section comes from exceedingly small x. For large x, where $x_P \le x \le 1$, it suffices to approximate the proton structure function by

$$F_2^p(x, Q^2) = \frac{F_P}{x_P^{\alpha(Q^2)}(1 - x_P)^3} x^{\alpha(Q^2)}(1 - x)^3, \tag{7}$$

where the exponent $\alpha(Q^2)$ is chosen so that the first derivatives of Eqs. (5) and (7) are equal at $x = x_P$. This choice satisfies the spectator valence quark counting rule [13] $F_2^p(x) \to 0$ as $(1 - x)^3$ as $x \to 1$. Numerical analysis shows that this choice has the important consequence that the integral of the proton structure function over x is nearly constant over an enormous Q^2 range, i.e.,

$$\int_0^1 F_2^p(x, Q^2)dx \approx 0.16, \qquad 0.1 \le Q^2 \lesssim 10^5\ {\rm GeV}^2. \tag{8}$$

The constant 0.16 is compatible with results that show that quarks carry ~50% of the momentum in a proton.

The description of $F_2^p(x, Q^2)$ by Eqs. (5)–(8) yields a high-quality fit to the HERA inclusive deep-inelastic data for all x and Q^2.

VII. "WEE PARTON" PICTURE

We obtain the quark distribution functions in Eq. (1) from a wee parton model for very small Bjorken x, having the following features:

(i) there are essentially *only* sea quarks at small enough x, with *negligible* valence quark contributions (for earlier use, see Ref. [8]), i.e., we set $u_v(x, Q^2) = d_v(x, Q^2) = 0$.

(ii) all sea quarks give equal contribution (i.e., equipartition), $U(x, Q^2) = u_s(x, Q^2) = \bar{u}_s(x, Q^2) = d_s(x, Q^2) = \bar{d}_s(x, Q^2) = s_s(x, Q^2) = \bar{s}_s(x, Q^2) = c_s(x, Q^2) = \bar{c}_s(x, Q^2)$.

If only two families contribute (u, d, c, and s),

$$F_2^p(x, Q^2) = \sum_i e_i^2 x[q_i(x, Q^2) + \bar{q}_i(x, Q^2)], \; i = 1, \ldots 4, \tag{9}$$

or, alternatively,

$$xU(x, Q^2) = \frac{9}{20} F_2^p(x, Q^2), \tag{10}$$

for $x < x_{\max}$, where $x_{\max} \sim 10^{-3}$–10^{-4}. If we had used *only one family* of quarks—u, d—or *three families*—u, d, c, s, t, b—instead of two families—u, d, c, s— we would also find that $xq(x, Q^2) = x\bar{q}(x, Q^2) = \frac{9}{10} F_2^p(x, Q^2)$, so that Eq. (1) for charged currents is independent of the number of families. A similar result is true for the neutral current cross section. Employing this picture, we find that accurate knowledge of $F_2^p(x, Q^2)$ at small x and large Q^2 provides the ingredients necessary to calculate the charged and neutral current neutrino cross sections. The fitted form of Eq. (5) is sufficiently accurate to furnish us with quark distribution functions having the needed precision. Using the full squared error matrix for the structure function determination [12], we find that $F_2^p(x = 10^{-8}, Q^2 = 6400\ \text{GeV}^2) = 24.84 \pm 0.17$, a fractional *statistical* accuracy of only ~0.7%. This very small uncertainty due to *parameter errors* assumes, of course, the validity of our $\ln^2(1/x)$ model at very small x.

VIII. CHARGED CURRENT CROSS SECTION EVALUATION

For our model, $xq_{\text{CC}}(x, Q^2) = x\bar{q}_{\text{CC}}(x, Q^2) = 2xU(x, Q^2)$. Thus $xU(x, Q^2) = \frac{9}{20} F_2^p(x, Q^2)$ and Eq. (1) simplifies to

$$\frac{d^2\sigma_{\text{CC}}}{dxdy}(E_\nu) = \frac{2G_F^2 m E_\nu}{\pi}\left(\frac{M_W^2}{Q^2 + M_W^2}\right)^2 \times \left[\frac{9}{10} F_2^p(x, Q^2)\right](2 - 2y + y^2), \tag{11}$$

with $F_2^p(x, Q^2)$ given by Eq. (5) for $0 \le x \le x_P$ and Eq. (7) for $x_P < x \le 1$.

Results of a direct double integration of Eq. (11), with $Q^2_{\min} = 0.01\ \text{GeV}^2$, for the neutrino energy range $10 \le E_\nu \le 10^{14}$ GeV, are given in Table I and shown in Fig. 1 as the solid curve. Also shown, for comparison, are the results of Gandhi *et al.* [7] for the CC cross section with the quark distributions from CTEQ4-DIS [14]. The Gandhi *et al.* curve—the long dash curve—covers the energy range $10 \le E_\nu \le 10^{12}$ GeV. The agreement up to neutrino energies $\lesssim 10^8$ GeV is striking.

TABLE I. Neutrino CC and NC total cross sections, with neutrino energy E_ν energy in GeV and cross sections in cm^2.

E_ν	σ_{CC}	σ_{NC}	E_ν	σ_{CC}	σ_{NC}
10^1	$5.93\ 10^{-38}$	$1.96\ 10^{-38}$	10^8	$4.49\ 10^{-33}$	$1.83\ 10^{-33}$
10^2	$5.51\ 10^{-37}$	$1.82\ 10^{-37}$	10^9	$8.90\ 10^{-33}$	$3.70\ 10^{-33}$
10^3	$5.01\ 10^{-36}$	$1.67\ 10^{-36}$	10^{10}	$1.58\ 10^{-32}$	$6.63\ 10^{-33}$
10^4	$3.80\ 10^{-35}$	$1.32\ 10^{-35}$	10^{11}	$2.57\ 10^{-32}$	$1.09\ 10^{-32}$
10^5	$1.91\ 10^{-34}$	$7.03\ 10^{-35}$	10^{12}	$3.92\ 10^{-32}$	$1.67\ 10^{-32}$
10^6	$6.87\ 10^{-34}$	$2.65\ 10^{-34}$	10^{13}	$5.68\ 10^{-32}$	$2.44\ 10^{-32}$
10^7	$1.94\ 10^{-33}$	$7.74\ 10^{-34}$	10^{14}	$7.92\ 10^{-32}$	$3.40\ 10^{-32}$

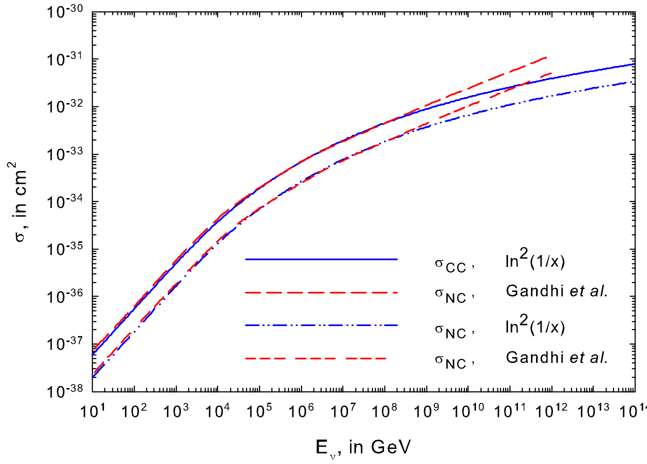

FIG. 1 (color online). Charged (CC) and neutral (NC) current neutrino cross sections in cm^2 vs E_ν, the neutrino energy in GeV. The solid and dash-dot-dot curves are our CC and NC cross sections, respectively, for $10 \le E_\nu \le 10^{14}$ GeV, based on a proton structure function that varies as $\ln^2(1/x)$ for small x. The long dash curve and the dash-dash-dash curve are the Gandhi *et al.* [7] CC and NC cross sections, respectively, for $10 \le E_\nu \le 10^{12}$ GeV, based on the CTEQ4-DIS quark distributions.

IX. NEUTRAL CURRENT CROSS SECTION EVALUATION

For our model, the NC quark distributions in Eq. (1) are

$$\begin{aligned} xq_{\text{NC}}(x, Q^2) &= x\bar{q}_{\text{NC}}(x, Q^2) \\ &= 2xU(x, Q^2) \times (L_u^2 + L_d^2 + R_u^2 + R_d^2) \\ &= 4\left(1 - 2\sin^2\theta_w + \frac{20}{9}\sin^4\theta_W\right) xU(x, Q^2) \\ &= 2.65xU(x, Q^2) = 1.19F_2^p(x, Q^2), \end{aligned} \tag{12}$$

where Eq. (10) is used in the last step. The neutral current cross section simplifies considerably. For direct comparison with the charged current cross section of Eq. (11), it can be rewritten as

$$\frac{d^2\sigma_{\text{NC}}}{dxdy}(E_\nu) = \frac{2G_F^2 m E_\nu}{\pi}\left(\frac{M_Z^2}{Q^2 + M_Z^2}\right)^2 \times [0.298F_2^p(x, Q^2)](2 - 2y + y^2). \tag{13}$$

To the extent that the Z propagator is somewhat less restrictive as a cutoff than the W propagator, comparison of Eq. (11) and (13) shows that the *ratio* of the NC cross section to the CC cross section is $\gtrsim 0.298/0.9 = 0.33$, independent of energy. Numerical evaluation gives 0.40 at $E_\nu = 10^7$ GeV, slightly higher because of the Z propagator. Our NC cross section for isoscalar nucleons is given in Table I and shown in Fig. 1 as the dash-dot-dot curve, plotted in the energy interval $10 \leq E_\nu \leq 10^{14}$ GeV. The Gandhi *et al.* [7] NC cross section, for $10 \leq E_\nu \leq 10^{12}$ GeV, is the dash-dash-dash curve. Again, the agreement is excellent up to $E_\nu \sim 10^8$ GeV.

X. ROBUSTNESS OF CROSS SECTIONS

The differential cross sections were evaluated numerically in Mathematica and found to be numerically stable, essentially independent of $Q^2_{\rm min}$ and the methods of integration. The dependence of the cross sections on the functional form of $F_2^p(Q^2, x)$ for $1 \geq x \geq x_P$ was tested by setting $F_2^p(Q^2, x) \sim x(1-x)^3$ for large x, and the change was found to be ~2% at $E_\nu = 10^8$ and ~0 at $E_\nu = 10^{12}$ GeV. If we set $F_2^p(Q^2, x) = 0$ for $1 \geq x \geq x_P$, an extreme case, we find the changes to be 6% at $E_\nu = 10^8$ GeV and ~0 at $E_\nu = 10^{12}$ GeV. We tested our equipartition hypothesis by changing the strengths of the heavy sea quark distributions such that

$$s_s(x, Q^2) = \bar{s}_s(x, Q^2) = 0.96U(x, Q^2)$$
$$c_s(x, Q^2) = \bar{c}_s(x, Q^2) = 0.80U(x, Q^2), \tag{14}$$

similar to the distributions used by CTEQ. This change gives us cross sections that are ~6% greater at $E_\nu = 10^8$ GeV and ~3% greater at $E_\nu = 10^{12}$ GeV. These variations are negligible compared to the very large differences with respect to the cross sections of Gandhi *et al.* [7] at the highest neutrino energies. Our calculations are numerically stable with regard to our choice of $x_{\rm min}$ in the integration, and thus, insensitive to our choice of $Q_{\rm min} = 0.01$ GeV^2

XI. CONCLUSIONS

We compute ultrahigh energy neutrino cross sections based on an extrapolation to very small Bjorken x of the logarithmic Froissart dependence in x shown previously to provide an excellent fit to the measured proton structure function $F_2^p(x, Q^2)$ over a broad range of the virtuality Q^2. In order to devise expressions for the neutral current and the charged current cross sections, we first extract quark and antiquark contributions based on a simple equipartition wee parton picture valid for $x_{\rm max} \lesssim 10^{-3}$–$10^{-4}$ or $E_\nu^{\rm min} \gtrsim 3 \times 10^6$–$3 \times 10^7$ GeV. However, it is gratifying to see in Fig. 1 that we are in excellent agreement with calculations based on CTEQ4-DIS parton densities over the much larger energy range $10 \leq E_\nu \leq 10^8$ GeV. The two sets of expectations diverge for $E_\nu \gtrsim 10^8$ GeV, as may be expected since our proton structure functions agree with those from CTEQ only for x-values greater than 10^{-3} [12]. The increasing differences for $x < 10^{-3}$ reflect the fundamental difference in the assumed functional forms for the x dependence, in our case a form that is constrained to increase no more rapidly than $\ln^2(1/x)$, in contrast to the inverse power growth in the CTEQ case. For large neutrino energies—above 10^9 GeV—where much smaller x is sampled, our Froissart-bound-model neutrino cross sections are as much as a *decade smaller* than those based on a pQCD extrapolation, a consequence of the fact that our structure function $F_2^p(x, Q^2)$ is significantly smaller at small x. The very small x region is also the region where our wee parton picture is most robust.

The region of very small x is a region of growing interest theoretically. It is a region in which nonperturbative physics is expected to set in [10] and in which linear DGLAP pQCD evolution is not expected to hold. While we cannot claim that logarithmic dependence on x will result from a first-principles solution to small x dynamics, neither can we expect an inverse power form to survive. The logarithmic form we use provides an excellent fit to data over the range in x and Q^2 where it has been tested. Its extrapolation to energies relevant in UHE neutrino studies provides estimates for event rates that should be taken into serious consideration for the planning and data analysis of new experiments.

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[1] A. Achterberg *et al.* (IceCube Collaboration), Phys. Rev. D **76**, 027101 (2007); Phys. Rev. D **76**, 042008 (2007); G. Anassontzis *et al.* (NESTOR Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A **479**, 439 (2002); J. A. Aguilar *et al.* (Antares Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A **570**, 107 (2007); C. Spiering *et al.* (Bailkal collaboration), Nucl. Phys. B, Proc. Suppl. **138**, 175 (2005).

[2] I. Kravchenko *et al.* (RICE Collaboration), Phys. Rev. D **73**, 082002 (2006); S.W. Barwick *et al.* (ANITA Collaboration), Phys. Rev. Lett. **96**, 171101 (2006); N. Letinen *et al.* (FORTE Collaboration), Phys. Rev. D **69**,

013008 (2004); P. Gorham *et al.* (GLUE Collaboration), Phys. Rev. Lett. **93**, 041101 (2004).
[3] W. Deng *et al.* (HiRes Collaboration), Proceedings of the 29th International Cosmic Ray Conference (ICRC 2005), Pune, India, 2005; X. Bertou *et al.*, Astropart. Phys. **17**, 183 (2002).
[4] P. Gorham *et al.*, Phys. Rev. D **72**, 023002 (2005); H. Landsman (AURA Collaboration), Nucl. Phys. B, Proc. Suppl. **168**, 268 (2007).
[5] S. Bottai (EUSO Collaboration), Proceedings of the 27th International Cosmic Ray Conference, Hamburg, 2001, p. 848.
[6] Yu. Andreev, V. Berezinsky, and A. Smirnov, Phys. Lett. B **84**, 247 (1979); M. H. Reno and C. Quigg, Phys. Rev. D **37**, 657 (1988); R. Gandhi, C. Quigg, M. H. Reno, and I. Sarcevic, Astropart. Phys. **5**, 81 (1996).
[7] R. Gandhi, C. Quigg, M. H. Reno, and I. Sarcevic, Phys. Rev. D **58**, 093009 (1998).
[8] D. W. McKay and J. P. Ralston, Phys. Lett. B **167**, 103 (1986); G. M. Frichter, D. W. McKay, and J. P. Ralston, Phys. Rev. Lett. **74**, 1508 (1995); **77**, 4107(E) (1996).
[9] V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. **15**, 438 (1972); **15**, 675 (1972); Yu. L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977); G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).
[10] L. V. Gribov *et al.*, Phys. Rep. **100**, 1 (1983); A. Capella *et al.*, Phys. Rev. D **63**, 054010 (2001); G. Soyez, Phys. Rev. D **71**, 076001 (2005); R. C. Brower *et al.*, J. High Energy Phys. 12 (2007) 005; arXiv:0707.2408; Y. Hatta, E. Iancu, and A. H. Mueller, J. High Energy Phys. 01 (2008) 026.
[11] M. Froissart, Phys. Rev. **123**, 1053 (1961).
[12] M. M. Block, E. L. Berger, and C-I Tan, Phys. Rev. Lett. **97**, 252003 (2006); E. L. Berger, M. M. Block, and C-I Tan, Phys. Rev. Lett. **98**, 242001 (2007).
[13] R. Blankenbecler and S. J. Brodsky, Phys. Rev. D **10**, 2973 (1974); J. F. Gunion, Phys. Rev. D **10**, 242 (1974); S. J. Brodsky and G. P. Lepage, in Proceedings of 1979 Summer Institute on Particle Physics, SLAC.
[14] H. Lai *et al.* (CTEQ Collaboration), Phys. Rev. D **55**, 1280 (1997).